UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498-970 SÃO PAULO - SP
BRASIL

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FINITE-SIZE SCALING BEHAVIOUR OF THE SU(2) LATTICE GAUGE THEORY

Nelson A. Alves
Instituto de Física, Universidade de São Paulo

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Finite-size scaling behaviour of the SU(2) lattice gauge theory

Nelson A. Alves

Departamento de Física Matemática, Instituto de Física
Universidade de São Paulo, Caixa Postal 20516
01498 - São Paulo, SP, Brazil

Abstract

We apply spectral density reweighting techniques to study the deconfinement transition of the SU(2) lattice gauge theory. We have included scaling corrections in our finite-size analysis of the order parameter susceptibility $\chi(P)$. The stability of finite-size scaling fits of $\chi(P)$ is investigated in the $\{\gamma/\nu,\ w\}$ parameter space, where w is the critical exponent related to the leading irrelevant scaling field.

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During the last years new numerical techniques and the improvement of earlier methods of data analysis have led to a better determination of physical quantities in Monte Carlo (MC) calculations. In particular, reweighting techniques and optimization in combining MC samplings, because of their efficiency, have received considerable attention [1-9]. In [10] we have further elaborated on the reweighting technique ideas and introduced a new procedure to combine ("patching") overlaping MC data from simulations at various β_0 couplings. There, we have shown how spectral density methods greatly increase accuracy and facilitate finite-size scaling (FSS) calculations for the SU(3) deconfining phase transition. This have allowed us to obtain more accurate MC estimates for the location of the peak of relevant thermodynamic functions, i.e, specific heat c_v and the Polyakov loop susceptibility $\chi(P)$.

Here, we also apply the patching procedure to the SU(2) lattice gauge theory to study its well known second order deconfinement transition. To draw conclusions about the evaluation of the ratio γ/ν of critical exponents, we investigate the stability of FSS fits of $\chi(P)$ in the $\{\gamma/\nu,\ w\}$ parameter space. Here w is the critical exponent related to the leading irrelevant scaling field [11,12] of the model.

We simulated the SU(2) Wilson action in a four-dimensional lattice,

$$S = \sum_p S_p \quad \text{with} \quad S_p = \frac{1}{2} Tr(U_p) \ , \tag{1}$$

where U_p is the ordered product of link matrices around the plaquette p of the $L_t L^3$ lattice, for $L_t = 4$. For each simulation, a certain number of initial sweeps (10000) were discarded for thermalization, and due to the continuous nature of SU(2) action density it was convenient to store all measurements after every sweep through the lattice. This storage procedure provides us with the full empirical time series for each MC simulation (120000 measurements in general, for L=16 and $\beta_0 = 2.300$ we have used twice this statistics), which allows a precise application of reweighting and patching techniques. With the available time series, the reweighting technique allows us to calculate an estimator $\overline{f}(\beta)$ for the physical observable f, in a neighborhood $\triangle\beta$ of the simulated point β_0. $\triangle\beta$ is the range of validity for a spectral density obtained from a MC simulation. We have defined in [10] a convenient way to estimate this range by applying the concept of q-tiles s_q [13] on our empirical action density distribution. This β-range is used to check the validity in finding out the maximum of $\chi(P)$ when carrying out the extrapolation in β. Besides that we can define better overlapping regions among different MC runs at β_0^i values $(\beta_0^{i+1} > \beta_0^i\ ,\ \ i = 1, ..., P)$ for P runs. This led us to a new procedure of combining different histograms to improve our estimate for $\overline{f}(\beta)$,

$$\overline{f} = \sum_{i=1}^{P} w_i \overline{f}_i \ , \tag{2}$$

where the optimal choice for the normalized weight factors w_i turns out to be the inverse variance of $\overline{f}_i$, which can be estimated as the empirical error bars,

$$w_i \ \sim \ \frac{1}{(\triangle \overline{f}_i)^2} \ , \tag{3}$$

from each MC simulation at β_0^i. The overall constant is fixed by the normalization condition $\sum_{i=1}^{P} w_i = 1$.

