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DEFORMED GAUSSIAN ORTHOGONAL ENSEMBLE ANALYSIS OF THE INTERACTING BOSON MODEL

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Abstract

A Deformed Gaussian Orthogonal Ensemble (DGOE) which interpolates between the Gaussian Orthogonal Ensemble and a Poissonian Ensemble is constructed. This new ensemble is then applied to the analysis of the chaotic properties of the low lying collective states of nuclei described by the Interacting Boson Model (IBM). This model undergoes a transition order-chaos-order from the $SU(3)$ limit to the $O(6)$ limit. Our analysis shows that the quantum fluctuations of the IBM Hamiltonian, both of the spectrum and the eigenvectors, follow the expected behaviour predicted by the DGOE when one goes from one limit to the other.

It is widely assumed that Random Matrix Theories (RMT) [1] provide a basis to study quantum chaotic systems. In particular, it is expected that fluctuation properties of fully chaotic systems with time reversal symmetry follow the Gaussian Orthogonal Ensemble (GOE) whereas non-chaotic ones follow the Poissonian Ensemble. Some physical systems however may exhibit statistics intermediate between these two limits, as recent investigations have shown in the case of the excitation spectra and intensities of deformed [2][3] and spherical [4] nuclei. The analysis that has been performed on these systems have a more or less empirical character in the sense that they are not based directly on an ensemble of RMT. It is therefore important to test the reliability of the RMT predictions in these intermediate situations.

Recently, two of us have constructed a Deformed Gaussian Orthogonal Ensemble (DGOE) using the maximum entropy principle applied to generic random matrices subjected to appropriate constraints[5]. The constraints imposed were such that the ensemble obtained goes from a pure GOE to a combination of two GOE's. This corresponds to the case of $SU(2)$ symmetry breaking, when a quantum number which can have only two values, e.g. isospin, is partially conserved[6]. To deal with the more general case of the GOE-Poisson transition an extension of the DGOE of Ref.[5] is re-

quired. In this paper we provide this extended DGOE and apply it to the analysis of the transition chaos-order exhibited by the Interacting Boson Model (IBM) Hamiltonian.

We start using projection operators $P_i = \mid i >< i \mid$, $Q_i = 1 - P_i$, where $\mid i >$, with $i = 1, 2, ..., N$, are N abstract basis vectors, to split a generic Hamiltonian operator H in its diagonal and off-diagonal parts, H_0 and H_1. Namely,

$$H_0 = \sum_{i=1}^{N} P_i H P_i \ , \tag{1}$$

$$H_1 = \sum_{i=1}^{N} P_i H Q_i \tag{2}$$

where H_0 and H_1 satisfy the identity $H = H_0 + H_1$.

Following the discussion of Ref.[5] we add to the usual GOE constraints

$$< Tr H^2 >= \int dH\, P(H) Tr\, H^2 = \mu \tag{3}$$

$$< 1 >= \int dH\, P(H) = 1 \tag{4}$$

the additional one

$$< Tr\, H_1^2 >= \sum_{i=1}^{N} \int dH\, P(H) Tr(P_i\, H\, Q_i\, H\, P_i) = \nu. \tag{5}$$

Maximizing then the entropy subjected to the above conditions we get, after normalization, the probability distribution:

$$P(H) = P_{GOE}(H) exp[-\beta \sum_i Tr(P_i H Q_i H P_i)](1 + \frac{\beta}{\alpha})^{\frac{N}{4}(N-1)} \tag{6}$$

where α and β are Lagrange multipliers and

$$P_{GOE}(H) = 2^{-N/2} \left(\frac{\pi}{2\alpha}\right)^{-\frac{1}{4}N(N-1)} exp(-\alpha\, Tr\, H^2) \tag{7}$$

is the GOE distribution.

It is clear from Eq. 6 that when $\beta \to 0$ we recover the GOE distribution while with the increase of β the system becomes less chaotic; in the limit $\beta \to \infty$ we recover the Poissonian Ensemble .

