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EFFECT OF THE DIPOLE ZERO POINT MOTION ON THE ELASTIC ELECTRON SCATTERING AND ON DIPOLE SUM RULES

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ABSTRACT

We investigate the effect of ground state correlations due to the zero point energy of dipole oscillations on the elastic electron scattering and on dipole sum rules. We describe the ground state correlations in the framework of the generator coordinate method where the choice of the generator coordinate states is guided by the Goldhaber-Teller model of the giant dipole resonance.

In this framework we show that the effect of the ground state correlation is only to change the amplitude of the relative motion of the protons as a whole against the neutrons as a whole. This effect can make a large change in the inverse energy weighted photoabsorption cross section, as suggested by experiment, with only a small change in the form factor for elastic electron scattering.

Since our model can be considered as a restricted RPA, the same effect should appear qualitatively in the more sophisticated calculations.

I - INTRODUCTION

In a very interesting paper Dellafiore and Brink [1] investigate the effect of the residual neutron-proton interaction on the moments of the photonuclear cross section and on the elastic scattering form factor.

To give an outline of the work of reference 1 consider the p^{th} moment of the photonuclear cross section [1]

$$\sigma_p = 4\,\pi^2\,\frac{e^2}{\hbar c}\sum_{n\neq 0} E_{n0}^{p+1}\,|\langle\psi_n|\hat{D}|\psi_0\rangle|^2 \tag{1}$$

where E_{n0} is the nuclear excitation energy and $\hat{D}$ is the dipole moment operator

$$\hat{D} = \sum_{i=1}^{A}\left(\hat{x}_3\,(i) - \hat{R}_3\right)\,\frac{(1+\hat{\tau}_3\,(i))}{2} \doteq \frac{NZ}{A}\,\hat{r}_3 \tag{2}$$

In eq (2), $\hat{\vec{r}}$ is the relative coordinate between the center of mass of the protons and the center of mass of the neutrons

$$\begin{aligned}
\hat{\vec{r}} &= \hat{\vec{R}}_Z - \hat{\vec{R}}_N \\
\hat{\vec{R}}_Z &= \frac{1}{Z}\sum_{i=1}^{A}\hat{\vec{x}}\,(i)\,\frac{(1+\hat{\tau}_3\,(i))}{2} \\
\hat{\vec{R}}_N &= \frac{1}{N}\sum_{i=1}^{A}\hat{\vec{x}}\,(i)\,\frac{(1-\hat{\tau}_3\,(i))}{2} \quad ,
\end{aligned} \tag{3}$$

$\hat{\vec{R}}$ is the operator of the center of mass coordinate

$$\hat{\vec{R}} = \frac{1}{A}\sum_i \hat{\vec{x}}\,(i)$$

and $\hat{\tau}_3\,(i) = 2\hat{t}_3\,(i)$ is the z-component of the isotopic spin operator.

As is well-known σ_0 and σ_{-1} are given by

$$\begin{aligned}
\sigma_0 &= 4\,\pi^2\,\frac{e^2}{\hbar c}\,\frac{1}{2}\,\langle\psi_o|[\hat{D},[\hat{H},\hat{D}]]|\psi_0\rangle \\
&= 4\,\pi^2\,\frac{e^2}{\hbar c}\,S_1
\end{aligned} \tag{4}$$

$$\begin{aligned}
\sigma_{-1} &= 4\,\pi^2\,\frac{e^2}{\hbar c}\,\langle\psi_0|\hat{D}^2|\psi_0\rangle \\
&= 4\,\pi^2\,\frac{e^2}{\hbar c}\,S_0
\end{aligned} \tag{5}$$

In eqs (4) and (5), S_1 and S_0 are the energy weighted (EWSR) and non-energy weighted (NEWSR) sum rules respectively.

S_1 can be evaluated in a model independent way, apart from the uncertainty due to the charge-exchange forces

$$\sigma_0 = \sigma_0^{cl}\,(1+k) \quad , \tag{6}$$

where σ_0^{cl} is the classical sum rule (Thomas-Reiche-Kuhn value)

$$\sigma_0^{cl} = 4\,\pi^2\,\frac{e^2}{\hbar c}\,\frac{\hbar^2}{2\mu}\,\left(\frac{NZ}{A}\right)^2 \quad , \tag{7}$$

μ is the reduced mass of the proton-neutron system

$$\mu = \frac{NZ}{A}m$$

and k is the enhancement factor

$$k = \frac{2\mu}{\hbar^2}\,\left(\frac{A}{NZ}\right)^2\,\frac{1}{2}\,\langle\psi_0|\left[\hat{D},[\hat{V},\hat{D}]\right]|\psi_0\rangle \tag{8}$$

In equation (5) S_0 can be related to the proton and neutron root mean square radius plus two-body correlations which are known to have a large effect [1]. In the case that we approximate the ground state of the nucleus by the intrinsic state of a (non-spurious) Slater determinant of harmonic oscillator wave functions, which is a very good approximation for light double closed-shell nuclei, σ_{-1} becomes equal to

$$\sigma_{-1}^{HO} = \frac{\sigma_0^{cl}}{\hbar\,\omega_0} \tag{9}$$

where

$$\hbar\,\omega_0 = \frac{\hbar^2}{m a_0^2} \tag{10}$$

and a_0 is the oscillator size parameter which is determined by the elastic electron scattering form factor.

However the experimental data is not well reproduced by eq (9), roughly for medium-heavy nuclei [1,2]

$$\sigma_{-1}^{HO} \simeq 2\,\sigma_{-1}^{\exp} \quad .$$

Instead the experimental data is much better reproduced if in eq (9) we replace $\hbar\omega_0$ by $\hbar\omega_R$, where $\hbar\omega_R$ is the energy of the dipole resonance [1,2].

