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THERMAL MATTER AND RADIATION IN A GRAVITATIONAL FIELD

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Abstract

We study the one-loop contributions of matter and radiation to the gravitational polarization tensor at finite temperatures. Using the analytically continued imaginary-time formalism, the contribution of matter is explicitly given to next-to-leading (T^2) order. We obtain an exact form for the contribution of radiation fields, expressed in terms of generalized Riemann zeta functions. A general expression is derived for the physical polarization tensor, which is independent of the parametrization of graviton fields. We investigate the effective thermal masses associated with the normal modes of the corresponding graviton self-energy.

I. INTRODUCTION

lany properties of plasmas in thermal field theories can be understood from the study e polarization tensor evaluated at finite temperature [1–5]. This tensor, which is the oint correlation function, describes phenomena such as the propagation of waves and ing of fields in the plasma. In thermal quantum gravity, the behavior of the polarization r is also of interest, especially in connection with cosmological applications. If the erature T is well below the Planck scale, perturbation theory can be used to calculate hermal Green functions. Thus, one obtains loop-diagrams in which the internal lines sent matter and radiation in thermal equilibrium, and the external lines represent the

gravitational fields. There has been a lot of work on hot quantum field theory in the presence of a gravitational field [6–9]. Thus far these investigations have been mainly restricted to the study of the hard thermal loops contributions, which are obtained in the high temperature limit.

The purpose of this work is to study the behavior of the graviton polarization tensor at all temperatures, which might be useful in some applications. Since these calculations are considerably more complicated than those performed at high temperatures, we have restricted for definiteness to work to one-loop order with thermal bosonic fields, which may be of spin 0 or 1. The method we use is that of reference [9], where the Green functions are related to a momentum integral of the forward scattering amplitude of thermal particles in a gravitational field. Then, the temperature-dependent part of the graviton polarization tensor can be written at all temperatures in the form:

$$\Pi^{\mu\nu,\alpha\beta}(k) = \frac{1}{(2\pi)^3}\int \frac{d^3q}{2Q}\frac{1}{\exp(Q/T)-1}F^{\mu\nu,\alpha\beta}(q,k)\,. \tag{1.1}$$

Here $q_\mu \equiv (Q,\vec{q})$ represents the on-shell momenta of a thermal particle with mass m and energy $Q = \sqrt{|\vec{q}|^2 + m^2}$. $F^{\mu\nu,\alpha\beta}(q,k)$ is the forward scattering amplitude, summed over the polarizations of thermal particles, which is a covariant function of q and the external momenta k. This temperature-independent amplitude is weighted in (1.1) by the Bose distribution factor. Because of the angular integrations, $\Pi^{\mu\nu,\alpha\beta}$ is no longer a Lorentz covariant function. It depends on the time-like vector u^μ, representing the local rest frame of the plasma. For simplicity, we work in the comoving coordinate system where $u^\mu = \delta^\mu_0$. The above method simplifies very much the calculations in the present case.

In Sec.II we consider the contribution of matter particles described by the scalar field ϕ, coupled to a gravitational field. The coupling characterized by the term $\xi R\phi^2$ is included, where ξ is a numerical factor and R denotes the Ricci scalar. We verify that $\Pi^{\mu\nu,\alpha\beta}$ satisfies the Ward identity which reflects the invariance of the action under general coordinate transformations. We obtain a general expression for the leading (T^4) and next-to-leading (T^2) contributions to the graviton polarization tensor. The special case when $\xi = -1/6$ and

$m = 0$ is of particular interest, since then the scalar action is also invariant under conformal transformations [10]. Due to this invariance, $\Pi^{\mu\nu,\alpha\beta}$ satisfies in this case a Weyl identity which is explicitly verified.

In Sec.III we discuss the coupling of radiation fields which may be photons or gluons, to a gravitational field. This coupling is also invariant under general coordinate transformations as well as under conformal transformations. We remark that the thermal contributions associated with internal gauge fields represent gauge-invariant quantities. The Ward and Weyl identities determine uniquely the (T^4) contributions, which are the same for all thermal particles, apart from numerical factors which count the number of degrees of freedom. Using general properties of the forward scattering amplitude, we show that all other contributions can be expressed in terms of just 2 parameters which are not fixed by the Ward and Weyl identities. Rather, these parameters depend specifically on the nature of thermal particles.

In Sec.IV we obtain a closed form expression for the contributions of thermal radiation fields to the graviton polarization tensor. We show that these can be expressed in terms of generalized Riemann zeta functions $\zeta(-n,t)$ [11] for natural values of n, t being a ratio of external momenta and the temperature. In the high temperature limit, this expression yields a series of decreasing powers in the temperature, which includes leading (T^4) and-next-to-leading (T^2) contributions. Some technical aspects which arise in the calculations are discussed in the Appendices.

In Sec.V we analyze the dependence on the parametrization of the graviton fields, of the one-particle irreducible (1PI) contributions to the graviton polarization tensor. This behavior occurs generally because of the non-vanishing of the thermal graviton 1-point function. We show that the physical polarization tensor, identified with the graviton self-energy, is described by a traceless function which includes contributions from thermal 1-point functions. A general expression for the physical self-energy at finite temperature is derived, which is independent of the graviton parametrization.

In Sec.VI we discuss the effective graviton propagator, obtained by iterative insertions in the free propagator of the physical self-energy. We analyze, in the static limit, the

corresponding poles which describe three normal modes of dynamical screening. While on of the modes remains unshielded, a non-vanishing screening mass $m_s^2 = 32\pi G\rho/3$ appears i the spatially transverse one, where ρ is the thermal energy density. The spatially longitudina mode is characterized by an imaginary mass $m_J^2 = -32\pi G\rho$, similar to the classical Jean mass, indicating an instability of thermal quantum gravity.

II. MATTER CONTRIBUTIONS TO THE POLARIZATION TENSOR

We consider here thermal matter represented by scalar particles of mass m, coupled t the gravitational field via the Lagrangian:

$$\mathcal{L}(x) = -\frac{1}{2}\sqrt{-g(x)}\left[g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi - \left(m^2 + \xi R\right)\phi^2\right], \tag{2.1}$$

and expand the metric tensor $g_{\mu\nu}$ in terms of the deviation from the Minkowski metric $\eta_{\mu\nu}$

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa\, h_{\mu\nu}, \tag{2.2}$$

where $\kappa = \sqrt{32\pi G}$. In order to derive the one-particle irreducible (1PI) contributions t the thermal graviton 2-point function, we consider the Feynman graphs shown in Fig.1. According to Eq. (1.1), these can be expressed in terms of the forward scattering amplitud of on-shell scalar particles, as indicated in Fig.2. The corresponding contributions to the amplitude can be expressed in terms of a basis of 14 independents tensors $T_i^{\mu\nu,\alpha\beta}(q,k)$, which are symmetric under the interchanges $(\mu \leftrightarrow \nu)$, $(\alpha \leftrightarrow \beta)$ and $(\mu,\nu) \leftrightarrow (\alpha,\beta)$. These tensors are covariant functions of q and k, being polynomials of maximum degree 4 in the momenta. They can be obtained from Table I, replacing the vectors (X,Y) by the pair (q,k). With help of the Feynman rules given in Appendix A, it is straightforward to obtain for the forward scattering amplitude the expression:

$$F^{\mu\nu,\alpha\beta}(q,k) = \frac{\kappa^2}{(k^2+2k\cdot q)}\,\Big[-\Big(\frac{1}{8}\xi\,k^4+\frac{1}{4}\xi\,k^2\,k\cdot q\Big)\,T_1^{\mu\nu,\alpha\beta}-\Big(\frac{1}{4}\,k^2+\frac{1}{2}\,k\cdot q\Big)\,T_2^{\mu\nu,\alpha\beta}+$$
$$T_3^{\mu\nu,\alpha\beta}+\Big(\frac{1}{4}\,\xi\,k^4+\xi^2\,k^4-\frac{1}{2}\xi\,k^2\,k\cdot q+\frac{1}{4}(k\cdot q)^2\Big)\,T_4^{\mu\nu,\alpha\beta}+$$
$$\Big(\frac{1}{4}k^2+\xi\,k^2\Big)\,T_5^{\mu\nu,\alpha\beta}+\frac{1}{2}T_7^{\mu\nu,\alpha\beta}+\Big(\frac{1}{8}\xi\,k^2+\frac{1}{4}\xi\,k\cdot q\Big)\,T_8^{\mu\nu,\alpha\beta}-$$
$$\xi T_9^{\mu\nu,\alpha\beta}+\frac{1}{4}T_{10}^{\mu\nu,\alpha\beta}-\frac{1}{2}\xi T_{11}^{\mu\nu,\alpha\beta}+\xi^2 T_{12}^{\mu\nu,\alpha\beta}-$$
$$\Big(\frac{1}{4}\xi\,k^2+\xi^2\,k^2\Big)\,T_{13}^{\mu\nu,\alpha\beta}+\Big(\frac{1}{2}\xi\,k^2-\frac{1}{4}k\cdot q\Big)\,T_{14}^{\mu\nu,\alpha\beta}\Big]$$
$$+(k\leftrightarrow -k)\,. \tag{2.3}$$

To obtain the leading (T^4) and the next to leading (T^2) contributions to the polarization tensor, we need to expand the energy $Q=\sqrt{|\vec{q}|^2+m^2}$ in powers of $(m^2/|\vec{q}|^2)$, as well as the Feynman denominators:

$$\frac{1}{k^2+2k\cdot q}=\frac{1}{2k\cdot q}-\frac{k^2}{(2k\cdot q)^2}+\frac{k^4}{(2k\cdot q)^3}-\frac{k^6}{(2k\cdot q)^4}+\cdots. \tag{2.4}$$

The T^4 contributions come from terms in the forward amplitude (2.3) which are homogeneous functions of q of degree 2. These are given by:

$$-T_2^{\mu\nu,\alpha\beta}(q,k)-\frac{k^2}{(k\cdot q)^2}T_3^{\mu\nu,\alpha\beta}(q,k)+\frac{1}{k\cdot q}T_7^{\mu\nu,\alpha\beta}(k,q)=$$
$$-\Big(\eta^{\nu\beta}q^\mu q^\alpha+\eta^{\nu\alpha}q^\mu q^\beta+\eta^{\mu\beta}q^\nu q^\alpha+\eta^{\mu\alpha}q^\nu q^\beta\Big)-\frac{k^2}{(k\cdot q)^2}q^\mu q^\nu q^\alpha q^\beta\,. \tag{2.5}$$
$$\frac{1}{k\cdot q}\Big(q^\mu q^\nu q^\alpha k^\beta+q^\mu q^\nu k^\alpha q^\beta+q^\mu k^\nu q^\alpha q^\beta+k^\mu q^\nu q^\alpha q^\beta\Big)$$