Using an idea suggested by Dyson [7], Alhassid and Levine[8] discussed an intermediate ensemble as a solution of a non-equilibrium problem defined by an appropriate stochastic Langevin equation. The non-integrability is characterized by a parameter ϵ which is the ratio between the variances of the non-diagonal to the diagonal elements of H. We easily find the relation $\epsilon = \frac{1}{\sqrt{1+\beta/\alpha}}$ showing that ϵ goes from zero to unity as β varies from infinity to zero.

At this point, we remark that by using the projectors P_i and Q_i we can alternatively decompose the Hamiltonian H as

$$H = H_0 + \epsilon \sum_{i=1}^{N} P_i H_G Q_i \tag{8}$$

where $H_G = H(\beta = 0)$ showing that the problem may be reformulated in such a way that the parameter ϵ appears as a coupling constant. This relation makes a connection

of our ensemble to the one discussed recently by Lenz and Haake[9].

We present now the numerical results. In Ref.[8] a new parameter $\tilde{\epsilon}$ was introduced such that the GOE limit is expected to be approached when $\tilde{\epsilon} \to 1$. This parameter may be expressed as

$$\tilde{\epsilon} = \sqrt{\frac{2\nu}{\mu - \nu}} = \epsilon\sqrt{N-1} \tag{9}$$

and for N=2 $\tilde{\epsilon} = \epsilon$ while for large N, $\tilde{\epsilon} \sim \sqrt{N}\epsilon$. To test this idea we have plotted in Fig.1 the parameter ω of the Brody distribution that fits the level spacing as a function of the size N of the matrix, keeping ϵ or $\tilde{\epsilon}$ fixed. We fixed the value of $\tilde{\epsilon}$ to be that at N=50, the lowest dimension in the calculation. It is clear that ω saturates quite rapidly when $\tilde{\epsilon}$ is kept constant.

In Fig.2, we have considered matrices of size N=200 and calculated the level spacing P(s), the spectral rigidity $\Delta_3(L)$ and the distribution P($\ln y$), where y is the square of the component normalized with respect to its average ($y = c^2/ < c^2 >$), for several values of the parameter $\tilde{\epsilon}$. We see that indeed we are very close to GOE when $\tilde{\epsilon} = 1$. The level spacing distribution exhibits the universality law which says that the level repulsion only disappears in the Poisson limit, $\tilde{\epsilon} = 0$ or $\beta = \infty$. We remark also that our P(s) are practically identical to those obtained by Lenz and Haake with N = 500 matrices. With respect to P($\ln y$), the distributions get broader as $\tilde{\epsilon}$ decreases but we cannot fit them with only one P_ν. Actually, as in the previous case of the DGOE for two GOE's, an excellent fit is obtained if we use instead two P_ν's. This behavior may be understood as a consequence of the fact that in the transition from the GOE to the

Poissonian case, the components of the eigenvectors go from a situation in which they are equally distributed among the basis states, to a limit situation in which only one intensity, say y_i, is different from zero. Near this limit, the average of the components becomes equal to $< y > \simeq 1 - y_i$ and this explains why the distribution shift to the left in Fig.2 . This fact, together with the splitting of the intensities in two sets fitted by two different P_ν's, seem to be universal features of the eigenvector statistics in the transition from a chaotic to a regular regime.

The Interacting Boson Model (IBM) has been successfully applied to the phenomenological description of low-lying collective states of atomic nuclei[10]. In Ref.[2] the fluctuation properties of such states were analyzed in the framework of IBM.We turn now to a detailed analysis of the statistical properties of the IBM Hamiltonian, using the above intermediate ensemble. The IBM Hamiltonian may be written as

$$H = \eta\, n_d - (1-\eta) Q^\chi \cdot Q^\chi \tag{10}$$

where n_d is the number of d bosons and Q^χ is the quadrupole operator

$$n_d = d^\dagger \cdot \tilde{d} \tag{11}$$

$$Q^\chi = (d^\dagger \times s + s^\dagger \times \tilde{d})^{(2)} + \chi (d^\dagger \times \tilde{d})^{(2)}, \tag{12}$$

with the relation $\tilde{d}_\mu = (-)^\mu d_{-\mu}$. The six bosons s and d_μ span a six-dimensional Hilbert space which has $U(6)$ symmetry. The IBM Hamiltonian has three dynamical

