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CORRELATORS IN NON-CRITICAL SUPERSTRINGS INCLUDING THE SPINOR EMISSION VERTEX

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Correlators in non-critical superstrings including the spinor emission vertex.

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Abstract

We discuss the structure of correlators involving the spinor emission vertex in non critical $N = 1$ superstring theory. The technique used in the computation is the zero mode integration to arrive at the integral representation, and later an analysis of the pole structure of the integrals which are thus obtained. Our analysis has been done primarily for the 5-point functions. The result confirms previous expectations and prepares ground for a comparison with computations using matrix models techniques.

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The recent discovery of the double scaling limit[1] in matrix models has led to precise computations of higher genus correlation functions in non critical strings propagating in $d \leq 2$-dimensional target spaces. From the point of view of the theory in the continuum space, however, such calculations have been carried out only for the sphere (genus zero). The computations have been performed either by integrating the Liouville zero mode[2,3] with the subsequent evaluation of the corresponding integrals, or more recently by use of the ground ring structure[4], or also by quantum group technique[5]. The agreement of all such approaches (see [2,6] for comparison) encourage us to generalize such correlators on the sphere to the case of the superstring. By use of the zero mode technique this has been done in [7,8,9] by some groups, at least in the Neveu-Schwarz sector in the $N = 1$ case. The $N = 2$ non-critical theory is discussed in [10]. Almost simultaneously, a supersymmetric version of the one matrix model has appeared in the literature[11], which describes super (p,q) minimal models with $p = 4$, $q = 4m$, $m = 1,2,\cdots$ coupled to two dimensional supergravity. An identification of scaling operators and comparison analogous to [12] can be successfully carried[13], but for a more precise comparison in the Ramond sector we have to calculate further correlators involving the spinor emission vertex, thus generalizing the calculations of [3]. The aim of this letter is to calculate some of such correlators missing in the literature.

The supergravity part of the superstring is described by an $N = 1$ super Liouville theory, defined by the action (see e.g. [14])

$$S_{SL} = \frac{1}{4\pi}\int d^2zd^2\theta\hat{E}\left(\frac{1}{2}D_\alpha\Phi_{SL}D^\alpha\Phi_{SL} - Q\hat{Y}\Phi_{SL} - 4i\mu e^{\alpha_+\Phi_{SL}}\right) \tag{1}$$

where $\hat{E}$ is the superdeterminant of the superzweibein E_{ab}, and $\hat{Y}$ is the supercurvature, and Φ_{SL} is the super Liouville superfield, which defines the super world sheet dynamics. The matter sector is given by an $N = 1$ superfield Φ_M, with the action

$$S_M = \frac{1}{4\pi}\int d^2zd^2\theta\hat{E}\left(\frac{1}{2}D_\alpha\Phi_M D^\alpha\Phi_M - Q\hat{Y}\Phi_M\right) \tag{2}$$

The matter sector has central charge $\hat{c}_M = 1 - 8\alpha_0^2$, and Q, $\alpha_\pm$ are defined by

$$Q = 2\left(1+\alpha_0^2\right)^{\frac{1}{2}}, \tag{3a}$$

$$\alpha_\pm = -\frac{1}{2}Q \pm |\alpha_0| \tag{3b}$$

The particle content of the 2D-superstring consists of a scalar (NS-sector) and a spinor (R-sector) particle in the space time. Both are massless. The Neveu-Schwarz vertex (or scalar emission vertex) is simply the supersymmetric extension of a planar wave (the "tachyon"), being defined by

$$T_k = \int d^2zd^2\theta\hat{E}e^{ik\Phi_M+\beta\Phi_{SL}} = \int d^2z e^{ikX+\beta\phi}\left(\beta\psi + ik\xi\right)\left(\beta\overline{\psi} + ik\overline{\xi}\right) \tag{4}$$

where ψ, ξ are partners of ϕ, X, forming the fermionic and bosonic components of Φ_{SL}, Φ_M. The dressing $\beta(k)$ is given by

$$E = \beta(k) + \frac{Q}{2} = |k - \alpha_0| \,. \tag{5}$$

It will be necessary for our purposes to have also another form of (4) which corresponds to the gauge fixed ($\theta = 0$) version of the NS-vertex, $T_k^{(-1)}$, given by

$$T_k^{(-1)} = \int d^2 z e^{-\sigma + ikx + \beta\phi} \,, \tag{6}$$

where σ is related to the original supersymmetry ghosts via bosonization (see [15] for details), and the (holomorphic part of the) propagator is given by

$$\langle \sigma(z)\sigma(w) \rangle = -\ln(z-w). \tag{7}$$

Note that the definition of β in (5) is still the same since $e^{-\sigma}$ has the same conformal weight of $d^2\theta$ (in order to check, one can use $\Delta\left(e^{a\sigma}\right) = -a(a+2)/2$).