This approach is now applied to the lattice average of the Polyakov loop $P = L^{-3} \sum_{\mathbf{x}} P_{\mathbf{x}}$. Here $P_{\mathbf{x}}$ stands for the product of all link variables $U_{(t,\mathbf{x});0}$ along the time direction at a fixed spatial lattice point $\mathbf{x}$, closed by the imposed periodicity of the lattice. The expectation value of $P_{\mathbf{x}}$ is an order parameter for the deconfinement transition quite similar to the magnetization in a Z_2 spin system [14]. Recall that the universality conjecture [15] classifies the SU(2) lattice gauge theory and the three dimensional Ising model in the same class.

In this letter we present our data analysis for the order parameter susceptibility which, due to a spin flip like system, is most conveniently defined in terms of the modulus of its lattice average [16,17],

$$\chi(P) \ = \ L^3(< P^2 > - < |P| >^2). \tag{4}$$

In table 1 we present our numerical results for the maximum of the susceptibility density $L^{-3}\chi_{max}(P)$ in the validity β range, for each of our main data sets at couplings β_0, and the most significative patching results. The relative weights of patched data sets at β_{max} are presented in the last column. Here, β_{max} corresponds to a L-dependent series, defined by the condition

$$\chi(P;L)(\beta) \ = \ maximum, \ \ \text{for} \ \beta \ = \ \beta_{\max}(\mathrm{L}) \ . \tag{5}$$

The leading scaling behavior for the maximum of the susceptibility in a second order phase transition determines the ratio between the critical exponents γ and ν. If analytic contributions dominate, the susceptibility may scale as

$$\chi(P;L)(\beta_c(L)) \ = \ a_1 L^{\gamma/\nu} + a_2 \ . \tag{6}$$

For small systems, we may have, beyond corrections to finite-size scaling, corrections due to the leading irrelevant field or nonlinearities of the scaling variables [11,12] as we stay away from the truly asymptotic regime. This regime is expected to be atttained at the infinite-volume critical coupling $\beta_c \equiv \beta_c(L = \infty)$. In fact, our β_{max} is a finite-size estimate for the critical coupling, $\beta_c(L) = \beta_{max}$, obtained from rather small lattice sizes. Correspondingly, if we take into account the effect of these possible corrections, we can obtain a more reliable estimate for $\chi(P)$ through the scaling fit [18,19]

$$\chi(P;L)(\beta_c(L)) \ = \ a_1 L^{\gamma/\nu} + a_2 L^{\gamma/\nu + w}, \tag{7}$$

where w is a negative exponent. It stands for an effective correction exponent to account for the above mentioned possible sources for corrections. For large lattice sizes we expect that finite size effects will not give relevant corrections, although we may have them from the leading irrelevant field [20].

The leading irrelevant exponent has been calculated for the 3D Ising model by high-temperature series expansions as the first confluent correction exponent [20,21], which corresponds to $w \simeq -0.80$. Monte Carlo calculations give the values $w \simeq -0.25$ [19] (for the susceptibility, and values close to 1.0 for other thermodynamic quantities), and $w \simeq -0.75$ [18].

We now proceed to evaluate the critical exponents γ/ν and w. Equation (7) corresponds to a multi-parameter fit. Hence, it is convenient to study its consistency under the input parameter w by monitoring the goodness-of-fit Q, $0 \leq Q \leq 1$ [22]. As a natural assumption, a very small value for Q would mean that the fit is probably not acceptable for the available data. To have a more stable fit we looked at eq. (7) as a two parameter fit for a_1 and a_2 in the $\{\gamma/\nu,\ w\}$ parameter space.