symmetry limits which describe vibrational nuclei for $\eta = 1$, rotational nuclei for $\eta = 0$ and $\chi = -\sqrt{7}/2$, and, finally, γ-unstable nuclei for $\eta = 0$ and $\chi = 0$. It was shown recently [2] that among these three limiting cases, there is a region where the Hamiltonian becomes chaotic. In particular, taking $\eta = 0$ and various values of χ in the range $-\sqrt{7}/2 \leq \chi \leq 0$, namely going from rotational nuclei to γ-unstable ones we encounter the following results shown in Fig.3. With regards to the level spacing and the Δ_3 we are just reproducing the results of Ref.[2]. However, instead of presenting the BE(2) distributions as was done in Ref.[2], we show in the first column of the figure the statistics of the components. This is necessary in order to compare with the DGOE statistics presented in Fig.2. Besides, the components were calculated with respect to the basis of the nearest regular cases.

The IBM spacings and Δ_3 distributions show a clear Poisson-GOE-Poisson transition as one varies χ from -1.25 to -0.1. The two transitions considered separately have a behavior which compares well with the description given by ensemble. However, insofar as the component distributions are concerned, there is a marked difference, which seems to arise from what we may call the intermediate structure that modulates the components distribution. This intermediate structure was refered to as secular variations in Ref.[2]. Therefore in order to make a sensible comparison we still have to extract from the components this secular variation. This can be performed by introducing a local average defined by an appropriate moving window. Following the procedure of Ref.[2] we have used a Gaussian window and have taken the labels of the components as variables. Explicitly, we have used

$$\bar{y}(a,b) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\left(c_i^j\right)^2 \exp\left(-\frac{(a-i)^2}{2\gamma^2}\right)\exp\left(-\frac{(b-j)^2}{2\gamma^2}\right)}{\sum_{i=1}^{N}\sum_{j=1}^{N}\exp\left(-\frac{(a-i)^2}{2\gamma^2}\right)\exp\left(-\frac{(b-j)^2}{2\gamma^2}\right)} \tag{13}$$

and the normalized components are defined as $y_a^b = \frac{\left(c_a^b\right)^2}{\bar{y}(a,b)}$. In fig.4 we show the resulting distributions. In the column on the left the we have the distributions for the DGOE in the case of 200×200 matrices whereas the other two present the IBM results. The remaining two columns show the transitions order-chaos starting near the rotational limit ($\chi = -1.25$, bottom) and starting near the $\gamma-$unstable limit ($\chi = -0.10$, bottom), respectively. We can see that the distributions for the two transitions of the IBM Hamiltonian follow the pattern described by the ensemble.

In conclusion we have shown in this paper that the extended version of the DGOE of Ref.[5], can indeed describe well the statistical behavior of a realistic nuclear model such as the IBM. Similar observations can be made with regard to the study of high-spin states [3], and the analysis of spherical nuclei made in Ref.[4] We have reasons to believe that any physical system is said to exhibit order-chaos transition when its statistics follows rigorously that of a extended DGOE.

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Figure Captions:

Fig.1-The Brody parameter, ω, of the adjusted spacing distributions, vs. the size of the matrix. The diamonds correspond to $\epsilon = 0.01$, whereas the crosses to $\epsilon = \tilde{\epsilon}/\sqrt{N-1} = 0.07/\sqrt{N-1}$.

Fig.2-The distributions, P($\ln y$), P(s), and Δ_3 of the DGOE for N=200 and various values of ϵ. See text for details.

Fig.3-Same as Fig.2, for J=6^+ IBM states. See text and Ref.[2] for details.

Fig.4-The components distributions after normalization, the lhs column presents the DGOE 's and the other two the IBM ones . See text for details.