Note that terms involving the parameter ξ do not contribute to (2.5). These contribute only to next-to-leading (T^2) order, which result from terms of degree zero in q in the forward amplitude. In order to find these contributions, we perform the $|\vec{q}|$ integration in (1.1) using the formulas:

$$\int_0^\infty \frac{|\vec{q}|\,d\,|\vec{q}|}{[\exp(|\vec{q}|/T)-1]}=\frac{\pi^2T^2}{6}, \tag{2.6}$$

$$\int_0^\infty \frac{|\vec{q}|^4\,d\,|\vec{q}|}{Q\,[\exp(Q/T)-1]}=\frac{\pi^4T^4}{15}-\frac{\pi^2m^2T^2}{4}+\cdots. \tag{2.7}$$

The angular integrals can be done using the methods described in [9]. The result can expressed in terms of the basis of 14 tensors $T_i^{\mu\nu,\alpha\beta}(u,K)$, obtained from Table I, where replace the pair (X,Y) by (u,K). Here $u^\mu=\delta_0^\mu$ and

$$K^\mu\equiv\frac{k^\mu}{|\vec{k}|}=\left(\frac{k_0}{|\vec{k}|},\,\hat{k}\right)\equiv\left(r,\,\hat{k}\right). \tag{2}$$

Then the 1PI contributions to the polarization tensor can be written up to the next leading order in the form:

$$\Pi^{\mu\nu,\alpha\beta}(k,m,\xi)=\sum_{i=1}^{14}\Pi_i(r,K,\xi)\,T_i^{\mu\nu,\alpha\beta}(u,K), \tag{2}$$

where:

$$\Pi_i(r,K,\xi)=\frac{\kappa^2}{30}\left[\pi^2T^4\,l_i(r,K)+\left|\vec{k}\right|^2T^2\,n_i(r,K,\xi)+m^2T^2\,s_i(r,K,\xi)\right]+\cdots. \tag{2.}$$

The explicit form of the dimensionless functions $l_i(r,K)$, $n_i(r,K,\xi)$ and $s_i(r,K,\xi)$ are gi in Appendix B. These exhibit, apart from a logarithmic dependence in r, a polynon behavior in K of maximum degree 10. The coefficients $l_i(r,K)$ which contribute to leading (T^4) order, have been obtained previously [8] and are included here for completen

As a consequence of the invariance of the theory under general coordinate transfor tions, the 1PI graviton 2-point function satisfies the Ward identity:

$$\frac{2}{\kappa}k_\nu\,\Pi^{\mu\nu,\alpha\beta}(k)=k^\mu\,\Gamma^{\alpha\beta}-k_\sigma\left(\Gamma^{\alpha\sigma}\,\eta^{\beta\mu}+\Gamma^{\beta\sigma}\,\eta^{\alpha\mu}\right). \tag{2.}$$

Here $\Gamma^{\alpha\beta}$ denotes the thermal graviton 1-point function, which is given by:

$$\Gamma^{\alpha\beta}=\frac{\pi^2T^4\kappa}{180}\left(4\,\delta_0^\alpha\delta_0^\beta-\eta^{\alpha\beta}\right)+\frac{m^2T^2\kappa}{48}\left(\eta^{\alpha\beta}-2\,\delta_0^\alpha\delta_0^\beta\right)+\cdots. \tag{2.}$$

With the help of the expression given by Eqs. (2.9) and (2.10), the Ward identity (2.11) be explicitly verified to this order. It is well known [10] that in the conformally coupled c when $\xi=-1/6$ and $m=0$, the action is also invariant under conformal transformati given by:

$$\bar{g}_{\mu\nu}(x)=\Omega^2(x)\,g_{\mu\nu}, \tag{2}$$

$$\bar{\phi}(x) = \Omega^{-1}(x)\,\phi(x)\,. \tag{2.14}$$

In consequence of this invariance, the 1PI graviton 2-point function will also satisfy the Weyl identity [8,9]:

$$\frac{1}{\kappa}\Pi^{\mu\nu,\,\sigma}{}_{\sigma}(k) = -\Gamma^{\mu\nu}. \tag{2.15}$$

This identity is explicitly verified by our expression for $\Pi^{\mu\nu,\alpha\beta}(k)$ [Eq. (2.9)] and for $\Gamma^{\mu\nu}$ [Eq. (2.12)] evaluated at $\xi = -1/6$ with $m = 0$.

III. RADIATION FIELDS CONTRIBUTIONS TO THE POLARIZATION TENSOR

In this section we analyze the contributions of spin 1 gauge fields, which may be photons or gluons. Since for our purpose the self-interactions of the Yang-Mills particles can be neglected, there is no loss of generality in considering only the contribution of an Abelian field A^{μ}. For non-Abelian fields the contributions are the same, up to an overall color factor. The coupling of the gauge field A^{μ} is described by the Lagrangian:

$$\mathcal{L}_A = -\frac{1}{4}\sqrt{-g(x)}\,g^{\mu\nu}\,g^{\alpha\beta}\left(\partial_\mu A_\alpha - \partial_\alpha A_\mu\right)\left(\partial_\nu A_\beta - \partial_\beta A_\nu\right). \tag{3.1}$$

It is convenient for computational purposes to fix the gauge by choosing:

$$\mathcal{L}_{fix} = -\frac{1}{2\alpha}\sqrt{-g(x)}\left(\nabla_\mu A^\mu\right)\left(\nabla_\nu A^\nu\right), \tag{3.2}$$

where ∇_μ is the covariant derivative. The corresponding Faddeev Popov Lagrangian is given by:

$$\mathcal{L}_{FP} = g^{\mu\nu}\sqrt{-g(x)}\,\left(\partial_\mu\bar{\chi}\right)\left(\partial_\nu\chi\right), \tag{3.3}$$

where χ and $\bar{\chi}$ are the ghost fields. The form of the above interactions is such that the theory is invariant under local coordinate transformations, as well as under conformal transformations given by:

$$\bar{A}_\mu(x) = A_\mu(x); \qquad \bar{g}_{\mu\nu}(x) = \Omega^2(x)\,g_{\mu\nu}. \tag{3.4}$$

As we have seen, these invariances ensure the 1PI graviton 2-point function to satisfy the Ward and Weyl identities given respectively by Eqs. (2.11) and (2.15). Here $\Gamma^{\alpha\beta}$ is obtained multiplying (2.12) by a factor 2 and setting $m = 0$.

With help of the Feynman rules listed in Appendix A, we can evaluate the 1PI graphs contributing to $\Pi^{\mu\nu,\alpha\beta}$, which are shown in Fig.3. The diagrams contributing to the corresponding forward amplitude are represented in Fig.4. It is important to note that the thermal contributions from internal gauge fields represent gauge-independent quantities. We have verified this independence explicitly, performing all computations in the general class of covariant gauges defined by (3.2). The dependence on the gauge parameter α cancels in the final expression of the forward scattering amplitude, which is given by:

$$\begin{aligned} F^{\mu\nu,\alpha\beta}(q,k) = \frac{2\kappa^2}{k^2 + 2k\cdot q}\,\Bigg(&\frac{1}{4}(k\cdot q)^2\,T_1^{\mu\nu,\alpha\beta}(q,k) - \frac{k\cdot q}{2}T_2^{\mu\nu,\alpha\beta}(q,k) + T_3^{\mu\nu,\alpha\beta}(q,k) \\ &-\frac{1}{4}(k\cdot q)^2\,T_4^{\mu\nu,\alpha\beta}(q,k) - \frac{k^2}{4}T_5^{\mu\nu,\alpha\beta}(q,k) + \frac{k^4}{8}T_6^{\mu\nu,\alpha\beta}(q,k) \\ &+\frac{1}{2}T_7^{\mu\nu,\alpha\beta}(q,k) + \frac{1}{2}T_9^{\mu\nu,\alpha\beta}(q,k) - \frac{k^2}{8}T_{14}^{\mu\nu,\alpha\beta}(q,k)\Bigg) \\ &+(k\leftrightarrow -k) \end{aligned} \tag{3.5}$$

At this point, it is interesting to compare (3.5) with the amplitude corresponding to the scalar case [Eq. (2.3)], evaluated for $m = 0$ and $\xi = -1/6$. We see that in both amplitudes the coefficients of the tensors $T_i^{\mu\nu,\alpha\beta}$ $(i = 2, 3, 7)$ which contribute in the high temperature limit [cf. Eq. (2.5)] are the same, up to a factor of 2 which counts the degrees of freedom of a physical gauge particle. On the other hand, all other coefficients seen to be different in general.

In order to understand this behavior we consider now the consequences of the Ward (2.11) and Weyl (2.15) identities on the structure of the forward scattering amplitudes. To this end, we use the following representation of the graviton one-point function:

$$\Gamma^{\alpha\beta} = \frac{\kappa}{(2\pi)^3}\int\frac{d^3q}{Q}\frac{1}{\exp(Q/T)-1}\,q^\alpha\,q^\beta. \tag{3.6}$$

Then, we find from Eq. (1.1) that the Ward and Weyl identities ensure the forward scattering amplitudes to obey respectively the relations:

$$\frac{1}{\kappa^2}k_\nu F^{\mu\nu,\alpha\beta}(q,k) = k^\mu q^\alpha q^\beta - k\cdot q\left(q^\alpha\eta^{\mu\beta}+q^\beta\eta^{\mu\alpha}\right), \tag{3.7}$$

$$\frac{1}{\kappa^2}F^{\mu\nu,\alpha}{}_{\alpha}(q,k) = -2\,q^\mu q^\nu \tag{3.8}$$

We will now investigate the constraints imposed by the relations (3.7) and (3.8) on the general form of the amplitudes $F^{\mu\nu,\alpha\beta}$. Since these are Lorentz covariant functions of q and k, they can be expressed in terms of the tensor basis $T_i^{\mu\nu,\alpha\beta}(q,k)$ as follows:

$$F^{\mu\nu,\alpha\beta}(q,k) = \kappa^2\sum_{i=1}^{14}F_i\left(k^2,k\cdot q\right)T_i^{\mu\nu,\alpha\beta}(q,k), \tag{3.9}$$

where F_i are invariant functions of k^2 and $k\cdot q$. Inserting (3.9) into the Ward identity (3.7) and identifying the coefficients of the independents tensor structures yields 10 relations among the F_i. Similarly the use of the Weyl identity (3.8) gives 4 more relations. However, not all of these relations are independent, so that we can express 11 functions F_i in terms of the remaining 3 as follows:

$$F_1 = \frac{3k^4}{4}F_{12} + k\cdot qF_{14} \tag{3.10a}$$

$$F_2 = -\frac{1}{2} - \frac{k^4}{2k\cdot q}F_{11} + \frac{k^2}{k\cdot q}F_{14} \tag{3.10b}$$

$$F_3 = -\frac{k^2}{2(k\cdot q)^2} - \frac{2k^6}{(k\cdot q)^3}F_{11} + \frac{k^4}{(k\cdot q)^3}F_{14} \tag{3.10c}$$

$$F_4 = -\frac{k^4}{2}F_{12} - (k\cdot q)F_{14} \tag{3.10d}$$

$$F_5 = -\frac{k^2}{(k\cdot q)}F_{14} \tag{3.10e}$$

$$F_6 = \frac{k^2}{2}F_{11} - F_{14} \tag{3.10f}$$

$$F_7 = \frac{1}{2k\cdot q} + \frac{2k^4}{(k\cdot q)^2}F_{11} - \frac{k^2}{(k\cdot q)^2}F_{14} \tag{3.10g}$$

