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Radiatively induced electron and electron-neutrino masses

F. Pisano, V. Pleitez

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900 – São Paulo, S.P.
Brazil

and

M.D. Tonasse

Instituto de Física
Universidade de São Paulo
Caixa Postal 20516
01498-970 – São Paulo, S.P.
Brazil

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900 – São Paulo, S.P.
Brazil

Telephone: 55 (11) 251-5155
Telefax: 55 (11) 288-8224
Telex: 55 (11) 31870 UJMFBR
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Instituto de Física Teórica

Universidade Estadual Paulista

Rua Pamplona, 145

01405-900- São Paulo, SP

Brazil

M.D. Tonasse

Instituto de Física da Universidade de São Paulo

01498-970 C.P. 20516-São Paulo, SP

Brazil

We consider in the context of an $SU(3)\otimes U(1)$ electroweak model the generation of the neutrino and charged lepton masses. In the minimal model at zeroth order neutrinos and one of the charged leptons are massless and the other two charged leptons are mass degenerate. In order to obtain the right mass spectrum it is necessary to add right-handed neutrinos. However, the masses of the electron and its neutrino partner arise via radiative corrections.

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It is well known that in renormalizable theories, some masses or mass differences vanish at tree level if there are some symmetries in the theory which forbid them. This is maintained in higher order. Or, if the respective higher order corrections are infinite the introduction of the counter term necessary to remove the infinity leaves these masses or mass differences as free parameters. Notwithstanding, in theories with spontaneously broken symmetry if a mass and mass counter term are forbidden by gauge structure, then higher order corrections are finite and calculable [1].

In this spirit, many mechanisms for finding fermion masses as radiative corrections have been considered in the literature [2].

Recently, it was proposed an electroweak model based on the $SU(3)_L\otimes U(1)_N$ gauge symmetry [3,4]. Leptons are treated democratically with the three generations transforming as $(\mathbf{3},0)$ but with one quark generation (it does not matter which one) transforming differently from the other two. This condition arises because the model in order to be anomaly free must contain the same number of triplets as antitriplets. Hence, the number of generations is related to the number of quark colors.

In Refs. [4,5] the spontaneous symmetry breaking and fermion masses generation are assumed to arise from the vacuum expectation values (VEV) of three scalars η,ρ and χ which are triplets under $SU(3)$ and give mass to the quarks, and a sextet S which gives mass to the charged leptons. In the minimal model the neutrinos remain massless since there is a global symmetry which prevents them to get a mass. This symmetry implies the conservation of the quantum number $F=L+B$, where L is the total lepton number $L=L_e+L_\mu+L_\tau$ and B is the baryon number [6]. Here we will see that if we allow this symmetry to be explicitly broken one of the charged leptons and one neutrino get mass through radiative corrections.

Let us introduce the following Higgs scalars, $\eta=(\eta^0,\eta_1^-,\eta_2^+)^T$, $\rho=(\rho^+,\rho^0,\rho^{++})^T$ and $\chi=(\chi^-,\chi^{--},\chi^0)^T$ which transform, under $SU(3)\otimes U(1)$ as $(\mathbf{3},0),(\mathbf{3},1)$ and

$(\mathbf{3},-1)$, respectively.

The leptonic triplets are $\psi_{aL} = (\nu''_a, l''_a, l''^c_a)^T \sim (\mathbf{3},0)$, where the double primed fields denote weak eigenstates, $l''_a = e'', \mu'', \tau''$ and $\nu''_a = \nu''_e, \nu''_\mu, \nu''_\tau$.

The lepton mass term transforms as $(\mathbf{3}\otimes\mathbf{3}) = \mathbf{3}^*_A \oplus \mathbf{6}_S$. Thus, we can introduce a triplet, like η, or a symmetric antisextet $S = (\mathbf{6}^*_S, 0)$. In the former case one of the charged leptons remains massless, and the other two are mass degenerate. For this reason it was chosen in Ref. [5] the latter one in order to obtain arbitrary mass for charged leptons.

