ARE THE LIGHT SCALAR MESONS FOUR-QUARK STATES?

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We use the QCD sum rule approach to study the two point-function for the scalar mesons σ, κ, $f_0(980)$ and $a_0(980)$ as diquak-antidiquark states. We find that the masses are consistent with existing experimental data, supporting the four-quark structure for the light scalar mesons.

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The experimental proliferation of light scalar mesons is consistent with two nonets, one below 1 GeV region and another one near 1.5 GeV. If the light scalars (the isoscalars $\sigma(500)$, $f_0(980)$, the isodoublet κ and the isovector $a_0(980)$) form an SU(3) flavor nonet, in the naive quark model the flavor structure of these scalars would be:

$$\begin{aligned} &\sigma = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \qquad f_0 = s\bar{s}, \\ &a_0^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \qquad a_0^+ = u\bar{d}, \qquad a_0^- = d\bar{u}, \\ &\kappa^+ = u\bar{s}, \qquad \kappa^0 = d\bar{s}, \qquad \bar{\kappa}^0 = s\bar{d}, \qquad \kappa^- = s\bar{u}. \end{aligned} \tag{1}$$

Although with this model it is difficult to understand the mass degeneracy of $f_0(980)$ and $a_0(980)$, and it is hard to explain why σ and κ are broader than $f_0(980)$ and $a_0(980)$, its use is not yet discarded [1–5]. Some alternative models allow a mixing between the isoscalars. However, different experimental data lead to different mixing angles [6–8].

On the other hand, the scalar mesons in the 1.3 – 1.7 GeV mass region (the isoscalars $f_0(1370)$), $f_0(1500)$, the isodoublet $K_0^*(1430)$ and the isovector $a_0(1450)$) may be easily accommodated in an SU(3) flavor nonet. Therefore, theory and data are now suggesting that QCD forces are at work but with different dynamics dominating below and above 1 GeV mass. Below 1 GeV the phenomena point clearly towards an $S-$wave attraction among two quarks and two anti-quarks, while above 1 GeV the preferred configuration is $P-$wave $q\bar{q}$ [9].

Below 1 GeV the inverted structure of the four-quark dynamics in S-wave is revealed with $f_0(980)$, $a_0(980)$, κ and σ symbolically given by [10]

$$\begin{aligned} &\sigma = ud\bar{u}\bar{d}, \qquad f_0 = \frac{1}{\sqrt{2}}(us\bar{u}\bar{s} + ds\bar{d}\bar{s}), \\ &a_0^- = ds\bar{u}\bar{s}, \qquad a_0^0 = \frac{1}{\sqrt{2}}(us\bar{u}\bar{s} - ds\bar{d}\bar{s}), \qquad a_0^+ = us\bar{d}\bar{s}, \\ &\kappa^+ = ud\bar{d}\bar{s}, \qquad \kappa^0 = ud\bar{u}\bar{s}, \qquad \bar{\kappa}^0 = us\bar{u}\bar{d}, \qquad \kappa^- = ds\bar{u}\bar{d}. \end{aligned} \tag{2}$$

This is supported by a recent lattice calculation [11], and by QCD large N_c scaling [12]. In this four-quark scenario for the light scalars, the mass degeneracy of $f_0(980)$ and $a_0(980)$ is natural, and the mass hierarchy pattern of the nonet is understandable. Besides, it is easy to explain why σ and κ are broader than f_0 and a_0. The decays $\sigma \to \pi\pi$, $\kappa \to K\pi$ and f_0, $a_0 \to KK$ are OZI superallowed without the need of any gluon exchange, while $f_0 \to \pi\pi$ and $a_0 \to \eta\pi$ are OZI allowed as it is mediated by one gluon exchange. Since $f_0(980)$ and $a_0(980)$ are very close to the $\bar{K}K$ threshold, the $f_0(980)$ is dominated by the $\pi\pi$ state and $a_0(980)$ is governed by the $\eta\pi$ state. Consequently, they are narrower than σ and κ.

In this work we use the method of QCD sum rules (QCDSR) [13] to study the two-point functions of the scalar mesons, considered as four-quark states. The first evaluation of the $f_0(980)$ as a four-quark state in the QCDSR formalism was done in [14] for the meson-vacuum decay constant, and in [15] for the hadronic coupling contants (for a review see [16]). We extend these works by considering all the scalar mesons in the nonet and by using different currents.

