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Abstract

We show that chiral symmetry is responsible for large cancellations in the two-pion exchange nucleon-nucleon interaction, which are similar to those occuring in free pion-nucleon scattering.

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The two-pion exchange nucleon-nucleon potential ($\pi\pi$E-NNP) in the framework of chiral symmetry has recently attracted considerable attention, especially as far as the restricted pion-nucleon sector is concerned [1–5]. This process is closely related to the pion-nucleon scattering amplitude, as pointed out many years ago by Brown and Durso [6]. In the non-linear realization of chiral symmetry, one possibility is to write the pion-nucleon interaction Lagrangian as a sum of scalar and pseudoscalar terms, as follows:

$$\mathcal{L}_{PS+S} = \cdots - g\overline{N}\left[\sqrt{f_\pi^2 - \phi^2} + i\boldsymbol{\tau}\cdot\boldsymbol{\phi}\gamma_5\right]N + \cdots,\tag{1}$$

where g is the πN coupling constant, f_π is the pion decay constant and N and ϕ are the nucleon and pion fields, respectively.

In the case of pion-nucleon scattering, this Lagrangian yields a tree amplitude which contains poles in the s and u channels, as well as a scalar contact interaction, which is the signature of chiral symmetry. At low energies, this last term cancels a large part of the pole amplitude, giving rise to a final result which is much smaller than the individual contributions.

As far as nucleon-nucleon scattering is concerned, the two-pion exchange amplitude to order g^4 is given by five diagrams, usually named box, crossed box, triangle (twice) and bubble [5]. The first two diagrams contain only nucleon propagators and are independent of chiral symmetry, whereas the triangles and the bubble involve the scalar interaction and hence are due to the symmetry. When one considers the potential instead of the amplitude, the iterated OPEP has to be subtracted from the box diagram.

In this work we show that, as in pion-nucleon scattering, there are large cancellations among the various individual contributions to the interaction, that yield a relatively small net result, and thus prevent the perturbative explosion of the amplitude.

Using the potential in coordinate space produced recently [5] and parametrized in Ref. [7], it is possible to notice two important cancellations within the scalar-isoscalar sector of the $\pi\pi$E-NNP. The first of them happens between the triangle and bubble contributions, as shown in Fig. 1. It is worth noting that the scale of this figure is given in thousands of MeV.

The other one occurs when the remainder from the previous cancellation (S) is added to the sum of the box and crossed box diagrams (PS). In this last case, the direct inspection of the profile functions for the potential, given in Fig 2, provides just a rough estimate of the importance of the cancellation, since the iterated OPEP is not included. Therefore the second cancellation can be better studied directly in the NN scattering problem, by considering the singlet waves, where the influence of chiral symmetry is stronger.

It is well known that a full chiral calculation of the potential in momentum space requires the inclusion of undetermined counterterms in the Lagrangian, involving higher orders of the relevant momenta [1]. In configuration space, on the other hand, these counterterms become delta functions which affect only the origin and hence are effective just for waves with low orbital momentum. Therefore, in order to avoid these undetermined short range effects, we consider the behaviour of the $^1D_2$, $^1G_4$, $^1F_3$, and $^1H_5$ waves.

For each channel, we decompose the full NN potential V as

$$V = U_\pi + U_{PS} + U_S + U_C \;, \tag{2}$$

where U_π is the OPEP, U_C represents the short ranged core contributions, U_{PS} is due to the box and crossed box diagrams whereas U_S is associated with the chiral triangle and bubble interactions. Using the variable phase method, it is possible to write the phase shift for angular momentum L as [8,9]

$$\delta_L = -\frac{m}{k} \int_0^\infty dr \, V \, P_L^2 \;. \tag{3}$$

In this expression, the structure function P_L is given by

$$P_L = \hat{j}_L \cos D_L - \hat{n}_L \sin D_L \;, \tag{4}$$

where $\hat{j}_L$ and $\hat{n}_L$ are the usual Bessel and Neumann functions multiplied by their arguments and D_L is the variable phase. Using the decomposition of the potential given in Eq. 2, one writes the perturbative result