Here we will not introduce the sextet S but only the triplets η, ρ and χ; the respective VEV will be denoted by v_η, v_ρ and v_χ. Hence, the Yukawa interaction in the leptonic sector is

$$\mathcal{L}_{l\eta} = -\frac{i}{2}\sum_{a,b=e,\mu,\tau}\epsilon^{ijk} f_{ab}\overline{(\psi_{Lai})^c}\psi_{Lbj}\eta_k + H.c. \tag{1}$$

The Yukawa couplings f_{ab} must be antisymmetric due to Fermi statistics. Explicitly we have

$$\mathcal{L}_{l\eta} = -i(v_\eta + \eta'^0)\overline{l''_{Ra}}F_{ab}l''_{Lb} + \frac{i}{2}\overline{l''_{Ra}}F_{ab}\nu''_{Lb}\eta_1^- + \frac{i}{2}\overline{\nu''^c_{Ra}}F_{ab}l''_{Lb}\eta_2^+ + H.c., \tag{2}$$

where

$$F_{ab} = \begin{pmatrix} 0 & -f_{e\mu} & -f_{e\tau} \\ f_{e\mu} & 0 & -f_{\mu\tau} \\ f_{e\tau} & f_{\mu\tau} & 0 \end{pmatrix}. \tag{3}$$

The matrix F_{ab} may be diagonalized by an orthogonal matrix O, $l'' = Ol'$, $\nu' = O^T\nu''$. Hence, all terms in (2) become diagonal in the primed fields basis: $(e', \mu', \tau'), (\nu'_e, \nu'_\mu, \nu'_\tau)$. We have the following mass spectrum for charged leptons $(0, m, -m)$ where $m = fv_\eta$, $f^2 = f^2_{e\mu} + f^2_{e\tau} + f^2_{\mu\tau}$. Notice that one of the mass is negative. To get positive mass eigenvalues we let m be positive and redefine the

respective field with a γ^5 factor, i.e., $\tau' \to \gamma^5\tau'$. Because of this γ^5 the CP of τ' is -1. An alternative formalism is to choose a matrix O' of the form $O'' = O'\Phi$ with $\Phi = diag(1,1,i)$. Such phases, it is already known to appear in models with massive neutrinos. In the later case the phase does not imply CP violation in the weak interactions since the (apparent) CP violation can be transformed away by redefining the Majorana fields [7]. In the present case as we need to introduce the phase (or the γ^5 factor) in the charged lepton sector we cannot use Majorana fields. It means that it is not possible to avoid a genuine explicit CP violation.

Then, the charged leptons at this stage are massless ("electron") and mass degenerate ("muon" and "tau") while neutrinos are all massless.

Let

$$\begin{aligned} V(\eta,\rho,\chi) = {} & \mu_1^2\eta^\dagger\eta + \mu_2^2\rho^\dagger\rho + \mu_3^2\chi^\dagger\chi + \lambda_1(\eta^\dagger\eta)^2 \\ & + \lambda_2(\rho^\dagger\rho)^2 + \lambda_3(\chi^\dagger\chi)^2 + (\eta^\dagger\eta)\left[\lambda_4(\rho^\dagger\rho)\right. \\ & \left.+\lambda_5(\chi^\dagger\chi)\right] + \lambda_6(\rho^\dagger\rho)(\chi^\dagger\chi) + \lambda_7(\rho^\dagger\eta)(\eta^\dagger\rho) \\ & + \lambda_8(\chi^\dagger\eta)(\eta^\dagger\chi) + \lambda_9(\rho^\dagger\chi)(\chi^\dagger\rho) \\ & + [\lambda_{10}(\eta^\dagger\chi)(\eta^\dagger\rho) + \lambda'\epsilon^{ijk}\eta_i\rho_j\chi_k + H.c.]. \end{aligned} \tag{4}$$

This is the most general $SU(3)\otimes U(1)$ gauge invariant renormalizable Higgs potential for the three triplets.

