UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498-970 SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-1028

STRING INSPIRED EFFECTIVE LAGRANGIAN AND INFLATIONARY UNIVERSE

E. Abdalla and **A.C.V.V. de Siqueira**
Instituto de Física, Universidade de São Paulo

January/1993

String inspired effective Lagrangian and Inflationary Universe

E. Abdalla† and A.C.V.V. de Siqueira*

Instituto de Física da Universidade de São Paulo

Abstract

We consider a string inspired effective Lagrangian for the graviton and dilaton, containing Einstein gravity at the zero slope limit. The numerical solution of the problem shows asymptotically an inflationary universe. The time is measured by the dilaton, as one expects. The result is independent of the introduction of ad-hoc self interactions for the dilaton field.

† Work partially supported by CNPq.

* Work supported by CAPES; permanent address, Universidade Católica de Recife, Pernambuco, Brazil.

The inflationary scenario solves important problems posed by the big bang cosmology[1,2,3]. Indeed, the observed isotropy of the cosmic background radiation, as well as the unstable value $\Omega \sim 1$ of the observed ration of the density of the universe and the critical density can only be explained by the exponential growth of the radius of the universe[2,3] at an early stage. This is known as the inflationary scenario and the way to achieve this description is by means of the introduction of the Coleman-Weinberg potential[2,4,5] which develops a false vacuum as the primordial temperature lowers. The decay of that metastable state develops the required growth of the radius.

We aim at verifying whether a string inspired cosmological model could develop that kind of behavior. In fact, a modified Einstein equation might lead to such a behavior[6]. In fact, study of strings moving on a background lead to new equations for the gravitational fields, with quantum corrections for the Einstein equations, as shown in [7] (see also [8],[9]). Due to the way the α - parameter appears in the solution, it is convenient to start the discussion by the effective action obtained as a consequence of the requirement of conformal invariance of string in a background field[7,8]

$$S = \int d^4x\sqrt{-g}e^{-2\phi}\left[R^\mu_\mu + 4(\nabla\phi)^2 + \frac{1}{4}\alpha R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right] \tag{1}$$

The field equations derived from eq.(1) are obtained after some tedious but simple algebra, and read

$$\begin{aligned}\beta^g_{\mu\nu} = {} & R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi + \frac{1}{2}\alpha R_{\mu\beta\gamma\delta}R_\nu{}^{\beta\gamma\delta} \\ & + \frac{1}{2}\alpha\left[2\nabla_\gamma\nabla_\pi\phi - 4\nabla_\gamma\phi\nabla_\pi\phi + 4\nabla_\gamma\phi\nabla_\pi - \nabla_\gamma\nabla_\pi\right]\left(R_\mu{}^{\gamma\pi}{}_\nu + R_\nu{}^{\gamma\pi}{}_\mu\right) = 0 \qquad (2a)\end{aligned}$$

$$\beta^\phi = \nabla^2\phi - (\nabla\phi)^2 + \frac{1}{4}R^\mu_\mu + \frac{1}{16}\alpha R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 0 \qquad (2b)$$

We wish to obtain the consequences of equations (2) for the big bang cosmology. Therefore, we consider a Robertson-Walker type metric, defined by the line element ds^2 as

$$ds^2 = dx^{0\,2} - R(t)^2\left[\frac{dr^2}{1-kr^2} + r^2 d\omega^2\right] \quad . \tag{3}$$

Using the above metric, it is not difficult to go back to (2) and obtain the resulting differential equations

$$\begin{aligned} & 3\frac{\ddot R}{R} - 2\frac{\partial^2\phi}{\partial t^2} - 3\alpha\left(\frac{\ddot R}{R}\right)^2 - 6\alpha\frac{\ddot R\dot R}{R^2}\frac{\partial\phi}{\partial t} + 3\alpha\frac{\dot R}{R}\left[\frac{d}{dt}\left(\frac{\ddot R}{R}\right) + 2\frac{\dot R}{R}\left(\frac{\ddot R}{R} - \frac{k+\dot R^2}{R^2}\right)\right] = 0 \\ & \ddot\phi + 3\frac{\dot R}{R}\dot\phi - \dot\phi^2 - \frac{3}{2}\left[\frac{\ddot R}{R} + \frac{k+\dot R^2}{R^2}\right] + \frac{3}{2}\alpha\left[\left(\frac{\ddot R}{R}\right)^2 + \left(\frac{k+\dot R^2}{R^2}\right)^2\right] = 0 \end{aligned} \tag{4}$$