In order to reconcile the elastic electron scattering and the photoabsorption data, Dellafiore and Brink [1] points out that if $|\phi_0^{HO}\rangle$ is a Slater determinant of harmonic oscillator wave functions it is the ground state of the harmonic oscillator hamiltonian

$$\hat{H} = \sum_{i=1}^{A} \frac{\hat{\vec{p}}(i)^2}{2m} + \frac{1}{2} m\,\omega_0^2\,\hat{\vec{x}}(i)^2 \tag{11}$$

which can be written as [3]

$$\hat{H} = \hat{H}_{CM} + \hat{H}_r + \hat{H}_{ZN}$$

where $\hat{H}_{CM}$ is the hamiltonian of the center of mass motion

$$\hat{H}_{CM} = \frac{\hat{\vec{P}}^2}{2\,Am} + \frac{1}{2} m\,A\,\omega_0^2\,\hat{\vec{R}}^2 \quad , \tag{12}$$

where $\hat{\vec{R}}$ and $\hat{\vec{P}}$ are the coordinate and momentum of the center of mass, $\hat{H}_r$ is the hamiltonian of the relative motion of the protons as a whole against the neutrons as a whole

$$\hat{H}_r = \frac{\hat{\vec{p}}^2}{2\mu} + \frac{1}{2}\mu\,\omega_0^2\,\hat{\vec{r}}^2 \tag{13}$$

where $\hat{\vec{p}}$ is the momentum conjugate to $\hat{\vec{r}}$,

$$\hat{\vec{p}} = \frac{N}{A}\,\hat{\vec{P}}_Z - \frac{Z}{A}\,\hat{\vec{P}}_N \tag{14}$$

$$\begin{aligned} \hat{\vec{P}}_Z &= \sum_{i=1}^{A} \hat{\vec{p}}(i)\,\frac{(1+\hat{\tau}_3(i))}{2} \\ \hat{\vec{P}}_N &= \sum_{i=1}^{A} \hat{\vec{p}}(i)\,\frac{(1-\hat{\tau}_3(i))}{2} \end{aligned}$$

and $\hat{H}_{ZN}$ depends only on the proton-proton and neutron-neutron intrinsic degrees of freedom.

These considerations imply that the ground state of the hamiltonian eq (11) can be written as

$$\begin{aligned} |\phi_o^{HO}\rangle &= |\phi_0^{CM}\rangle|\phi_0^{int}\rangle \\ |\phi_o^{int}\rangle &= |\phi_0^{r}\rangle|\phi_0^{ZN}\rangle \quad , \end{aligned} \tag{15}$$

where $|\phi_0^r\rangle$ and $|\phi_0^{ZN}\rangle$ are the ground state wave functions of $\hat{H}_r$ and $\hat{H}_{ZN}$ respectively. In this case σ_{-1} becomes

$$\begin{aligned} \sigma_{-1} &= 4\pi^2\,\frac{e^2}{\hbar c}\,\langle\phi_0^r|\hat{D}^2|\phi_0^r\rangle \\ &= 4\,\pi^2\,\frac{e^2}{\hbar c}\,\frac{1}{2}\left(\frac{NZ}{A}\right)^2 b_0^2 \\ &= \frac{\sigma_0^{cl}}{\hbar\omega_0} \quad , \end{aligned} \tag{16}$$

where b_0 is the size parameter of the harmonic oscillator hamiltonian of the relative motion of the protons and the neutrons, eq (13)

$$b_0 = \sqrt{\frac{\hbar}{\mu\,\omega_0}} \tag{17}$$

Eq (16) shows that σ_{-1} depends only on $|\phi_o^r\rangle$ the wave function of the relative motion of protons and neutrons.

On the other hand, the elastic electron scattering form factor is

$$\begin{aligned} F(q) &= \frac{1}{Z}\,\langle\phi_0^{\rm int}|\sum_{i=1}^{A} e^{-i\vec{q}.\left(\hat{\vec{r}}(i)-\hat{\vec{R}}\right)}\,\frac{\left(1+\hat{\tau}_3^{(i)}\right)}{2}\,|\phi_0^{\rm int}\rangle \\ &= F_Z(q)\,C(q) \end{aligned} \tag{18}$$

where

$$
\begin{aligned}
F_Z(q) &= \frac{1}{Z}\left\langle \phi_0^{ZN} | \sum_{i=1}^{A} e^{-i\vec{q}\cdot(\hat{\vec{r}}(i)-\hat{\vec{R}})} \frac{(1+\hat{\tau}_3(i))}{2} |\phi_0^{ZN}\right\rangle \\
C(q) &= \langle \phi_0^r | \, e^{-i\frac{N}{A}\vec{q}\cdot\hat{\vec{r}}} | \phi_0^r \rangle \qquad (19)\\
&= e^{-\frac{1}{4}\left(\frac{N}{A}\right)^2 q^2 b_0^2}
\end{aligned}
$$

The elastic electron scattering form factor depends essentially on $F_Z(q)$ since $C(q)$ has a smooth dependence on q. Since $F_Z(q)$ depends only on the distribution of the protons relative to the center of mass of the protons, it depends only on $|\phi_0^{ZN}\rangle$. On the other hand, σ_{-1} depends only on the wave function of the relative motion of the protons against the neutrons, $|\phi_0^r\rangle$. In order to describe the photoabsorption data without affecting the elastic electron scattering data Dellafiore and Brink suggest that the effect of the residual neutron-proton interaction is to increase the dipole energy to $\hbar\omega_R$. In a collective picture, the residual neutron-proton interaction must give a smaller average separation between the proton and neutron center of mass. This has the effect of decreasing σ_{-1} without any major change on the elastic electron scattering form factor.

To show this we replace, following reference 1, $\hat{H}_r$ in eq (13) by

$$
\hat{H}_r' = \frac{\hat{\vec{p}}^2}{2\mu} + \frac{1}{2}\mu\,\omega_r^2\,\hat{\vec{r}}^2. \qquad (20)
$$

leaving $\hat{H}_{CM}$ and $\hat{H}_{ZN}$ unchanged. This replacement only changes the wave function of the relative motion of the protons as a whole against the neutrons as a whole and it modifies the σ_{-1} sum rule to

$$
\begin{aligned}
\sigma_{-1}(\omega_R) &= 4\pi^2 \frac{e^2}{\hbar c}\left\langle \phi_0^{r'} | \hat{D}^2 | \phi_0^{r'} \right\rangle \\
&= 4\pi^2 \frac{e^2}{\hbar c}\frac{1}{2}\left(\frac{NZ}{A}\right)^2 b_R^2 \qquad (21)\\
&= \frac{\sigma_0^{cl}}{\hbar\omega_R}
\end{aligned}
$$

where

$$
b_R = \sqrt{\frac{\hbar}{\mu\,\omega_R}}
$$

On the other hand the elastic electron scattering form factor changes to

$$
F'(q) = F_Z(q)\,C'(q) \qquad (22)
$$

where

$$
\begin{aligned}
C'(q) &= \langle \phi_0^{r'} | e^{-1\frac{N}{A}\vec{q}\cdot\hat{\vec{r}}} | \phi_0^{r'} \rangle \\
&= e^{-\frac{1}{4}\left(\frac{N}{A}\right)^2 q^2 b_R^2} \qquad (23)
\end{aligned}
$$

The relationship between $F'(q)$ and $F(q)$ is

$$
F'(q) = F(q)\, e^{-\frac{1}{4}\frac{N^2}{A^2}q^2(b_R^2-b_0^2)} \quad , \qquad (24)
$$

which shows that the change depends only very smoothly on q [1].

