UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498-970 SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-1037

INCLUSIVE BREAK-UP OF EXOTIC NUCLEI

M.S. Hussein
Instituto de Física, Universidade de São Paulo

F. Carstoiu
Institute of Atomic Physics
P.O. Box MG6
Bucharest, Romania

Fevereiro/1993

Inclusive Break-up of Exotic Nuclei

M.S. Hussein*
Nuclear Theory and Elementary Particle Phenomenology Group, Instituto de Física, Universidade de São Paulo, C.P. 20516, São Paulo, S.P., Brazil

F. Carstoiu
Institute of Atomic Physics, P.O. Box MG6
Bucharest, Romania

December 7, 1992

Abstract

The inclusive break-up of neutron-rich nuclei is discussed. Using an extension of the Kox empirical formula for the total reaction cross-section of heavy ions, the nuclear component of the break-up of exotic nuclei is calculated as a function of bombarding energy. Results are presented for $^{11}Li + ^{12}C$ and $^{11}Li + ^{208}Pb$ systems in the laboratory energy range $B_c \leq E_{Lab.} \leq 60$ MeV/n, where B_c is the height of the Coulomb barrier. Comparison with experimental results for ^8He, ^{11}Li, ^{14}Be fragmentation reactions on Be, Ni and Au targets at 30 MeV/n are made. The one-neutron removal cross-section in ^{11}Be-induced reactions is also calculated.

*Supported in part by CNPq

The determination of the electromagnetic dissociation cross-section (EDC) of neutron-rich nuclei is of great importance for the inference of the low energy response. The extraction of this responce allows the study of the so-called "soft giant resonances" (SGR). In order to obtain this response from the experimental data, it is essential to have a reliable estimate of the nuclear contribution [1]. Having in mind the upsurge of interest in low energy exotic beam physics, a through discussion of this question is required.

Several publications have addressed the question of claculating the nuclear component of the 2n removal cross-section in ^{11}Li-induced reactions. These range from Glauber, polarization potential and phenomenological. Here, we suggest an extension of the theory of Hussein and McVoy [2] that allows the calculation of σ_{-2n}^N within an optical model description. Similar ideas were developed by Ogawa [3] and collaborators. Our method, to be described below, allows the calculation of σ_{-2n}^N in the low energy range $B_c \leq E \leq 60$ MeV/n, where the Glauber model, used by Ogawa et al [3]. would not work.

In order to similify the presentation we present in what follows a qualitative account of the HM theory. Denoting the survival probability of a fragment i by $Pi(b)$, where b is the impact parameter, we can write the inelastic nuclear inclusive probability, $P^{N,inel}(b)$, of detecting fragment b in the break-up of the projectile a into $x + b$

$$P_b^{N,inel}(b) = (1 - P_x(b))\, P_b(b) \tag{1}$$

Eq.(1) measures the probability that b survives where as x interacts non-elastically with the target.

The other important break-up process is the elastic nuclear break-up, whose probability is given by

$$P_b^{N,el}(b) = P_b(b)\, P_x(b) - P_a(b) \tag{2}$$

Summing (1) and (2) gives the total nuclear inclusing break-up probability

$$P_b^N(b) = P_b^{N,el}(b) + P_b^{N,inel}(b) \tag{3}$$

$$= P_b(b) - P_a(b) \tag{4}$$

The corresponding cross-section is just

$$\sigma_b^N = 2\pi \int b db\, (P_b(b) - P_a(b)) \tag{5}$$

$$\equiv \sigma_{Int}(a) - \sigma_{Int}(b) \quad (6)$$

In Eq.(6), $\sigma_{Int}(i)$ is the interaction cross-section of fragment i. Eq.(6) has been derived within the Glauber theory by Ref.3). Here, we suggest that this equation is valid in general. The next step is to decide upon a Model for $\sigma_{Int}(i)$. We now assume that $\sigma_{Int}(a)-\sigma_{Int}(b)$ is equal to $\sigma_R(a)-\sigma_R(b)$, where $\sigma_R(i)$ is the total reaction cross-section of fragment i. Such an approximation implies that the inelastic excitations of the target by a and b are iqual. Then a very simple recipe can be devised for σ^N_{-2n} based on known facts about $\sigma_R(i)$.