As we said before let us define the *lepto-baryon* number $F = L + B$, which is additively conserved. As usually $B(l, \nu_l) = 0$ for any lepton l, ν_l, and $L(q) = 0$ for any quark q, $F(l) = F(\nu_l) = +1$. In order to make F a conserved quantum number in the Yukawa sector, we also assign to the scalar fields the following values $-F(\chi^-) = -F(\eta_2^+) = F(\rho^{++}) = -F(\chi^{--}) = +2$, and with all the other scalar fields carrying $F = 0$.

Notice that the F-conservation forbids the quartic term $\lambda_{10}(\eta^\dagger\chi)(\eta^\dagger\rho)$ in (4) [8].

Hence, assuming that the λ_{10} does exist we are violating explicitly the F symmetry. We must stress that if we had introduced the sextet scalar and allow F to be broken there will be additional terms involving the sextet and the triplets as well. In this case it is not possible to maintain neutrinos with calculable masses unless a fine tune is imposed [6].

The λ_{10} term has interactions like $\rho^0\chi^0\eta_1^-\eta_2^+$, $\eta^0\rho^0\eta_1^-\chi^+$, and we have mixing between η_1^- and η_2^+, etc. In fact the mass matrix in the single charged scalars sector (in the $\eta_1^-, \rho^-, \eta_2^-, \chi^-$ basis) is

$$v_\chi^2 \begin{pmatrix} eba^{-1} & e-\lambda_7 ab & \lambda_{10}b & \lambda_{10}ab \\ e-\lambda_7 ab & eab^{-1}-\lambda_7 a^2 & \lambda_{10}a & \lambda_{10}a^2 \\ \lambda_{10}b & \lambda_{10}a & eba^{-1}-\lambda_8 & eb-\lambda_8 a \\ \lambda_{10}ab & \lambda_{10}a^2 & eb-\lambda_8 a & eab-\lambda_8 a^2 \end{pmatrix}, \tag{5}$$

where $e=\lambda'/v_\chi$, $a=v_\eta/v_\chi$, $b=v_\rho/v_\chi$. Hence we have a mixing among all single charged scalars. The spectrum from (5) will be given elsewhere but here we must stress that it has two Goldstone bosons. Notice that if there is no λ_{10} term the mixing occurs between η_1^-, ρ^- and between η_2^-, χ^-.

Due to the mixing of η_1^- and η_2^- diagrams like the one showed in Fig. 1 exist and they are finite. Yukawa couplings are those in Eq. (2) but in terms of the primed basis. The mass insertion means an m factor and it is the same for ν'_μ and ν'_τ. Hence, both neutrinos are mass degenerate with a mass $\delta m_\nu \propto \lambda_{10}m(v_\rho v_\chi/m_\eta^2)\ln(m_{\eta_1}^2/m_{\eta_2}^2)$ where $m=fv_\eta$, and $m_{\eta_1}^2, m_{\eta_2}^2$ are typical masses in the scalar sector and m_η^2 is the greatest of them. Hence, the neutrino mass matrix is at this stage

$$M^L = diag(0, \delta m_\nu, \delta m_\nu). \tag{6}$$

If $\lambda_{10}v_\rho v_\chi/m_\eta^2 \ll 1$ the neutrino mass could be arbitrarily small.

The electron neutrino is still massless. Higher order diagrams exist but they maintain this mass spectrum for leptons because at this stage all couplings in the model are diagonal. Although this is not the true mass spectrum of the leptonic sector, at least in charged sector, it is an interesting prediction of the minimal 3-3-1 model with λ_{10} term and with no sextet Higgs scalar.

In order to get the "right" mass spectrum for leptons we can choice: i) add the sextet, or ii) add at least one right-handed neutrino. In the former case neutrinos can remain massless in perturbation theory. As we are interested in massive neutrinos we will consider two modifications of the latter case.

In the first case we introduce three right-handed neutrinos. So now we have another Yukawa interaction

$$\mathcal{L}'_{l\eta} = -\sum_{a,b=e,\mu,\tau} h_{ab}\overline{\psi_{La}}\nu'_{Rb}\eta - \frac{1}{2}M_{ab}^R\overline{(\nu'_{Ra})^c}\nu'_{Rb} + H.c. \tag{7}$$

where h_{ab} is an arbitrary matrix and M_{ab}^R a symmetric matrix (Majorana mass) for the right-handed neutrinos.