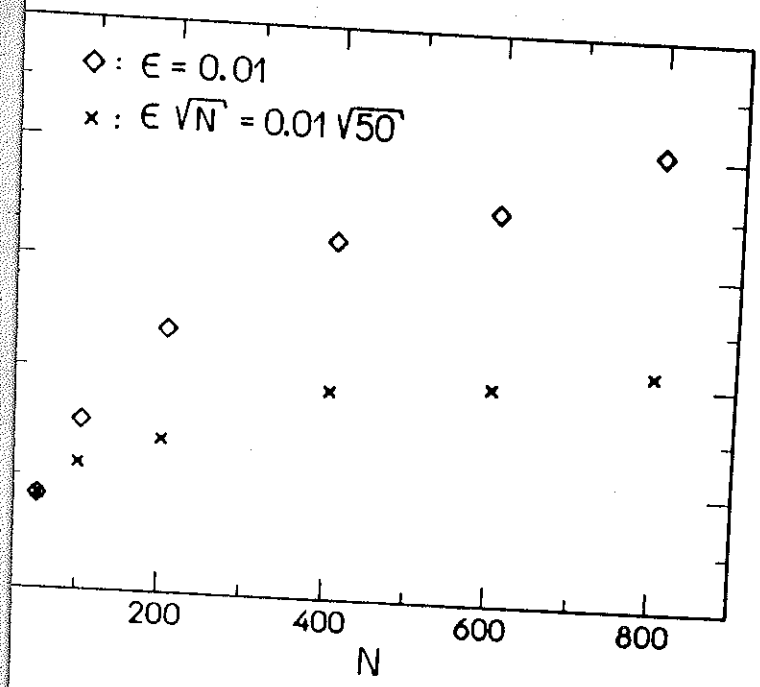
◇ : ϵ = 0.01
× : ϵ √N = 0.01 √50
200
400
600
800
N

Fig 1

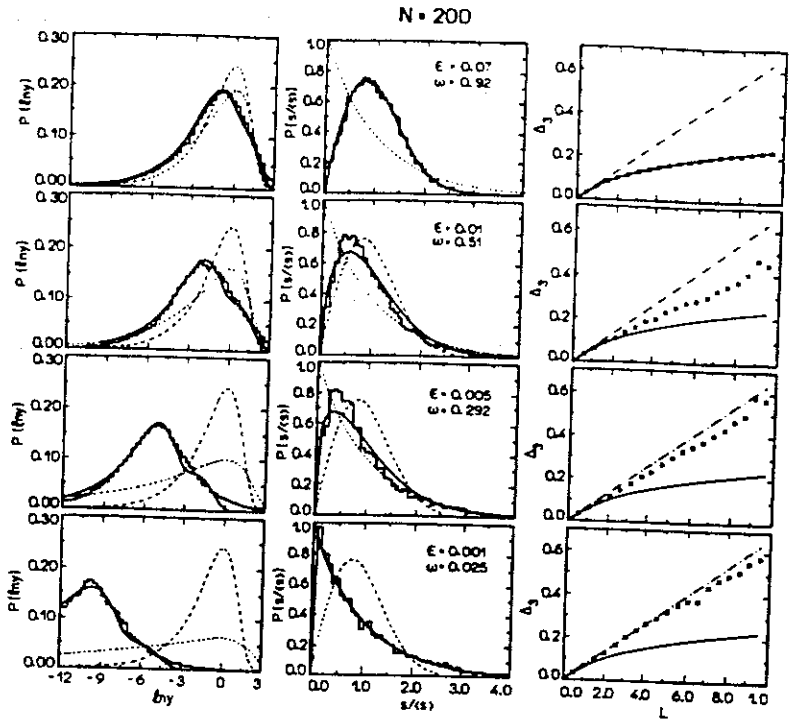
N = 200
ϵ = 0.07
ω = 0.92
ϵ = 0.01
ω = 0.51
ϵ = 0.005
ω = 0.292
ϵ = 0.001
ω = 0.025
P (fny)
P (s/⟨s⟩)
Δ3
fny
s/⟨s⟩
L