In fig. 1 and 2 we show the Q-surfaces performed for the ranges $L = 6-26$ and $L = 8-26$, respectively, for our patched data from table 1 and data for $L = 18$ and 26 from ref. [16]. A simple inspection of table 1 shows that patched data are clearly more precise than the ones obtained as a simple error propagation average over single data sets. A figure similar to fig. 2 is also obtained for $L = 10-26$. Fig. 1 shows how stable is the γ/ν determination ($\gamma/\nu \simeq 1.98$) on a rather large range for w, while its position decreases to $\gamma/\nu \simeq 1.94$ for the next L range. In table 2 we illustrate numerically some γ/ν estimates for the above mentioned values for w, with the corresponding values for Q. As a matter of fact we can not select values for w as given the most probable γ/ν estimation, due to its large acceptable range. It seems we need to study in addition larger lattice sizes to observe the above correction. This is because the fit seems to be fully compatible with the condition $w = -\gamma/\nu$, which means we are just fitting eq. (6). Actually, eq. (7) is only valid for $\gamma/\nu > |w|$, else corrections coming from the nonsingular part of the free energy will dominate the first correction term. This is in analogy with the specific heat FSS behavior, where the leading correction is a constant [12,18]. In fig. 3 we show the FSS fit eq. (6) for all available data. The estimates given in table 2, with acceptable Q values, are in good agreement with the recent ones obtained for SU(2) lattice gauge model in ref. [16,17].

We now consider the β_c evaluation. Under the above considerations we should take into account the correction term to estimate β_c as a fitting in L [18,19],

$$\beta_c(L) \;=\; \beta_c + aL^{-1/\nu} + bL^{-1/\nu+w} \,. \tag{8}$$

However this is a 5-parameter fit, which decreases the number of degrees of freedom. On input conditions $b = 0$ and $1/\nu = 1.587$ it is possible to obtain an estimate for the critical coupling. For our patched data we obtain $\beta_c = 2.2973(4)$ ($Q = 0.01$) for the range $L = 6-16$, and $\beta_c = 2.2979(4)$ ($Q = 0.13$) for the range $L = 8-16$. From standard error propagation data we obtain $\beta_c = 2.2978(9)$ ($Q = 0.43$) for $L = 8-16$. These values can be compared with the estimate $\beta_c = 2.2985(6)$ of ref. [16,17].

In conclusion, we have discussed the dificulties in estimating the critical exponents γ/ν and w. The Q-surfaces give an overview of acceptable values for the critical exponents when one uses the criterium of highest goodness of the FSS fit. Our FSS analysis for lattice sizes from $L = 6-8$ up to $L = 26$ showed how the γ/ν evaluation dependends on the w-range. Since a conclusive evaluation of w can not be obtained for the available number of lattice sizes, it appears that our results for w also include finite size effects beyond the one expected from an irrelevant scaling field, as shown by the different Q-surface shapes.

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Table 1
Single runs and patching for the Polyakov loop susceptibility

L	β_0	$L^{-3}\chi_{max}(P)$	β_{max}	weights
4	2.200, 2.300	none		
6	2.300	none		1.00
6	2.332	7.58 (12) $\times 10^{-3}$	2.345 (66)	1.00
6	2.300, 2.332	7.712 (95)$\times 10^{-3}$	2.346 (11)	0.37, 0.63
8	2.282	none		1.00
8	2.300	5.142 (93)$\times 10^{-3}$	2.3063 (18)	1.00
8	2.320	5.37 (11) $\times 10^{-3}$	2.3109 (15)	1.00
8	2.282, 2.300, 2.320	5.236 (63)$\times 10^{-3}$	2.3102 (19)	0.24, 0.40, 0.36
10	2.287	4.148 (89)$\times 10^{-3}$	2.3018 (16)	1.00
10	2.300	4.05 (14)$\times 10^{-3}$	2.3049 (16)	1.00
10	2.314	4.09 (14)$\times 10^{-3}$	2.3047 (24)	1.00
10	2.287, 2.300, 2.314	4.094 (69)$\times 10^{-3}$	2.3032 (11)	0.35, 0.29, 0.36
12	2.290	3.008 (95)$\times 10^{-3}$	2.3021 (19)	1.00
12	2.300	3.34 (12)$\times 10^{-3}$	2.3027 (14)	1.00
12	2.310	3.39 (13)$\times 10^{-3}$	2.3043 (13)	1.00
12	2.290, 2.300, 2.310	3.215 (78)$\times 10^{-3}$	2.3056 (19)	0.37, 0.37, 0.26
14	2.292	none		1.00
14	2.300	2.80 (13)$\times 10^{-3}$	2.3028 (12)	1.00
14	2.307	2.720 (82)$\times 10^{-3}$	2.3021 (07)	1.00
14	2.292, 2.300, 2.307	2.806 (63)$\times 10^{-3}$	2.3020 (07)	0.23, 0.40, 0.37
16	2.293	none		1.00
16	2.300	2.433 (65)$\times 10^{-3}$	2.3013 (06)	1.00
16	2.307	2.51 (16)$\times 10^{-3}$	2.3006 (15)	1.00
16	2.293, 2.300, 2.307	2.457 (58)$\times 10^{-3}$	2.3013 (05)	0.16, 0.71, 0.13