We follow ref. [17] and consider that the lowest lying scalar mesons are S-wave bound states of a diquark-antidiquark pair. As suggested in ref. [18] the diquark is taken to be a spin zero colour anti-triplet, flavour anti-triplet. Therefore, the $(q)^2(\bar{q})^2$ states make a flavour $SU(3)$ nonet. The corresponding interpolating fields are:

$$\begin{aligned} j_\sigma &= \epsilon_{abc}\epsilon_{dec}(u_a^T C\gamma_5 d_b)(\bar{u}_d\gamma_5 C\bar{d}_e^T), \\ j_{f_0} &= \frac{\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}\left[(u_a^T C\gamma_5 s_b)(\bar{u}_d\gamma_5 C\bar{s}_e^T) + u \leftrightarrow d\right], \\ j_{a_0} &= \frac{\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}\left[(u_a^T C\gamma_5 s_b)(\bar{u}_d\gamma_5 C\bar{s}_e^T) - u \leftrightarrow d\right], \end{aligned}$$

$$j_\kappa = \epsilon_{abc}\epsilon_{dec}(u_a^T C\gamma_5 d_b)(\bar{q}_d\gamma_5 C\bar{s}_e^T), \quad \bar{q} = \bar{u}, \bar{d}, \tag{3}$$

where a, b, c, ... are colour indices and C is the charge conjugation matrix. The other members of the nonet are easily constructed.

The coupling of the scalar meson S, to the scalar current j_S, can be parametrized in terms of the meson decay constant f_S as [14]:

$$\langle 0|j_S|S\rangle = \sqrt{2} f_S m_S^4 \,. \tag{4}$$

In order to compute this parameter with QCDSR, we consider the two-point correlation function

$$\Pi(q) = i\int d^4x\, e^{iq.x}\langle 0|T[j_S(x)j_S^\dagger(0)]|0\rangle. \tag{5}$$

In the QCD side we work at leading order and consider condensates up to dimension six. We deal with the strange quark as a light one and consider the diagrams up to order m_s. In the phenomenological side we consider the usual pole plus continuum contribution. Therefore, we introduce the continuum threshold parameter s_0 [19]. In the $SU(2)$ limit the f_0 and a_0 states are, of course, mass degenerate, and we get the same decay constant for them. After doing a Borel tranform the two-point sum rules are given by:

$$\begin{aligned}
2f_\sigma^2 m_\sigma^8 e^{-m_\sigma^2/M^2} &= \frac{M^{10}E_4}{2^9 5\pi^6} + \frac{\langle g^2G^2\rangle M^6 E_2}{2^{10}3\pi^6} + \frac{\langle\bar{q}q\rangle^2 M^4 E_1}{12\pi^2},\\
2f_\kappa^2 m_\kappa^8 e^{-m_\kappa^2/M^2} &= \frac{M^{10}E_4}{2^9 5\pi^6} - \frac{m_s(2\langle\bar{q}q\rangle - \langle\bar{s}s\rangle)}{2^6 3\pi^4}M^6E_2 + \frac{\langle g^2G^2\rangle M^6 E_2}{2^{10}3\pi^6} + \frac{(\langle\bar{q}q\rangle^2 + \langle\bar{q}q\rangle\langle\bar{s}s\rangle)}{24\pi^2}M^4E_1\\
&+ \frac{m_s\langle\bar{s}g\sigma.Gs\rangle}{2^8 3\pi^4}M^4E_1 + \frac{m_s\langle\bar{q}g\sigma.Gq\rangle}{2^7\pi^4}\left(\frac{3}{2} - \ln(M^2/\Lambda_{QCD}^2)\right)M^4E_1,\\
2f_{f_0}^2 m_{f_0}^8 e^{-m_{f_0}^2/M^2} &= \frac{M^{10}E_4}{2^9 5\pi^6} - \frac{m_s(2\langle\bar{q}q\rangle - \langle\bar{s}s\rangle)}{2^5 3\pi^4}M^6E_2 + \frac{\langle g^2G^2\rangle M^6 E_2}{2^{10}3\pi^6} + \frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{12\pi^2}M^4E_1\\
&+ \frac{m_s\langle\bar{s}g\sigma.Gs\rangle}{2^7 3\pi^4}M^4E_1 + \frac{m_s\langle\bar{q}g\sigma.Gq\rangle}{2^6\pi^4}\left(\frac{3}{2} - \ln(M^2/\Lambda_{QCD}^2)\right)M^4E_1,
\end{aligned} \tag{6}$$

where

$$E_n \equiv 1 - e^{-s_0/M^2}\sum_{k=0}^{n}\left(\frac{s_0}{M^2}\right)^k\frac{1}{k!}\,, \tag{7}$$

which accounts for the continuum contribution.