fig 2

J = 6

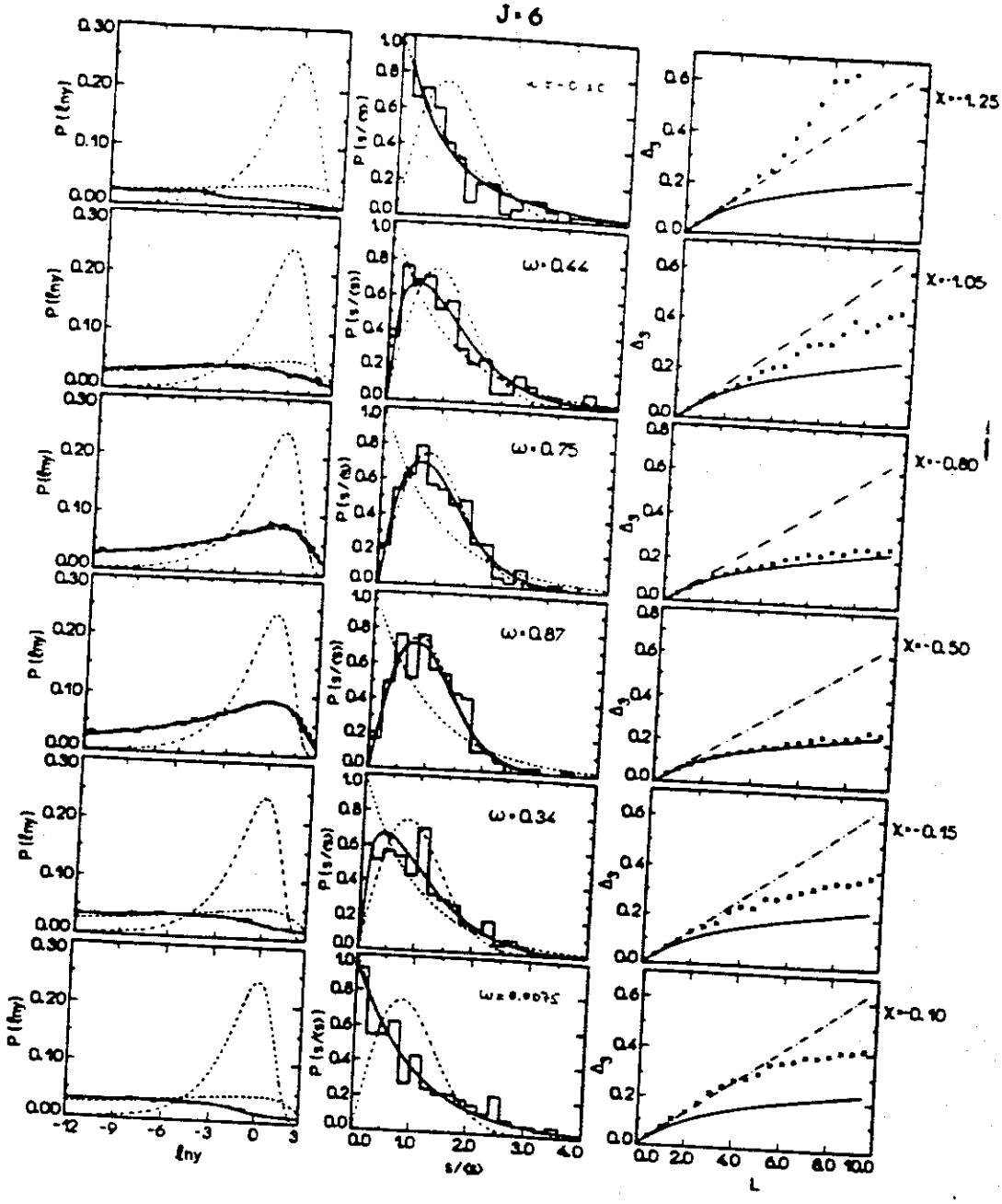
P(ℓny)
ℓny
P(s/ω)
s/ω
Δ3
L
ω = 0.44
ω = 0.75
ω = 0.87
ω = 0.34
X = -1.25
X = -1.05
X = -0.80
X = -0.50
X = -0.15
X = -0.10