Before presenting our numerical results we first present a derivation of Eq.(6) using the theory of inclusive break-up reaction developed by Hussein and McVoy (HM). We also extend this to take into account the elastic break-up contribution not considered in HM. According to HM, the inclusive enelastic break-up reaction

$$a + A \rightarrow b + (x + A) \quad (7)$$

$$a = b + x$$

is described by the following cross-section

$$\begin{aligned} \frac{d\sigma}{d\Omega_b dE_b} &= -\frac{2}{\hbar v_a}\rho(E_b)(\hat{\psi}_x^{(+)} \mid W_{xA} \mid \psi_x^{(+)}) \\ \rho(E_b) &= \frac{M_b k_b}{(2\pi)^3\hbar^2} \ , \end{aligned} \quad (8)$$

where W_{xA} is the imaginary potential of the $x - A$ system and the overlap wave function $\mid \psi_x^{(+)}$ is just $(\chi_b^{(-)} \mid \phi_a \chi_a^{(+)} >$. If the internal wave function of a, ϕ_a, is taken to be a Gaussian, then the integration of (2) gives the primary inelastic yield of the observed spectator fragment b,

$$\sigma_b^{N,inel} = \frac{\pi}{\sigma^2}\sum_{j=0}^{\infty} \mid S_j^b \mid^2 (1- \mid S_j^x \mid^2) \quad (9)$$

where σ is the momentum width of ϕ_a and $\mid S_j^i \mid^2$ is given by

$$\mid S_i^j \mid^2 \equiv \frac{(\sigma^2)^{j+i}}{j!}\int_0^\infty b_i^{2j} db_i^2 e^{-\sigma^2 b_i^2} \mid S_i(b_i) \mid^2 \quad (10)$$

In (4) $S_i(b_i)$ is the elastic S-matrix of fragment i. Therefore $\mid S_j^i \mid^2$ measures the Fermi motion modified survival probabability of fragment i. Note, if σ is very small, as would be the case of a loosely bound $x - b$ system,

$$\begin{aligned} \sigma_b^{N,inel}(\sigma \rightarrow 0) &= 2\pi \int bdb \mid S_b(b) \mid^2 (1- \mid S_x(b) \mid^2) \ , \\ P_i(b) &\equiv \mid S_i(b) \mid^2 \end{aligned} \quad (11)$$

Eq.(5) is the formula used by Esbensen and Bertsch [4] and by Ogawa et al [3] to represent the inclusive inelastic break-up yield. It is obvious from the discussion above that such an expression is reasonable only when σ is very small or, equivalently, the binding energy of the cluster is very small, such as in $^{11}Li(E_{2n} \simeq 0.3$ MeV).

We turn now to elastic break-up contribution. The cross-section for this process in the prior representation is given by

$$\begin{aligned} \frac{d^2\sigma_b^{N,el}}{d\Omega_b dE_b} &= \frac{2\pi\rho(E_b)}{\hbar v_a}\sum_{k_x} \mid< \chi_x^{(-)}\chi_b^{(-)} \mid (U_{xA} + U_{bA} - U_{aA}) \mid \phi_a \chi_a^{(+)} >\mid^2 \\ &\quad . \ \delta(E - E_x - E_b - \varepsilon_0) \end{aligned} \quad (12)$$

In Eq.(12), U_{iA} is the complex optical for the $i + A$ system and ε_0 is the Q-value (separation energy) that determines the spatial extention of the internal wave function of the projectile ϕ_a. The total elastic break-up is obtained easily by writing

$$\sigma_b^{N,el} \equiv \int \frac{d^2\sigma_b^{N,el}}{d\Omega_b dE_b} d\Omega_b dE_b = \int \frac{d^2\sigma_b^{N,el}}{d\Omega_b dE_b}\frac{d\vec{k}_b}{(2\pi)^3}\frac{1}{\varrho(E_b)} \quad (13)$$

thus, approximately

$$\begin{aligned} \sigma_b^{N,el} &= \frac{2\pi}{\hbar v_a}\int \frac{d\vec{k}_b}{(2\pi)^3}\int \frac{d\vec{k}_x}{(2\pi)^3} \mid< \chi_x^{(-)}\chi_b^{(-)} \mid U_{xA} + U_{bA} \mid \chi_x^{(+)}\chi_b^{(+)}\phi_a > \\ &- < \chi_a^{(-)} \mid U_{aA} \mid \chi_a^{(+)}\phi_a >\mid^2 \delta(E - E_x - E_b - \varepsilon_0) \end{aligned} \quad (14)$$