Thus Dellafiore and Brink show that by an appropriate change of the amplitude of the zero point motion of the dipole mode they could explain the photoabsorption data and the elastic scattering data. This change comes from the effect of ground state correlations introduced by the residual neutron-proton interaction.

In this paper we are going to present a microscopic dynamical model for the effect suggested in reference 1. Our model is based on the generator coordinate method where the choice of generator coordinate wave functions is guided by the Goldhaber-Teller model of the giant dipole resonance which is known to be a good description of dipole oscillations especially for light nuclei [4]. In the framework of the model we are going to show that the only effect of the ground state correlations is to change the amplitude of the zero point motion of the dipole mode as suggested in reference 1.

There exists in the literature papers which use the same model of our paper to describe the properties of the dipole resonance in light double closed shell nuclei [4,5]. However since our interest is to exhibit in the clearest way possible a microscopic dynamical mechanism for the effect proposed in reference 1, our formulation and treatment of the model differ considerably from those papers.

All the above considerations have been obtained taking advantage of the properties of the harmonic oscillator hamiltonian and wave functions. However the picture of the nucleus that they imply should have a more general validity. First the microscopic approach used in the description of nuclear vibrations is the RPA. As it is well known, the generator coordinate method is equivalent to the RPA if we take as generator wave functions the family of all Slater determinants not orthogonal to the Hartree-Fock Slater determinant and make the small amplitude approximation [6]. The choice made in our paper can be seen as a restricted RPA, appropriate to the description of dipole oscillations in light double closed shell nuclei. Another important ingredient in the argument of reference 1 is also of general validity. It is the existence of a canonical transformation to proton and neutron intrinsic degrees of freedom, degrees of freedom of the relative motion between protons and neutrons and center of mass degrees of freedom.

Our paper is organized as follows: in section II we discuss the choice of the generator wave functions appropriate to the description of the Goldhaber-Teller mode and the properties of the collective subspace generated by this choice. To investigate the properties of the correlated ground state we diagonalize the hamiltonian, in the small amplitude approximation, in the collective subspace. In section III we evaluate the dipole sum rules and the elastic electron scattering form factor paying special attention to the effects of ground state correlations and in section IV we present our concluding remarks.

II - THE MICROSCOPIC GOLDHABER-TELLER MODEL

In the original paper of Goldhaber and Teller [7] they suggested that the dipole state is

a harmonic vibration of the protons as a whole relative to the neutrons as a whole. In other words it is assumed that the equilibrium spherically symmetric proton and neutron densities gets displaced rigidly in opposite directions. In what follows we will present a microscopic approach to describe dipole oscillations in the framework of the generator coordinate method where the choice of the generator wave functions is guided by the Goldhaber-Teller model (G-T model) of the giant dipole resonance.

We discuss the choice of the generator wave functions appropriate to the description of the G-T mode and the properties of the collective subspace selected by this choice. To investigate the properties of the correlated ground state we diagonalize the hamiltonian, in the small amplitude approximation, in the collective subspace.

II-1 The Generator Wave Function

In a microscopic description of the G-T mode we consider all the many-body wave functions such that the protons and the neutrons are rigidly displaced from equilibrium in opposite directions. Besides we allow the protons and the neutrons to have a net momentum is opposite directions. A family of many-body wave functions which have these properties can be parametrized as follows,

$$|\vec{\alpha},\vec{\beta}\rangle = e^{i(\vec{\beta}.\hat{\vec{r}}-\vec{\alpha}.\hat{\vec{p}})}|\phi_0\rangle \quad . \tag{25}$$

In eq (25) $\hat{\vec{r}}$ and $\hat{\vec{p}}$ are canonical operators

$$[\hat{r}_k,\hat{p}_j] = i \ \delta_{kj}$$

and they are the relative coordinate and momentum of the protons as a whole relative to the neutrons as a whole, as shown in eqs (3) and (14). In the case of a $N=Z$ nuclei they are equal to

$$\hat{\vec{r}} = \frac{2}{A}\sum_i \hat{\vec{x}}(i)\,\hat{\tau}_3(i)$$

$$\hat{\vec{p}} = \frac{1}{2}\sum_i \hat{\vec{p}}(i)\,\hat{\tau}_3(i) \tag{26}$$

$|\phi_0\rangle$ is a reference state and as was explained before it is a Slater determinant of harmonic oscillator wave functions which is the ground state of the harmonic oscillator hamiltonian. Thus, it can be written as in (15) which shows that it is the vacuum of the boson operators,

$$\hat{b}_i^+ = \frac{1}{\sqrt{2}}\left(\frac{\hat{r}_i}{b_0} - i\,\hat{p}_i\,b_0\right)$$

$$\hat{b}_i = \frac{1}{\sqrt{2}}\left(\frac{\hat{r}_i}{b_0} + i\,\hat{p}_i\,b_0\right) \tag{27}$$

$$\left[\hat{b}_i, \hat{b}_j^+\right] = \delta_{ij}$$

$$\hat{b}_i|\phi_0\rangle = 0 \tag{28}$$

To verify that the states (25) actually describe the motion of the nucleus in the G-T mode notice that since the operators $\hat{\vec{r}}$ and $\hat{\vec{p}}$ are one-body operators, the states $|\vec{\alpha},\vec{\beta}\rangle$ are, by Thouless theorem, Slater determinants . Therefore its one-body density can be easily calculated,

$$\rho_p^{(\vec{\alpha},\vec{\beta})}(\vec{x}) = \rho_{0_p}\left(\vec{x} - \frac{\vec{\alpha}}{2}\right)$$

$$\rho_n^{(\vec{\alpha},\vec{\beta})}(\vec{x}) = \rho_{0_n}\left(\vec{x} + \frac{\vec{\alpha}}{2}\right) \tag{29}$$

where $\rho_{0_p}(\vec{x})$ and $\rho_{0_n}(\vec{x})$ are the equilibrium proton and neutron one-body densities respectively. We can also easily show that

$$\langle\vec{\alpha}\,\vec{\beta}|\hat{\vec{R}}_Z|\vec{\alpha}\,\vec{\beta}\rangle = \frac{\vec{\alpha}}{2} \qquad \langle\vec{\alpha}\,\vec{\beta}|\hat{\vec{R}}_N|\vec{\alpha}\,\vec{\beta}\rangle = -\frac{\vec{\alpha}}{2}$$

$$\langle\vec{\alpha}\,\vec{\beta}|\hat{\vec{P}}_Z|\vec{\alpha}\,\vec{\beta}\rangle = \vec{\beta} \qquad \langle\vec{\alpha}\,\vec{\beta}|\hat{\vec{P}}_N|\vec{\alpha}\,\vec{\beta}\rangle = -\vec{\beta} \tag{30}$$

Eqs (29) and (30) show clearly that the states $|\vec{\alpha},\vec{\beta}\rangle$ describe the motion of the nucleus in the GT mode.