fig. 3

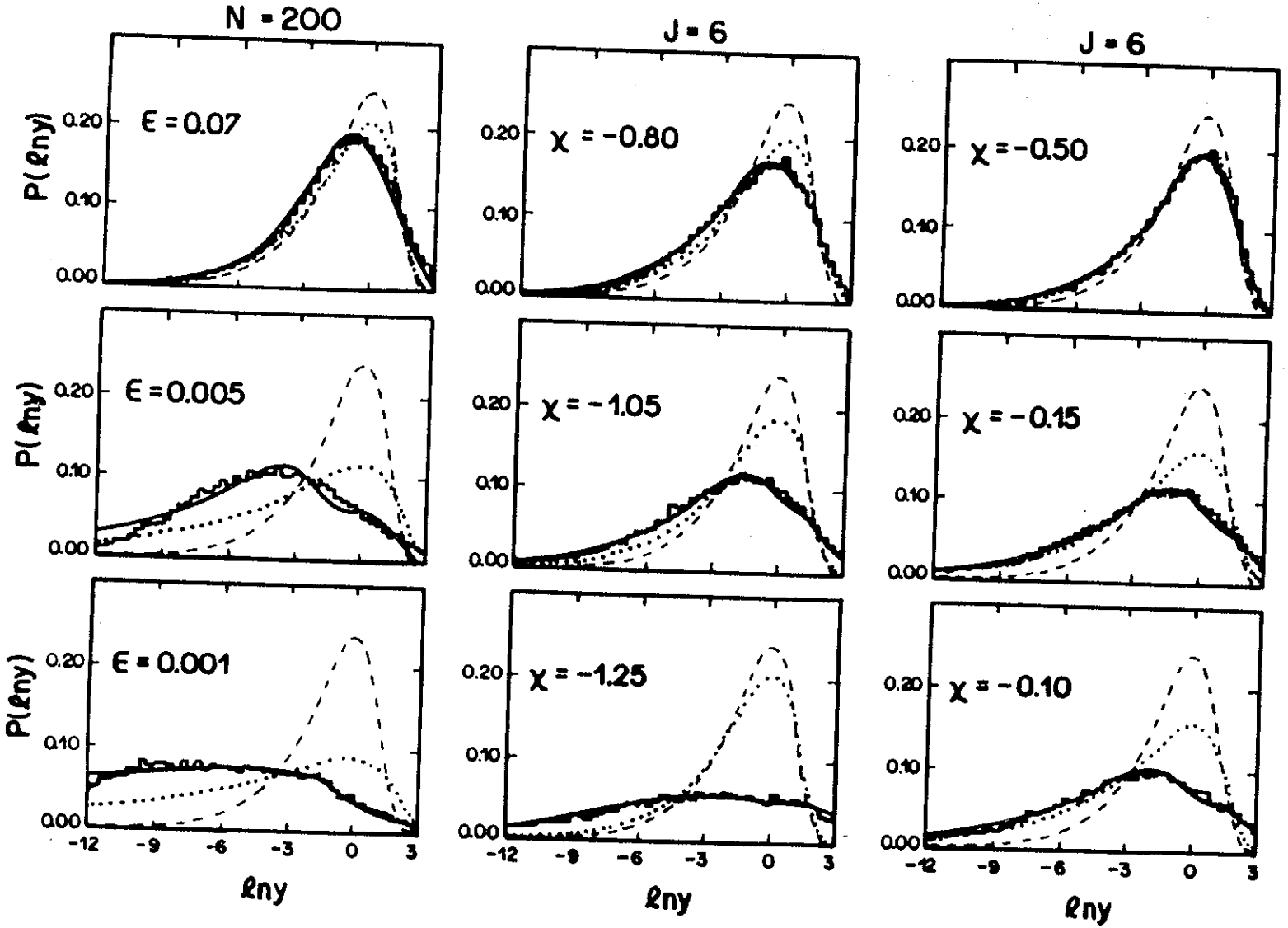
N = 200
J = 6
J = 6
ε = 0.07
χ = −0.80
χ = −0.50
ε = 0.005
χ = −1.05
χ = −0.15
ε = 0.001
χ = −1.25
χ = −0.10
P(ℓny)
0.20
0.10
0.00
−12
−9
−6
−3
0
3
ℓny

fig. 4