$$\delta_L = -\frac{m}{k} \int_0^\infty dr \left\{ U_\pi \hat{j}_L^2 + \left[U_\pi \left(P_L^2 - \hat{j}_l^2 \right) + U_{PS} P_l^2 + U_S P_L^2 \right] + U_C P_L^2 \right\}$$

$$\equiv \delta_L)_{\pi L} + [\delta_L)_{\pi I} + \delta_L)_{PS} + \delta_L)_S] + \delta_L)_C \;. \tag{5}$$

In this expression, the first term represents the perturbative long range OPEP (πL), the second the iterated OPEP (πI), the third the part due to the box and crossed-box diagrams (PS), the fourth the contribution from chiral symmetry (S). The last one is due to the core and vanishes for waves with $L \neq 0$. In Fig. 3 we show the partial contributions to the $^1D_2$ and $^1F_3$ phase shifts as functions of energy. There it is possible to see large cancellations of the medium range contributions, represented by the terms within square brackets in Eq. 5. In the case of the $^1F_3$ wave one notes a contribution from the iterated OPEP, the cancellation is almost complete and the total phase shift is very close to that due to the long-OPEP term. The same patterns also holds for the $^1G_4$ and $^1H_5$ waves.

These results show that chiral symmetry is responsible for large cancellations in the two-pion exchange interaction. This process is therefore similar to threshold pion-nucleon or pion-deuteron [10] scattering amplitudes, where the main role of the symmetry is to set the scale to the problem to be small.

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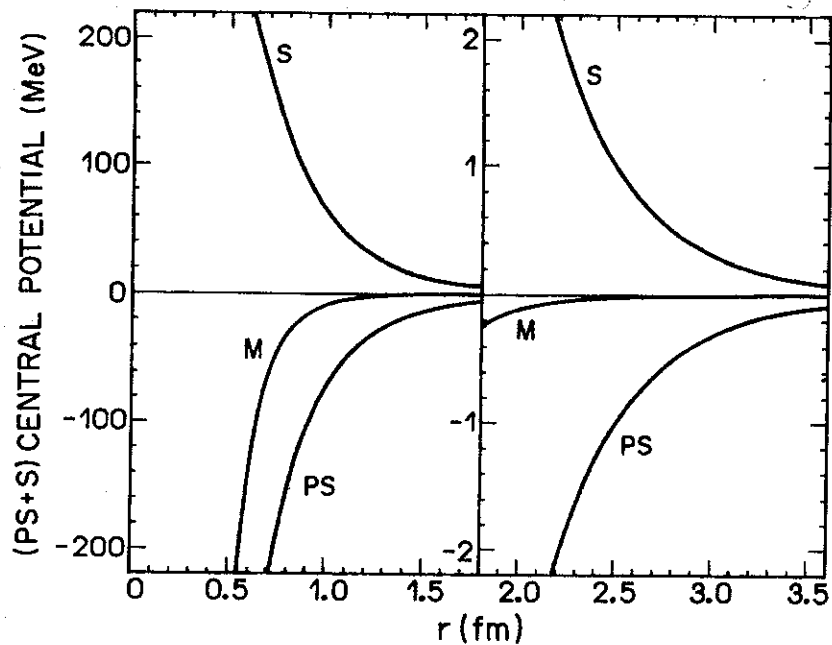
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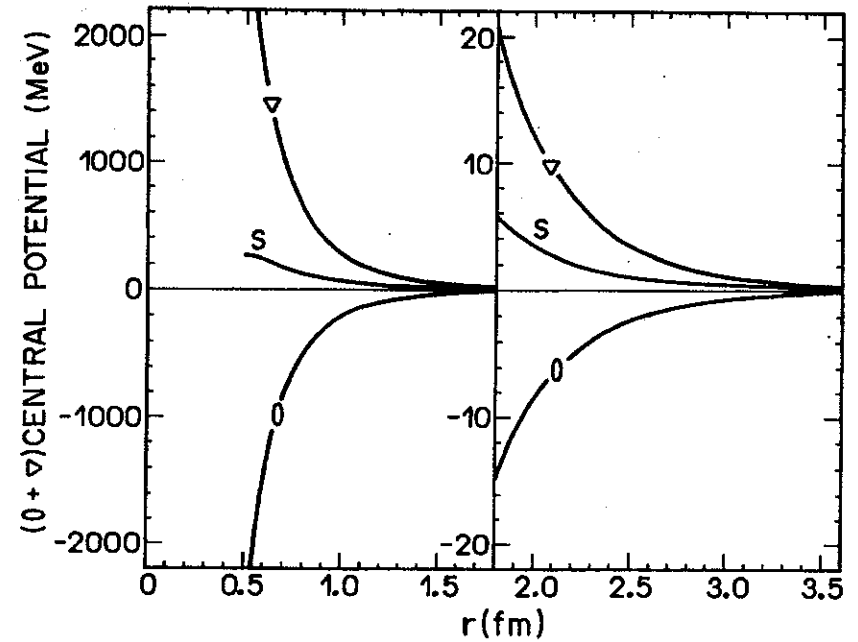
FIGURE 1
from paper "*Two-pion exchange nuclear potential - Chiral cancellations*"
J.L. Ballot, M.R. Robilotta and C.A. da Rocha