$$F_8 = -\frac{k\cdot q}{2}F_{11} - \frac{3\,k^2}{4}F_{12} \tag{3.10h}$$

$$F_9 = -\frac{2k^2}{k\cdot q}F_{11} + \frac{2}{k\cdot q}F_{14} \tag{3.10i}$$

$$F_{10} = -\frac{3k^2}{2k\cdot q}F_{11} \tag{3.10j}$$

$$F_{13} = \frac{k^2}{2}F_{12} \tag{3.10k}$$

We see that the invariance of the theory under local coordinate and conformal transformations does not fix the functions F_{11}, F_{12} and F_{14}. Further constraints are provided by the property of the forward scattering amplitude of being a function with dimension of (momenta)^2, which is even under $(k\longleftrightarrow -k)$. Furthermore, to one loop order in perturbation theory this amplitude can have at most one denominator involving $(k^2\pm 2k\cdot q)$. For instance, these general properties require the functions F_3 and F_7 to have the structure:

$$F_3 = c_3\left(\frac{1}{k^2+2k\cdot q}+\frac{1}{k^2-2k\cdot q}\right), \tag{3.11}$$

$$F_7 = c_7\left(\frac{1}{k^2+2k\cdot q}-\frac{1}{k^2-2k\cdot q}\right), \tag{3.12}$$

where c_3 and c_7 are constants. Furthermore, it follows that F_2 must be an even function of k, having the structure:

$$F_2 = c_2k\cdot q\left(\frac{1}{k^2+2k\cdot q}-\frac{1}{k^2-2k\cdot q}\right) + c_2'k^2\left(\frac{1}{k^2+2k\cdot q}+\frac{1}{k^2-2k\cdot q}\right), \tag{3.13}$$

where c_2 and c_2' are constants. Similar structures can be found for all other functions appearing in Eqs. (3.10). These structures yield a set of relations which must be satisfied identically in Eqs. (3.10), for all values of q and k. In this way, we find that the constants c_2, c_3 and c_7 are uniquely determined as:

$$c_2 = -\frac{1}{2};\quad c_3 = 1;\quad c_7 = \frac{1}{2}. \tag{3.14}$$

Note that the functions F_3 and F_7, as well as the part of F_2 which determine the T^4 contributions [cf. Eq.(2.5)] are now uniquely fixed. This is in accordance with the argument [9] that all hard thermal particles should contribute the same, up to a weight factor. The above relations imply further the equation:

$$F_{14} = 2k^2 F_{11} + \frac{k^2 (k\cdot q)}{2\left[k^4 - 4(k\cdot q)^2\right]}. \tag{3.15}$$

Using (3.15), we see from Eqs. (3.10) that the only independent functions left over are F_{11} and F_{12}. From the general properties of the forward scattering amplitude, these functions must have the structure:

$$F_{11} = c_{11}\left(\frac{1}{k^2 + 2k\cdot q} + \frac{1}{k^2 - 2k\cdot q}\right), \tag{3.16}$$

$$F_{12} = c_{12}\left(\frac{1}{k^2 + 2k\cdot q} - \frac{1}{k^2 - 2k\cdot q}\right), \tag{3.17}$$

where c_{11} and c_{12} are constants which depend specifically on the nature of the thermal particles. For instance, in the scalar case we get:

$$c_{11} = \frac{1}{12}; \qquad c_{12} = \frac{1}{36}, \tag{3.18}$$

whereas in the case of internal gauge fields we find that:

$$c_{11} = c_{12} = 0. \tag{3.19}$$

The above relations explain the features of the forward scattering amplitudes described by Eq. (2.3) [at $\xi = -1/6$ and $m = 0$] and by Eq. (3.5).

IV. EXACT EVALUATION OF RADIATION FIELDS CONTRIBUTIONS

We will now evaluate all finite-temperature contributions in closed form, using the techniques described in the first paper of reference [12]. To this end, we express the 1PI graviton 2-point function in terms of the tensor basis $T_i^{\mu\nu,\alpha\beta}(u,K)$ in a way analogous to (2.9):

$$\Pi^{\mu\nu,\alpha\beta}(k) = \sum_{i=1}^{14} \Pi_i(r,K)\, T_i^{\mu\nu,\alpha\beta}(u,K). \tag{4.1}$$

According to the discussion of the last section [cf. Eq.(2.10)], we can write the functions $\Pi_i(r,K)$ as follows:

$$\Pi_i(r,K) = \kappa^2\left[\frac{\pi^2 T^4}{15} l_i(r,K) + |\vec{k}|^2 T^2 N_i(r,K)\right], \tag{4.2}$$

where the functions $l_i(r,K)$ are given in appendix (B). Our task is to determine the functions $N_i(r,K)$, which should be non-leading in the high temperature limit. For this, it is convenient to consider first the projections of the graviton 2-point function into the tensor basis $T_i^{\mu\nu,\alpha\beta}$:

$$\bar{P}_i(r,K) = \frac{1}{8}\Pi^{\mu\nu,\alpha\beta}(k)\, T_{i\,\mu\nu,\alpha\beta}(u,K). \tag{4.3}$$

Once we find these (see next), the functions Π_i in (4.2) can be determined by the relation:

$$\Pi_i(r,K) = 8(T_i^{\mu\nu,\alpha\beta} T_{j\,\mu\nu,\alpha\beta})^{-1}\bar{P}_j(r,K) \equiv 8\,(T_{ij})^{-1}\,\bar{P}_j(r,K), \tag{4.4}$$

where $(T_{ij})^{-1}$ denotes the inverse of the matrix $T_{ij} \equiv T_i^{\mu\nu,\alpha\beta} T_{j\,\mu\nu,\alpha\beta}$.

We now proceed with the evaluation of the functions $\bar{P}_j(r,K)$ in 4.3. From Table I we see that for $j = 4,5,\cdots,14$ these involve the contraction of $\Pi^{\mu\nu,\alpha\beta}(k)$ with $\eta^{\mu\nu}$, $\eta^{\alpha\beta}$ or with the external momenta. Using the Ward (2.11) and Weyl (2.15) identities, the corresponding functions $\bar{P}_j(r,K)$ will be given by a linear combination of graviton 1-point functions. These are proportional to T^4 [cf. Eq. (2.12) with $m = 0$], and so will contribute only to the functions $l_i(r,K)$ in (4.2). The functions $N_i(r,K)$ are determined from the contributions corresponding to $\bar{P}_j(r,K)$ $(j = 1,2,3)$, which are not proportional to T^4. These contributions, which we denote by P_j $(j = 1,2,3)$ can be found from Eq. (4.3) by using for the graviton 2-point function the expression (1.1). Substituting here the expression (3.5) for the forward scattering amplitude, we are lead to integrals of the form:

$$I_S(k,T) = \frac{1}{(2\pi)^3}\int \frac{d^3q}{2Q}\frac{Q^S}{\exp(Q/T)-1}\left(\frac{1}{k^2+2k\cdot q} + \frac{1}{k^2-2k\cdot q}\right), \tag{4.5}$$

where $S = 0,2,4$ and $Q = |\vec{q}|$. In terms of $x = \cos(\theta)$, where θ is the angle between $\vec{k}$ and $\vec{q}$, we find that the above expression becomes:

$$I_S(k,T) = -\frac{1}{(2\pi)^2}\frac{k^2}{4}\int_{-1}^{1}\frac{dx}{\left(k_0 - |\vec{k}|x\right)^2}\int_0^\infty dQ \frac{Q^{S+1}}{\exp(Q/T)-1}\frac{1}{Q^2 + (2\pi T y)^2}, \tag{4.6}$$

vhere,

$$y \equiv \frac{1}{4i\pi T}\frac{k^2}{k_0 - |\vec{k}|x}. \tag{4.7}$$

\part from simple functions, the integration in (4.6) can be reduced to the basic integral 11]:

$$I(y) = \int_0^\infty \frac{QdQ}{Q^2 + (2\pi T y)^2}\frac{1}{\exp(Q/T) - 1} = \frac{1}{2}\Theta[\mathrm{Re}(y)]\left(\ln y - \frac{1}{2y} - \psi(y)\right) + (y \leftrightarrow -y), \tag{4.8}$$

vhere $\psi(y) = \frac{d}{dy}\ln\Gamma(y)$ denotes the Euler psi function. The real-time limit of the Green's unction can be obtained from the analytically continued imaginary-time formalism via the ›rescription [13] $k_0 = (1 + i\varepsilon)K_0$, where $\varepsilon \to 0^+$ and K_0 is real. With the presence of the $i\varepsilon$ actor being understood, we find in (4.8) that $\mathrm{Re}(y) = \varepsilon'\mathrm{Re}(k_0)$, with $\varepsilon' \to 0^+$.

Many of the angular integrations in (4.6) can be easily done in terms of elementary unctions, after changing variables from x to y. The most difficult one involve an integrand :ontaining $\psi(y)$ multiplied by a power of y^n, for $n = 0, 1, 2, 3, 4$. The relevant integrals can ›e put in the form:

$$J_n \equiv \Theta[\mathrm{Re}(k_0)]\left[J_n(t(k_0)) - J_n(-t(-k_0))\right] + [k_0 \leftrightarrow -k_0], \tag{4.9}$$

vhere

$$J_n(t) = \left(\frac{i\pi T}{|\vec{k}|}\right)^{n-1}\int_C^t y^n\left[\ln(y) - \frac{1}{2y} - \psi(y)\right]dy. \tag{4.10}$$

The choice of C is immaterial, since any constant is irrelevant for our purposes because t cancels out in the expression (4.9). Here $t(k_0)$ and $-t(-k_0)$ denote the limits of the r-integration corresponding respectively to $x = 1$ and $x = -1$ in Eq. (4.7). Hence:

$$t(k_0) = \frac{k_0 + |\vec{k}|}{4i\pi T}. \tag{4.11}$$

These integrals can be expressed in closed form in terms of derivatives of generalized Riemann zeta functions $\zeta(-n, t)$ for natural values of $n = 0, 1, 2, 3, 4$. For instance we have hat:

$$J_0(t) = \frac{|\vec{k}|}{i\pi T}\int^t\left[\ln(y) - \frac{1}{2y} - \psi(y)\right]dy = \frac{|\vec{k}|}{i\pi T}\left[t\ln t - \zeta'(0,t) - t - \frac{1}{2}\ln t\right], \tag{4.12}$$

$$J_1(t) = \int^t y\left[\ln(y) - \frac{1}{2y} - \psi(y)\right]dy = t\zeta'(0,t) - \zeta'(-1,t) + \frac{t^2}{2}\ln t - \frac{3}{4}t^2. \tag{4.13}$$

