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## STUDYING MEDIUM EFFECTS WITH THE OPTIMIZED δ EXPANSION

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# Studying medium effects with the optimized $\delta$ expansion

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## Abstract

The possibility of using the optimized $\delta$ expansion for studying medium effects on hadronic properties in quark or nuclear matter is investigated. The $\delta$ expansion is employed to study density effects with two commonly used models in hadron and nuclear physics, the Nambu–Jona-Lasinio model for the dynamical chiral symmetry breaking and the Walecka model for the equation of state of nuclear matter. The results obtained with the $\delta$ expansion are compared to those obtained with the traditional Hartree-Fock approximation. Perspectives for using the $\delta$ expansion in other field theoretic models in hadron and nuclear physics are discussed.



## I. INTRODUCTION

The study of possible modifications of hadron properties in the nuclear medium is one of the central problems of contemporary nuclear physics. In principle, these and related phenomena in nuclear physics are governed by the fundamental theory of the strong interactions, quantum chromodynamics (QCD). However, although QCD has been very successful in explaining a large class of hadronic processes at high energy and large momentum transfer, typical nuclear phenomena at lower energies cannot be derived from QCD with the theoretical tools presently available. The difficulty of using QCD for phenomena at the nuclear scale is related to the nonperturbative nature of these. Due to the asymptotic freedom property of QCD, high energy processes are calculable by perturbative techniques in the quark-gluon coupling constant. On the other hand, since there are no reliable systematic approximation schemes in field theory for performing nonperturbative calculations, the construction of models is an important aspect of low energy QCD. While there is considerable optimism that eventually one will be able to solve QCD numerically on the lattice using supercomputers, the development of analytical approximation methods are in urgent need to make contact with the wealth of data on nonperturbative phenomena, presently available, or that will be available when the new experimental facilities under construction start operating. The $\delta$ expansion [1] is an example of a method recently developed with the aim of studying nonperturbative phenomena in field theory.

The idea of the $\delta$ expansion is to perturb the original theory by the introduction of an artificial expansion parameter $\delta$, absent in the original theory. The parameter $\delta$ is introduced in such a way that it interpolates between the theory one wants to solve and another theory that one knows how to solve. The $\delta$ expansion can be formulated in two different forms, the logarithmic $\delta$ expansion [1] and the linear $\delta$ expansion [2]- [4]. In this paper we consider the linear form. Specifically, let $S$ be the action of the theory one wants to solve, and $S_0$ the action of the soluble theory. Then, the interpolating action $S(\delta)$ is defined as

$$S(\delta) = (1-\delta)S_0 + S\delta, \tag{1}$$

so that $S(0) = S_0$ and $S(1) = S$. The next step involves the evaluation of desired physical quantities as a perturbation series in powers of $\delta$, which is then set equal to 1 at the

end. A crucial aspect of the method is the recognition that $S_0$ involves arbitrary unknown (dimensionful and/or dimensionless) parameters. If one were able to solve the new theory to all orders in $\delta$, the unknown parameters would not play any role, since no physical observable would depend on them. However, since we will be able to solve the interpolated theory only to a finite order in $\delta$, there will remain a residual dependence of the results on the parameters of $S_0$. These arbitrary parameters must therefore be determined according to some criterion and in fact there are many ways in which this can be done [2]- [6]. One physically appealing way to fix the unknown parameters, which is the one adopted here, is the principle of minimal sensitivity (PMS) introduced in Ref. [5]. This principle amounts to the requirement that physical quantities should be at least locally independent of the parameters. In the original applications of the method, the unknown parameters were set to be equal to unity. The $\delta$ expansion, together with the criterion of the PMS of physical observables, is known as the *optimized $\delta$ expansion*. The convergence of the optimized $\delta$ expansion has been proved in Ref. [7].

The different forms of the $\delta$ expansion have been successfully applied to many different problems in quantum mechanics [6], particle theory [8,9], statistical physics [10] and lattice field theory [3,11]. Motivated by these successes, in this paper we investigate the possibility of employing the linear $\delta$ expansion to study medium effects in hadron and nuclear physics using typical field theoretic models. In the next section, we consider the use of the $\delta$ expansion in the study of density effects on the chiral symmetry breaking in the Nambu–Jona-Lasinio (NJL) model. The PMS criterion for the typical chiral quantities is applied following previous experience with the method. We also investigate an alternative way of fixing the parameters by applying the PMS to the energy density of the system. In Section III we consider the Walecka model and study the effective nucleon mass in nuclear matter. Both problems can be treated by obtaining the propagators of the fields involved. Traditionally, the propagators are obtained in the Hartree-Fock (HF) approximation. In the case of the NJL model this approximation amounts to neglecting the corrections to the four-point vertex. In the Walecka model, the HF approximation consists in obtaining self-consistently the nucleon propagator with bare meson propagators, and neglecting corrections to the meson-nucleon vertices. An additional approximation in the Walecka model is the