With (7) the neutrinos gain an arbitrary mass. Taking into account all mass contributions the 6×6 mass matrix for the neutrinos reads

$$M_\nu = \begin{pmatrix} M^L & D \\ D^T & M^R \end{pmatrix} \tag{8}$$

where M^L is the Majorana mass arisen from radiative corrections (6) and $D=h_{ab}v_\eta$ (M^R) the Dirac (Majorana) mass coming from (7).

The second possibility is the following. We can also introduce just a single right-handed neutrino [10]. In this case we have a mass term contribution like

$$\mathcal{L}_\nu = -\sum_{a=e,\mu,\tau} h_a\bar{\nu}'_{La}\nu'_R - \frac{1}{2}M\overline{(\nu'_R)^c}\nu'_R + H.c., \tag{9}$$

where $h_a = v_\eta \hat{h}_a$, and $\hat{h}_a$ are arbitrary dimensionless parameters. From (6) and (9) we obtain the following mass matrix

$$M'_\nu = -\frac{1}{2}\bar{N}\begin{pmatrix} 0 & 0 & 0 & a_e \\ 0 & \delta m_\nu & 0 & a_\mu \\ 0 & 0 & \delta m_\nu & a_\tau \\ a_e & a_\mu & a_\tau & M \end{pmatrix} N^c + H.c., \tag{10}$$

where $N = (\nu'_{eL}, \nu'_{\mu L}, \nu'_{\tau L}, \nu'^{c}_{R})^T$. Diagonalyzing the mass matrix in Eq. (10) we obtain four massive Majorana neutrinos. Denoting these physical states $\nu_1, \nu_2, \nu_P, \nu_F$, we obtain [11]

$$m_1 = \delta m_\nu, \quad m_2 = \mathcal{M}_1 - \frac{1}{2}Re\mathcal{M}_2 + \sqrt{3}Im\mathcal{M}_2,$$

$$m_{\nu_P} = \mathcal{M}_1 - \frac{1}{2}Re\mathcal{M}_2 - \sqrt{3}Im\mathcal{M}_2, \; m_{\nu_F} = \mathcal{M}_1 + Re\mathcal{M}_2 \tag{11}$$

where we have defined

$$\mathcal{M}_1 = \frac{1}{3}(M + m_1), \quad \mathcal{M}_2 = (H_1 + H_2)^{\frac{1}{3}} \tag{12}$$

with

$$H_1 = -\frac{1}{6}(M + m_1)(Mm_1 - A^2) - \frac{1}{2}h_e^2 m_1 + \frac{1}{27}(M + m_1)^3,$$

$$H_2 = i\frac{\sqrt{3}}{18}[M^4 m_1^2 + 2M^3 m_1 g_1 + M^2 g_2 + 2m_1 M g_3 + g_4]^{\frac{1}{2}} \tag{13}$$

$$g_1 = m_1^2 + A^2 - 2h_e^2, \quad g_2 = m_1^2(6h_e^2 - 8A^2 - m_1^2) - A^4,$$

$$g_3 = A^2 m_1^2 + 5A^4 + 3h_e^2 m_1^2 - 9h_e^2 A^2,$$

$$g_4 = -18h_e^2 A^2 m_1^2 - 4A^6 + 27h_e^4 m_1^2 - 4h_e^2 m_1^4. \tag{14}$$

where $A^2 = h_e^2 + h_\mu^2 + h_\tau^2$. In the approximation $\delta m_\nu = 0$, the two of the eigenvalues in (11) vanish and the other two become [10]

$$m_{\nu_P} = \frac{1}{2}[(M^2 + 4A^2)^{\frac{1}{2}} - M], \; m_{\nu_F} = \frac{1}{2}[(M^2 + 4A^2)^{\frac{1}{2}} + M].$$

In this approximation, constraints on the masses m_{ν_P}, m_{ν_F} and the mixing angle $\sin^2\alpha \approx m_{\nu_P}/(m_{\nu_P} + m_{\nu_F})$ coming from the measured Z^0 invisible width were considered in Ref. [11].

