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THREE-BODY FADDEEV CALCULATION FOR <sup>11</sup>Li WITH A SEPARABLE *p*-WAVE NEUTRON-<sup>9</sup>Li POTENTIAL

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### ABSTRACT

The two neutron separation energy and mean square radius of  $^{11}$ Li are calculated using the Faddeev formalism in a model in which the  $^{11}$ Li consists of a structureless  $^{9}$ Li core and two neutrons. The  $n^{-9}$ Li interaction is described by a separable potential which acts only on the  $p_{1/2}$  wave and is adjusted to reproduce the resonance observed in the  $n^{-9}$ Li system. For the  $n^{-n}$  interaction a separable s-wave potential that describes low energy scattering is used.

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21.60.-n

 $11.80.\mathrm{Jy}$ 

27.20.+n

#### I - INTRODUCTION

Recently many calculations have been made treating the nucleus <sup>11</sup>Li as a three-body system composed of a <sup>9</sup>Li core plus two valence neutrons [1–6]. Although neither two-body subsystem of <sup>11</sup>Li is able separately to form a bound state, nevertheless the three-body system is known to have a bound state of about 0.3 MeV [7–8]. This particular situation in which one has two nucleons interacting in a low density medium suggests the possibility of applying the minimal coupling three-body model developed earlier and which proved to be quite effective in the description of the structure of <sup>6</sup>Li, <sup>6</sup>He, <sup>18</sup>O and <sup>18</sup>F [9,15].

We shall assume that the n- $^9$ Li interaction is dominated by a  $p_{1/2}$  single particle resonance. According to the experimental data of Wilcox et al. [10] a resonance corresponding to  $^{10}$ Li occurs at  $0.80 \pm 0.25$  MeV and has a width of  $1.20 \pm 0.30$  MeV. According to more recent work [11] it is split into a  $J^{\pi}=2^+$  state situated at 0.42 MeV and a  $1^+$  state at 0.80 MeV. One can consider these resonances as the splitting, due to a spin-spin n- $^9$ Li interaction, of a single particle resonance at 0.66 MeV [4], the centroid of the  $2^+$  and  $1^+$  resonances.

#### II - THE TWO BODY INTERACTION

As in Ref. [15], we shall use separable potentials. Thus the n- $^9\mathrm{Li}$  interaction in momentum representation will be written as

$$\langle \mathbf{p}_{i} | V_{i} | \mathbf{p}_{i}' \rangle = -\frac{\Lambda}{2m} \, v \left( p_{i} \right) \, v \left( p_{i}' \right) \, \sum_{\mu} \, \langle \hat{\mathbf{p}}_{i} | y_{1 \frac{1}{2} \mu} \rangle \, \langle y_{1 \frac{1}{2} \mu} | \hat{\mathbf{p}}_{i}' \rangle \quad , \qquad (i = 1, 2) \quad , \tag{1}$$

where m is the mass of neutron and,  $\mathbf{p}_i$  is the momentum of neutron i and

$$\langle \hat{\mathbf{p}}_i | y_{lj\mu} \rangle = \sum_{m,m} \left( l \, m_l \, \frac{1}{2} \, m_s | j \, \mu \right) \, Y_l^{m_l} \, (\hat{\mathbf{p}}_i) \left| \frac{1}{2} \, m_s \right\rangle \quad . \tag{2}$$

For simplicity in the three-body kinematics we consider the mass of the core to be infinite.

The form factor v(Q) was chosen to be

$$v(Q) = Q \exp(-\beta^2 Q^2/2) \tag{3}$$

The parameter  $\beta$  was fixed as follows. Since the spin-orbit splitting of the single particle  $1p_{1/2}$  and  $1p_{3/2}$  levels in  $^{10}$ Li is about 5 MeV [12], we assume that the  $1p_{3/2}$  level has a binding energy of 4 MeV. This single particle state can be reproduced by a separable potential of the form

$$-\frac{\Lambda_{p\frac{3}{2}}}{2m}v_{p\frac{3}{2}}(Q)v_{p\frac{3}{2}}(Q')\sum_{\mu}\langle\hat{\mathbf{Q}}|y_{1\frac{3}{2}\mu}\rangle\langle y_{1\frac{3}{2}\mu}|\hat{\mathbf{Q}}'\rangle . \tag{4}$$