The property that $|\phi_0\rangle$ is the vacuum of $\hat{b}_i$ allow us to rewrite the states $|\vec{\alpha},\vec{\beta}\rangle$, eq (25), in a more convenient way. Using eqs (27) we have

$$|\vec{\alpha},\vec{\beta}\rangle = e^{-\frac{1}{2}\sum_k |Z_k|^2}|\vec{Z}\rangle \tag{31}$$

where we have introduced the coherent states $|\vec{Z}\rangle$, defined by [8,9]

$$|\vec{Z}\rangle = e^{\sum_{k=1}^{3} Z_k\,\hat{b}_k^+}|\phi_0\rangle \tag{32}$$

The coherent states have the following properties, which will be helpful in later developments [8]

$$\hat{b}_i|\vec{Z}\rangle = Z_i|\vec{Z}\rangle \tag{33}$$

$$\langle\vec{Z}|\vec{Z}'\rangle = e^{\sum_{k=1}^{3} Z_k^* Z_k'} \tag{34}$$

$$\int d\mu\left(\vec{Z}\right)|\vec{Z}\rangle\langle\vec{Z}|\vec{Z}'\rangle = |\vec{Z}'\rangle \tag{35}$$

where $d\mu(\vec{Z})$ is given by

$$d\mu(\vec{Z}) = \prod_i d\mu(Z_i)$$

$$d\mu(Z_i) = \frac{e^{-|Z_i|^2}}{\pi}\,d\,R_e Z_i\,d\,\mathrm{Im}\,Z_i$$

II-2 The Eigenvalue Problem in the Collective Subspace

The GHW ansatz is [9]

$$|f\rangle = \int f(\vec{Z})|\vec{Z}\rangle d_\mu(\vec{Z}) \tag{36}$$

where $f(\vec{Z})$ is an entire function of $\vec{Z}^*$ [9]. The subspace of the Hilbert space selected by the ansatz (36) is the collective subspace and a projection operator in this collective subspace is given by [9],

$$\hat{S} = \int d\mu\,(\vec{Z})|\vec{Z}\rangle\langle\vec{Z}| \tag{37}$$

That $\hat{S}$ is a projection operator can be easily seen using eqs (33–35).

We could as well use an orthornormal representation for $\hat{S}$. To do so we can write the coherent state (32) as

$$|\vec{Z}\rangle = \sum_{n_1,n_2,n_3} \frac{(Z_1)^{n_1}}{\sqrt{n_1!}} \frac{(Z_2)^{n_2}}{\sqrt{n_2!}} \frac{(Z_3)^{n_3}}{\sqrt{n_3!}} |n_1\, n_2\, n_3\rangle \tag{38}$$

where the states $|n_1\, n_2\, n_3\rangle$ are orthornormal states given by

$$\begin{aligned} |\{n\}\rangle &= |n_1\, n_2\, n_3\rangle \\ &= \frac{1}{\sqrt{n_1! n_2! n_3!}} \left(\hat{b}_1^+\right)^{n_1} \left(\hat{b}_2^+\right)^{n_2} \left(\hat{b}_3^+\right)^{n_3} |\phi_0\rangle \end{aligned} \tag{39}$$

Using eq (38), the projection operator $\hat{S}$ can be written as

$$\hat{S} = \int d\mu(\vec{Z})|\vec{Z}\rangle\langle\vec{Z}| = \sum_n |\{n\}\rangle\langle\{n\}| \tag{40}$$

Other useful representation is the coordinate representation

$$|\vec{r}\rangle = \sum_{n_1,n_2,n_3} \phi^*_{n_1}\left(\frac{r_1}{b_0}\right) \phi^*_{n_2}\left(\frac{r_2}{b_0}\right) \phi^*_{n_3}\left(\frac{r_3}{b_0}\right) |n_1\, n_2\, n_3\rangle \tag{41}$$

where ϕ_n is the harmonic oscillator wave function of order n. The states $|\vec{r}\rangle$, which are eigenstates of the operator $\hat{\vec{r}}$,

$$\hat{\vec{r}}|\vec{r}\,'\rangle = \vec{r}\,'|\vec{r}\,'\rangle \quad , \tag{42}$$

are normalized to

$$\langle\vec{r}|\vec{r}\,'\rangle = \delta\,(\vec{r} - \vec{r}\,') \tag{43}$$

and the projection operator $\hat{S}$ can be written as

$$\hat{S} = \int d^3\vec{r}\,|\vec{r}\rangle\langle\vec{r}| \quad . \tag{44}$$

The transformations between the various representations considered above can be easily obtained and the one which well be needed below is the transformation between the coherent state representation and the coordinate representation [8]

$$\begin{aligned} \langle\vec{r}|\vec{Z}\rangle &= \sum_{n_1,n_2,n_3} \frac{(Z_1)^{n_1}}{\sqrt{n_1!}}\,\phi_{n_1}\left(\frac{r_1}{b_0}\right) \frac{(Z_2)^{n_2}}{\sqrt{n_2!}}\,\phi_{n_2}\left(\frac{r_2}{b_0}\right) \frac{(Z_3)^{n_3}}{\sqrt{n_3!}}\,\phi_{n_3}\left(\frac{r_3}{b_0}\right) \\ &= A_1\,(Z_1, r_1)\; A_1\,(Z_2, r_2)\; A_1\,(Z_3, r_3) \end{aligned} \tag{45}$$

where

$$A_1(Z,r) = \left(\frac{1}{\sqrt{\pi}\,b_0}\right)^{1/2} e^{\left\{-\frac{1}{2}\left(Z^2 + \frac{r^2}{b_0^2}\right) + \sqrt{2}\,\frac{r}{b_0}\,Z\right\}} \tag{46}$$