In the next step we turn to the spinor emission vertex, which is more complicated. For future convenience, we first bosonize the two fermionic (Majorana) components ψ and ξ into a free massless boson h, following the usual rules for the Dirac fermions $\psi \pm \xi$ (we omit cocycles)

$$\psi \pm i\xi = \sqrt{2} e^{\mp ih}. \tag{8}$$

The propagator $\langle h(z)h(w) \rangle = -\ln(z-w)$ gives rise to the usual fermionic propagators

$$\langle \psi(z)\psi(w) \rangle = \langle \xi(z)\xi(w) \rangle = \frac{1}{z-w}$$
$$\langle \psi(z)\xi(w) \rangle = 0 \,,$$

analogous results hold for the anti-holomorphic part. Following [15] we notice that states in the the Neveu-Schwarz (resp. Ramond) sector have periodic (antiperiodic) boundary conditions, and the spinor emission vertex which will correspond later to the emission of a fermionic particle in the two dimensional embedding space, must interchange such boundary conditions. Therefore, the spinor emission vertex must include a field S_ϵ ($\epsilon = \pm 1$) –called spin field– which introduces a cut at the point of the emission, such that after a rotation of 2π one gets a minus sign, in other words,

$$\psi^\mu(z) S_\epsilon(w) \sim \frac{(\gamma^\mu)_\epsilon^{\ \beta} S_\beta(z)}{(z-w)^{\frac{1}{2}}} \tag{9}$$

where γ^μ are the two dimensional gamma matrices and $\psi^0 = \psi$, $\psi^1 = \xi$. The solution to the above OPE constraint is given by

$$S_\epsilon = e^{\frac{i}{2}\epsilon h}$$

Due to supersymmetry, we need also a spinor field corresponding to the ghost σ,

$$\Sigma_{-\frac{1}{2}} = e^{-\frac{1}{2}\sigma} \,,$$

Therefore we have

$$V_{-\frac{1}{2}}(k,\epsilon) = \int d^2 z e^{-\frac{1}{2}\sigma + \frac{i}{2}\epsilon h + ikx + \beta\phi} \tag{10}$$

It is not difficult to check that S_ϵ, as well as $V_{-\frac{1}{2}}(k,\epsilon)$ behave like a Weyl spinor under $SO(1,1)$ generators $:\psi^\mu\psi^\nu:$. Imposing BRST invariance of $V_{-\frac{1}{2}}(k,\epsilon)$ we have two different dispersion relations for the different spin components $\epsilon = \pm 1$, as expected for a Weyl spinor in two dimensions[1]:

$$E = \beta + \frac{Q}{2} = -\epsilon(k - \alpha_0) \tag{11}$$

We are now in position to compute mixed correlation functions:

$$\mathcal{A}_{\mathcal{N}}^{(n,\mathcal{N}-n)}(k_1 \cdots k_n, \epsilon_1 \cdots \epsilon_m) = \left\langle \prod_{i=1}^{m} V_{-\frac{1}{2}}(k_i,\epsilon_i) \prod_{j=m+1}^{m} T(k_j) \right\rangle_\mu . \tag{12}$$

After integrating over the matter (X_0) and Liouville (ϕ_0) bosonic zero modes we have[2,3]

$$\mathcal{A}_{\mathcal{N}}^{(n,\mathcal{N}-n)} = \Gamma(-s) \left(\frac{i\mu}{\pi}\right)^s \left\langle \prod_{i=1}^{n} V_{-\frac{1}{2}}(k_i,\epsilon_i) \prod_{j=n+1}^{\mathcal{N}} T(k_j) \left(\int d^2\theta d^2 z e^{\Phi}\right)^s \right\rangle_{\mu=0} , \tag{12}$$

where the momentum conservation law and the definition of s are, respectively,

$$\sum_i k_i = 2\alpha_0,$$
$$\sum_i \beta_i = -Q - \alpha_+ s, \tag{13}$$

As in the previous calculations[8,9], the strategy is to assume that s is a positive integer and to analytically continue the result for arbitrary real s. For integer s, it is enough to use free propagators to write (12) in an integral representation. In the case of no spinor emission vertex ($n = 0$) these amplitudes have been calculated in ref. [9] for arbitrary value of $\mathcal{N}$ by successive derivatives of the $\mathcal{N} = 3$ point function with respect to the cosmological constant μ, the $\mathcal{N} = 3$ point integral was calculated on its turn in ref.[7,8]. In the case where we include the spinor emission vertex $V_{-1/2}^\epsilon$ we have not been able to accomplish the integrals corresponding to the three point function[2] $\langle V_{-1/2}^{\epsilon_1} V_{-1/2}^{\epsilon_2} T_k^{-1} \rangle$ for non-vanishing integer s and therefore we cannot use the same strategy as above to obtain higher point