We have to solve these (highly non linear) equations. An analytical solution of the above system was beyond our abilities. However, we could easily obtain a numerical solution. We first chose unities such that $\alpha = 1$. We also allowed for a dilaton effective self interaction[9]

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}g\phi^4 \quad . \tag{5}$$

However, the results were not much sensitive to the above constants. We have obtained results as follows. In general there is a transient region, for small values of the time, where the solution depends very much on the initial conditions. However, one quickly runs into the asymptotic region, where the radius grows exponentially, and the dilaton is linear on time. Some case examples are shown in figure 1. We have selected several initial conditions in order to illustrate the fact that the result is quite robust.

This is generally the case for $k = 1$. For $k = 0, -1$ the transient region persisted for a longer time, and we could not arrive to it in most cases as shown in examples in fig. 2, although for $k = 0$ we expect the same behavior as for $k = 1$ on the ground of the arguments shown below. Once the asymptotic region has been reached, it is not difficult to see that the Hubble constant defined in

$$R = R_0 e^{\chi t} \tag{6}$$

and the relation between time and the dilaton

$$\dot{\phi} = \xi = \dot{\phi}(t = 0) \tag{7}$$

are uniquely determined in terms of the string constant. Indeed, equations (4) imply (7) once (6) has been used, in the asymptotic region ($t >> 1/\chi$). The constants χ and ξ are solutions to the algebraic equations

$$\begin{aligned} \alpha\chi^2 + 2\xi\alpha\chi - 1 = 0 \\ \xi^2 - 3\chi\xi(1 - \alpha\chi^2) + \frac{3}{2}\chi^2 = 0. \end{aligned} \tag{8}$$

In principle it is possible to obtain $\chi < 0$, but this solution is incompatible with the approximations made (in fact, we expect that case to describe a shrinking universe, although this claim is rather speculative; in an oversimplified model shown below, in eq. (11), this is the case). Thus we obtain

$$\chi = -\xi + \sqrt{\xi^2 + \frac{1}{\alpha}}. \tag{9}$$

Therefore we arrive also at an algebraic equation for ξ:

$$12\alpha\xi^4 + 2\xi^2 - \frac{3}{2\alpha} + 3(1 - 4\alpha\xi^2)\xi\sqrt{\xi^2 + \frac{1}{\alpha}} = 0 \tag{10}$$

It is easy to verify that $\xi = \tilde{\xi}/\sqrt{\alpha}$ where $\tilde{\xi}$ obeys a α-independent equation. For infinite string tension ($\alpha \to \infty$), $\xi \to 0$. It is interesting to verify that the oversimplified equation for the graviton

$$R_{\mu\nu} + \frac{1}{2}\alpha R_{\mu\rho\pi\sigma}R_\nu{}^{\rho\pi\sigma} = 0 \tag{11}$$

has also a solution of type (3),(6), with $\chi = \pm 1/\sqrt{\alpha}$. Indeed, the large distance behavior is governed by the above equation. The doubling of the sign is also noteworthy, since we have not only the case of an inflationary universe, but also a compactified type dimension. Thus all richness seeked as desired properties of string theories seem to show up already in these oversimplified models.

Some numerical examples are worthwhile noticing. First, the results are independent of the parameters m^2 and g, unless they are higher than 100, showing that the result is very robust. In order to freeze the dilaton field, we need $m^2, g \sim 200$. Moreover, for $k = 1$, the result is also very regular. We arrive at $\xi \sim -.5$, and $\chi = 1.5$. However, for $k = 0, -1$ the results show instability, as we have shown in fig.2.

Further details as well as inclusion of further modes, such as antisymmetric tensor fields as well as supersymmetry inspired fields are under investigation[10].

Figure captions.

Figure 1: diagrams showing the time dependence of $\dot{\phi}$ ln R, and ln $\dot{R}$ for $k = 1$. The numbers above the diagram concern α, k, m^2, g, ϕ_0, $\dot{\phi}_0$, R_0, $\dot{R}_0$, $\ddot{R}_0$, respectively; Notice that we chose α to be always unity.
Figure 2: diagrams obtained for $k = 0$.