The eigenvalue problem in the collective subspace is given by

$$\hat{S}\,\hat{H}\,\hat{S}|f_n\rangle = E_n|f_n\rangle \quad , \tag{47}$$

which using the coherent state representation, eq (36) becomes

$$\int \langle\vec{Z}|H|\vec{Z}\,'\rangle\, f_n\left(\vec{Z}\,'\right)\, d_\mu\left(\vec{Z}\,'\right) = E_n\, f_n(\vec{Z}) \tag{48}$$

In the small amplitude approximation $\langle\vec{Z}|H|\vec{Z}\,'\rangle$ is given by [9]

$$\langle\vec{Z}|H|\vec{Z}\,'\rangle = \langle\vec{Z}|\vec{Z}\,'\rangle \left(E_0 + \frac{1}{2}\sum_{i=1}^{3}\left(B\left(Z_i'^2 + Z_i^{*^2}\right) + 2A\, Z_i'\, Z_i^*\right)\right) \tag{49}$$

where

$$\begin{aligned} E_0 &= \langle\phi_0|H|\phi_0\rangle \\ A &= \langle\phi_0|\hat{b}_3\, H \hat{b}_3^+|\phi_0\rangle - \langle\phi_0|H|\phi_0\rangle \\ B &= \langle\phi_0|H\,\hat{b}_3^{+2}|\phi_0\rangle = \langle\phi_0|\hat{b}_3^2\, H|\phi_0\rangle \end{aligned} \tag{50}$$

When we use Hartree-Fock wave functions, the absence of linear terms in (49) is due to its stationarity property. In our case since we use harmonic oscillator wave functions, the absence of linear terms is due to the fact that $|\phi_o\rangle$ and $\hat{H}$ are scalar under rotations and $\hat{b}_i$ and $\hat{b}_i^+$ are vector operators. These properties are also responsible for the absence of cross quadratic terms and for the independence of the coefficients of the diagonal quadratic terms with the orientation.

Using the property, eq (33)

$$\hat{b}_i|\vec{Z}\rangle = Z_i|\vec{Z}\rangle \tag{51}$$

we can rewrite the equation (49) as

$$\langle\vec{Z}|H|\vec{Z}\,'\rangle = \left\langle\vec{Z}|E_0 + \frac{1}{2}\sum_{i=1}^{3}\left(B\left(\hat{b}_i^{+2}+\hat{b}_i^2\right)+2A\,\hat{b}_i^+\hat{b}_i\right)|\vec{Z}\,'\right\rangle \tag{52}$$

Therefore in the small amplitude approximation $\hat{S}\hat{H}\hat{S}$ is equal to

$$\hat{S}\hat{H}\hat{S} = \hat{S}\left(E_0 + \frac{1}{2}\sum_{i=1}^{3}\left(B\left(b_i^{+2}+\hat{b}_i^2\right)+2A\,b_i^+b_i\right)\right)\hat{S} \tag{53}$$

The hamiltonian (53) can be diagonalized by a canonical transformation, to new boson operators [9]

$$\begin{aligned}\hat{B}_i &= X\hat{b}_i - Y\hat{b}_i^+\\ \hat{B}_i^+ &= X\hat{b}_i^+ - Y\hat{b}_i\\ X^2 &- Y^2 = 1\end{aligned} \tag{54}$$

Imposing that the transformation (59) diagonalizes (53) we have.

$$\hat{S}\hat{H}\hat{S} = \hat{S}\left(E_0 + \frac{3}{2}\epsilon - \frac{3}{2}A + \sum_i \epsilon\hat{B}_i^+\hat{B}_i\right)\hat{S}$$

where

$$\begin{aligned}\epsilon &= \sqrt{A^2-B^2}\\ X+Y &= \frac{1}{X-Y} = \left(\frac{A-B}{A+B}\right)^{1/4}\end{aligned} \tag{55}$$

and X and Y are such that $X,(X+Y)$ and $X-Y$ are greater than zero

The equation for the correlated ground state is

$$\hat{B}_i|\phi_0^c\rangle = 0 \tag{56}$$

Since $|\phi_0^c\rangle$ is in $\hat{S}$ it can be written as

$$|\phi_0^c\rangle = \int d_\mu(\vec{Z}\,)\,f_o(\vec{Z}\,)|\vec{Z}\rangle \tag{57}$$

and eq (56) leads to the following equation for $f_0(\vec{Z}\,)$

$$\begin{aligned}&f_0(\vec{Z}\,) = f_0(Z_1)\,f_0(Z_2)\,f_0(Z_3)\\ &X\,\frac{d}{d\,Z_i^*}\,f_0(Z_i) - Y\,Z_i^*\,f_0(Z_i) = 0\end{aligned} \tag{58}$$

The solution of the equation (58) is

$$f_o(Z_i) = \frac{1}{\sqrt{X}}\,e^{\frac{1}{2}\left(\frac{Y}{X}\right)Z_i^{*2}} \tag{59}$$

where the normalization of $f_0(Z_i)$ was chosen such that $|\phi_0^c\rangle$ is normalized to one.

Using equation (59) the correlated ground state $|\phi_0^c\rangle$ is

$$|\phi_0^c\rangle = \frac{1}{X^{3/2}}\int d_\mu(\vec{Z}\,)\,e^{\frac{1}{2}\frac{Y}{X}\sum_{i=1}^{3}Z_i^{*2}}|\vec{Z}\rangle \tag{60}$$

Performing the integral in the complex plane in eq (60) we have

$$|\phi_0^c\rangle = \frac{1}{X^{3/2}}\,e^{\frac{Y}{2X}\sum_k b_k^{+2}}|\phi_0\rangle \tag{61}$$

Using eqs (27) we can write the hamiltonian (53) as

$$\hat{S}\,\hat{H}\,\hat{S} = \hat{S}\left(E_0 - \frac{3}{2}A + \frac{\hat{\vec{p}}^{\,2}}{2\,M_R} + \frac{1}{2}\,M_R\omega_R^2\,\hat{\vec{r}}^{\,2}\right)\hat{S} \tag{62}$$

where the correlated mass M_R and frequency ω_R are given by

$$\begin{aligned}M_R &= \frac{\hbar^2}{(A-B)b_0^2}\\ \hbar\omega_R &= \sqrt{A^2-B^2}\end{aligned} \tag{63}$$