[1] The BRST invariance automatically guarantees supersymmetry for $V_{-\frac{1}{2}}(k,\epsilon)$. Notice that this is not explicit, as in the case of $T(k)$ (see [15]).

[2] The only non vanishing correlators are those with a net -2 charge for the σ-ghost which cancels the background +2 charge.

amplitudes from the 3-point amplitude, hence we restrict ourselves in this paper to the so called bulk amplitudes ($s = 0$) where we take the finite part of $\Gamma(-s)\mu^s$ when $s \to 0$, namely $\ln \mu$. The simplest non trivial bulk amplitude to be calculated (see ref.[3]) is the two spinor-two scalar scattering four-point amplitude:

$$\mathcal{A}_4^{(2,2)} = \ln \mu \left\langle V_{-1/2}^{\epsilon_1}(k_1) V_{-1/2}^{\epsilon_2}(k_2) T_{k_3}^{-1} T_{k_4} \right\rangle \quad . \tag{14}$$

It is convenient at this point to rewrite T_{k_4} from (4) in the bosonized form, using (8), to obtain

$$T_{k_4} = \frac{1}{2} \int d^2 z_4 e^{ik_4 x + \beta_4 \phi} \left(\alpha_+ e^{ih(z_4)} + 2\alpha_- m_4 e^{-ih(z_4)} \right) \left(\alpha_+ e^{ih(\bar{z}_4)} + 2\alpha_- m_4 e^{-ih(\bar{z}_4)} \right) \tag{15}$$

where $m_i = \frac{1}{2}(\beta_i^2 - k_i^2)$. From the conservation law derived from the integration of the zero mode associated with the massless field h we must have $\epsilon_1 = \epsilon_2$ in order to have $\mathcal{A}_4^{(2,2)} \neq 0$; we choose for convenience $\epsilon_1 = \epsilon_2 = +1$. In this case if we follow Seiberg[16] and work only with positive energy particles ($E > 0$) we use (11) to obtain $k_1, k_2 \geq \alpha_0$. The remaining two momenta k_3 and k_4 are chosen such that $k_3 \geq \alpha_0$ and $k_4 \leq \alpha_0$, thus after fixing the residual $\widehat{SL}(2, \Re)$ invariance with $z_1 = 0, z_2 = 1, z_3 = \infty$ and calling $z_4 = z$ we have

$$\begin{aligned} \mathcal{A}_4^{(2,2)} =& 2 \ln \mu \, \alpha_-^2 m_4^2 \int d^2 z |z|^{2\theta_{14} - 1} |1 - z|^{2\theta_{24} - 1} \\ =& 2\pi \ln \mu \, \alpha_-^2 m_4^2 \Delta(\theta_{14} + \frac{1}{2}) \Delta(\theta_{24} + \frac{1}{2}) \Delta(-\theta_{14} - \theta_{24}) \end{aligned} \tag{16}$$

where $\theta_{ij} = k_i k_j - \beta_i \beta_j$ and $\Delta(x) = \Gamma(x)/\Gamma(1-x)$. Using (13) with $s = 0$, we can rewrite $\mathcal{A}_4^{(2,2)}$ in the kinematic region $k_i \geq \alpha_0$, $i = 1, 2, 3$, $k_4 \leq \alpha_0$:

$$\mathcal{A}_4^{(2,2)} = 2\pi \ln \mu \alpha_-^2 \Delta(m_1) \Delta(m_2) \Delta(m_3 + 1/2) \quad . \tag{17}$$

This result agrees with refs.[3] and suggests again the redefinition[3)]:

$$\begin{aligned} T_k &\to T_k / \Delta(m + 1/2) \\ V_{-1/2}^{\epsilon}(k) &\to V_{-1/2}^{\epsilon}(k) / \Delta(m) \quad . \end{aligned} \tag{18}$$