In Eq.(14), we have made use of the fact that U_{aA} depends on $\vec{r}_{aA}$, U_{xA} depends on $\vec{r}_{xA}$ and U_{bA} on $\vec{r}_{bA}$. With the aid of the Glauber-type approximation employed by HM, Eq.(14) can be reduced to the form (for very small momentum width of Φ_a).

$$\sigma_b^{N,el} = 2\pi \int bdb[| S_b(b) |^2 | S_x(b) |^2 - | S_a(b) |^2] \qquad (15)$$

Having verified the reaction theoretical bais of Eq.(6), with σ_{int} replaced by σ_R, (see discussion following Eq.6) we now turn to applications. In order to calculate Eq.(6) we use the well-known Kox formula for σ_R [5]. We have

$$\sigma_R = \frac{\hbar\pi r_o^2\omega}{2\pi E}\{A_1^{1/3}+A_2^{1/3}+b\frac{A_1^{1/3}+A_2^{1/3}}{A_1^{1/3}A_2^{1/3}}-a+D\}\ell n(1+exp\frac{2\pi}{\hbar\omega}(E-B_c)) \quad (16)$$

In Eq.(16) we have replaced the barrier factor $(1-\frac{B_c}{E})$ in the original Kox formula by the more exact Wong factor $\frac{\hbar\omega}{2\pi E}\ell n(1-exp[\frac{2\pi}{\hbar\omega}(E-B_c)])$ with $\hbar\omega \sim 3.0$ MeV.

When applied to non-exotic nuclei Eq.(16) gives the following values for the parameters
$r_0 = 1.1$ fm , $b = 1.85$, $a = 0.65$, $D = 0$ and $B_c = \frac{Z_1Z_2e^2}{1.3(A_1^{1/3}+A_2^{1/3})}$. In the calculation we present below for the break-up of neutron-rich nuclei, we use the exact values for their radii.

In figure 1 we show σ_{-2n}^N vs. E_{Lab} for the system ^{11}Li + ^{12}C in the energy range 6.5 MeV/n < E_{Lab} < 60 MeV/n. We believe our model to be adequate in this range since mostly geometrical features of the system dictates the value of σ_{-2n}^N. At highes energies, a description using the nucleon-nucleon cross-section is more appropriate. We see in the figure that σ_{-2n}^N rises with energy and reaches a maximum at about 10 MeV/n after which it starts slowly decreasing. The average value of the cross-section in the energy range 10 MeV/n < E_L < 50 MeV/n is roughly $\pi(R_{^{11}Li}+R_{Target})^2-\pi(R_{^9Li}+R_{Target})^2$. The Coulomb dissociation cross-section for this system is calculated using the formulation of Ref.6 within the usual Weissacher- Williams picture, and assuming a cluster structure of the exotic nucleus [7] with $E_{2n} = 0.35$ MeV. We show this also in Fig.1. The Coulomb contribution also peaks but at a much smaller energy, $E \cong 2$ MeV/n. Then it starts dropping, much faster than the nuclear contribution. At $E \sim 58$ MeV/n it amounts to just about 3% of σ_{-2n}^N , as one expects due to the small charges envolved. We mention here that our results for σ_{-2n}^N at $E_{Lab} \sim 60$ MeV/n is very close to those Ref.3 and to Canto et al.[8]. The result of Ref.8 at lower energy, however, overestimates σ_{-2n}^N.

In figure 2 we show the result of our calculation for the system ^{11}Li + ^{208}Pb. Here the Coulomb dissociation cross-section is quite large and dominant. Again we see the peaking in σ_{-2n}^N. We should remind the reader that our calculated Coulomb dissociation cross-section within the cluster model is an overestimate [1],[9]. We include it here for completeness.