The hamiltonian (62), which is the hamiltonian in the collective subspace S, is analogous to the hamiltonian eq (20), suggested by Dellafiore and Brink, for the relative motion of the protons and neutrons. However there is one important difference in the fact that the effect of correlations not only changes the frequency but also the inertial mass. The change of the inertial mass comes from the presence of exchange forms. To verify this point we can rewrite the expressions for A and B, eqs (50), using the property that $|\phi_0\rangle$ is the vacuum of the boson operators $\hat{b}_i$ as

$$\begin{aligned} A &= \frac{2}{b_0^2}\langle\phi_0|\hat{r}_3\,\hat{H}\,\hat{r}_3|\phi_0\rangle - \langle\phi_0|\hat{H}|\phi_o\rangle \\ &= 2b_0^2\langle\phi_0|\hat{p}_3\,\hat{H}\,\hat{p}_3|\phi_0\rangle - \langle\phi_0|\hat{H}|\phi_0\rangle \end{aligned} \tag{64}$$

$$\begin{aligned} B &= \frac{\langle\phi_0|\hat{r}_3^2\,\hat{H} + \hat{H}\,\hat{r}_3^2|\phi_0\rangle}{b_0^2} - \langle\phi_0|\hat{H}|\phi_0\rangle \\ &= \langle\phi_0|\hat{H}|\phi_0\rangle - b_0^2\langle\phi_0|\hat{p}_3^2\,\hat{H} + \hat{H}\,\hat{p}_3^2|\phi_0\rangle \end{aligned} \tag{65}$$

Therefore M_R and ω_R, eqs (63) are equal to

$$\begin{aligned} M_R &= \frac{\hbar^2}{\langle\phi_0|\left[\hat{r}_3\left[\hat{H},\hat{r}_3\right]\right]|\phi_0\rangle} \\ \hbar\omega_R &= \left(\langle\phi_0|\left[\hat{r}_3,\left[\hat{H},\hat{r}_3\right]\right]\phi_0\rangle\langle\phi_0|\left[\hat{p}_3,\left[\hat{H},\hat{p}_3\right]\right]|\phi_0\rangle\right)^{1/2} \end{aligned} \tag{66}$$

Using the value of the EWSR given by eq (6), M_R is easily seen to be equal to

$$M_R = \frac{\mu}{1+k_0} \tag{67}$$

where k_0 is the enhancement factor, defined in eq (8) where the exact ground state $|\psi_0\rangle$ is replaced by the independent particle ground state $|\phi_0\rangle$. Eq (67) shows that the effect of the residual neutron-proton interactions is to decrease the value of the inertial mass by $(1+k_0)^{-1}$.

Since the correlated size parameter, b_R, is given by

$$b_R = \sqrt{\frac{\hbar}{M_R\,\omega_R}} \tag{68}$$

we see that its value depends on two opposite effects. One is that the residual neutron-proton interaction increases the value of ω_R with respect to ω_0 the other is that the residual neutron-proton interaction decreases the value of M_R with respect to the reduced mass, μ. Before we finish this section we would like to comment on the question of spurious center of mass motion. The state $|\phi_0\rangle$ is non-spurious that is, is given by the product of an intrinsic wave function and a wave function of the center of mass motion which is the ground state of the hamiltonian (12). Since the operators $\hat{\vec{r}}$ and $\hat{\vec{p}}$ in eq (25) do not change the center of mass wave function we see that the states in S are also non spurious. Therefore the center of mass wave function does not have any influence on the results.

III - DIPOLE SUM RULES AND THE ELASTIC ELECTRON SCATTERING FORM FACTOR

To discuss the dipole sum rules we would like first to find the collective coordinate representation of the correlated ground state.

Using the transformation property between the coherent state representation and the collective coordinate representation eqs 45 and 46 we have

$$\langle\vec{r}\,|\phi_0^c\rangle = I_1(r_1)I_1(r_2)I_1(r_3) \tag{69}$$

where

$$\begin{aligned} I_1(r_i) &= \frac{1}{\left(\sqrt{\pi}\,b_0X\right)^{1/2}}\int d_\mu(Z_i)\,e^{\frac{1}{2}\frac{Y}{X}Z_i^{*2}} \\ &\quad e^{\left\{-\frac{1}{2}\left(Z_i^2+\frac{r_i^2}{b_0^2}\right)+\sqrt{2}\,\frac{r_i}{b_0}\,Z_i\right\}} \end{aligned} \tag{70}$$

This integral can be easily evaluated following reference 8, with the result

$$I_1(r_i) = \frac{1}{\sqrt{\pi\,b_0(X+Y)}}\,e^{-\frac{1}{2}\frac{r_i^2}{b_0^2(X+Y)^2}} \tag{71}$$

Therefore

$$\langle \vec{r}\,|\phi_0^c\rangle = \phi_0\left(\frac{\vec{r}}{b_R}\right) \tag{72}$$

where

$$b_R = \sqrt{\frac{\hbar}{M_R\,\omega_R}} = b_0(X+Y) \tag{73}$$

and $\phi_0(\vec{r})$ is the ground state harmonic oscillator wave function.

The collective coordinate representation of the uncorrelated ground state is

$$\langle \vec{r}\,|\phi_0\rangle = \phi_0\left(\frac{\vec{r}}{b_o}\right) \tag{74}$$

and comparing with the expression for the correlated ground state we see that the effect of the correlations is only to change the size parameter of the oscillator wave function of the relative motion or which is the same the amplitude of the dipole zero point motion.

To evaluate the dipole sum rules we would like to show first that the NEWSR and EWSR are exhausted in the collective subspace [4], provided

$$\hat{S}|\psi_0\rangle = |\psi_0\rangle \quad . \tag{75}$$

The energy weighted sum rule is given by

$$S_1 = \frac{1}{2}\langle\psi_0|[\hat{D},[\hat{H},\hat{D}]]|\psi_0\rangle \tag{76}$$

Using the property that the ground state belongs to the collective subspace $\hat{S}$, eq (75), and that

$$[\hat{S},\hat{D}] = 0 \tag{77}$$

S_1 can be written as

$$\begin{aligned} S_1 &= \frac{1}{2}\langle\psi_0|[\hat{D},[\hat{S}\hat{H}\hat{S},\hat{D}]]|\psi_0\rangle \\ &= \sum_{n\neq 0}\left(E_n^{(S)}-E_0^{(S)}\right)|\langle\psi_0^{(S)}|\hat{D}|\psi_n^{(S)}\rangle|^2 \end{aligned} \tag{78}$$

where the superscript$^{(S)}$ means that the energies and the states are given by the diagonalization of the hamiltonian $\hat{H}$ in the collective subspace, $\hat{S}$.

