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QUANTUM LIMITS FOR MEASUREMENTS ON MACROSCOPIC BODIES: A DECOHERENCE ANALYSIS

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Quantum Limits for Measurements on Macroscopic Bodies : a Decoherence Analysis

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Abstract

We consider in this letter the quantum limits for measurements on macroscopic bodies, which are obtained in a novel way, employing the concept of decoherence coming from an analysis of the quantum mechanics of dissipative systems. Two cases are analysed, the free particle and the harmonic oscillator and for both systems, we compare our approach with previous treatments of such limits.

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One of the outstanding problems in contemporary physics is the detection of the gravitational waves predicted by Einstein in his General Relativity Theory [1]. The detection of such waves presents a challenge to the experimentalists who have to monitor extremely small fluctuations in the position of macroscopic bodies [2]. The sensitivity planned for the future generation of detectors, will bring us close to the limits imposed by quantum mechanics [3].

It is our purpose in this letter to examine in detail the obtention of quantum limits for free masses and for the harmonic oscillator, subjected to thermal fluctuations. We rely on the methods developed by several authors to analyse the quantum mechanics of a system in interaction with its environment [4].We differ in this respect from Braginsky [3] who uses a two step procedure in order to obtain the quantum limits, firstly considering the limits imposed by the uncertainty principle on an isolated system, not subjected to thermal fluctuations and then, in the second step, looking at the same system, this time classically behaved but interacting with the environment. This procedure is, in our view, unacceptable since it ignores that the system is always in contact with the environment. There is no need for this asymmetry, treating the system classically when subjected to thermal fluctuations, but ignoring the latter when viewing it quantum mechanically.

Following the approach of references [4], we are led to consider the master equation for the evolution of the reduced density matrix ρ in the coordinate representation, describing the state of the system under consideration

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] - \gamma(x - x')\left(\frac{\partial\rho}{\partial x} - \frac{\partial\rho}{\partial x'}\right) - \frac{2m\gamma k_B T}{\hbar^2}(x - x')^2\rho \qquad (1)$$

For our purposes the relevant term in Eq.(1) is the last one. We see that this term contains the thermal fluctuations in a quantum mechanical context, entailing a characteristic scale which gives the decoherence time.The decoherence time characterizes the vanishing of the off-diagonal elements of the density matrix, which provide a measure of the quantumness of the system [4]. From Eq.(1) the decoherence time scale is given by

$$\tau_D = \frac{\hbar^2}{2m\gamma k_B T(\Delta x)^2} \qquad (2)$$

In (1) and (2) γ^{-1} is the relaxation time of the system, m the mass and T the temperature. A proper interpretation of Eq. 2 indicates that $(\Delta x)^2$ is the mean square deviation of the position of the system. We will come back to this important point at the end of this letter.

Let us now apply the above concept in order to obtain the quantum limits for two systems, the free particle relevant to laser interferometric antennas, and the harmonic oscillator, relevant to mechanical bars.

Free Particle The quantum limit is obtained when $(\Delta x)^2$ in Eq.(2) becomes of the same magnitude as the uncertainty arising from Heisenberg's principle, when applied to successive measurement of the position of the free particle separated by a time interval τ . This quantum mechanical uncertainty is given by [3]

$$(\Delta x)^2 \approx \frac{\hbar\tau}{m} \tag{3}$$

Inserting (3) into the expression for τ_D (Eq.(2)), we obtain

$$\tau_D = \frac{\hbar}{2\gamma k_B T\tau} \tag{4}$$

If τ_D is greater than the time interval between two successive measurements, τ , the system must be treated quantum mechanically. When this happens we obtain the quantum limit

$$\hbar > 2\gamma k_B T\tau^2 \tag{5}$$

Since γ^{-1} is the relaxation time of the system (τ^*), Eq.(5) can be rewritten in the more usual form

$$\hbar > 2k_B T\frac{\tau^2}{\tau^*} \tag{6}$$

which is the limit obtained by Braginsky [3], who arrived at this result by a somewhat obscure path, as we will make explicit later on.

Harmonic Oscillator For a harmonic oscillator of fundamental frequency ω, the Heisenberg uncertainty principle gives

$$(\Delta x)^2 \approx \frac{\hbar}{2m\omega} \tag{7}$$

for a measurement time of the order of the period of the harmonic oscillator $\left(\tau \approx \frac{2\pi}{\omega}\right)$. Replacing (7) into (3) leads to

$$\tau_D = \frac{\hbar\omega}{\gamma k_B T} \tag{8}$$

Imposing $\tau_D > \tau$ gives the quantum limit for the oscillator

$$\hbar > \gamma\frac{k_B T\tau}{\omega} = \frac{k_B T}{\omega}\frac{\tau}{\tau^*} \tag{9}$$

A similar result was obtained by Braginsky [3], only differing from (9) by a numerical factor.

Having shown how to obtain the quantum limits in an internally consistent way, which takes into account both quantum and thermal fluctuations, we now raise one further objection to the derivation by Braginsky of the quantum limit for a free particle.

The authors of reference [5] use as the starting point, Nyquist's theorem which gives for the spectral density of the fluctuating force the result

$$\left\langle F_{fl}^2\right\rangle_\omega = 4k_B T\frac{\gamma}{m} \tag{10}$$

Then they use this fluctuating force properly integrated over a range of frequencies $\Delta\omega \approx \tau^{-1}$, to find the displacement of the particle, $x = x_0 + \frac{1}{2}at^2$ under a constant acceleration given by

$$a = \frac{\sqrt{\left\langle F_{fl}^2\right\rangle_\omega \tau^{-1}}}{m} \tag{11}$$

which then gives for the *displacement*

$$\Delta x = \sqrt{\frac{k_B T}{m}\frac{\tau^3}{\tau^*}} \tag{12}$$

This result is in disagreement with a standard treatment of the Brownian classical particle [6]. Of course the latter treatment refers to the mean square deviation of the position of the particle, which as a matter of fact is what is monitored in gravitational wave antennas [2], while Braginsky considers a displacement, to be later on compared with a fluctuation (from Heisenberg's principle). By sheer coincidence his result is the same as the one obtained by us. Finally we remark that when Braginsky considers the harmonic oscillator, the classical part is treated in the right way, as a fluctuating Brownian particle in the potential of a harmonic oscillator.

We finish this letter with a comment to the respect of the validity conditions of the master equation (1). It is normally referred in the literature as been derived in the high-temperature limit. The results of Zurek [7] indicate however its validity outside this domain. Next we plan to attack the problem of quantum non-demolition measurements [8] taking into account dissipative effects.

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