Choosing the form factor  $v_{p\frac{3}{2}}(Q)$  equal to the form factor given by Eq. (3) (with the same value of  $\beta$ ) and requiring the mean square radius of the bound state generated by the above potential to be equal to the mean square radius of the 1p state of a harmonic oscillator whose frequency is given by the prescription [13]

$$\hbar\omega = 45 A^{-1/3} - 25 A^{-2/3} \text{MeV} ,$$
 (5)

appropriate to light nuclei, we obtain the value  $\beta=0.931 {\rm fm}$ . For this value of  $\beta$  we show in Fig. 1 the total  $p_{1/2}$  cross section for neutron-9Li scattering calculated for different values of the parameter  $\Lambda$  adjusted to produce resonance energies 0.66 MeV, 0.80 MeV and 0.93 MeV.

For the n-n interaction which is assumed to act only in the s-wave we use the separable potential.

$$\langle \mathbf{p}|V_{12}|\mathbf{p}'\rangle = -\frac{\Lambda_0}{m} v_0(p) v_0(p') |00\rangle\langle 00|$$
 (6)

where **p** is the relative momentum  $(\mathbf{p}_1 - \mathbf{p}_2)/2$  of the neutrons and  $|00\rangle$  is the spin wave function of the singlet state. For  $v_0(p)$  the Yamaguchi form  $[p^2 + \alpha_0^2]^{-1}$  was chosen. The value of the parameters of the potential are  $\alpha_0 = 1.13 \mathrm{fm}^{-1}$  and  $\Lambda_0 = 0.323 \mathrm{fm}^{-3}$ 

which were determined assuming the values -17fm and 2.84fm for the scattering length and effective range of the singlet s-wave n-n scattering [14].

### III - THREE BODY FORMALISM

The formal treatment of  $^{11}$ Li as a three-body system is essentially the same as that applied to  $^{18}$ O and  $^{18}$ F [15]. The Hamiltonian of the system is  $H = H_0 + V_1 + V_2 + V_{12}$ , where  $H_0$  is the kinetic energy,  $V_1$  and  $V_2$  the neutron-core interaction and  $V_{12}$  the interaction between the two neutrons. The bound state wave function  $\Psi$  may be decomposed into a sum of three terms  $\Psi = \Psi^{(1)} + \Psi^{(2)} + \Psi^{(3)}$  which are determined through the coupled homogeneous Faddeev equations

$$\Psi^{(1)} = G_0 T_1 \left( \Psi^{(2)} + \Psi^{(3)} \right) ,$$

$$\Psi^{(2)} = G_0 T_2 \left( \Psi^{(3)} + \Psi^{(1)} \right) ,$$

$$\Psi^{(3)} = G_0 T_{12} \left( \Psi^{(1)} + \Psi^{(2)} \right) .$$
(7)

Here  $G_0 = (E + i0 - H_0)^{-1}$  is the Green's function and  $T_1, T_2$  and  $T_{12}$  are the two body T-matrices corresponding to the potentials  $V_1, V_2$  and  $V_{12}$  respectively. By factoring out the angular momentum part the functions,  $\Psi^{(i)}$  may be expressed as

$$\Psi^{(1)}(\mathbf{p}_1, \mathbf{p}_2) = \frac{2m}{2mE - p_1^2 - p_2^2} v(p_1) \frac{H^{(1)}(p_2)}{p_2} y_{1\frac{1}{2}, 1\frac{1}{2}; 00}(\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2) , \qquad (8)$$

$$\Psi^{(2)}(\mathbf{p}_1, \mathbf{p}_2) = \frac{2m}{2mE - p_1^2 - p_2^2} v(p_2) \frac{H^{(2)}(p_1)}{p_1} y_{1\frac{1}{2}, 1\frac{1}{2}; 00}(\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2) , \qquad (9)$$

$$\Psi^{(3)}(\mathbf{P},\mathbf{p}) = \frac{2m}{2mE - P^2/2 - 2p^2} \sqrt{4\pi} v_0(p) \frac{H^{(3)}(P)}{P} y_{00(0)0;00}(\hat{\mathbf{P}},\hat{\mathbf{p}}) , \quad (10)$$

where  $H^{(1)}, H^{(2)}$  and  $H^{(3)}$  are the spectator functions and the angular momentum func-