In the above expressions, the derivative is taken with respect to the first argument of the generalized zeta function. The functions $J_n(t)$ are discussed in more generality in Appendix C [cf. Eq. (C6)]. In this way, we find that the functions P_j ($j = 1, 2, 3$) in Eq. (4.3) are related to J_n in (4.9) as follows:

$$P_1(r, K) = -\frac{K^2}{96} - \frac{K^4}{32}J_0, \tag{4.14}$$

$$P_2 = \frac{K^2}{48}L(r) - \frac{K^4}{32}J_0 - r\frac{K^2}{4}J_1 - \frac{K^2}{2}J_2, \tag{4.15}$$

$$P_3 = -\frac{1}{576} + \frac{K^2}{192}L(r) - \frac{K^4}{256}J_0 - r\frac{K^2}{16}J_1 - \frac{2 + 3K^2}{8}J_2 - rJ_3 - J_4, \tag{4.16}$$

where we have defined:

$$L(r) = \frac{r}{2}\ln\frac{r+1}{r-1} - 1. \tag{4.17}$$

With help of these relations, the functions $N_i(r, K)$ can be explicitly determined from Eqs. (4.2) and (4.4). After a straightforward calculation we obtain that:

$$N_1(r, K) = P_1 + K^2P_2 + K^4P_3 \tag{4.18a}$$

$$N_2(r, K) = K^2P_1 + 2K^4P_2 + 5K^6P_3 \tag{4.18b}$$

$$N_3(r, K) = K^4P_1 + 5K^6P_2 + 35K^8P_3 \tag{4.18c}$$

$$N_4(r, K) = -P_1 - K^2P_2 + K^4P_3 \tag{4.18d}$$

$$N_5(r, K) = -K^2P_1 - K^4P_2 + 5K^6P_3 \tag{4.18e}$$

$$N_6(r, K) = -r\left(P_1 + 2K^2P_2 + 5K^4P_3\right) \tag{4.18f}$$

$$N_7(r, K) = -r\left(K^2P_1 + 5K^4P_2 + 35K^6P_3\right) \tag{4.18g}$$

$$N_8(r,K) = P_1 + \left(1+2K^2\right)P_2 + \left(4K^2+5K^4\right)P_3 \tag{4.18h}$$

$$N_9(r,K) = \left(2+K^2\right)P_1 + \left(6K^2+5K^4\right)P_2 + \left(30K^4+35K^6\right)P_3 \tag{4.18i}$$

$$N_{10}(r,K) = K^2P_1 + \left(3K^2+5K^4\right)P_2 + \left(30K^4+35K^6\right)P_3 \tag{4.18j}$$

$$N_{11}(r,K) = -r\left[P_1 + \left(2+5K^2\right)P_2 + \left(20K^2+35K^4\right)P_3\right] \tag{4.18k}$$

$$N_{12}(r,K) = P_1 + \left(4+5K^2\right)P_2 + \left(8+40K^2+35K^4\right)P_3 \tag{4.18l}$$

$$N_{13}(r,K) = -P_1 - K^2P_2 + \left(4K^2+5K^4\right)P_3 \tag{4.18m}$$

$$N_{14}(r,K) = r\left(P_1 + K^2P_2 + 5K^4P_3\right) \tag{4.18n}$$

At high temperatures, the Riemann zeta functions can be expanded in a power series in t, as shown in Eq. (C10). Then, in the high temperature domain, we can express the functions P_j $(j=1,2,3)$ as a series of powers of $(1/T)$. The dominant terms in these series are given by:

$$P_1 = -\frac{K^2}{96} \tag{4.19}$$

$$P_2 = \frac{K^2}{48}L(r) \tag{4.20}$$

$$P_3 = -\frac{1-3K^2L(r)}{576} \tag{4.21}$$

Although the above contributions are gauge invariant, they do not directly describe the physical properties of the plasma at finite temperatures in a gravitational field. This problem is related to the fact that the thermal graviton 2-point function depends on the choice of the basic graviton fields.

V. THE GRAVITON SELF-ENERGY AT FINITE TEMPERATURE

In thermal quantum field theory the 1PI contributions to the graviton 2-point function are in general dependent on the parametrization of the graviton fields. However, as shown in the second work of reference [12], the traceless quantity:

$$\bar{\Pi}^{\mu\nu,\alpha\beta}(k) \equiv \Pi^{\mu\nu,\alpha\beta}(k) - \frac{1}{4}\left(\eta^{\mu\alpha}\Pi^{\rho}{}_{\rho}{}^{\nu\beta} + \eta^{\mu\beta}\Pi^{\rho}{}_{\rho}{}^{\nu\alpha} + \eta^{\nu\alpha}\Pi^{\rho}{}_{\rho}{}^{\mu\beta} + \eta^{\nu\beta}\Pi^{\rho}{}_{\rho}{}^{\mu\alpha}\right), \tag{5.1}$$

represents at high temperatures a quantity which does not depend on the choice of basic graviton fields. In this domain, the masses can be effectively neglected so the theory is invariant under conformal transformations. The representation-independence of $\bar{\Pi}$ then follows in consequence of the Ward and Weyl identities. As we have seen, the contributions from internal massless particles, are invariant under local coordinate and conformal transformations. Consequently, the physical amplitude given by (5.1) can be identified in this case with the graviton self-energy even at finite temperature.

However for contributions from thermal matter, which are characterized by the presence of massive particles, $\bar{\Pi}^{\mu\nu,\alpha\beta}$ is no longer independent of the graviton field parametrization at finite temperatures. Our task is to generalize (5.1) in such a way that the corresponding quantity should represent a physical graviton self-energy at all temperatures. To achieve this we consider the effective action which generates the one-particle irreducible thermal Green functions. In the representation (2.2) for the h fields, we have that:

$$S_{eff} = \Gamma_h^{\alpha\beta}h_{\alpha\beta}(0) + \frac{1}{2}\int d^4k\,\Pi_h^{\mu\nu,\alpha\beta}(k)\;h_{\mu\nu}(k)\;h_{\alpha\beta}(-k) + \cdots. \tag{5.2}$$

Starting from these fields, the most general re-parametrization of the graviton fields can be written as:

$$h_r^{\mu\nu} = a\,h^{\mu\nu} + b\,h^{\lambda}{}_{\lambda}\,\eta^{\mu\nu} + c\,h^{\lambda}{}_{\lambda}\,h^{\mu\nu} + d\,h^{\mu\lambda}\,h^{\nu}{}_{\lambda} + e\left(h^{\lambda}{}_{\lambda}\right)^2\eta^{\mu\nu} + f\,h_{\alpha\beta}h^{\alpha\beta}\,\eta^{\mu\nu} + \cdots\;, \tag{5.3}$$

where a, b, c, d, e, and f denote arbitrary constants. For example, a basic graviton field often used in the literature which is defined by [9]

$$\sqrt{-g(x)}\,g^{\mu\nu} \equiv \eta^{\mu\nu} + \kappa h_r^{\mu\nu}, \tag{5.4}$$

corresponds to a special case of (5.3), with:

$$a=-1;\;\; b=\frac{1}{2};\;\; c=-\frac{1}{2};\;\; d=1;\;\; e=\frac{1}{8};\;\; f=-\frac{1}{4}. \tag{5.5}$$

In what follows we shall assume, without loss of generality that $a^2 = 1$. This can always be achieved by a further rescaling of the h_i fields in (5.3) [see ref. [12]]. Furthermore, we shall consider for simplicity the class of parametrizations characterized by the conditions

$$a\,b - a\,c + 2\,b\,d = 0; \qquad b + 2\,f = 0, \tag{5.6}$$

which are explicitly verified by all the graviton representations discussed in the literature [6-9].

Since the effective action is invariant under a general re-parametrization of the graviton fields, it can be written in terms of h_i as:

$$S_{eff} = \Gamma^{\alpha\beta}_{h_i} h_{i\alpha\beta}(0) + \frac{1}{2}\int d^4k \Pi^{\mu\nu,\alpha\beta}_{h_i}(k)\; h_{i\mu\nu}(k)\; h_{i\alpha\beta}(-k) + \cdots . \tag{5.7}$$

Identifying with the help of (5.3), the corresponding terms in (5.2) and (5.7), we obtain the following relations:

$$\Gamma^{\mu\nu}_{h} = a\,\Gamma^{\mu\nu}_{h_i} + b\,\eta^{\mu\nu}\Gamma^{\rho}_{h_i\,,\,\rho}, \tag{5.8}$$

$$\begin{aligned}\Pi^{\mu\nu,\alpha\beta}_{h}(k) \;=\; \Pi^{\mu\nu,\alpha\beta}_{h_i}(k) \;&+ a\,b\left(\Pi^{\mu\nu,\rho}_{h_i\,,\,\rho}\,\eta^{\alpha\beta} + \Pi^{\alpha\beta,\,\rho}_{h_i\,,\,\rho}\,\eta^{\mu\nu}\right) + b^2\,\Pi^{\sigma\,,\rho}_{h_i,\sigma\,,\,\rho}\eta^{\mu\nu}\,\eta^{\alpha\beta} \\ &+ c\left(\Gamma^{\mu\nu}_{h_i}\eta^{\alpha\beta} + \Gamma^{\alpha\beta}_{h_i}\eta^{\mu\nu}\right) \\ &+ \frac{d}{2}\left(\Gamma^{\mu\alpha}_{h_i}\eta^{\nu\beta} + \Gamma^{\nu\beta}_{h_i}\eta^{\mu\alpha} + \Gamma^{\nu\alpha}_{h_i}\eta^{\mu\beta} + \Gamma^{\mu\beta}_{h_i}\eta^{\nu\alpha}\right) \\ &+ 2e\,\Gamma^{\rho}_{h_i\,,\,\rho}\eta^{\mu\nu}\eta^{\alpha\beta} + f\,\Gamma^{\rho}_{h_i\,,\,\rho}\left(\eta^{\mu\beta}\eta^{\nu\alpha} + \eta^{\mu\alpha}\eta^{\nu\beta}\right)\end{aligned} \tag{5.9}$$

When the theory is invariant under conformal transformations, the following Weyl identity holds [12]:

$$\Pi^{\rho}_{h_i\,,\,\rho}{}^{,\mu\nu} = -\kappa\frac{a + 4c + 2d}{a\,(a + 4b)}\,\Gamma^{\mu\nu}_{h_i} \equiv -\tilde{\Gamma}^{\mu\nu}_{h_i}, \tag{5.10}$$

where $\tilde{\Gamma}^{\mu\nu}_{h_i}$ is a traceless function. In general, this is no longer true in the presence of thermal matter at finite temperature. In order to take this fact into account, we generalize (5.1) by considering the following traceless quantity:

$$\begin{aligned}\tilde{\Pi}^{\mu\nu,\alpha\beta}(k) \equiv \bar{\Pi}^{\mu\nu,\alpha\beta}(k) + \tfrac{1}{2}\Delta^{\lambda}_{\lambda}\;&\left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}\right) - \left(\eta^{\mu\nu}\Delta^{\alpha\beta} + \eta^{\alpha\beta}\Delta^{\mu\nu}\right) + \\ &\left(\eta^{\mu\alpha}\Delta^{\nu\beta} + \eta^{\mu\beta}\Delta^{\nu\alpha} + \eta^{\nu\alpha}\Delta^{\mu\beta} + \eta^{\nu\beta}\Delta^{\mu\alpha}\right)\end{aligned}, \tag{5.11}$$

where the tensor $\Delta^{\mu\nu}$ is given by:

$$\Delta^{\mu\nu} = \frac{1}{4}\left(\Pi^{\rho}_{\rho}{}^{,\mu\nu} + \tilde{\Gamma}^{\mu\nu}\right) - \frac{1}{32}\left(\Pi^{\rho}_{\rho}{}^{,\sigma}_{\sigma} + \tilde{\Gamma}^{\rho}_{\rho}\right)\eta^{\mu\nu}. \tag{5.12}$$

We remark that when the Weyl identity (5.10) is applicable, $\Delta^{\mu\nu}$ vanishes so that Eq. (5.11) reduces to (5.1) as expected. For this reason, only the contributions from thermal matter will appear in Eq. (5.12).