In both cases considered above there are contributions to the charged lepton masses showed in Fig. 2. Now the mass insertions are the neutrinos masses. Notice that the Yukawa couplings are not diagonal anymore since there is an additional mixing due to (9). In the second possibility considered above the diagram in Fig. 2 is a 2-loop one since the mass insertions include the 1-loop diagram in Fig. 1. In the primed basis the mass matrix of the charged leptons reads

$$\begin{pmatrix} \delta_{ee} & \delta_{e\mu} & \delta_{e\tau} \\ \delta_{e\mu} & m + \delta_{\mu\mu} & \delta_{\mu\tau} \\ \delta_{e\tau} & \delta_{\mu\tau} & m + \delta_{\tau\mu} \end{pmatrix}, \tag{15}$$

where δ_{ab} are the contributions coming from diagrams like the one in Fig. 2. From (15) we see that even if the non-diagonal entries are neglected, the "electron" has a mass arisen only via radiative correction and that these corrections also break the mass degeneration of the "muon" and "tau" leptons. Of course, we are assuming that the lightest leptons correspond to the first generation, this is not apriori obvious, since it depends on the mixing matrix. However as this matrix is not known at present our assumption is a reasonable one.

Notice that δ_{ab}, $a, b = e, \mu, \tau$ in (15) which come from contributions like that in Fig. 2 are proportional to the neutrino masses. However if $m \gg \delta_{ab}$ the eigenvalues of (15) will be greater than the neutrino masses. Anyway, since there is mixing in the lepton sector the current upper bound on neutrino masses are no longer valid [12].

We have shown that if one does not introduce the sextet S it is possible to give the right mass to all leptons in the model. In the case of three right-handed neutrino only the mass of the lightest charged lepton arises radiatively. More interesting, with only one right-handed neutrino the electron and two neutrinos get mass through radiatve corrections. In a supersimmetric version of the model it is also possible to give mass to the charged leptons without introducing the sextet [13].

Hence, in 3-3-1 models the smallness of the masses of the electron and its neutrino partner arise naturally.

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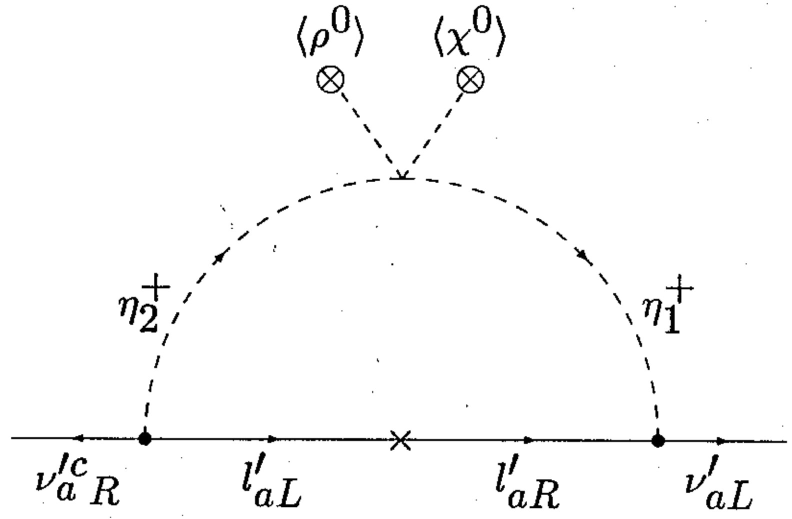
FIGURES

FIG. 1. One-loop contributions to the neutrinos mass matrix. This contribution is possible due to the mixing between η_1^- and η_2^-. The mass insertion on the the internal lepton line indicates the tree level charged leptons mass matrix elements $(0, m, m)$.



FIGURES

FIG. 2. Diagram inducing a mass contribution to the charged leptons. It breaks the degeneracy between μ' and τ' and give mass to the electron. The Γ_{ab} coefficients (not showed in the text) are the nondiagonal Yukawa interaction (2) in the primed basis when Eq. (7) or (9) is taken into account.