tions are

$$y_{lj,l'j';JM_J}(\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2) = \sum_{mm'} (jmj'm'|JM_J) y_{ljm}(\hat{\mathbf{p}}_1) y_{l'j'm'}(\hat{\mathbf{p}}_2) , \qquad (11)$$

$$y_{\Lambda\lambda(L)S;JM_J}\left(\hat{\mathbf{P}},\hat{\mathbf{p}}\right) = \sum_{M_LM_S} \left(LM_LSM_S|JM_J\right)$$

$$\left[\sum_{M_{\Lambda}m_{\lambda}} (\Lambda M_{\Lambda} \lambda m_{\lambda} | LM_{L}) Y_{\Lambda}^{M_{\Lambda}} (\mathbf{P}) Y_{\lambda}^{m_{\lambda}} (\mathbf{p})\right] |SM_{S}\rangle \quad . \quad (12)$$

Here  $\mathbf{p}_i(i=1,2)$  are the momenta of the neutrons relative to the <sup>9</sup>Li core, **P** is the total momentum  $\mathbf{p}_1 + \mathbf{p}_2$  of the two neutrons and **p** is the relative momentum  $(\mathbf{p}_1 - \mathbf{p}_2)/2$ . In addition, symmetry requirements impose the equality  $H^{(1)}(Q) = H^2(Q)$ .

Substitution of Eqs. (8)-(10) for the functions  $\Psi^{(i)}$  into Eq. (7) leads to a system of coupled integral equations connecting the spectator functions  $H^{(1)}$  and  $H^{(3)}$ , which are solved numerically.

#### IV - RADIAL PROBABILITY DENSITY

In order to obtain the radial probability density of a single neutron we expressed the functions  $\Psi^{(1)}, \Psi^{(2)}$  and  $\Psi^{(3)}$  in the coordinate representation by making the Fourier transformations [16]

$$\Omega^{(i)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{(2\pi)^3} \iint d\mathbf{p}_1 d\mathbf{p}_2 \Psi^{(i)}(\mathbf{p}_1, \mathbf{p}_2) e^{i(\mathbf{p}_1 \cdot \mathbf{r}_1 + \mathbf{p}_2 \cdot \mathbf{r}_2)} , \quad i = 1, 2 , \quad (13)$$

$$\Omega^{(3)}(\mathbf{R}, \mathbf{r}) = \frac{1}{(2\pi)^3} \iint d\mathbf{P} d\mathbf{p} \, \Psi^{(3)}(\mathbf{P}, \mathbf{p}) \, e^{i(\mathbf{P} \cdot \mathbf{R} + \mathbf{p} \cdot \mathbf{r})} , \qquad (14)$$

where  $\mathbf{r}=\mathbf{r_1}-\mathbf{r_2}$  and  $\mathbf{R}=\left(\mathbf{r_1}+\mathbf{r_2}\right)/2$  . Making these transformations one gets

$$\Omega^{(i)}(\mathbf{r}_1, \mathbf{r}_2) = F^{(i)}(r_1, r_2) y_{1\frac{1}{\alpha}, 1\frac{1}{\alpha}; 00}(\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2) , \quad i = 1, 2 , \qquad (15)$$

$$\Omega^{(3)}(\mathbf{R}, \mathbf{r}) = F^{(3)}(R, r) \frac{1}{4\pi} |00\rangle$$
 (16)

The function  $F^{(i)}$  are given by

$$F^{(1)}(r_1, r_2) = -\frac{2}{\pi} \int_0^\infty dp_2 p_2 H^{(1)}(p_2) j_1(p_2 r_2) R^E(r_1, p_2) , \qquad (17)$$

$$F^{(2)}(r_1, r_2) = -\frac{2}{\pi} \int_0^\infty dp_1 p_1 H^{(2)}(p_1) j_1(p_1 r_1) R^E(r_2, p_1) , \qquad (18)$$

$$F^{(3)}(R,r) = \frac{2}{\pi} \int_0^\infty dP P H^{(3)}(P) j_0(PR) \sqrt{4\pi} R_0^E(P,r) , \qquad (19)$$

where

$$R^{E}(r_{1}, p_{2}) = 2m \int_{0}^{\infty} dp_{1} \frac{p_{1}^{2} v(p_{1}) j_{1}(p_{1}r_{1})}{2mE - p_{1}^{2} - p_{2}^{2}} , \qquad (20)$$

$$R_0^E(P,r) = 2m \int_0^\infty dp \, \frac{p^2 \, v_0(p) \, j_0(pr)}{2mE - P^2/2 - 2p^2} \quad . \tag{21}$$