In the same way using equations (75) and (77) we can show

$$\begin{aligned} S_0 &= \langle\psi_0|\hat{D}^2|\psi_0\rangle = \langle\psi_0|\hat{D}\,\hat{S}\,\hat{D}|\psi_0\rangle \\ &= \sum_{n\neq 0}|\langle\psi_0^{(S)}|\hat{D}|\psi_n^{(S)}\rangle|^2 \end{aligned} \tag{79}$$

Now we are ready to evaluate the dipole sum rules.

The EWSR is given by

$$\begin{aligned} S_1 &= \frac{1}{2}\langle\phi_0^c|[\hat{D},[\hat{H},\hat{D}]]|\phi_0^c\rangle \\ &= \frac{1}{2}\langle\phi_0^c|[\hat{D},[\hat{S}\hat{H}\hat{S},\hat{D}]]|\phi_0^c\rangle \\ &= \frac{1}{2}\left(\frac{A}{4}\right)^2\langle\phi_0^c|\left[\hat{r}_3,\left[\hat{S}\hat{H}\hat{S},\hat{r}_3\right]\right]|\phi_0^c\rangle \end{aligned} \tag{80}$$

Using the expression (62) for the hamiltonian in the collective subspace we obtain

$$S_1 = \left(\frac{A}{4}\right)^2\frac{\hbar^2}{2M_R} = \left(\frac{A}{4}\right)^2\frac{\hbar^2}{2\mu}(1+k_0) \tag{81}$$

Using eq (4) we have

$$\sigma_0 = \sigma_0^{cl}(1+k_0) \tag{82}$$

The value of S_1 given by equation (81) is identical to the value that we obtain if we use in eq (80) the uncorrelated ground state $|\phi_o\rangle$ instead of the correlated one $|\phi_0^c\rangle$.

In the case of the NEWSR we have

$$
\begin{aligned}
S_0 &= \langle \phi_o^c | \hat{D}^2 | \phi_0^c \rangle \\
&= \left(\frac{A}{4}\right)^2 \langle \phi_0^c | \hat{r}_3^2 | \phi_0^c \rangle \\
&= \frac{1}{2}\left(\frac{A}{4}\right)^2 b_R^2 \\
&= \frac{S_1}{\hbar \omega_R}
\end{aligned} \tag{83}
$$

The above result gives for σ_{-1} the value

$$
\sigma_{-1} = \frac{\sigma_0}{\hbar \omega_R} = \frac{\sigma_0^{cl}(1+k_0)}{\hbar \omega_R} \tag{84}
$$

which differs from the uncorrelated value given by eq (9).

The results shown above indicates that the effect of ground state correlations is such that S_1 is unchanged whereas S_0 suffers a large change. This is in agreement to the known fact that correlations are much more important in S_0 than in S_1 [10].

To evaluate the elastic scattering form factor, eq (18), notice that eqs (15) and (60) show that

$$
|\phi_0^c\rangle = |\phi_0^{\mathrm{int};c}\rangle |\phi_0^{cm}\rangle \tag{85}
$$

where

$$
|\phi_0^{\mathrm{int};c}\rangle = |\phi_0^{r;c}\rangle |\phi_0^{ZN}\rangle \tag{86}
$$

with

$$
|\phi_0^{r;c}\rangle = \frac{1}{X^{3/2}} \int d_\mu(\vec{Z}\,)\, e^{\frac{1}{2}\frac{Y}{X}\sum_{i=1}^{3} Z_i^{*2}}\, e^{\sum_k Z_k \hat{b}_k^{+}} |\phi_0^r\rangle \tag{87}
$$

Therefore we see that, as suggested in ref 1, the correlation changes only the wave function of the proton-neutron relative motion. As shown in eq (72) this change amounts only to the replacement of the uncorrelated size parameter, b_0, by the correlated one, b_R eq (73).

Thus the elastic scattering form factor is given by an expression identical to the one suggested by Brink and Dellafiore,

$$
F(q) = F_Z(q)\, C'(q)
$$

where

$$
\begin{aligned}
C'(q) &= \langle \phi_0^{r;c} | e^{-i \frac{N}{A} \vec{q}.\hat{\vec{r}}} | \phi_0^{r;c} \rangle \\
&= e^{-\frac{1}{4}\frac{N}{A} q^2 b_R^2}
\end{aligned}
$$

Our discussion up to know have shown that in the framework of our model the ground state correlations is a change of scale in the wave function of the relative motion of the protons and the neutrons. This change of scale is accomplished by the action of the dilatation operator.

To verify this point we should notice that the canonical transformation, eq (54), is a unitary transformation

$$
\hat{B}_i = e^{-i \ln(X+Y)\hat{S}_c}\, \hat{b}_i\, e^{i \ln(X+Y)\hat{S}_c} \tag{88}
$$

whose generator is the dilatation operator $\hat{S}_c$

$$
\begin{aligned}
\hat{S}_c &= \frac{1}{2}\left(\hat{\vec{r}}.\hat{\vec{p}} + \hat{\vec{p}}.\hat{\vec{r}}\right) \\
&= \frac{i}{2}\sum_{k=1}^{3}\left(\hat{b}_k^{+2} - \hat{b}_k^2\right)
\end{aligned} \tag{89}
$$

Using eq (56) for the correlated ground state and the unitary transformation (88) we see that the relationship between the correlated and uncorrelated ground state is

$$
\begin{aligned}
|\phi_0^c\rangle &= e^{-i \ln(X+Y)\hat{S}_c} |\phi_0\rangle \\
&= e^{\frac{1}{2}\ln(X+Y)\sum_{k=1}^{3}\left(\hat{b}_k^{+2} - \hat{b}_k^2\right)} |\phi_0\rangle
\end{aligned} \tag{90}
$$

Eq (90) is easily seen to be equal to eq (61). The easiest way is to recognize that $\frac{1}{2}\hat{b}_k^{+2}, \frac{1}{2}\hat{b}_k^2$ and $\frac{1}{2}\left(\hat{b}_k^{+}\hat{b}_k + \hat{b}_k\hat{b}_k^{+}\right)$ are the generators of the group $Sp(1,R)$ and that $|\phi_0\rangle$ is a lowest weight state and use the desintangle formula for this group [11].