FIG. 1. Profile functions for the bubble (()) and triangle (∇) scalar-isoscalar potentials and for their chiral sum (S), showing a strong cancellation between these two contributions.

FIGURE 2
from paper *"Two-pion exchange nuclear potential - Chiral cancellations"*
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FIG. 2. Profile functions for the chiral sum (S - same as in Fig. 1) and pseudoscalar (PS)

scalar-isoscalar potentials and for their sum, representing the full medium ranged potential (M).

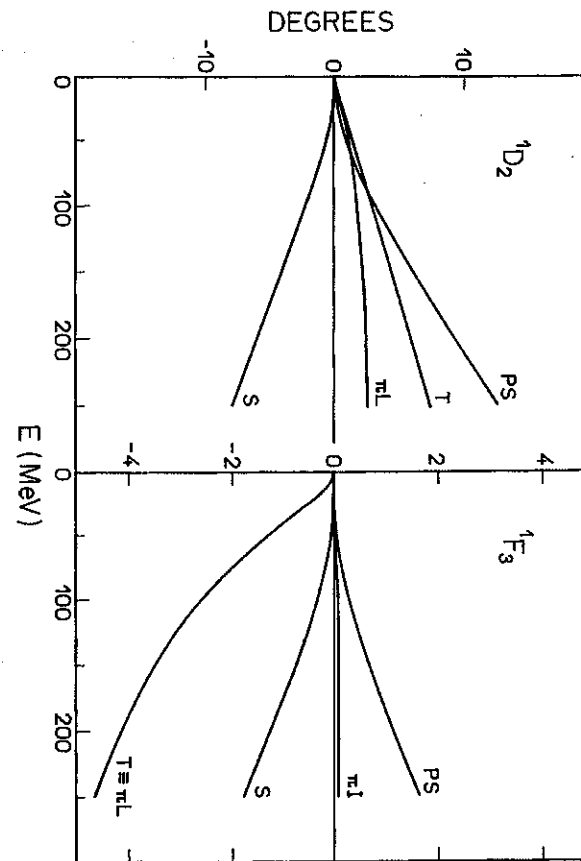One sees another strong cancellation between these two contributions.

FIG. 3. Contributions for the long-OPEP (πL), iterated OPEP (πI), pseudoscalar (PS) and

chiral (S) terms of the potential to the phase shifts for $^1D_2$ and $^1F_3$ waves. The total phase shifts

are indicated by (T).