This is in agreement (up to a factor $\pi\alpha_-^2$) with the conjectured result (for $\mathcal{N} = 4$ and $s = 0$):

$$\mathcal{A}_{\mathcal{N}}^{(n, \mathcal{N}-n)} = \frac{\partial^{\mathcal{N}-3}}{\partial \mu^{\mathcal{N}-3}} \mu^{s + \mathcal{N} - 3} \tag{19}$$

The next simplest amplitude to calculate is $\left\langle V_{\epsilon_1}^{-1/2} V_{\epsilon_2}^{-1/2} V_{\epsilon_3}^{-1/2} V_{\epsilon_4}^{-1/2} \right\rangle$ which cannot be calculated directly since the corresponding integral does not converge in the kinematic region $k_1, k_2 \geq \alpha_0$ $k_3, k_1 \leq \alpha_0$ which is required by selection rule imposed by the zero mode of h. Therefore, we treat instead the case of the 4 spinor - 1 scalar scattering ($\mathcal{A}_5^{(4,1)}$),

$$\mathcal{A}_5^{(4,1)} = \left\langle V_{-\frac{1}{2}}^{\epsilon_1}(k_1) \cdots V_{-\frac{1}{2}}^{\epsilon_4}(k_4) T_{k_5} \right\rangle$$

Due to the h zero mode selection rule we need three spinors with the same polarization and the last one with opposite polarization to those three; thus we choose $\epsilon_1 = +1 = -\epsilon_2 = -\epsilon_3 = -\epsilon_4$, and the kinematic region $k_i \geq \alpha_0$, $(i = 1, 2, 3, 4)$, $k_5 \leq \alpha_0$. Fixing the gauge $z_2 = 1$, $z_4 = \infty$, $z_5 = 0$ and defining $z_1 = w$, $z_3 = z$ we have:

$$\begin{aligned} \mathcal{A}_5^{(4,1)} =& \ln \mu \, \alpha_+^2 \int d^2 z d^2 w \times \\ & \times |z|^{2(2m_3 - 1)} |w|^{2(2m_1)} |1 - z|^{2(-m_2 - m_3)} |1 - w|^{2(-m_2 - m_1 - 1/2)} |z - w|^{2(-m_3 - m_1 - 1/2)} \end{aligned} \tag{20}$$

In general, we do not know how to calculate such integrals, but in this specific case we find that after a convenient shift in m_1 the above integral can be cast into the form of the bulk five point correlator of scalars in the bosonic non-critical string, and such integral has been indirectly calculated in [3]; using that result, we arrive at

$$\mathcal{A}_5^{(4,1)} = \ln \mu \alpha_+^2 \Delta(m_1 + \frac{1}{2}) \prod_{i=2}^{4} \Delta(m_i) \, . \tag{21}$$

The above result also confirms formula (19) (up to α_+^2) after the redefinitions (18).

Now we come to the main computation of this paper, namely the two spinors-three scalars scattering amplitude:

$$\mathcal{A}_5^{(2,3)} = \left\langle V_{-\frac{1}{2}}^{\epsilon_1} V_{-\frac{1}{2}}^{\epsilon_2} T_{k_3}^{(-1)} T_{k_4} T_{k_5} \right\rangle \tag{22}$$

Due to the h zero mode, the spinors must have opposite helicity, and we choose $\epsilon_1 = +1 = -\epsilon_2$, and the kinematic region $k_1 \leq \alpha_0$, $k_i \geq \alpha_0$, $i = 2, 3, 4, 5$. Fixing the gauge $z_1 = 0$, $z_2 = \infty$ and $z_3 = 1$, and defining $z_4 = z$, $z_5 = w$, it is easy to arrive at the following integral representation :

$$\mathcal{A}_5^{(2,3)} = \int d^2 w \int d^2 z |z|^{2(2m_4 - \frac{1}{2})} |w|^{2(2m_5 - \frac{1}{2})} |1 - z|^{2(-m_3 - m_4)} |1 - w|^{2(-m_3 - m_5)} \times \vartheta . \tag{23}$$