The last column gives the relative weights of patched data sets ordered by increasing β_0. Single MC runs are labelled with "weights" corresponding to 1.00 . The result "none" means that maximum of the susceptibility is either unrealible or out of the β-range. All error bars are calculated with respect to twenty jackknife bins and corrected for the bias.

Table 2
FSS fits of $\chi_{max}(P)$

L range	$-w$	γ/ν	a_1	a_2	Q
6 – 26	0.25 [a]	1.906 (22)	0.0448 (50)	0.012 (04)	5×10^{-4}
6 – 26	0.75 [b]	2.208 (02)	0.0128 (02)	0.071 (01)	0.09
6 – 26	1.00 [a]	2.127 (02)	0.0213 (02)	0.092 (02)	0.19
6 – 26	γ/ν [c]	1.983 (28)	0.0392 (35)	0.291 (66)	0.37
8 – 26	0.25	1.931 (26)	0.0425 (30)	0.010 (06)	0.37
8 – 26	0.75	1.979 (03)	0.0375 (03)	0.030 (02)	0.41
8 – 26	1.00	1.960 (84)	0.041 (13)	0.033 (43)	0.41
8 – 26	γ/ν	1.936 (47)	0.0459 (71)	0.11 (16)	0.42

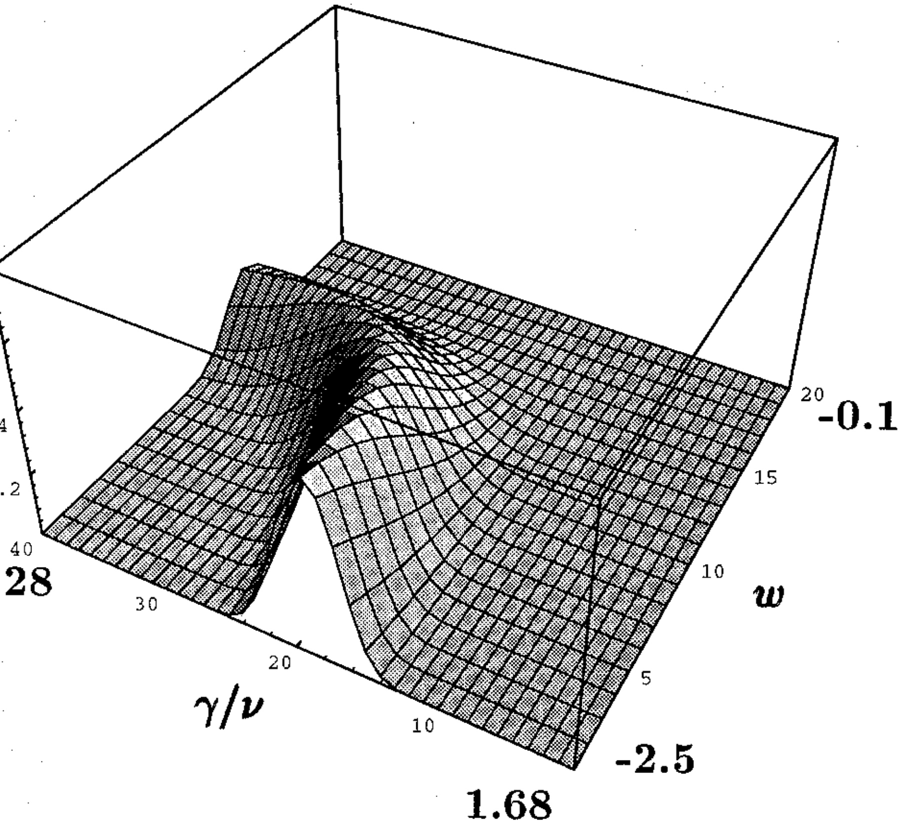
The quoted values for w are taken from FSS analysis for the 3D Ising model, respectively, according to the references ref. [19] [a], ref. [18] [b], and (c) means we fitted the susceptibility maxima to eq. (6).