The function  $\Omega^{(3)}$  is expanded in partial waves as

$$\Omega^{3}(\mathbf{R}, \mathbf{r}) = \Omega^{(3)}\left(\frac{\mathbf{r}_{1} + \mathbf{r}_{2}}{2}, \mathbf{r}_{1} - \mathbf{r}_{2}\right) = \sum_{l} f_{l}\left(r_{1}, r_{2}\right) P_{l}\left(\cos \theta_{12}\right) |00\rangle ,$$
(22)

which may be written as

$$\Omega^{(3)}(\mathbf{R}, \mathbf{r}) = \sum_{l=0}^{\infty} \sum_{j=|l-1/2|}^{|l+1/2|} \sqrt{j + \frac{1}{2}} (-1)^l \frac{4\pi}{2l+1} f_l(r_1, r_2) y_{lj, lj; 00} (\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2) . \tag{23}$$

Thus the term  $\Omega^{(3)}$  of the total three-body wave function  $\Omega(\mathbf{r}_1, \mathbf{r}_2)$  is cast into the same form as  $\Omega^{(1)}$  and  $\Omega^{(2)}$  in equation (15) and we may write

$$\Omega\left(\mathbf{r}_{1},\mathbf{r}_{2}\right) = \sum_{a} \Gamma_{aa;0}\left(r_{1},r_{2}\right) y_{a,a;oo}\left(\hat{\mathbf{r}}_{1},\hat{\mathbf{r}}_{2}\right) \quad , \qquad a = (l,j) \quad , \tag{24}$$

where

$$\Gamma_{1\frac{1}{2},1\frac{1}{2},0}(r_1,r_2) = -\frac{4\pi}{3} f_1(r_1,r_2) + F^{(1)}(r_1,r_2) + F^{(2)}(r_1,r_2)$$
 (25)

and

$$\Gamma_{lj,lj;0}(r_1,r_2) = (-1)^l \sqrt{j+\frac{1}{2}} \frac{4\pi}{2l+1} f_l(r_1,r_2), (l,j) \neq \left(1,\frac{1}{2}\right)$$
 (26)

The radial probability density  $W(r_1)$  for finding a valence neutron between  $r_1$  and  $r_1 + dr_1$  is given by

$$W(r_1) = \frac{r_1^2}{N^2} \int_0^\infty dr_2 \, r_2^2 \int d\hat{\mathbf{r}}_1 \, d\hat{\mathbf{r}}_2 \Omega(\mathbf{r}_1, \mathbf{r}_2)^{+} \, \Omega(\mathbf{r}_1, \mathbf{r}_2)$$
 (27)

and the mean square radius of the valence neutron is written

$$\langle r_1^2 \rangle = \int_0^\infty r_1^2 W(r_1) dr_1$$
 (28)

Using expansion (24) we write the mean square radius as a sum of contributions from different partial wave

$$\langle r_1^2 \rangle = \sum_a \langle r_1^2 \rangle_a \tag{29}$$

where

$$\langle r_1^2 \rangle_a = \mathcal{N}^{-2} \int_0^\infty dr_1 r_1^4 \int_0^\infty dr_2 r_2^2 |\Gamma_{a,a;0}(r_1, r_2)|^2$$
 (30)

The square of the normalization factor N may be written as

$$\mathcal{N}^2 = \sum_a \mathcal{N}_a^2 \quad , \tag{31}$$

where

$$\mathcal{N}_{a}^{2} = \int_{0}^{\infty} dr_{1}, r_{1}^{2} \int_{0}^{\infty} dr_{2} r_{2}^{2} |\Gamma_{a,a;0} (r_{1}, r_{2})|^{2} . \tag{32}$$

#### V - RESULTS

In Table 1 we are presenting the values of the coupling constant  $\Lambda$  of the n- $^9$ Li interaction (eq. (1)), which correspond to chosen values of the resonance energy of the n- $^9$ Li system in the neighborhood of 0.80 MeV. The value of  $\beta$  was taken as 0.931fm. The corresponding resonance widths, the  $^{11}$ Li 2n separation energies, the mean square

radii of the valence neutron and the <sup>11</sup>Li matter radii are given. The <sup>11</sup>Li radius was calculated following the expression

$$\langle r^2 \rangle_{^{11}\text{Li}} = \frac{2}{11} \left\langle r_1^2 \right\rangle + \frac{9}{11} \left\langle r^2 \right\rangle_{^{9}\text{Li}} \quad , \tag{33}$$

where  $[\langle r^2 \rangle_{^9\text{Li}}]^{1/2} = 2.32 \text{fm}$  according Ref. [7].