It is now straightforward to verify, with the help of the relations (5.6), (5.8) and (5.9) that:

$$\tilde{\Pi}^{\mu\nu,\alpha\beta}_{h}(k) = \tilde{\Pi}^{\mu\nu,\alpha\beta}_{h_i}(k). \tag{5.13}$$

This equation shows that the graviton self-energy given by the relations (5.11) and (5.12) is invariant under re-parametrizations of graviton fields at all temperatures. In order to understand the mechanism which enforces the above property, using the relations (5.11) (5.11) and (5.12), we write the expression for the graviton self-energy in the form:

$$\tilde{\Pi}^{\mu\nu,\alpha\beta}(k) = \Pi^{\mu\nu,\alpha\beta}(k) + \Pi^{\mu\nu,\alpha\beta}_{tad}(k), \tag{5.14}$$

where $\Pi^{\mu\nu,\alpha\beta}_{tad}$ is given by:

$$\begin{aligned}\Pi^{\mu\nu,\alpha\beta}_{tad}(k) \equiv \frac{1}{4}\;&\left(\eta^{\mu\alpha}\tilde{\Gamma}^{\nu\beta} + \eta^{\mu\beta}\tilde{\Gamma}^{\nu\alpha} + \eta^{\nu\alpha}\tilde{\Gamma}^{\mu\beta} + \eta^{\nu\beta}\tilde{\Gamma}^{\mu\alpha}\right) \\ &-\eta^{\mu\nu}\Delta^{\alpha\beta} - \eta^{\alpha\beta}\Delta^{\mu\nu}\end{aligned}. \tag{5.15}$$

As mentioned before, Δ vanishes in the case when the thermal fields can be considered as being effectively massless. The contributions associated with the graviton 1-point functions in (5.15) can be represented diagrammatically as shown in Fig. 5. Both Π and $\tilde{\Gamma}$ depend individually on the choice of basic graviton fields in a way that ensures $\tilde{\Pi}$ to be independent of these parametrizations. Hence, in order to obtain a physical self-energy, one must consider in addition to 1PI graviton 2-point function, also the corresponding "tadpole" contributions. Since the graviton self-energy (5.11) is parametrization-independent, it may be conveniently evaluated in the representation (2.2) of the gravitational fields, from the contributions of thermal matter and radiation fields given respectively by Eqs. (2.9) and (4.1).

VI. THE EFFECTIVE GRAVITON PROPAGATOR

In order to investigate the thermal mass of gravitons, we will study the properties of he poles in the effective graviton propagator. This is obtained by iterative insertions of the hysical self-energy in the classical graviton propagator $k^{-2}P^{\mu\nu}_{\alpha\beta}$, where:

$$P^{\mu\nu}_{\alpha\beta} = \frac{\delta^\mu_\alpha\delta^\nu_\beta + \delta^\mu_\beta\delta^\nu_\alpha - \eta^{\mu\nu}\eta_{\alpha\beta}}{2} \equiv \mathbb{I}^{\mu\nu}_{\alpha\beta} - \frac{\eta^{\mu\nu}\eta_{\alpha\beta}}{2} \tag{6.1}$$

hich is insensitive to changes of parametrizations [6,12].

As we have seen, because of the inclusion of graviton 1-point functions, the graviton self-nergy (5.14) is also independent of the parametrizations of graviton fields. These properties nsure that physical quantities such as masses are independent of the choice of basic graviton elds. Using the fact that the physical self-energy is traceless, P behaves effectively like the lentity when acting on $\tilde{\Pi}$. Hence, the effective graviton propagator can be written in the rm:

$$\tilde{D}^{\mu\nu}_{\alpha\beta}(k) = \frac{1}{k^2}\left[P^{\mu\nu}_{\alpha\beta} + \frac{1}{k^2}\tilde{\Pi}^{\mu\nu}_{\alpha\beta}(k) + \left(\frac{1}{k^2}\right)^2 \tilde{\Pi}^{\mu\nu}_{\rho\sigma}(k)\tilde{\Pi}^{\rho\sigma}_{\alpha\beta}(k) + \ldots\right] \tag{6.2}$$

he right-hand side of this equation sums up to a geometric series, giving the relation:

$$\left(k^2 P^{\mu\nu}_{\rho\sigma} - \tilde{\Pi}^{\mu\nu}_{\rho\sigma}\right)\tilde{D}^{\rho\sigma}_{\alpha\beta} = \mathbb{I}^{\mu\nu}_{\alpha\beta} \tag{6.3}$$

The effective propagator satisfies certain fundamental constrains. In view of the traceless roperty of $\tilde{\Pi}$, Eq. (6.2) requires that:

$$\tilde{D}^{\rho\ ,\mu\nu}_{\rho} = -\frac{\eta_{\mu\nu}}{k^2} \tag{6.4}$$

urthermore, the Ward identity (2.11) expressing gauge invariance requires a longitudinal ontribution in $\tilde{\Pi}$ connected with the background energy-momentum tensor [cf. Eq. (2.16) i the second paper of Ref. [12]]. Considering for definiteness the high-temperature limit, it then straightforward to verify that Eq. (6.2) implies:

$$k_\alpha k_\beta \tilde{D}^{\alpha\beta}_{\mu\nu} = \frac{k_\mu k_\nu}{k^2} - \frac{1}{2}\eta_{\mu\nu} + \frac{\kappa^2\rho}{12}(\eta^{\alpha\beta} - 4\delta^\alpha_0\delta^\beta_0)(k^2\mathbf{1} - \tilde{\Pi})^{-1}_{\alpha\beta,\,\mu\nu} \tag{6.5}$$

where the energy density ρ is given by:

$$\rho = \frac{\omega\pi^2 T^4}{30}. \tag{6.6}$$

Here ω denotes the total number of degrees of freedom of the thermal particles.

In what follows we shall be interested only in determining the effective dynamical masses in the static case $k_0 = 0$, which are relevant in the process of dynamical screening. To this end, we project the corresponding contributions of $\tilde{\Pi}$ into the following traceless normal modes:

$$T_J^{\mu\nu,\alpha\beta} = \left[-\frac{4}{3}T_3(u) - \frac{1}{12}T_4(u) + \frac{1}{3}T_5(u)\right]^{\mu\nu,\alpha\beta} \tag{6.7}$$

$$T_S^{\mu\nu,\alpha\beta} = \left[-\frac{1}{2}T_2(u) + 2T_3(u)\right]^{\mu\nu,\alpha\beta}, \tag{6.8}$$

where the tensors $T_i^{\mu\nu,\alpha\beta}(u)$ $(i = 1,\cdots,5)$ are obtained from the corresponding ones in Table I by replacing X^α with $u^\alpha \equiv \delta^\alpha_0$. The normal modes T_J and T_S are idempotent (up to a minus sign) and orthogonal to each other. While the mode T_J is three-dimensionally longitudinal, the mode T_S is spatially-transverse in the sense that:

$$k_i k_j T_S^{i\,j,\,\mu\nu} = 0; \qquad (i,\,j = 1,2,3). \tag{6.9}$$

In terms of these tensors, we can decompose the self-energy as follows:

$$\tilde{\Pi}^{\mu\nu,\alpha\beta}(k_0 = 0) = \kappa^2\rho\left(T_J^{\mu\nu,\alpha\beta} - \frac{1}{3}T_S^{\mu\nu,\alpha\beta}\right) \tag{6.10}$$

It is then easy to invert Eq. (6.3), yielding for the effective graviton propagator the result:

$$\tilde{D}^{\mu\nu,\alpha\beta}(k_0 = 0) = \frac{1}{\vec{k}^2}T_U^{\mu\nu,\alpha\beta} + \frac{1}{\vec{k}^2 + \frac{\kappa^2\rho}{3}}T_S^{\mu\nu,\alpha\beta} + \frac{1}{\vec{k}^2 - \kappa^2\rho}T_J^{\mu\nu,\alpha\beta} \tag{6.11}$$

where the normal mode T_U given by:

$$T_U^{\mu\nu,\alpha\beta} = \left[-\frac{1}{2}T_1(u) + \frac{1}{2}T_2(u) - \frac{2}{3}T_3(u) + \frac{7}{12}T_4(u) - \frac{1}{3}T_5(u)\right]^{\mu\nu,\alpha\beta} \tag{6.12}$$

is orthogonal to the modes T_J and T_S.

We see that in the normal mode T_V, the gravitational plasma is unscreened. This is mewhat similar to the spatially-transverse mode in the QCD plasma. On the other hand, ıon-vanishing screening mass appears in the mode T_S:

$$m_S^2 = \frac{\kappa^2 \rho}{3} = \frac{32\pi G\rho}{3} \tag{6.13}$$

alogously to the behavior shown by the spatially-longitudinal mode in the QCD plasma. The mode T_J is characterized by an imaginary mass:

$$m_J^2 = -32\pi G\rho \tag{6.14}$$

ıich is similar to the classical Jeans mass. This anti-screening mode indicates a gravita- ınal instability for density fluctuations with wavelength larger than $|m_J|^{-1}$, owing to the tractive nature of gravity. One may generalize this calculation by including internal gravi- ns in thermal equilibrium at high temperatures [9]. Their contributions will not affect the ove conclusion, since these change only the weight factor w appearing in ρ [cf. Eq.(6.6)] ıich counts the total number of degrees of freedom.

In conclusion we emphasize that this work, like that of Ref. [6], has been concerned with avitational perturbations at finite temperatures around the Minkowski background, in an ymptotically flat space. Using a different approach, based on the study of small distur- nces around the solutions of Einstein equations, Rebhan [8] has performed a rather com- ete investigation of the gravitational instabilities. These metric perturbations are relevant high temperatures in the context of a radiation dominated Robertson-Walker universe.

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APPENDIX A

In this appendix we present the Feynman rules for the couplings and propagators involving scalar, gauge and graviton fields. These rules can be obtained from the respective Lagrangians given in Eq. (2.1) and Eqs. (3.1), (3.2) and (3.3). In a perturbative calculation we first have to expand all the metric dependent quantities up to some given order in the graviton field h. These expansions and the subsequent reading of the momentum space Feynman rules is a straightforward procedure (but a very tedious task for humans) which was accomplished using a algebraic computer algorithm written in Mathematica. Here we will only present the results for the vertices involving up to two gravitons, which are relevant for the calculation of the graviton polarization tensor. We will also restrict only to the Abelian couplings of the gauge fields [cf. Eq. (3.1)].