We have also calculated σ_{-2n}^N for other systems such as those studied by Riisager et al. at $E_{Lab} = 30$ MeV/n [10]. We present in table 1 some representative cases. The measured values if the total 2n-renoval cross-section are also given. The agreement between our model calculation and the experimental data is reasonable. The two-neutron separation energies in ^8He, ^{11}Li and ^{14}Be were taken to be 2.137 MeV, o.35 MeV and 1.34 NeV, respectively. The experimental value of σ_{-2n} for ^{11}Li+ Au, namely 10.8 ± 1.9 barns would yield a Coulomb dissociation cross-section of $\simeq 9.0 \pm 1.9$ barns if we subtract our calculated nuclear component. We find this value a bit too large considering the result of Ref.9 which gives for σ_{-2n}^C of $^{11}Li+^{208}Pb$ at E_{Lab} =28 MeV/n, a quite similar system, the value of 3.6 ± 0.4 b. Finally, we have performed calculation of the one-neutron removal cross-section in ^{11}Be-induced reactions. Here, the one-neutron separation energy is about 0.5 MeV and one would expect similar behaviour for σ_{-n} as that of σ_{-2n}. At $E_{Lab} = 30$ MeV/n, we obtain for the system $^{11}Be+^{28}Si$ $\sigma_{-n}^N = 568$ mb, $\sigma_{-n}^C = 82$ mb, where as for $^{11}Be+^{238}U$ we find $\sigma_{-n}^N = 1050$ mb and σ_{-n}^C =2463 mb.

In conclusion, we have developed in this paper a model for the nuclear component of the inclusive break-up cross-section of exotic nuclei valid at low energies where geometrical features dominate. We applied our model to the $^{11}Li+^{12}C$ and $^{11}Li+^{208}Pb$ systems in the energy range $B_c \leq E_{Lab}$ < MeV/n. The cross-section is found to rise sharply as the energies is increased above these barrier then it saturates at the value $\pi(R_{^{11}Li}+R_{Target})^2-\pi(R_{^9Li}+R_{Target})^2$. At energies above 50 MeV/n the cross-section starts dropping. The one-neutron removal cross-section in ^{11}Be-induced reactions is also calculated. We also compared our results with available data at $E_L = 30$ MeV/n. The systems considered are ^8He, ^{11}Li and ^{14}Be on Be, Ni and Au targets. The agreement with the data was found to be quite reasonable.

Acknowledgements

The authors express their thanks to their collegues at GANIL where part of this work was performed.

References

[1] For a recent review see e.g., C.A. Bertulani, L.F. Canto and M.S. Hussein, *Physics Reports*, in press.

[2] M.S. Hussein and K.W. McVoy, *Nucl. Phys.* A445, 124 (1985).

[3] Y. Ogawa, K. Yabana and Y. Suzuki, *Nucl. Phys.* A543, 722 (1992).

[4] H. Esbensen and G.F. Bertsch, *Phys. Rev.* C46, 1552 (1992).

[5] S. Kox et al. *Phys. Lett.* 159B, 15 (1985); S. Kox et al. *Phys. Rev.* C35, 1678 (1985).

[6] M.S. Hussein, M.P. Pato and C.A. Bertulani, *Phys. Rev.* C44, 2219 (1991).

[7] C.A. Bertulani, G. Baur and M.S. Hussein, *Nucl. Phys.* A526, 751 (1991).

[8] L.F. Canto, R. Donangelo, M.S. Hussein and M.P. Pato, *Nucl. Phys.* A542, 131 (1992).

[9] The recent measurement of σ^C_{-2n} for $^{11}Li + ^{208}Pb$ at E = 28 MeV/n by K. Ieki et al., *MSUCL-851*, July 1992, gave $\sigma^C_{-2n} \sim 3.6 \pm 0.4$ barns. Our cluster calculation gives $\sigma^C_{-2n} = 4.8$ barns with $\varepsilon_{2n} = 0.35$ MeV.

[10] K. Riisager et al., *Nucl. Phys.* A540, 365 (1992).

Table Captions

Table 1. Comparison between calculated cross-sections and the experimental data taken from Ref.10. See text for details.