References

[1] A.H. Guth, Phys. Rev. **D23** (1981)347-356.
[2] A.D. Linde, Phys. Lett. **108B** (1982)389-393.
[3] A. Zee, "Unity of Forces in the Universe", World Scientific Publishing, 1982.
L.F. Abbott, So-Young Pi, "Inflationary Cosmology", World Scientific Publishing, 1986.
[4] A.D. Linde, Rep. Prog. Phys. **42**,389(1979)
[5] S. Coleman and E. Weinberg, Phys. Rev. **D7** (1973)1888.
[6] A. Starobinski, Phys. Lett. **91B** (1980)99-102.
[7] C.G. Callan, I.R. Klebanov and M.J. Perry, Nucl. Phys. **B278** (1986)78.
C.G. Callan, E.J. Martinec, M.J. Perry and D. Friedan, Nucl. Phys. **B262** (1985)593.
A.A. Tseytlin, Class. Q. Grav.**9** (1992)979.
[8] M.B. Green, J.H. Schwarz and E. Witten, "Superstring Theory", Cambridge University Press, 1987.
[9] A.A. Tseytlin, Proceeding of the International Workshop on String, Quantum Gravity and Physics at the Plank Energy Scale, Erice, 1992, DAMTP-92-36.
[10] E. Abdalla, A.C.V.V. Siqueira, to appear.