In all the expressions which follows we will always denote the graviton momenta and indices by $[k_1, (\mu, \nu)]$ and $[k_2, \alpha, \beta)]$. The momenta of scalars and ghosts are denoted by p_1 and p_2. The gluon momenta and indices are denoted by $[p_1, \rho]$ and $[p_2, \sigma]$. Using this notation, we obtain from the Lagrangian (2.1) the *scalar-scalar-graviton* interaction vertex

$$\frac{2}{\kappa} V^{1\,scalar}_{\mu\nu}(k_1; p_1, p_2) = p_{1\mu}\, p_{2\nu} + p_{1\nu}\, p_{2\mu} - p_1 \cdot p_2\, \eta_{\mu\nu} - m^2\, \eta_{\mu\nu} + 2\,\xi \left(k_{1\mu}\, k_{1\nu} - k_1^2\, \eta_{\mu\nu} \right) \tag{A1}$$

and the *scalar-scalar-graviton-graviton* interaction vertex

$$\begin{aligned} \frac{16}{\kappa^2}\, V^{2\,scalar}_{\mu\nu,\alpha\beta}(k_1, k_2; p_1, p_2) = {} & -8\, p_{1\nu}\, p_{2\beta}\, \eta_{\alpha\mu} + 2\, p_1 \cdot p_2\, \eta_{\alpha\mu}\, \eta_{\beta\nu} + 4\, p_{1\alpha}\, p_{2\beta}\, \eta_{\mu\nu} - \\ & p_1 \cdot p_2\, \eta_{\alpha\beta}\, \eta_{\mu\nu} + m^2\, (2\, \eta_{\alpha\mu}\, \eta_{\beta\nu} - \eta_{\alpha\beta}\, \eta_{\mu\nu}) + \\ & \xi\, (4\, k_{1\mu}\, k_{1\nu}\, \eta_{\alpha\beta} + 4\, k_{1\mu}\, k_{2\nu}\, \eta_{\alpha\beta} - 4\, k_{1\beta}\, k_{1\nu}\, \eta_{\alpha\mu} - \\ & 6\, k_{1\beta}\, k_{2\nu}\, \eta_{\alpha\mu} - 4\, k_{1\beta}\, k_{1\mu}\, \eta_{\alpha\nu} + 2\, k_{1\beta}\, k_{2\mu}\, \eta_{\alpha\nu} - \\ & 4\, k_{1\alpha}\, k_{1\nu}\, \eta_{\beta\mu} - 2\, k_{1\alpha}\, k_{2\nu}\, \eta_{\beta\mu} + 2\, k_1 \cdot k_2\, \eta_{\alpha\nu}\, \eta_{\beta\mu} - \\ & 4\, k_{1\alpha}\, k_{1\mu}\, \eta_{\beta\nu} - 8\, k_{1\mu}\, k_{2\alpha}\, \eta_{\beta\nu} + 2\, k_{1\alpha}\, k_{2\mu}\, \eta_{\beta\nu} + \\ & 8\, k_1^2\, \eta_{\alpha\mu}\, \eta_{\beta\nu} + 4\, k_1 \cdot k_2\, \eta_{\alpha\mu}\, \eta_{\beta\nu} + 8\, k_{1\alpha}\, k_{1\beta}\, \eta_{\mu\nu} + \\ & 4\, k_{1\beta}\, k_{2\alpha}\, \eta_{\mu\nu} - 4\, k_1^2\, \eta_{\alpha\beta}\, \eta_{\mu\nu} - 2\, k_1 \cdot k_2\, \eta_{\alpha\beta}\, \eta_{\mu\nu}) \,, \end{aligned} \tag{A2}$$

where in the expression above one has to perform a symmetrization over the graviton indices and permutation of the scalar particles.

From the ghost Lagrangian we obtain the *ghost-ghost-graviton* interaction vertex

$$\frac{2}{\kappa}V^{1\,ghost}_{\mu\nu}(k_1;\,p_1,p_2) = -p_{1\nu}\,p_{2\mu} - p_{1\mu}\,p_{2\nu} + p_1\cdot p_2\,\eta_{\mu\nu}, \tag{A3}$$

and the *ghost-ghost-graviton-graviton* interaction vertex

$$\frac{8}{\kappa}V^{2\,ghost}_{\mu\nu,\,\alpha\beta}(k_1,k_2;\,p_1,p_2) = -4\,p_{1\mu}\,p_{2\nu}\,\eta_{\alpha\beta} + 8\,p_{1\beta}\,p_{2\nu}\,\eta_{\alpha\mu} - 2\,p_1\cdot p_2\,\eta_{\alpha\mu}\,\eta_{\beta\nu} + p_1\cdot p_2\,\eta_{\alpha\beta}\,\eta_{\mu\nu}. \tag{A4}$$

Expression (A4) has to be symmetrized over the graviton indices. Notice that, as can be easily seen from Eqs. (2.1) and (3.3), the interaction vertices of ghosts or scalars with the graviton field, differ only by a minus sign when $\xi = m = 0$. The corresponding propagators are given by

$$D(k) = \begin{cases} \dfrac{1}{k^2} & \text{for the ghost,} \\ \dfrac{1}{m^2-k^2} & \text{for the scalar} \end{cases} . \tag{A5}$$

From Eqs. (3.1) and (3.2) we can obtain the gauge fields Feynman rules in the general covariant gauge characterized by the gauge fixing parameter α. In this class of gauges the gauge field propagator is given by

$$D_{\mu\nu}(k) = \frac{1}{k^2}\left[\eta_{\mu\nu} - (1-\alpha)\,\frac{k_\mu\,k_\nu}{k^2}\right]. \tag{A6}$$

The interaction vertices will also depend on the parameter α. The *gauge-gauge-graviton* coupling is

$$\begin{aligned}\frac{4}{\kappa}V^{1\,gauge}_{\mu\nu;\rho,\sigma}(k_1;p_1,p_2) = {} & -p_{1\sigma}\,p_{2\rho}\,\eta_{\mu\nu} + p_{1\sigma}\,p_{2\nu}\,\eta_{\mu\rho} + p_{1\nu}\,p_{2\rho}\,\eta_{\mu\sigma} + \\ & p_{1\sigma}\,p_{2\mu}\,\eta_{\nu\rho} + p_{1\mu}\,p_{2\rho}\,\eta_{\nu\sigma} - 2\,p_1\cdot p_2\,\eta_{\mu\rho}\,\eta_{\nu\sigma} - \\ & 2\,p_{1\mu}\,p_{2\nu}\,\eta_{\rho\sigma} + p_1\cdot p_2\,\eta_{\mu\nu}\,\eta_{\rho\sigma} + \\ & \frac{1}{\alpha}\left(-2\,p_{1\rho}\,p_{1\sigma}\,\eta_{\mu\nu} - p_{1\rho}\,p_{2\sigma}\,\eta_{\mu\nu} - 2\,p_{1\nu}\,p_{2\sigma}\,\eta_{\mu\rho} + \right. \\ & \left. 4\,p_{1\nu}\,p_{1\rho}\,\eta_{\mu\sigma} + 2\,p_{1\rho}\,p_{2\nu}\,\eta_{\mu\sigma}\right),\end{aligned} \tag{A7}$$

and the *gauge-gauge-graviton-graviton* vertex is

$$\begin{aligned}\frac{16}{\kappa^2}V^{2\,gauge}_{\mu\nu,\,\alpha\beta;\,\rho,\sigma}(k_1,k_2;p_1,p_2) = {} & 2\,p_{1\sigma}\,p_{2\rho}\,\eta_{\alpha\mu}\,\eta_{\beta\nu} - 4\,p_{1\sigma}\,p_{2\nu}\,\eta_{\alpha\mu}\,\eta_{\beta\rho} - 4\,p_{1\nu}\,p_{2\rho}\,\eta_{\alpha\mu}\,\eta_{\beta\sigma} + \\ & 4\,p_{1\mu}\,p_{2\nu}\,\eta_{\alpha\rho}\,\eta_{\beta\sigma} - p_{1\sigma}\,p_{2\rho}\,\eta_{\alpha\beta}\,\eta_{\mu\nu} + 2\,p_{1\sigma}\,p_{2\beta}\,\eta_{\alpha\rho}\,\eta_{\mu\nu} + \\ & 2\,p_{1\beta}\,p_{2\rho}\,\eta_{\alpha\sigma}\,\eta_{\mu\nu} + 2\,p_{1\sigma}\,p_{2\alpha}\,\eta_{\beta\rho}\,\eta_{\mu\nu} + 2\,p_{1\alpha}\,p_{2\rho}\,\eta_{\beta\sigma}\,\eta_{\mu\nu} - \\ & 4\,p_1\cdot p_2\,\eta_{\alpha\rho}\,\eta_{\beta\sigma}\,\eta_{\mu\nu} - 4\,p_{1\alpha}\,p_{2\nu}\,\eta_{\beta\sigma}\,\eta_{\mu\rho} - 4\,p_{1\sigma}\,p_{2\beta}\,\eta_{\alpha\mu}\,\eta_{\nu\rho} - \\ & 4\,p_{1\beta}\,p_{2\mu}\,\eta_{\alpha\sigma}\,\eta_{\nu\rho} + 8\,p_1\cdot p_2\,\eta_{\alpha\mu}\,\eta_{\beta\sigma}\,\eta_{\nu\rho} - 4\,p_{1\beta}\,p_{2\rho}\,\eta_{\alpha\mu}\,\eta_{\nu\sigma} + \\ & 4\,p_{1\alpha}\,p_{2\beta}\,\eta_{\mu\rho}\,\eta_{\nu\sigma} + 8\,p_{1\nu}\,p_{2\beta}\,\eta_{\alpha\mu}\,\eta_{\rho\sigma} - 2\,p_1\cdot p_2\,\eta_{\alpha\mu}\,\eta_{\beta\nu}\,\eta_{\rho\sigma} - \\ & 4\,p_{1\alpha}\,p_{2\beta}\,\eta_{\mu\nu}\,\eta_{\rho\sigma} + p_1\cdot p_2\,\eta_{\alpha\beta}\,\eta_{\mu\nu}\,\eta_{\rho\sigma} + \\ & \frac{1}{\alpha}\left(16\,k_{1\nu}\,p_{1\rho}\,\eta_{\alpha\sigma}\,\eta_{\beta\mu} - 8\,k_{1\sigma}\,p_{1\rho}\,\eta_{\alpha\mu}\,\eta_{\beta\nu} - 2\,p_{1\rho}\,p_{2\sigma}\,\eta_{\alpha\mu}\,\eta_{\beta\nu} + \right. \\ & 2\,k_{1\rho}\,k_{2\sigma}\,\eta_{\alpha\beta}\,\eta_{\mu\nu} + 4\,k_{1\sigma}\,p_{1\rho}\,\eta_{\alpha\beta}\,\eta_{\mu\nu} + p_{1\rho}\,p_{2\sigma}\,\eta_{\alpha\beta}\,\eta_{\mu\nu} - \\ & 8\,k_{1\sigma}\,p_{1\beta}\,\eta_{\alpha\rho}\,\eta_{\mu\nu} - 4\,p_{1\beta}\,p_{2\sigma}\,\eta_{\alpha\rho}\,\eta_{\mu\nu} - 4\,k_{1\rho}\,k_{2\beta}\,\eta_{\alpha\sigma}\,\eta_{\mu\nu} - \\ & 8\,k_{1\beta}\,p_{1\rho}\,\eta_{\alpha\sigma}\,\eta_{\mu\nu} - 4\,p_{1\rho}\,p_{2\beta}\,\eta_{\alpha\sigma}\,\eta_{\mu\nu} - 4\,k_{1\nu}\,k_{2\sigma}\,\eta_{\alpha\beta}\,\eta_{\mu\rho} + \\ & 8\,p_{1\beta}\,p_{2\sigma}\,\eta_{\alpha\nu}\,\eta_{\mu\rho} + 8\,k_{1\nu}\,k_{2\beta}\,\eta_{\alpha\sigma}\,\eta_{\mu\rho} + 8\,p_{1\nu}\,p_{2\beta}\,\eta_{\alpha\sigma}\,\eta_{\mu\rho} - \\ & 8\,k_{1\nu}\,p_{1\rho}\,\eta_{\alpha\beta}\,\eta_{\mu\sigma} + 8\,k_{1\beta}\,p_{1\rho}\,\eta_{\alpha\nu}\,\eta_{\mu\sigma} + 8\,p_{1\rho}\,p_{2\beta}\,\eta_{\alpha\nu}\,\eta_{\mu\sigma} + \\ & \left. 16\,k_{1\nu}\,p_{1\beta}\,\eta_{\alpha\rho}\,\eta_{\mu\sigma} + 8\,k_{1\alpha}\,p_{1\rho}\,\eta_{\beta\nu}\,\eta_{\mu\sigma}\right).\end{aligned} \tag{A8}$$