In the following we shall give additional results corresponding to the two neutron separation energy of 0.25 MeV, which is close to the experimental value [8]. In Figs. 2 and 3 the spectator functions  $H^{(1)}$  and  $H^{(3)}$  (Eqs. (8-10)) are plotted. The neutron probability density function  $W(r_1)$  is presented in Fig. 4. Notice that this function has an exponential tail

$$W(r_1) \sim A \exp(-br_1)$$
 ,  $b = 0.347 \text{fm}^{-1}$  . (34)

In Table 2 we give the contributions of the partial waves of the expansion of  $\Omega$  (Eq. (24)) to the mean square radius of the external neutrons and also the contribution which comes from  $\Omega^{(1)} + \Omega^{(2)}$  alone. The results of this table show that a substantial contribution to the mean square radius comes from the term  $\Omega^{(3)}$  of the wave function. We remark here that this term has the structure of dineutron coupled to the <sup>9</sup>Li core.

A question which arises is whether the contributions from the  $s_{1/2}$  and  $p_{3/2}$  partial waves violate the Pauli Principle with respect to the orbits occupied in the <sup>9</sup>Li core. We estimated this effect by calculating the overlap of the  $s_{1/2}$  and  $p_{3/2}$  partial waves in Eq. (23) with the single particle wave functions of the  $1s_{1/2}$  and  $1p_{3/2}$  orbits, for which we took harmonic oscillator wave functions. The results shows that the  $1s_{1/2}$  forbidden state contributes about 0.7% to the  $s_{1/2}$  partial wave and that forbidden  $1p_{3/2}$  state contributes only 0.2% to the  $p_{3/2}$  partial wave. Therefore the main part of the contribution of the  $s_{1/2}$  and  $p_{3/2}$  partial waves to the mean square radius of the valence neutrons arises from Pauli allowed components.

As explained in section 2, the above results were obtained by fixing the parameter  $\beta$  (somewhat arbitrarily) by utilizing the  $p_{1/2} - p_{3/2}$  spin orbit splitting and the radius of the  $1p_{3/2}$  single particle orbit. Another possibility is to fix instead, the resonance energy  $E_R$  of the n- $^9$ Li system and the  $^{11}$ Li 2n separation energy  $S_{2n}$ . In table 3 we give the results for  $E_R = 0.66$  MeV and  $E_R = 0.80$  MeV using  $S_{2n} = 0.25$  MeV. One finds that the parameter  $\beta$  does change at most by 20% from that of Table 1. Recent experimental values of the  $^{11}$ Li radius are  $(3.10 \pm 0.17)$ fm [7] and  $(3.02 \pm 0.2)$ fm [17]. We can see that by making the adjustment corresponding to Table 3 we were able to get a better radius than the value 2.77fm for  $S_{2n} = 0.25$  MeV of Table 1.

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# TABLE CAPTIONS

- 1) Calculated mean square radius of the external neutrons and of the  $^{11}$ Li nucleus for  $\beta=0.931$ fm and several values of  $\Lambda$ .  $E_R$  and  $\Gamma$  are the resonance energy and width of the n- $^9$ Li system and  $S_{2n}$  is the 2n separation energy of  $^{11}$ Li.  $E_R=0.80$  MeV corresponds to the data of Wilcox et al. [10] and  $E_R=0.66$  MeV is the centroid of the  $1^+$  and  $2^+$  resonances according to Ref. [11].
- 2)  $\mathcal{N}_{lj}^2$  and  $\langle r_{lj}^2 \rangle$  are the contributions of the (l,j) partial wave to the square of the normalization factor and to the mean square radius. (a) Only the contributions from  $\Omega^{(1)}$   $(\mathbf{r}_1, \mathbf{r}_2) + \Omega^{(2)}$   $(\mathbf{r}_1, \mathbf{r}_2)$  with the same norm  $\mathcal{N}$ .
- 3) Calculated mean square radius of the external neutrons and of the  $^{11}{\rm Li}$  nucleus by fixing the resonance energy  $E_R$  of the n- $^9{\rm Li}$  system and the  $^{11}{\rm Li}$  2n separation energy  $S_{2n}$ .