Where the fermionic correlators (ϑ) are given by the expression

$$\begin{aligned} \vartheta =& \frac{1}{4} \left\langle e^{\frac{i}{2} h(0)} e^{-\frac{i}{2} h(\infty)} \left(\beta_4 \psi(z) + i k_4 \xi(z) \right) \left(\beta_5 \psi(w) + i k_5 \xi(w) \right) \right\rangle \times \left[h.c. \right] = \\ =& \frac{1}{4} \left\langle e^{\frac{i}{2} h(0)} e^{-\frac{i}{2} h(\infty)} \left(e^{ih(z)} \underbrace{(\beta_4 - k_4)}_{\alpha_+} + e^{-ih(z)} \underbrace{(\beta_4 + k_4)}_{2\alpha_- m_4} \right) \times \right. \\ & \left. \left(e^{ih(w)} \underbrace{(\beta_5 - k_5)}_{\alpha_+} + e^{-ih(w)} \underbrace{(\beta_5 + k_5)}_{2\alpha_- m_5} \right) \right\rangle \times \left[h.c. \right] \\ =& \left[(m_4 + m_5) \left(m_4 |z|^{-1} |w| + m_5 |z| |w|^{-1} \right) |z - w|^{-2} - m_4 m_5 |z|^{-1} |w|^{-1} \right] \end{aligned} \tag{24}$$

[3)] Note that due to the kinematics, $m_4 = -\frac{1}{2}$, and the factor $\Delta(0)$ we would be dividing by corresponds to $\Gamma(-s = 0)$, which has been combined with μ^s in order to produce $\ln \mu$.

Therefore we have,

$$\mathcal{A}_5^{(2,3)} = \{[m_4(m_4+m_5)I_1(m_3,m_4,m_5) + m_5(m_4+m_5)I_1(m_3,m_5,m_4)] - m_4m_5I_2\} \tag{25}$$

where the first integral is given by the expression

$$I_1 = \int d^2z \int d^2w |z|^{2(2m_4-1)}|w|^{2(2m_5)}|1-z|^{2(-m_3-m_4)}|1-w|^{2(-m_3-m_5)}|z-w|^{2(-m_4-m_5-1)} \tag{26}$$

while the second one has already appeared in bosonic computations[3]:

$$I_2 = \int d^2z \int d^2w |z|^{2(2m_4-1)}|w|^{2(2m_5-1)}|1-z|^{2(-m_3-m_4)}|1-w|^{2(-m_3-m_5)}|z-w|^{2(-m_4-m_5)} \tag{27}$$

In general the amplitude $\mathcal{A}_5^{2,3}$ is a combination of two unknown integrals, however it is very fortunate that in the case where we place one of spinor vertices at the infinity we end up with the two integrals above where I_2 has been calculated (indirectly) before in ref.[3] with the result:

$$I_2 = \pi^2\Delta(m_3)\Delta(m_4)\Delta(m_5)\Delta(1-m_3-m_4-m_5) = \pi^2\Delta(m_3)\Delta(m_4)\Delta(m_5)\Delta(1/2+m_2) \tag{28}$$

where we have used the kinematic relation $m_2+m_3+m_4+m_5 = 1/2$. At this point it is quite surprising to have the result (28) contributing to $\mathcal{A}_5^{(2,3)}$ since the role of the factors $\Delta(m)$ and $\Delta(m+1/2)$ seem to be interchanged which might point to some non universality of the vertices redefinition (18). We will convince the reader by calculating I_1 that this is actually not the case and the misplaced poles of I_2 are cancelled by corresponding poles of I_1. In order to obtain I_1 we follow a by now standard procedure. First of all it is convenient to rewrite I_1 using a change of variables after which we have:

$$I_1(m_3,m_4,m_5) = \int d^2z d^2w \times$$
$$\times |z|^{2(2m_3-1)}|w|^{2(-m_3-m_5)}|1-z|^{2(2m_4-1)}|1-w|^{2(-m_4-m_5-1)}|z-w|^{2(2m_5)} \tag{29}$$

Using a technique developed by Dotsenko and Fatteev[21] it is easy to derive from (26) and (29) the asymptotic behaviour:

$$\begin{aligned} I_1(m_3, m_4 \to \infty, m_5) &\sim m_4^{-2-2(m_3+m_5)} \\ I_1(m_3,m_4,m_5\to\infty) &\sim m_5^{-2(m_3+m_4)} \\ I_1(m_3\to\infty,m_4,m_5) &\sim m_3^{-2(m_4+m_5)} \end{aligned} \tag{30}$$