Similarly as in the scalar vertices, one has to symmetrize the gauge field vertices over the graviton indices and include the permutations of the gluons. In all interaction vertices there is momentum conservation, with all momenta inwards.

From the expressions above we can perform the explicit computation of the scattering amplitudes shown in the figures 2 and 4. This was done using these Feynman rules as an input to a algebraic computer program.

APPENDIX B

In this appendix we present the leading and next-to-leading structure functions for the matter contribution to the polarization tensor. The leading structure functions presented here are the same for all thermal particles. The explicit result for the functions $l_i(r,K)$, $n_i(r,K)$ and $s_i(r,K)$ appearing in Eq. (2.10) is the following:

$$l_1 = \frac{1}{6} - \frac{K^2}{24} + \frac{K^4 L}{8}$$
$$n_1 = \frac{-5K^4}{192} - \frac{5K^2\xi}{16} + \frac{5K^6 L}{64} \quad , \tag{B1}$$
$$s_1 = -\frac{5}{8} + \frac{5K^2 L}{16}$$

$$l_2 = -\frac{1}{3} + \frac{K^2}{12} - \frac{5K^4}{24} + \frac{5K^6 L}{8}$$
$$n_2 = \frac{-25K^6}{192} + \frac{5K^6 L}{32} + \frac{25K^8 L}{64} \quad , \tag{B2}$$
$$s_2 = \frac{5}{8} - \frac{5K^2}{16} + \frac{15K^4 L}{16}$$

$$l_3 = \frac{-K^2}{3} + \frac{7K^4}{12} - \frac{35K^6}{24} + \frac{35K^8 L}{8}$$
$$n_3 = \frac{-5K^6}{32} - \frac{175K^8}{192} + \frac{25K^8 L}{16} + \frac{175K^{10} L}{64} \quad , \tag{B3}$$
$$s_3 = \frac{5K^2}{8} - \frac{25K^4}{16} + \frac{75K^6 L}{16}$$

$$l_4 = \frac{-K^2}{24} + \frac{K^4 L}{8}$$
$$n_4 = \frac{-5K^2}{32} - \frac{5K^4}{192} - \frac{5K^2\xi}{8} + \frac{5K^4 L}{16} + \frac{5K^6 L}{64} + \frac{5K^4\xi L}{4} \quad , \tag{B4}$$
$$s_4 = \frac{5K^2 L}{16}$$

$$l_5 = \frac{K^2}{12} - \frac{5K^4}{24} + \frac{5K^6 L}{8}$$
$$n_5 = \frac{-5K^4}{32} - \frac{25K^6}{192} - \frac{5K^4\xi}{8} + \frac{5K^6 L}{8} + \frac{25K^8 L}{64} + \frac{15K^6\xi L}{8} \quad , \tag{B5}$$
$$s_5 = \frac{-5K^2}{16} + \frac{15K^4 L}{16}$$

$$l_6 = \left(\frac{-1}{12} + \frac{5K^2}{24} - \frac{5K^4 L}{8}\right) r$$
$$n_6 = \left(\frac{25K^4}{192} - \frac{5K^4 L}{32} - \frac{25K^6 L}{64}\right) r \quad , \tag{B6}$$
$$s_6 = \left(\frac{5}{16} - \frac{15K^2 L}{16}\right) r$$

$$l_7 = \left(\frac{1}{3} - \frac{7K^2}{12} + \frac{35K^4}{24} - \frac{35K^6 L}{8}\right) r$$
$$n_7 = \left(\frac{5K^4}{32} + \frac{175K^6}{192} - \frac{25K^6 L}{16} - \frac{175K^8 L}{64}\right) r \quad , \tag{B7}$$
$$s_7 = \left(\frac{-5}{8} + \frac{25K^2}{16} - \frac{75K^4 L}{16}\right) r$$

$$l_8 = -\frac{1}{12} - \frac{5K^2}{24} + \frac{K^2 L}{2} + \frac{5K^4 L}{8}$$
$$n_8 = \frac{-5K^2}{48} - \frac{25K^4}{192} + \frac{5\xi}{16} + \frac{5K^2 L}{32} + \frac{15K^4 L}{32} + \frac{25K^6 L}{64} \quad , \tag{B8}$$
$$s_8 = -\frac{5}{16} + \frac{5L}{8} + \frac{15K^2 L}{16}$$

$$l_9 = \frac{1}{6} - \frac{2K^2}{3} - \frac{35K^4}{24} + \frac{15K^4 L}{4} + \frac{35K^6 L}{8}$$
$$n_9 = \frac{-15K^4}{16} - \frac{175K^6}{192} + \frac{5K^2\xi}{8} + \frac{15K^4 L}{16} + \frac{125K^6 L}{32} + \frac{175K^8 L}{64} - \frac{15K^4\xi L}{8} \quad , \tag{B9}$$
$$s_9 = -\frac{5}{8} - \frac{25K^2}{16} + \frac{15K^2 L}{4} + \frac{75K^4 L}{16}$$

$$l_{10} = \frac{1}{6} - \frac{2K^2}{3} - \frac{35K^4}{24} + \frac{15K^4 L}{4} + \frac{35K^6 L}{8}$$
$$n_{10} = \frac{-5K^2}{32} - \frac{15K^4}{16} - \frac{175K^6}{192} + \frac{45K^4 L}{32} + \frac{125K^6 L}{32} + \frac{175K^8 L}{64} \quad , \tag{B10}$$
$$s_{10} = -\frac{5}{8} - \frac{25K^2}{16} + \frac{15K^2 L}{4} + \frac{75K^4 L}{16}$$

$$l_{11} = \left(\frac{1}{4} + \frac{35K^2}{24} - \frac{5K^2 L}{2} - \frac{35K^4 L}{8}\right) r$$
$$n_{11} = \left(\frac{65K^2}{96} + \frac{175K^4}{192} - \frac{5\xi}{8} - \frac{5K^2 L}{8} - \frac{25K^4 L}{8} - \frac{175K^6 L}{64} + \frac{15K^2\xi L}{8}\right) r \quad , \tag{B11}$$
$$s_{11} = \left(\frac{25}{16} - \frac{15L}{8} - \frac{75K^2 L}{16}\right) r$$

$$l_{12} = -\frac{13}{12} - \frac{35K^2}{24} + L + 5K^2 L + \frac{35K^4 L}{8}$$
$$n_{12} = -\frac{5}{24} - \frac{115K^2}{96} - \frac{175K^4}{192} + \frac{5\xi}{4} + \frac{15K^2 L}{8} + \frac{75K^4 L}{16} + \frac{175K^6 L}{64} - \frac{5\xi L}{2} - \frac{15K^2\xi L}{4} \quad , \tag{B12}$$
$$s_{12} = -\frac{25}{16} - \frac{5}{8K^2} + \frac{15L}{4} + \frac{75K^2 L}{16}$$

$$l_{13} = -\frac{1}{12} - \frac{5K^2}{24} + \frac{K^2 L}{2} + \frac{5K^4 L}{8}$$
$$n_{13} = \frac{-25K^2}{96} - \frac{25K^4}{192} - \frac{5K^2\xi}{8} + \frac{5K^2 L}{16} + \frac{15K^4 L}{16} + \frac{25K^6 L}{64} + \frac{5K^2\xi L}{8} + \frac{15K^4\xi L}{8} \quad , \tag{B13}$$
$$s_{13} = -\frac{5}{16} + \frac{5L}{8} + \frac{15K^2 L}{16}$$

$$l_{14} = \left(\frac{-1}{12} + \frac{5K^2}{24} - \frac{5K^4 L}{8}\right) r$$
$$n_{14} = \left(\frac{5K^2}{32} + \frac{25K^4}{192} + \frac{5K^2\xi}{8} - \frac{5K^4 L}{8} - \frac{25K^6 L}{64} - \frac{15K^4\xi L}{8}\right) r \quad . \tag{B14}$$
$$s_{14} = \left(\frac{5}{16} - \frac{15K^2 L}{16}\right) r$$

The dimensionless quantity L is a function of r given by:

$$L(r) = \frac{r}{2}\ln\frac{r+1}{r-1} - 1. \tag{B15}$$

APPENDIX C

re we calculate the integrals

$$J_n(t) = \left(\frac{i\pi T}{|\vec{k}|}\right)^{n-1} \int^t dy\, y^n \left[\ln(y) - \frac{1}{2y} - \psi(y)\right] \tag{C1}$$

ns of the generalized ζ function, defined as

$$\zeta(z,y) = \sum_{l=0}^{\infty} \frac{1}{(l+y)^z}. \tag{C2}$$

this end we express the ψ function as

$$\psi(y) = \lim_{\varepsilon\to 0}\left[\frac{1}{\varepsilon} - \zeta(1+\varepsilon, y)\right] \tag{C3}$$

e the formula

$$\int^t dy \zeta(z,y) = \frac{1}{1-z}\zeta(1-z,t), \tag{C4}$$

ın be easily verified from eq. (C2), and can be generalized to

$$\int^t dy y^n \zeta(z,y) = \sum_{l=0}^{n} \frac{(n-l)!}{n!}\frac{\Gamma(1-z)}{\Gamma(l+2-z)} t^n \zeta(z-l-1,t), \tag{C5}$$

is eq. (C4) integrated by parts n times.