# FIGURE CAPTIONS

- 1) Total  $p_{1/2}$  cross sections for neutron <sup>9</sup>Li scattering for  $\beta=0.931 {\rm fm}$  and the resonance energies at 0.66 MeV, 0.80 MeV an 0.93 MeV.
- 2) Spectator function  $H^{(1)}$  in momentum representation corresponding to a 2n separation energy  $S_{2n}=0.25$  MeV. The values of  $\beta$  and  $\Lambda$  are respectively 0.931fm, and  $1694 {\rm fm}^3$  (see Table 1).
- 3) Same as figure 1, for spectator function  $H^{(3)}$ .
- 4) Neutron probability density function W corresponding to  $S_{2n}=0.25$  MeV,  $\beta=0.931 {\rm fm}$  and  $\Lambda=1.694 {\rm fm}$  .

Table 1

| $E_R$ | Λ                  | Г     | $S_{2n}$ | $\langle r^2 \rangle_n^{1/2}$ | $\langle r^2 \rangle_{^{11}\mathrm{Li}}^{1/2}$ |
|-------|--------------------|-------|----------|-------------------------------|--|
| (MeV) | (fm <sup>3</sup> ) | (MeV) | (MeV)    | (fm)                          | (fm)   |
| 0.55  | 1.744              | 0.32  | 0.97     | 3.48                          | 2.57   |
| 0.66  | 1.729              | 0.42  | 0.75     | 3.61                          | 2.60   |
| 0.80  | 1.710              | 0.58  | 0.47     | 3.86                          | 2.67   |
| 0.93  | 1.694              | 0.74  | 0.25     | 4.24                          | 2.77   |
| 1.05  | 1.678              | 0.90  | 0.07     | 5.02                          | 3.00   |

Table 2

| l    | j             | $\mathcal{N}_{lj}^2/\mathcal{N}^2$ | $\langle r^2  angle_{lj} ~({ m fm}^2)$ |  |  |  |  |  |  |
|------|---------------|------------------------------------|--|--|--|--|--|--|--|
| 0    | 1/2           | 0.1983                             | 6.1282                                 |  |  |  |  |  |  |
| 1    | 1/2           | 0.7585                             | 10.6049                                |  |  |  |  |  |  |
| 1    | 3/2           | 0.0267                             | 0.7186                                 |  |  |  |  |  |  |
| 2    | (3/2 + 5/2)   | 0.0103                             | 0.3117                                 |  |  |  |  |  |  |
| 3    | (5/2 + 7/2)   | 0.0035                             | 0.1231                                 |  |  |  |  |  |  |
| 4    | (7/2 + 9/2)   | 0.0015                             | 0.0576                                 |  |  |  |  |  |  |
| 5    | (9/2 + 11/2)  | 0.0007                             | 0.0316                                 |  |  |  |  |  |  |
| 6    | (11/2 + 13/2) | 0.0003                             | 0.0170                                 |  |  |  |  |  |  |
| 7    | (13/2 + 15/2) | 0.0002                             | 0.0102                                 |  |  |  |  |  |  |
| 1(a) | 1/2           | 0.5956                             | 7.7728                                 |  |  |  |  |  |  |

Table 3

| β     | $E_R$ | Λ                  | Γ     | $S_{2n}$ | $\langle r^2 \rangle_n^{1/2}$ | $\langle r^2 \rangle_{^{11}\mathrm{Li}}^{1/2}$ |
|-------|-------|--------------------|-------|----------|-------------------------------|--|
| (fm)  | (MeV) | (fm <sup>3</sup> ) | (MeV) | (MeV)    | (fm)                          | (fm)   |
| 1.040 | 0.80  | 2.345              | 0.66  | 0.25     | 4.45                          | 2.83   |
| 1.185 | 0.66  | 3.452              | 0.56  | 0.25     | 4.84                          | 2.94   |