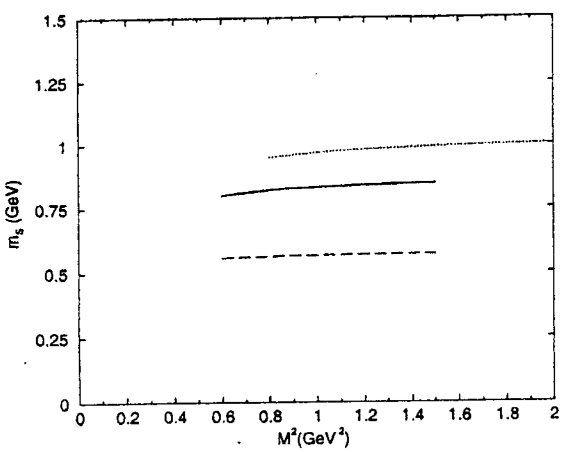

FIG. 1: The scalar meson masses as a function of the Borel mass. Dashed: m_σ; solid: m_κ; dotted: $m_{f_0} = m_{a_0}$.

In the numerical analysis of the sum rules, the values used for the strange quark mass and condensates are: $m_s = 0.13$ GeV, $\langle\bar{q}q\rangle = -(0.23)^3$ GeV^3, $\langle\bar{s}s\rangle = 0.8\langle\bar{q}q\rangle$, $\langle\bar{q}g\sigma.Gq\rangle = m_0^2\langle\bar{q}q\rangle$ with $m_0^2 = 0.8$ GeV^2, $\langle g^2G^2\rangle = 0.5$ GeV^4 and we take $\Lambda_{QCD} = 150$ MeV. For the continuum thresholds we use $s_0^\sigma = 0.8$ GeV^2, $s_0^\kappa = 1.1$ GeV^2, $s_0^{f_0} = 1.5$ GeV^2.

In order to get rid of the meson decay constant and extract the resonance mass, m_S, we first take the derivative of Eqs. (6) with respect to $1/M^2$ and then we divide it by Eqs. (6).

In Fig. 1 we show the masses of the mesons σ, κ and f_0, as a function of the Borel mass. Althought the result for m_σ is a little bigger than the experimental value [20]: $m_\sigma = 0.5$ GeV, the masses for the other resonances are in agreement with the experimental values: $m_\kappa = 0.8$ GeV, $m_{f_0} = 0.98$ GeV. For the three cases we get very stable results as a function of the Borel mass. The problem with these sum rules, as already noticed in ref. [14], is that the continuum contribution is only smaller than the pole contibution for small values of the Borel mass ($M^2 \leq 0.6\,\mathrm{GeV}^2$ for σ and κ and $M^2 \leq 0.8\,\mathrm{GeV}^2$ for f_0 and a_0). However, for these values of the Borel mass the perturbative contribution is smaller than the four-quark condensate contribution. The perturbative contribution will get bigger than the four-quark condensate contribution only for $M^2 \sim 1.5\,\mathrm{GeV}^2$ for σ and κ, and $M^2 \sim 2.\,\mathrm{GeV}^2$ for f_0 and a_0. Therefore, the Borel windows used represent a compromise between these two restrictions. For these Borel windows we get

$$m_\sigma = (0.56 \pm 0.01)\,\mathrm{GeV}, \quad m_\kappa = (0.82 \pm 0.02)\,\mathrm{GeV}, \quad m_{f_0} = (0.98 \pm 0.02)\,\mathrm{GeV}. \tag{8}$$