neglection of vacuum effects in the nucleon propagator. With the purpose of comparing the $\delta$ expansion method with the traditional HF approximation, we also neglect vertex corrections in both models, and neglect the vacuum in the Walecka model. Conclusions and discussions of the perspectives for future calculations appear in Section IV.

## II. NAMBU–JONA-LASINIO MODEL

In the limit of zero current quark masses, the two-flavor Lagrangian density of the Nambu–Jona-Lasinio (NJL) model [13] is given by

$$\mathcal{L}_{\rm NJL} = \bar{q}(i\partial\!\!\!/)q + G\left[(\bar{q}q)^2 - (\bar{q}\gamma_5\tau q)^2\right] , \qquad (2)$$

where the quark field operators $q = q(x)$ represent the doublet of $u$ and $d$ quarks. Since the model is non-renormalizable, one has a cutoff $\Lambda$ as an extra parameter of the model, besides $G$.

According to Eq. (1), one needs to introduce a Lagrangian density $\mathcal{L}_0$ such that

$$\mathcal{L}_{\rm NJL}(\delta) = (1-\delta)\mathcal{L}_0 + \delta\mathcal{L}_{\rm NJL} , \qquad (3)$$

where the equations of motion derived from $\mathcal{L}_0$ can be solved as exactly as possible. Since we are looking for solutions which break chiral symmetry, the natural choice for $\mathcal{L}_0$ is

$$\mathcal{L}_0 = \bar{q}(i\partial\!\!\!/ - \mu)q , \qquad (4)$$

where $\mu$ is an arbitrary mass parameter introduced for dimensional reasons. Therefore, the interpolated NJL Lagrangian density can be written as

$$\begin{aligned}\mathcal{L}_{\rm NJL}(\delta) &= (1-\delta)\left[\bar{q}(i\partial\!\!\!/ - \mu)q\right] + \delta\left\{\bar{q}(i\partial\!\!\!/)q + G\left[(\bar{q}q)^2 - (\bar{q}\gamma_5\tau q)^2\right]\right\} \\ &= \bar{q}(i\partial\!\!\!/ - \mu)q + \delta\left\{G\left[(\bar{q}q)^2 - (\bar{q}\gamma_5\tau q)^2\right] + \mu\bar{q}q\right\} . \end{aligned} \qquad (5)$$

The evaluation of physical quantities is performed using perturbation theory in the parameter $\delta$. The physical quantities of interest, whose values characterize the chiral symmetry breaking, are the constituent quark mass $M_q$, the quark condensate $< \bar{q}q >$ and the pion decay constant $f_\pi$. The quark condensate for a given flavor is given by

$$< \bar{q}q >= -i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[S(p)\right] \ , \tag{6}$$

where the trace is over spinor and color indices. The pion decay constant can be evaluated using the Pagels-Stokar [12] formula:

$$if_\pi q_\mu \delta^{ab} = g_{\pi qq} \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\gamma_\mu\gamma_5\tau^a S(q)\gamma_5 \frac{\tau^b}{2} S(q+p)\right] \ , \tag{7}$$

where the trace now is over spinor, flavor and color indices. The quark-pion coupling is obtained from the Goldberger-Treiman relation.

In order to calculate these quantities one needs the quark propagator $S(p)$, which can be obtained using Dyson's equation. Expressed in terms of the self-energy $\Sigma(p)$ the quark propagator reads

$$S^{-1}(p) = S_0^{-1}(p) - \Sigma(p), \tag{8}$$

where $S_0^{-1}(p)$ is the inverse of the quark propagator corresponding to $\mathcal{L}_0$:

$$S_0^{-1}(p) = \not{p} - \mu. \tag{9}$$

$\Sigma(p)$ is calculated as a power series in $\delta$, and $S(p)$ is then obtained by inverting Eq. (8). Since the self-energy is calculated perturbatively, this is still a perturbative scheme. It is the application of the PMS to physical quantities which furnishes the nonperturbative character to the $\delta$ expansion.