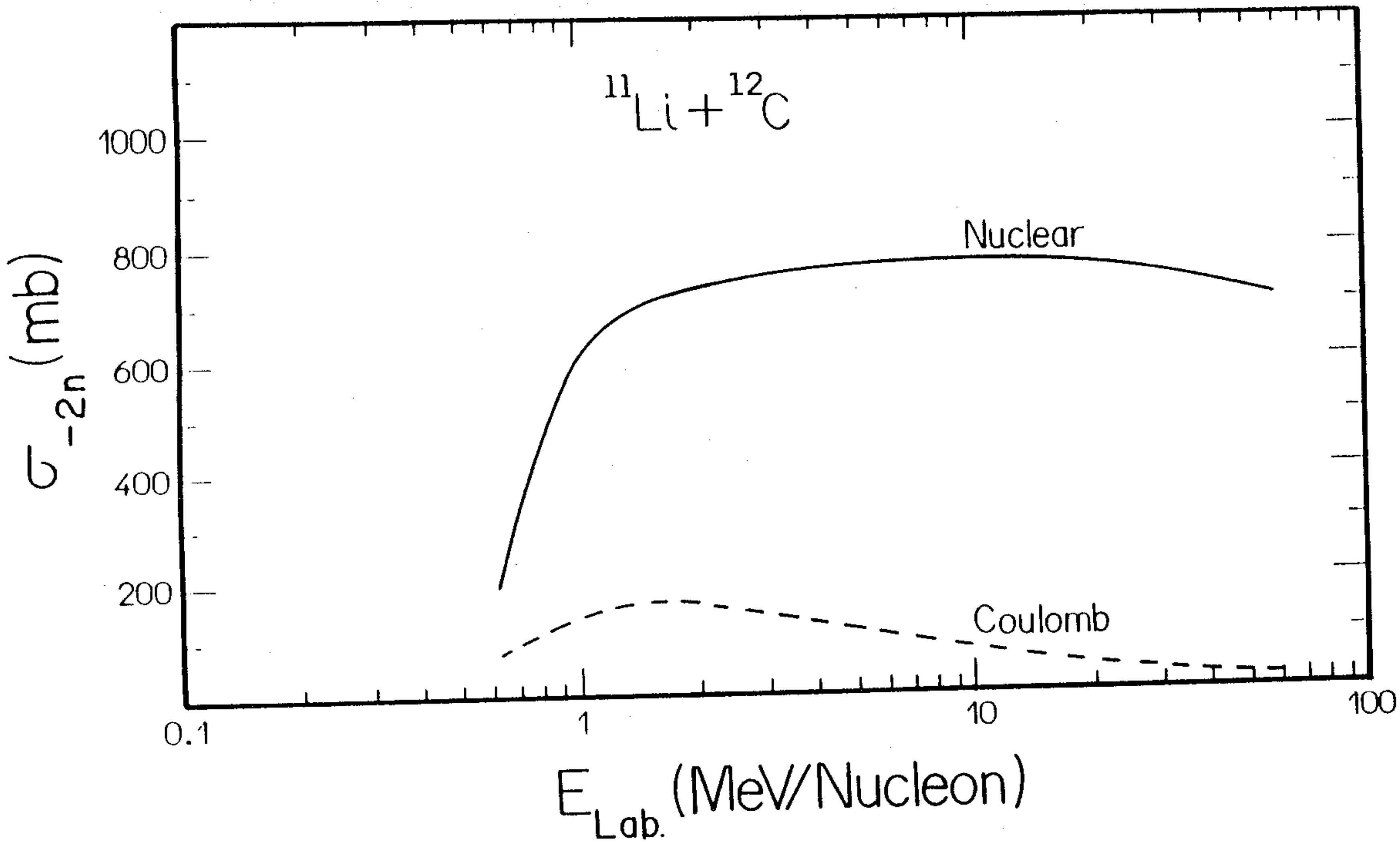
Figure Captions

Figure 1: The two-neutron removal cross-section for $^{11}Li +^{12} C$ vs. E_{Lab}, see text for details.

Figure 2: Same as in figure 1 for $^{11}Li +^{208} Pb$.