ɔstituting eq. (C3) in eq. (C1) and using eq. (C5) we can verify, using the properties ınction, that the divergent term as $\varepsilon \to 0$ cancels out, as expected. For $n \neq 0$ the ing terms give:

$$J_n(t) = \left(\frac{i\pi T}{|\vec{k}|}\right)^{n-1} \left\{ \frac{t^{n+1}}{n+1}\ln t - \frac{t^{n+1}}{(n+1)^2} - \frac{t^n}{2n} - \sum_{j=0}^{n} (-1)^j t^{n-j} \binom{n}{j} \zeta'(-j,t) + \right.$$
$$\left. \sum_{j=1}^{n} \frac{(-1)^j}{j+1}\binom{n}{j}\left(\sum_{k=1}^{j}\frac{1}{k}\right) t^{n-j} \mathrm{B}_{j+1}(t) \right\}, \tag{C6}$$

B_n are the Bernoulli polynomials [11]. For $n = 0$, $J_0(t)$ is given by Eq. (4.12).

w we discuss the behavior of the generalized zeta function for asymptotic values of the eter $t(k_0) = i\frac{k_0+|\mathbf{k}|}{4\pi T}$, which correspond to high temperature expansion. To this end, rt from the representation [11]:

$$\zeta(z,t) = \frac{1}{t^z} + \frac{1}{\Gamma(z)}\int_0^\infty \frac{x^{z-1}e^{-tx}}{e^x - 1} dx. \tag{C7}$$

Expanding (C7) in power series of t, making use of the integral representation of Riemann's zeta function and Euler's gamma function we find

$$\zeta(z,t) = \frac{1}{t^z} + \sum_{l=0}^{\infty} \frac{\Gamma(z+l)}{\Gamma(z)}\frac{(-t)^l}{l!}\zeta(z+l). \tag{C8}$$

Taking the derivative of (C8) with respect to z we obtain in terms of the psi function $\psi(z)$ that

$$\zeta'(z,t) = \sum_{l=0}^{\infty} \frac{(-t)^l}{l!}\frac{\Gamma(z+l)}{\Gamma(z)}\left\{[\psi(z+l) - \psi(z)]\,\zeta(z+l) + \zeta'(z+l)\right\} - t^z \ln(z). \tag{C9}$$

We are actually interested in the values of $\zeta'(z,t)$ for $z \to -n$ where n is a natural number. After a straightforward calculation we obtain

$$\zeta'(-n,t) = \sum_{l=0}^{n}\binom{n}{l}\left[\zeta'(l-n) - \zeta(l-n)\sum_{k=n-l+1}^{n}\frac{1}{k}\right] t^l - t^n\ln(z)$$
$$-\frac{t^{n+1}}{n+1}\left(\gamma - \sum_{k=1}^{n}\frac{1}{k}\right) + \sum_{l=n+2}^{\infty}(-1)^{n+1}t^l\frac{n!(l-n-1)!}{l!}\zeta(l-n). \tag{C10}$$

With help of this formula we can compute the functions J_n in Eq. (C6) and express P_n from Eqs. (4.14), (4.15) and (4.16) as a series of decreasing powers of T. Then, it is straightforward to arrive at Eqs. (4.19), (4.20) and (4.21).

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$$T_1^{\mu\nu,\alpha\beta}(X,Y) = \eta^{\alpha\nu}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\nu}$$

$$T_2^{\mu\nu,\alpha\beta}(X,Y) = X^\mu\left(X^\beta\eta^{\alpha\nu} + X^\alpha\eta^{\beta\nu}\right) + X^\nu\left(X^\beta\eta^{\alpha\mu} + X^\alpha\eta^{\beta\mu}\right)$$

$$T_3^{\mu\nu,\alpha\beta}(X,Y) = X^\alpha X^\beta X^\mu X^\nu$$

$$T_4^{\mu\nu,\alpha\beta}(X,Y) = \eta^{\alpha\beta}\eta^{\mu\nu}$$

$$T_5^{\mu\nu,\alpha\beta}(X,Y) = X^\mu X^\nu\eta^{\alpha\beta} + X^\alpha X^\beta\eta^{\mu\nu}$$

$$T_6^{\mu\nu,\alpha\beta}(X,Y) = X^\beta\left(Y^\nu\eta^{\alpha\mu} + Y^\mu\eta^{\alpha\nu}\right) + Y^\beta\left(X^\nu\eta^{\alpha\mu} + X^\mu\eta^{\alpha\nu}\right) + X^\alpha\left(Y^\nu\eta^{\beta\mu} + Y^\mu\eta^{\beta\nu}\right) + Y^\alpha\left(X^\nu\eta^{\beta\mu} + X^\mu\eta^{\beta\nu}\right)$$

$$T_7^{\mu\nu,\alpha\beta}(X,Y) = Y^\nu X^\alpha X^\beta X^\mu + Y^\mu X^\alpha X^\beta X^\nu + Y^\beta X^\alpha X^\mu X^\nu + Y^\alpha X^\beta X^\mu X^\nu$$

$$T_8^{\mu\nu,\alpha\beta}(X,Y) = Y^\beta Y^\nu\eta^{\alpha\mu} + Y^\beta Y^\mu\eta^{\alpha\nu} + Y^\alpha Y^\nu\eta^{\beta\mu} + Y^\alpha Y^\mu\eta^{\beta\nu}$$

$$T_9^{\mu\nu,\alpha\beta}(X,Y) = Y^\mu Y^\nu X^\alpha X^\beta + Y^\alpha Y^\beta X^\mu X^\nu$$

$$T_{10}^{\mu\nu,\alpha\beta}(X,Y) = \left(Y^\beta X^\alpha + Y^\alpha X^\beta\right)\left(Y^\nu X^\mu + Y^\mu X^\nu\right)$$

$$T_{11}^{\mu\nu,\alpha\beta}(X,Y) = Y^\beta Y^\mu Y^\nu X^\alpha + Y^\alpha Y^\mu Y^\nu X^\beta + Y^\alpha Y^\beta Y^\nu X^\mu + Y^\alpha Y^\beta Y^\mu X^\nu$$

$$T_{12}^{\mu\nu,\alpha\beta}(X,Y) = Y^\alpha Y^\beta Y^\mu Y^\nu$$

$$T_{13}^{\mu\nu,\alpha\beta}(X,Y) = Y^\mu Y^\nu\eta^{\alpha\beta} + Y^\alpha Y^\beta\eta^{\mu\nu}$$

$$T_{14}^{\mu\nu,\alpha\beta}(X,Y) = \left(Y^\nu X^\mu + Y^\mu X^\nu\right)\eta^{\alpha\beta} + \left(Y^\beta X^\alpha + Y^\alpha X^\beta\right)\eta^{\mu\nu}$$

TABLE I. A basis of 14 independent tensors $T_i^{\mu\nu,\alpha\beta}(X,Y)$

FIGURES

FIG. 1. Lowest order matter contributions to the thermal 1PI graviton two-point function. Curly lines denote the gravitational field and solid lines represent the scalar particle.

FIG. 2. The forward scattering graphs corresponding to Fig. 1. Crossed graphs with $(k \leftrightarrow -k)$ are to be understood.

FIG. 3. One-loop contributions of radiation fields to the graviton polarization tensor. Wavy lines denote the gauge field and broken lines represent ghost particles.

FIG. 4. Forward scattering diagrams containing ghost particles connected with Fig. 3. Crossed diagrams $(k \leftrightarrow -k)$ should be included.

FIG. 5. Lowest order contributions of graviton 1-point functions to Π_{tad}. The black dot represents terms proportional to η.

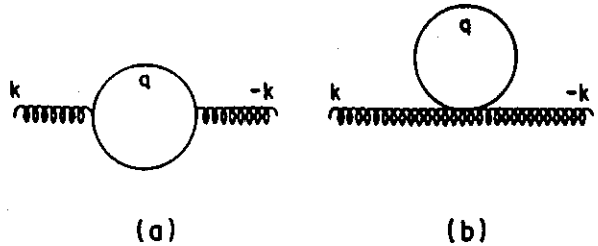

FIG. 1

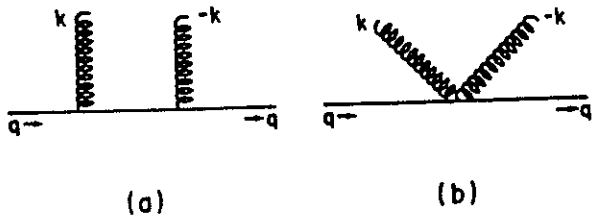

FIG. 2

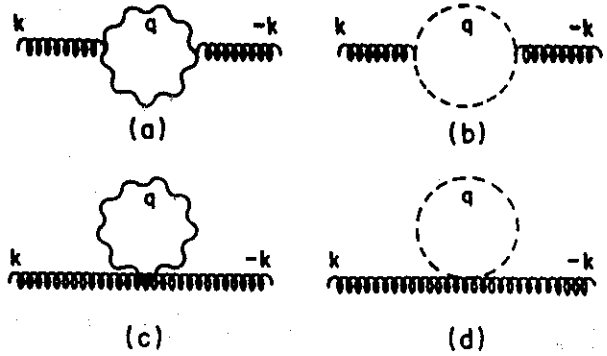

FIG. 3

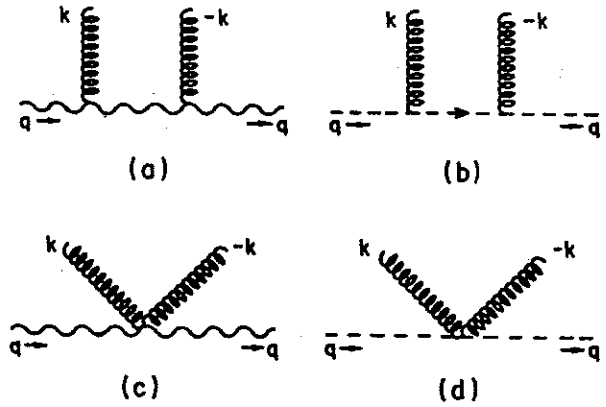

FIG. 4

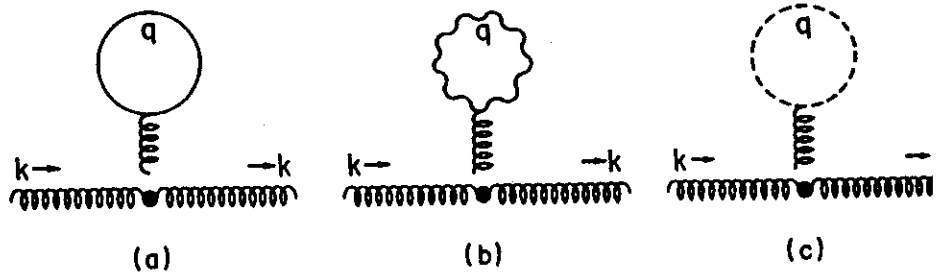

FIG. 5