Figure Captions:

Figure 1: Q-surface for eq. (7) in the $\{\gamma/\nu,\ w\}$ parameter space. This surface corresponds to $L = 6-26$ range.

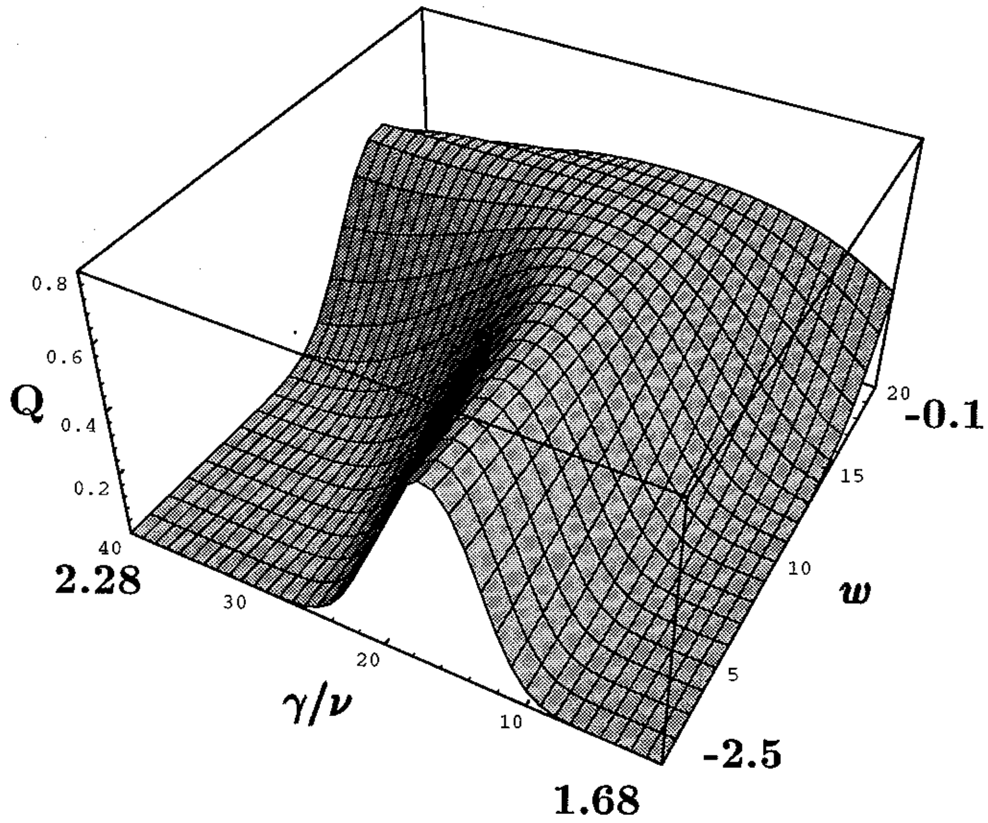
Figure 2: Q-surface for eq. (7) in the $\{\gamma/\nu,\ w\}$ parameter space for $L = 8-26$ range.

Figure 3: FSS fit eq. (6) for the susceptibility $\chi(P)$. We fitted our patched data in table 1 and data for $L = 18$ and 26 from ref. [16]. This fit shows that the dominant analytic contribution for $\chi(P)$ can not be excluded for the analysed data. It gives an acceptable fit with $Q = 0.37$.