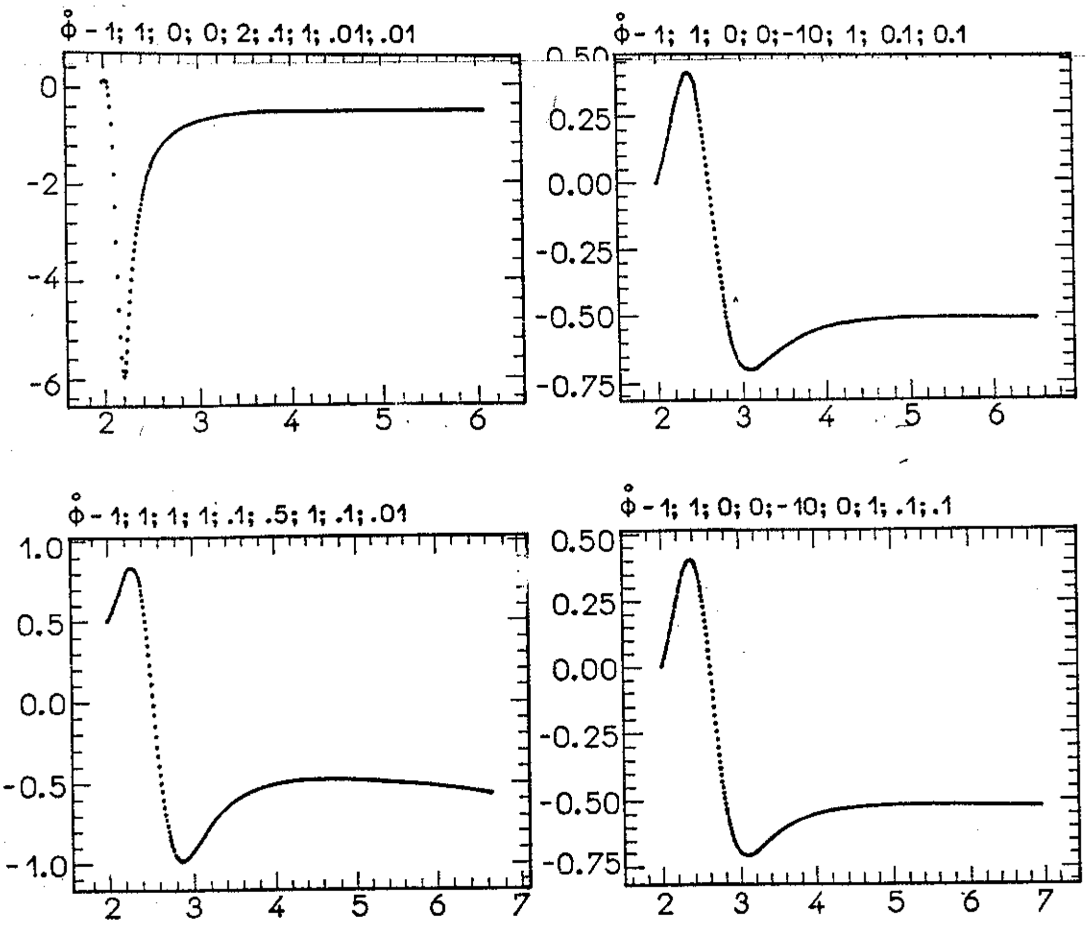
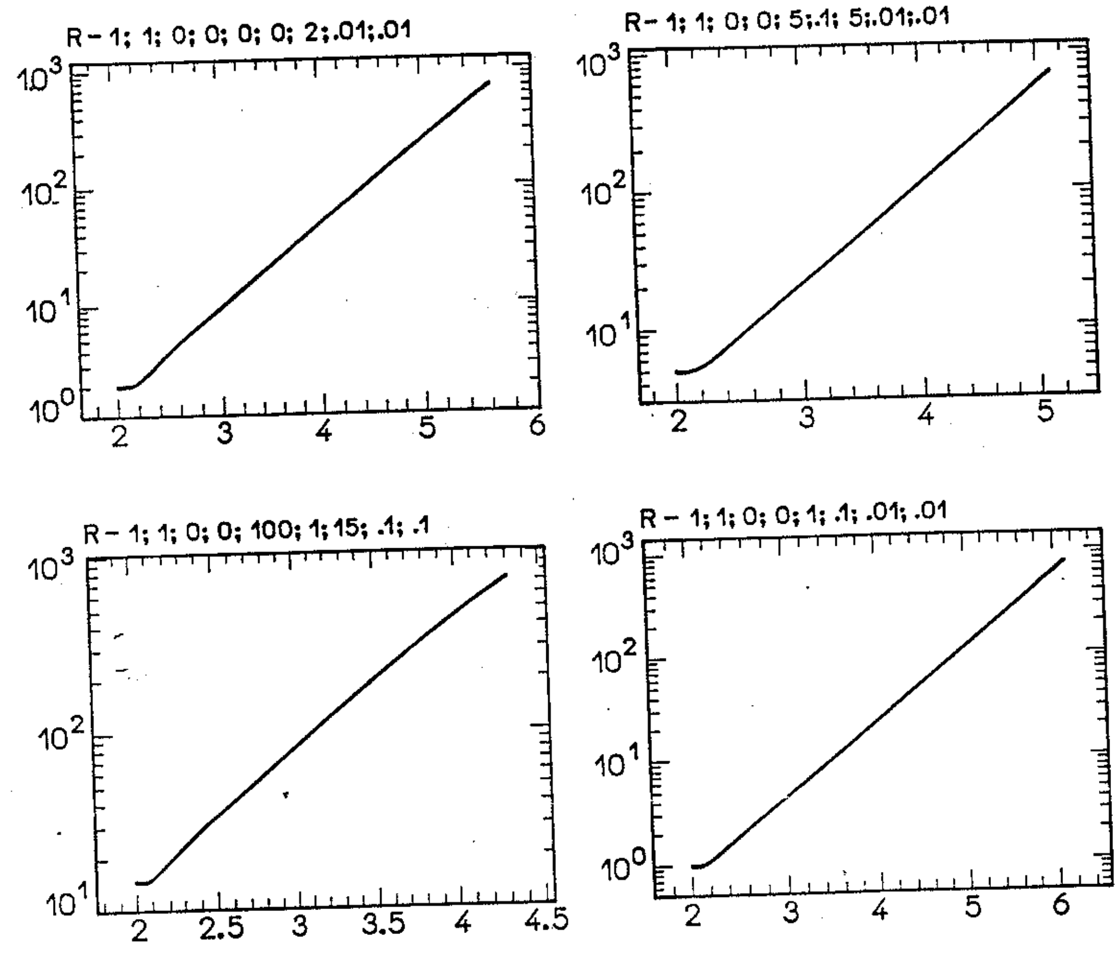

Φ̊ − 1; 1; 0; 0; 2; .1; 1; .01; .01
Φ̊ − 1; 1; 0; 0; −10; 1; 0.1; 0.1
Φ̊ − 1; 1; 1; 1; .1; .5; 1; .1; .01
Φ̊ − 1; 1; 0; 0; −10; 0; 1; .1; .1

FIG. 1


R − 1; 1; 0; 0; 0; 0; 2; .01; .01
R − 1; 1; 0; 0; 5; .1; 5; .01; .01
R − 1; 1; 0; 0; 100; 1; 15; .1; .1
R − 1; 1; 0; 0; 1; .1; .01; .01