In zeroth order in $\delta$, one has

$$\Sigma^{(0)}(p) = 0 \ . \tag{10}$$

Therefore, one can invert Eq. (8) by using the well-known modified Feynman $i\epsilon$ prescription for particles in a Fermi sea [16]. The in-medium quark propagator is then given by:

$$S^0(p) = \frac{\not{p}+\mu}{p^2-\mu^2+i\epsilon} + 2\pi i \frac{\not{p}+\mu}{2E_0(p)} \delta\left(p^0 - E_0(p)\right) \theta\left(P_F - |\mathbf{p}|\right) \ , \tag{11}$$

where $E_0(p) = (\mathbf{p}^2+\mu^2)^{\frac{1}{2}}$, and $P_F$ is the Fermi momentum, which relates to the density $\rho$ via $P_F = (\pi^2\rho/2)^{1/3}$.

At this zeroth order in $\delta$, no dynamical content from the model has been used. The dynamics of the model starts to show up at order $\delta$. At first order in $\delta$, the self-energy $\Sigma(p)$ is given by:



$$\Sigma^{(1)}(p) = -\delta\mu + 2i\delta G \int \frac{d^4q}{(2\pi)^4} \left\{\mathrm{Tr}\left[S_0(q)\right] - S_0(q) - \gamma_5\tau^a \mathrm{Tr}\left[\tau^a S_0(q)\gamma_5\right] + \gamma_5\tau^a S_0(q)\tau^a\gamma_5\right\} \ , \tag{12}$$

where a sum over the isospin index $a$ is implied.

Substituting Eq. (11) into this equation, we obtain for $\Sigma^{(1)}$ the following expression:

$$\Sigma^{(1)}(p) = -\delta\mu + M_1 - \gamma_0\Sigma_0 \ , \tag{13}$$

where

$$M_1 = \delta\frac{G}{\pi^2}\mu\left(N_cN_f + \frac{1}{2}\right)\left\{\Lambda\left(\Lambda^2+\mu^2\right)^{\frac{1}{2}} - P_F\left(P_F^2+\mu^2\right)^{\frac{1}{2}} - \mu^2 \ln\left[\frac{\Lambda + (\Lambda^2+\mu^2)^{\frac{1}{2}}}{P_F + (P_F^2+\mu^2)^{\frac{1}{2}}}\right]\right\} \ , \tag{14}$$

and

$$\Sigma_0 = -4\delta G \int \frac{d^3q}{(2\pi)^3}\theta(P_F - |\mathbf{q}|). \tag{15}$$

Since the effect of $\Sigma_0$ is just to shift the chemical potential [14] one may write the constituent quark mass to $O(\delta)$ as

$$M_q = \mu - \delta\mu + M_1. \tag{16}$$

Substituting Eq. (11) into Eq. (6) and Eq. (7), one gets for the order parameter per flavor and for the pion decay constant the following lowest order expressions:

$$< \bar{q}q >= -\frac{2N_c\mu}{(2\pi)^2}\left\{\Lambda\left(\Lambda^2+\mu^2\right)^{\frac{1}{2}} - P_F\left(P_F^2+\mu^2\right)^{\frac{1}{2}} - \mu^2\ln\left[\frac{\Lambda + (\Lambda^2+\mu^2)^{\frac{1}{2}}}{P_F + (P_F^2+\mu^2)^{\frac{1}{2}}}\right]\right\} \ , \tag{17}$$

and

$$f_\pi^2 = \frac{N_cN_f\mu^2}{(2\pi)^2}\left\{\ln\left[\frac{\Lambda + (\Lambda^2+\mu^2)^{\frac{1}{2}}}{P_F + (P_F^2+\mu^2)^{\frac{1}{2}}}\right] - \left(1+\frac{\mu^2}{\Lambda^2}\right)^{-\frac{1}{2}} + \left(1+\frac{\mu^2}{P_F^2}\right)^{-\frac{1}{2}}\right\} \ , \tag{18}$$

where the lowest order Goldberger-Treiman relation, $g_{\pi qq} = \mu/f_\pi$, has been used.