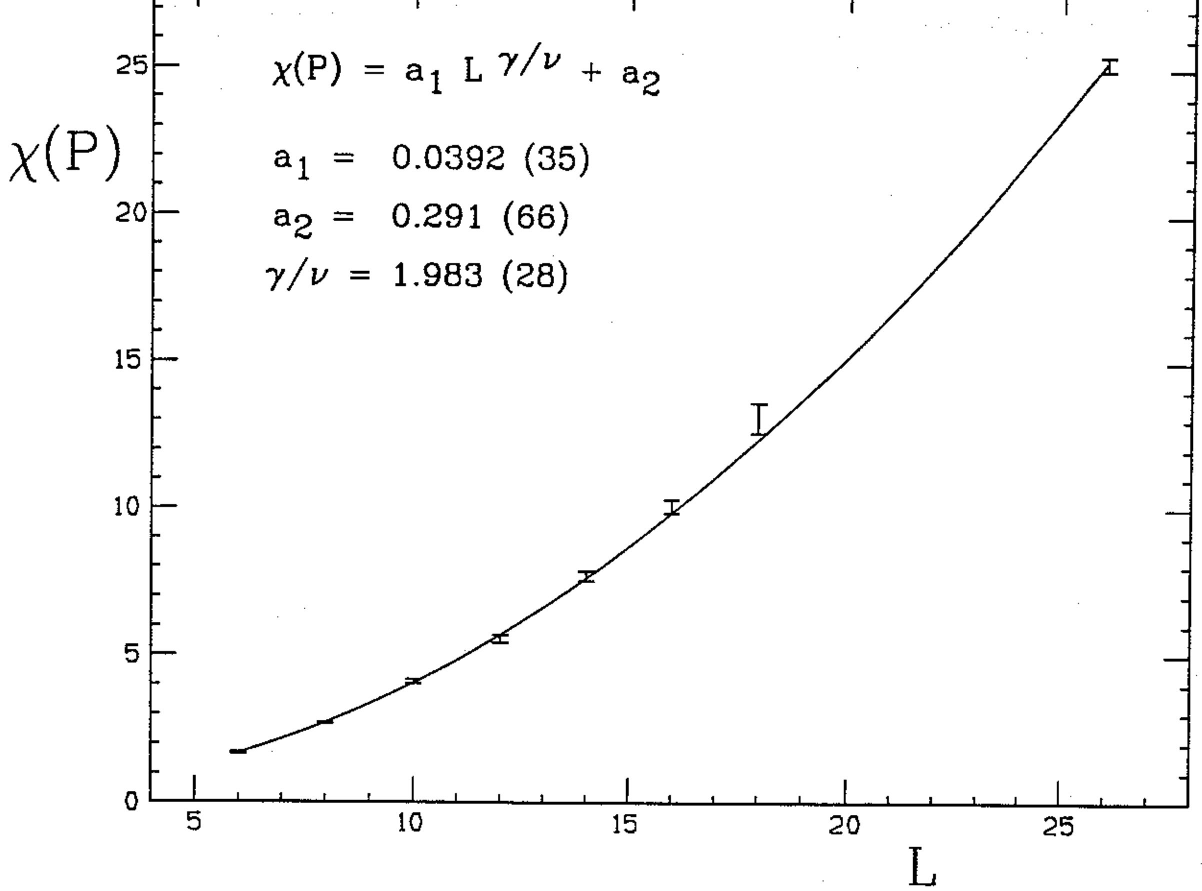
-0.1
20
15
10
5
w
40
28
30
20
10
γ/ν
1.68
-2.5

FIG. 1



0.8
0.6
Q
0.4
0.2
-0.1
20
15
10
5
w
40
2.28
30
20
10
γ/ν
1.68
-2.5

FIG. 2



$\chi(P) = a_1 L^{\gamma/\nu} + a_2$
$a_1 = 0.0392\ (35)$
$a_2 = 0.291\ (66)$
$\gamma/\nu = 1.983\ (28)$
$\chi(P)$
L
0
5
10
15
20
25

FIG. 3