Now we use the property that at the poles of intermediate states $\mathcal{A}_5$ factorizes in four point functions, for which we can use the Virasoro-Shapiro formula; in other words by

using $|z|^{-2+\epsilon} = \frac{\pi}{\epsilon}\delta^{(2)}(z)$ we get from (26) and (29):

$$\begin{aligned} I_1(m_4=\epsilon) &= \frac{\pi^2}{\epsilon}\Delta(m_5)\Delta(m_3)\Delta(1-m_3-m_5) \\ I_1(m_3=\epsilon) &= \frac{\pi^2 m_5^2}{\epsilon(m_4+m_5)^2}\Delta(m_4)\Delta(m_5)\Delta(1-m_4-m_5) \\ I_1(m_5=-1/2+\epsilon) &= \frac{-\pi^2}{\epsilon(m_4-1/2)^2}\Delta(1-m_3-m_4)\Delta(m_3+1/2)\Delta(m_4+1/2) \end{aligned} \tag{31}$$

Furthermore for $m_5 = 0$ the two integrals over z and w decouple, and we have (up to possible factors 1/2)

$$I_1(m_5=0) = \frac{-\pi^2}{m_4^2}\Delta(1/2-m_3-m_4)\Delta(m_3+1/2)\Delta(m_4+1/2) \tag{32}$$

Finally I_1 is also calculable at $m_4+m_5=\epsilon$ where it contains a double pole:

$$I_1(m_4+m_5=\epsilon) = -\frac{1}{\epsilon^2} \quad . \tag{33}$$

By taking formulae (31-33) and the result (28) for I_2 the reader can check that the poles at m_3, m_4, $m_5 \sim \epsilon$ and the double pole at $m_4, m_5 = \epsilon$ are canceled and do not appear in the final expression for $\mathcal{A}_5^{(2,3)}$, only the pole at $m_5 = -1/2+\epsilon$ survives which is in agreement with our expectations. The reader may object at this point that we have only checked the calculation of the first poles of the gamma functions of I_2, however by using the factorization properties, i.e., $|z|^{-6+\epsilon} \sim \frac{\pi}{\epsilon}(\partial_z\partial_{\bar z})^2\delta^{(2)}(z)$ we have also obtained after a long algebra:

$$I_1(m_3=-1+\epsilon) \approx -\frac{m_5^2}{6}(m_4+m_5-1)^2\Delta(m_4)\Delta(m_5)\Delta(-m_4-m_5) \quad . \tag{34}$$

Introducing the result above and (28) altogether in (25) one checks that the first excited pole at $m_3 = -1$ of I_2 does not appear in $\mathcal{A}_5$ and this is a very striking cancelation which leads us to suggest the following result for I_1:

$$\begin{aligned} I_1 = &\frac{\pi^2 m_5}{(m_4+m_5)^2}\Delta(m_3)\Delta(m_4)\Delta(m_5)\Delta(1-m_3-m_4-m_5) \\ &- \frac{\pi^2}{(m_4+m_5)^2}\Delta(m_3+1/2)\Delta(m_4+1/2)\Delta(m_5+1/2)\Delta(1/2-m_3-m_4-m_5) \end{aligned} \tag{35}$$

The above result is in agreement with all[4)] formulae we have derived so far from I_1 including the asymptotic behaviors (30) (one uses Stirling formula to check it). Using (28) and (35) in (25) we have as expected:

$$\mathcal{A}_5^{(2,3)} = -\pi^2\Delta(m_2)\Delta(m_3+1/2)\Delta(m_4+1/2)\Delta(m_5+1/2) \quad . \tag{36}$$

4) Formula (35) does not agree with $I_1(m_4+m_5=\epsilon)$ and $I_2(m_5=-1/2+\epsilon)$ up to factors 1/2 which we conjecture to be traced back to symmetry factors.

At last we should remark that although we have not been able to explicitly calculate the four spinor bulk ($s = 0$) scattering $\mathcal{A}_4^{(4,0)}$ it is clear that for $s = 1$ it corresponds to $\mathcal{A}_5^{(4,1)}$ with $k_1 = 0$. In order to calculate correlators involving more than four spinors we need the $V_{+1/2}$ vertex but this is out of the scope of this letter.

From the result (36) we are reasonably safe to conclude that the correlation functions factorize, and each bosonic external leg contributes a factor $\Delta(m + \frac{1}{2})$, while a Ramond vertex gives a contribution $\Delta(m)$.

The factorizable result is actually highly non-trivial. Although, as mentioned, some authors already present some of them, it is not clear at all how they should be obtained from the recent supermatrix model techniques[11], in such a way that a non-trivial check is also required. In some cases, the relevant spinor fields seem to contribute with a zero factor to several correlators (those involving more than two spinors) as opposite to our non-vanishing result (21), which involves four spinors. Nevertheless, there is a possibility of matching the results disentangling the analytical contributions to the fermionic correlators in a similar way as it was done in the bosonic matrix models[12].

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