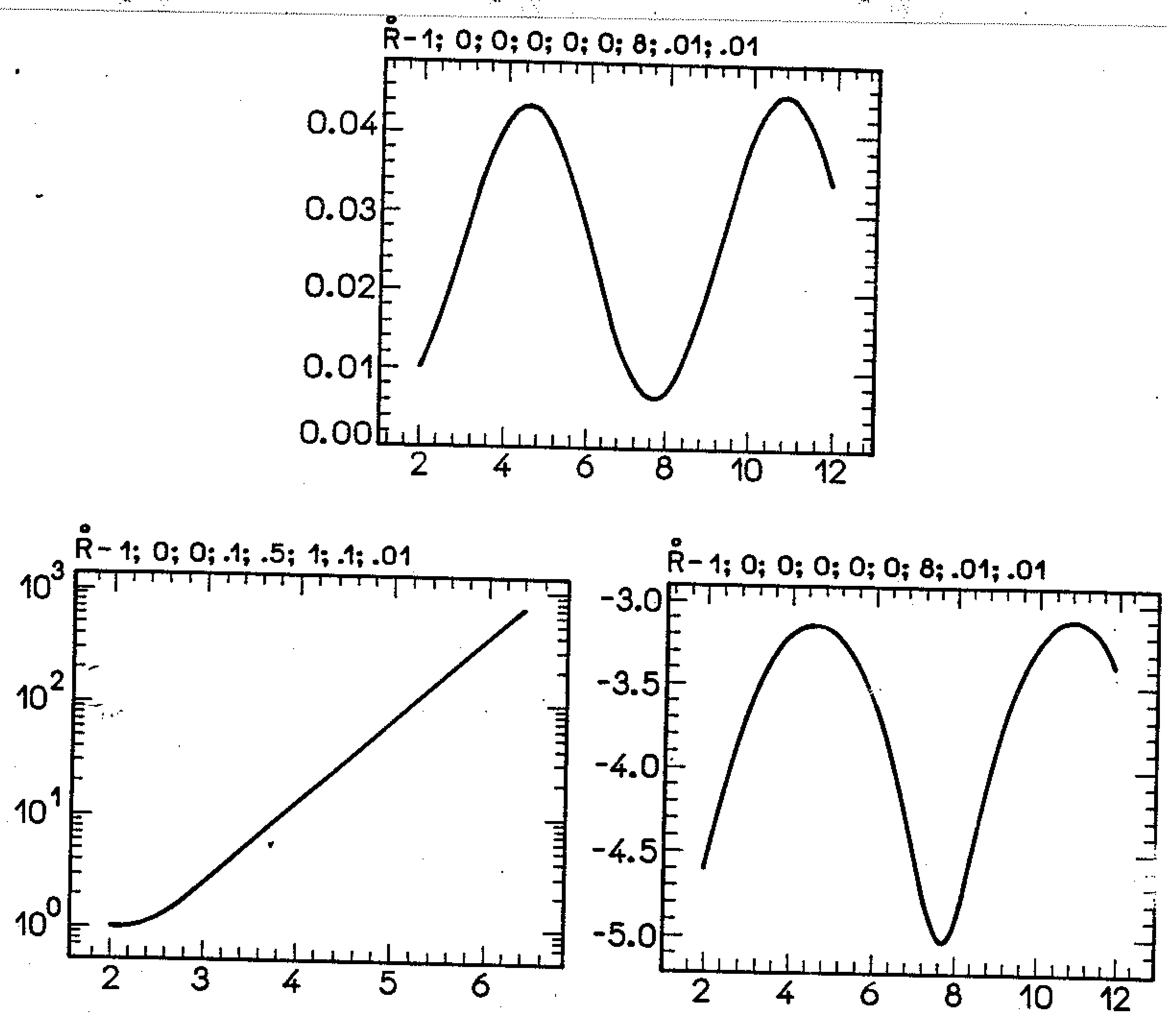

R̊-1; 0; 0; 0; 0; 0; 8; .01; .01
0.04
0.03
0.02
0.01
0.00
2
4
6
8
10
12
R̊-1; 0; 0; .1; .5; 1; .1; .01
10^3
10^2
10^1
10^0
2
3
4
5
6
R̊-1; 0; 0; 0; 0; 0; 8; .01; .01
-3.0
-3.5
-4.0
-4.5
-5.0
2
4
6
8
10
12

FIG. 2