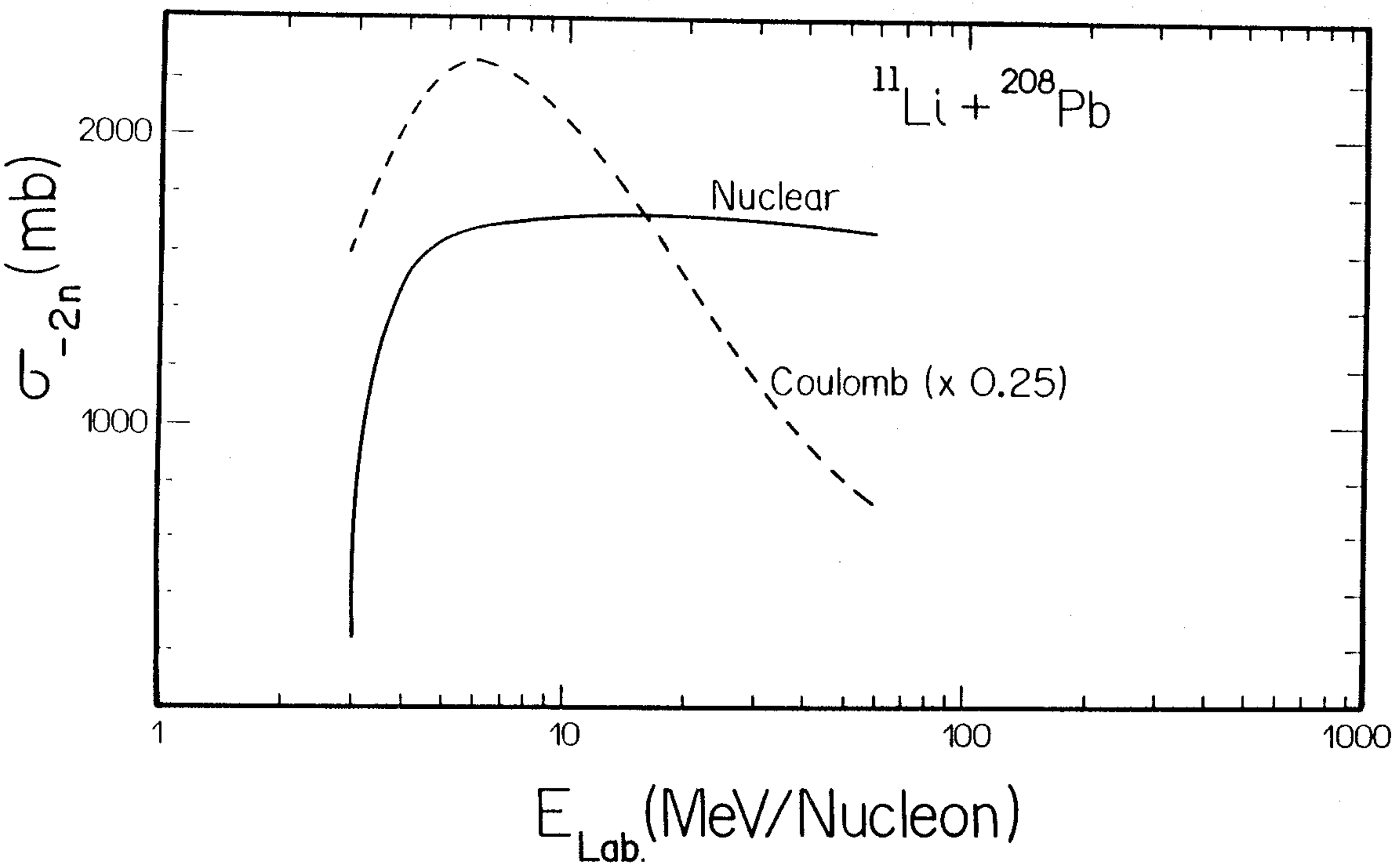
Table 1

Beam		Calculated Cross-Section (mb) Be	Ni	Au	Measured Cross-Section (barns) Be	Ni	Au
^{8}He	Nuclear:	310.295	555.23	832.50			
	Coulomb:	0.564	14.78	45.83			
	Sum :	310.858	570.01	878.33	0.41 ± 0.15	$1.5 {}^{+1.0}_{-0.5}$	$2.9 {}^{+5}_{-2.5}$
^{11}Li	Nuclear:	693.14	1181.02	1702.45			
	Coulomb:	15.61	645.55	4529.87			
	Sum :	708.75	1826.57	6212.32	0.47 ± 0.1	2.1 ± 0.4	10.8 ± 1.9
^{14}Be	Nuclear:	400.2	685.53	994.79			
	Coulomb:	1.91	55.40	219.78			
	Sum :	402.1	740.93	1214.57	0.44 ± 0.1	1.0 ± 0.3	2.1 ± 0.6



$^{11}Li + ^{12}C$
Nuclear
Coulomb
1000
800
600
400
200
σ_{-2n} (mb)
0.1
1
10
100
$E_{Lab.}$ (MeV/Nucleon)

Fig. 1



$^{11}Li + ^{208}Pb$
Nuclear
Coulomb (x 0.25)
2000
1000
1
10
100
1000
σ_{-2n} (mb)
$E_{Lab.}$(MeV/Nucleon)

Fig. 2