The next step in the process is to fix $\mu$. In a previous work [9] this arbitrary parameter was determined by requiring $f_\pi$, which is a fundamental quantity in the study of chiral symmetry breaking, to be stationary with respect to variations in $\mu$. This is also a convenient choice since, apart from having a well known empirical value, $f_\pi$ is the only one of the studied

quantities in Ref. [9] which has a well defined stationary point for finite values of $\mu$. To fix the noncovariant cutoff $\Lambda$ one uses the empirical value $f_\pi$ at zero density. If one applies the criterion of stationarity to $f_\pi$ in the vacuum, one obtains (for $N_c = 3$ and $N_f = 2$)

$$\mu = 0.97 \times \Lambda \ . \tag{19}$$

With the input $f_\pi = 93$ MeV, one finds $\Lambda = 571$ MeV, which implies $\mu = 553$ MeV and $< \bar{q}q >= -(250 \ \text{MeV})^3$. Using $G\Lambda^2 = 2.89$ as in Ref. [9] and setting $\delta = 1$ in Eq. (16) one finds the constituent quark mass to $O(\delta)$ to be $M_q = 574$ MeV. Figure 1 shows $\mu$ as a function of the density and has been obtained by applying the PMS to $f_\pi$ for different values of $P_F$. The results obtained for $M_q$ and $- < \bar{q}q >^{1/3}$, for different values of $P_F$ are shown in Figure 2 (solid and dashed lines respectively). In Figure 3 the solid line shows the $P_F$ dependence of $f_\pi$. Contrary to what happens at finite temperature [9] we find that our result are insensitive to whether $\mu(P_F)$ (as in Figure 1) or $\mu(0) = 533$ MeV is used.

A natural question which arises at this point is the uniqueness of the of the value of $\mu$. If one were to use other physical quantities to fix $\mu$, as for example the masses of the vector mesons, it is very likely that one would obtain a different value for $\mu$. In such a case, one would have a different quark propagator for each observable. Of course this would not be catastrophic if the spread of the values of $\mu$ determined with different observables is not too large. In order to avoid such potential uncertainties, we propose to fix $\mu$ by demanding that the energy density of the system be stationary with respect to variations of $\mu$. The energy density can always be written in terms of the propagators of the theory, and then it is natural to demand stationarity of the energy with respect to the unknown parameters of the propagators. Then, all physical observables are determined from the same quark propagator.

¿From the Lagrangian density, Eq. (2), we have that the energy-momentum tensor is given by:

$$\begin{aligned} T^{\mu\nu}_{\rm NJL} &= i\bar{q}\gamma^\mu\partial^\nu q - g^{\mu\nu}\mathcal{L}_{\rm NJL} \\ &= i\bar{q}\gamma^\mu\partial^\nu q - g^{\mu\nu}\left\{\bar{q}(i\partial\!\!\!/)q + G\left[(\bar{q}q)^2 - (\bar{q}\gamma_5\tau q)^2\right]\right\} \ . \end{aligned} \tag{20}$$

Note that we have not used the equation of motion for the quark field operator. The energy density is the volume integral of the expectation value of $T^{00}$ in the many-quark state. The



expectation value of the field operators can be evaluated using the usual Wick contraction technique. This leads to

$$\begin{aligned} \mathcal{E}_{\rm NJL} &= \frac{1}{V}\int d^3x \ < T^{00} > \\ &= -i\int \frac{d^4q}{(2\pi)^4} q^0 {\rm Tr}\left[\gamma^0 S(q)\right] + i\int \frac{d^4q}{(2\pi)^4}{\rm Tr}\left[\not{q}S(q)\right] - G\left\{-\left[\int \frac{d^4q}{(2\pi)^4}{\rm Tr}\left[S(q)\right]\right]^2\right. \\ &\quad + \int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}{\rm Tr}\left[S(q)S(k)\right] + \left[\int \frac{d^4q}{(2\pi)^4}{\rm Tr}\left[\tau\gamma_5 S(q)\right]\right]^2 \\ &\quad \left. - \int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}{\rm Tr}\left[\gamma_5\tau^a S(q)\gamma_5\tau^a S(k)\right]\right\} \ . \end{aligned} \tag{21}$$

Substituting Eq. (11) into the expression above, we obtain:

$$\mathcal{E}^{(0)}_{\rm NJL} = -2N_cN_f\int_{P_F}^{\Lambda}\frac{d^3q}{(2\pi)^3}\frac{\mathbf{q}^2}{E_0(q)} - 2GN_cN_f(2N_cN_f+1)\left[\int_{P_F}^{\Lambda}\frac{d^3q}{(2\pi)^3}\frac{\mu}{E_0(q)}\right]^2 \ . \tag{22}$$

The requirement that $\mathcal{E}$ be stationary with respect to variations in $\mu$ leads to

$$\mu = 4G\left(N_cN_f + \frac{1}{2}\right)\int_{P_F}^{\Lambda}\frac{d^3q}{(2\pi)^3}\frac{\mu}{E_0(q)} \ . \tag{23}$$

This is is the familiar Hartree-Fock gap equation of the model, where $\mu$ has the interpretation of the dynamically generated mass.