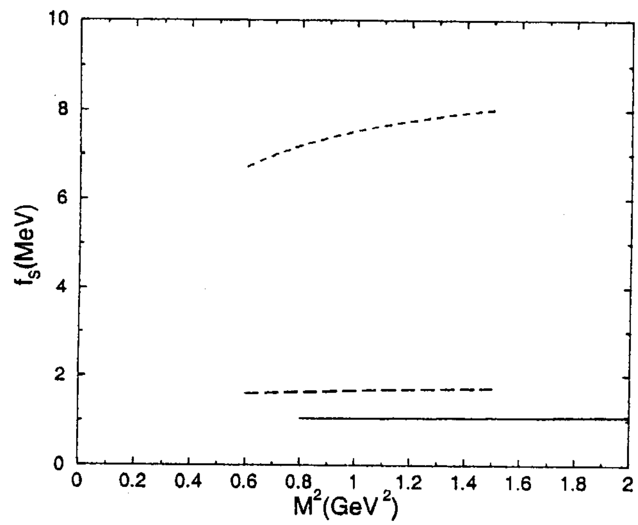

FIG. 2: The scalar meson decay constants as a function of the Borel mass. Solid: $f_{f_0} = f_{a_0}$; dashed: f_κ; dots: f_σ.

To estimate the decay constants we use the experimental values for the scalar meson masses. From Fig. 2 we see that we get a very stable result, as a function of the Borel mass, for f_{f_0} and f_κ. In the case of f_σ the stability is not so good, but it is still acceptable. Allowing a variation of $0.2\,\mathrm{GeV}^2$ in the continuum thresholds we arrive at the following values for the decay constants:

$$f_\sigma = (7.5 \pm 1.0)\,\mathrm{MeV}, \quad f_\kappa = (1.6 \pm 0.3)\,\mathrm{MeV}, \quad f_{f_0} = (1.1 \pm 0.1)\,\mathrm{MeV}. \tag{9}$$

We have presented a QCD sum rule study of the scalar mesons considered as diquark-antidiquark states. We have evaluated the mesons masses and decay constants. We found that the masses are consistent with existing experimental data. Therefore, we consider this result as one more point in favor of the four-quark structure of the light scalar mesons.

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[1] N.A. Törnqvist, *Phys. Rev. Lett.* **49**, 624 (1982); *Z. Phys.* **C68**, 647 (1995).
[2] E. van Beveren, G. Rupp and M.D. Scadron, *Phys. Lett.* **B495**, 300 (2000).

[3] H.G. Dosch, E.M. Ferreira, F.S. Navarra and M. Nielsen, *Phys. Rev.*, **D65**, 114002 (2002).
[4] F. de Fazio and M.R. Pennington, *Phys. Lett.* **B521**, 15 (2001).
[5] I. Bediaga and M. Nielsen, *Phys. Rev.* **D68**, 036001 (2003).
[6] I. Bediaga, F.S. Navarra, M. Nielsen, *Phys. Lett.* **B579**, 59 (2004).
[7] A. Gokalp, Y. Sarac, O. Yilmaz, hep-ph/0410380.
[8] H.-Y. Cheng, *Phys. Rev.* **D67**, 034024 (2003).
[9] F.E. Close and N.A. Törnqvist, *J. Phys.* **G28**, R249 (2002).
[10] R.L. Jaffe, *Phys. Rev.* **D15**, 267, 281 (1977); **D17**, 1444 (1978).
[11] M. Alford and R.L. Jaffe, *Nucl. Phys.* **B578**, 367 (2000).
[12] J.R. Peláez, *Phys. Rev. Lett.* **92**, 102001 (2004).
[13] M.A. Shifman, A.I. and Vainshtein and V.I. Zakharov, *Nucl. Phys.*, **B147**, 385 (1979).
[14] J.I. Latorre and P. Pascual, *Jour. Phys.* **G11**, L231 (1985).
[15] S. Narison, *Phys. Lett.* **B175**, 88 (1986).
[16] S. Narison, *Spectral Sum Rules*, World Scientific Lecture Notes in Physics **26**.
[17] L. Maiani, F. Piccinini, A.D. Polosa and V. Riquer, hep-ph/0407017.
[18] R.L. Jaffe and F. Wilczek, *Phys. Rev. Lett.* **91**, 232003 (2003).
[19] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 591(E) (1981).
[20] E791 Collaboration, E.M. Aitala et al. *Phys. Rev. Lett.* **86**, 765 (2001); *ibid.* **86**, 770 (2001); *ibid.* **89**, 121801 (2002).