NUMERICAL RESULTS

In our calculations we used the Skyrme force SIII [4]. In Table I we present the general properties of the dipole resonance in ^{16}O and ^{40}Ca. In Table II we present the value of σ_{-1} calculated according to our model (2nd column) compared to the experimental value [12] (third column). From those data we see that the effect of correlations goes in the right direction. Actually it overshoots the experimental value by about 10%.

In parentheses are the values of the evaluation of σ_{-1} according to the Hartree-Fock independent particle model (first column) and the RPA (2nd column) [12]. Those values are very similar to the ones calculated in our paper. This indicate that the harmonic oscillator wave functions are a very good approximation for the Hartree-Fock wave functions and that the effect of correlations is basically the one described in our paper.

CONCLUDING REMARKS

In our paper we have presented a dynamical mechanism for the effect suggested in reference 1.

In the framework of the GCM we have shown that the effect of the ground state correlations is only to change the amplitude of the zero point motion of the protons relative to the neutrons. This effect makes a large change in σ_{-1}, with only a small change in the elastic scattering form factor, as suggested by experiment. This change of amplitude amounts to the replacement on the wave function of the relative motion of the uncorrelated size parameter b_0, by the correlated one b_R. Furthermore we have shown that the net effect of the correlations is due to two opposite effects: one is that the residual neutron-proton interaction increases the value of the energy of the dipole resonance with respect to the independent particle value, the other is that it decreases the value of the dipole mass parameter

with respect to the reduced mass. Both the theoretical framework and the numerical results indicate that this simple mechanism should be the main effect in the more sophisticated calculations.

Although the subject addressed in this paper is not new, its interest has been renewed by the new generation of $(e, e'p)$ data in light nuclei [13]. The $(e, e'p)$ experiments on ^{12}C and ^{16}O have produced new data on GDR longitudinal and transverse form factors [13]. The simplicity and qualitative success of the model encourage its use to investigate the effect of correlations on the longitudinal and transverse form factor. This has already been done in an approach slightly different from ours [17]. Also we can assess the importance of two-body currents (exchange currents) in the calculation of the transverse form factor. An indication of its importance comes from the enhancement factor and from the new generation of medium energy (γ, N) data [14,15].

Before we finish I would like to point out that our model states have good angular momentum, in particular the correlated ground state, eq (90). This comes from the property that $\hat{S}$ is an eigenspace of the angular momentum operator, that is,

$$\left[\hat{J}_k, \hat{S}\right] = 0 \qquad k = 1, 2, 3$$

However since $\hat{b}_k^+$ is the 10 component of a isovector, it can be easily seen that $\hat{S}$ is not an eigenspace of the isospin operator

$$[\hat{T}_k, \hat{S}] \neq 0 \qquad k = 1, 2$$

As a consequence $|\phi_0^c\rangle$ does not have a good isospin quantum number. This violation of isospin has a purely kinematical origin, that is, comes from the ansatz eq (15) and not from the dynamics. To restore this broken-symmetry one can use projection techniques [16].

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REFERENCES

[1] A. Dellafiore and D.M. Brink, *Nucl. Phys.* **A286** (1977), 474
[2] B.L. Berman and S.C. Fultz, *Rev. Mod. Phys.* **47** (1975), 713
[3] D.M. Brink, *Nucl. Phys* **4** (1957), 215
[4] H. Flocard and D. Vautherin, *Nucl. Phys.* **A264** (1976), 197
[5] E.J.V. de Passos and F.F. de Souza Cruz, *Phys. Rev.* **C24** (1981), 690
[6] B. Jancovici and D.H. Schiff, *Nucl. Phys.* **58** (1964), 678
[7] M. Goldhaber and E. Teller, *Phys. Rev.* **74** (1948), 1046
[8] V. Bargmam, *Comm. on Pure and Appl. Math.* **14** (1961), 187
[9] D.M. Brink and A. Weiguny, *Nucl. Phys.* **A120** (1968), 59
[10] O. Bohigas, A.M. Lane, J. Martorell, *Phys. Rep.* **51** (1979), 267
[11] A.M. Perelomov, *Soviet Phys. Usp* **20** (1977), 703
[12] O. Bohigas, Supplement of the *Progress of Theoretical Physics* **74 & 75** (1983), 380
[13] University of New Hampshire Nuclear Physics Group Report (1991), unpublished (N. Zimmermann et al.)
[14] J. Ryckebusch, K. Heyde, L. Machenil, D. Ryckbosch, M. Vanderhaeghen and M. Waroquier, *Phys. Rev.* **C46** (1992), 829
[15] A.M. Saruis, *Phys. Rep.* **235** (1993), 57
[16] P. Ring and P. Schuck, *The Nuclear Many-Body Problem*, Springer-Verlag, 1980
[17] F.F. de Souza Cruz, M.M.W. de Moraes, J.R. Marinelli e M. Barbi, to be published

TABLE CAPTIONS

Table I. General properties of the dipole resonance. In our calculations we used the Skyrme force SIII. The numbers in parenthesis are the experimental values. See text for details.

Table II. Values of σ_{-1}. First column is the value according to the independent particle model with harmonic oscillator wave functions. Second column is the value calculated in our model and the third column is the experimental value (ref. 12). The values in parenthesis are the Hartree-Fock (first column) and RPA (second column) values (ref. 12).

TABLE I

	a_0 (fm)	$\sqrt{\langle r^2\rangle_c}$ (fm)	k_0	$\hbar\omega_R$ (MeV)	M_R/μ
^{16}O	1.76	2.71	0.33	24.5	0.75
		(2.67) [a]		(22–23) [c]	
^{40}Ca	1.97	3.48	0.37	20.5	0.73
		(3.49) [b]		(19–21) [d]	

a) H.A. Bentz, *Z. Naturforsch.* **A24** (1969), 858

b) R.F. Frosch et al, *Phys. Rev.* **174** (1968), 1380

c) M.N. Thompson and J.E.E. Baglin, *Phys. Lett.* **B25** (1967), 256

d) J. Ahrens et al, *Nucl. Phys.* **A251** (1975), 479

TABLE II

	σ_{-1}^{HO} (mb)	$\sigma_{-1}^{\mathrm{cal}}$ (mb)	$\sigma_{-1}^{\mathrm{exp}}$ (mb)
^{16}O	17.9	13.0	14.5
	(18.1)	(13.3)	
^{40}Ca	55.9	40.0	45.5
	(54.7)	(34.2)	