Next, we consider the first order self-energy. By inverting Dyson's equation, Eq. (8), one obtains the quark propagator:

$$S^{(1)}(p) = \frac{\not{p}_1 + M_1}{p_1^2 - M_1^2 + i\epsilon} + 2\pi i\frac{\not{p}_1 + M_1}{2E_1(p)}\delta\left(p_1^0 - E_1(p)\right)\theta\left(P_F - |\mathbf{p}|\right) \ , \tag{24}$$

where

$$p_1^\mu = (p_1^0, \mathbf{p}) = (p^0 + \Sigma_0, \mathbf{p}) \ , \tag{25}$$

$$E_1(p) = \left[\mathbf{p}^2 + (M_1)^2\right]^{\frac{1}{2}} \ . \tag{26}$$

The superscript (1) in $S^{(1)}$ indicates that the propagator has been obtained with a self-energy calculated to first order in $\delta$. Note that we are not expanding the propagator in powers of $\delta$. The process of obtaining the propagator by inverting Dyson's equation with a self-energy calculated in perturbation theory is known as the chain approximation.

Substituting Eq. (24) into Eq. (21), we obtain:

$$\mathcal{E}^{(1)}_{\rm NJL} = -2N_cN_f\int_{P_F}^{\Lambda}\frac{d^3q}{(2\pi)^3}\frac{\mathbf{q}^2}{E_1(q)} - 2GN_cN_f(2N_cN_f+1)\left[\int_{P_F}^{\Lambda}\frac{d^3q}{(2\pi)^3}\frac{M_1}{E_1(q)}\right]^2 , \qquad (27)$$

where $M_1$ is given by Eq. (14) and $E_1(q)$ is defined in Eq. (26).

Application of the PMS to $\mathcal{E}^{(1)}_{\rm NJL}$,

$$\frac{d\mathcal{E}^{(1)}_{\rm NJL}}{d\mu} = \frac{d\mathcal{E}^{(1)}_{\rm NJL}}{dM_1}\frac{dM_1}{d\mu} = 0 , \qquad (28)$$

leads to

$$M_1 = 4G\left(N_cN_f+\frac{1}{2}\right)\int_{P_F}^{\Lambda}\frac{d^3q}{(2\pi)^3}\frac{M_1}{E_1(q)} . \qquad (29)$$

Again, we have obtained the familiar Hartree-Fock gap equation for the dynamically generated mass.

If one proceeds to higher orders in $\delta$, in the scheme of neglecting vertex corrections, the higher order quark propagator will always be of the form of Eq. (24), with $M_1$ replaced by another constant, say $M$, which is a function of $\mu$. However, because of the PMS condition on $\mathcal{E}$, $M$ at each order will always be given by the same value. This value is the one that satisfies the usual gap equation:

$$M = 4G\left(N_cN_f+\frac{1}{2}\right)\int_{P_F}^{\Lambda}\frac{d^3q}{(2\pi)^3}\frac{M}{E(q)} , \qquad (30)$$

where

$$E(q) = \left(\mathbf{q}^2+M^2\right)^{\frac{1}{2}} . \qquad (31)$$

Therefore, the PMS condition on the energy density is equivalent to the usual Hartree-Fock solution for the dynamically generated mass, in the approximation of neglecting vertex corrections.

This result should be compared to the one presented in Ref. [4] where, in the context of the effective potential, it was found that the $\delta$ expansion and the $1/N$ expansion are identical in the large $N$ limit.

In Figure 2 we compare the results obtained for the quark mass and the quark condensate, when the two above described ways of applying the PMS are used. We call PMS1 the results obtained by imposing $f_\pi$ to be stationary with respect to $\mu$, and PMS2 the results



obtained when the PMS is imposed to the energy density. The solid and dashed lines give, respectively, $M_q$ and $-<\bar{q}q>^{1/3}$ obtained with PMS1, and the dotted and dot-dashed lines give, respectively, $M_q$ and $-<\bar{q}q>^{1/3}$ obtained with PMS2. In both cases we used the same set of parameters: $\Lambda = 571$ MeV and $G\Lambda^2 = 2.89$.

In Figure 3 we show the results obtained for $f_\pi$ with PMS1 (solid line), PMS2 (dotted line), both with the parameters given above, and with PMS2 (dashed line) with a new set of parameters: $\Lambda = 653$ MeV and $G\Lambda^2 = 1.98$. The last set of parameters was fixed by renormalizing $f_\pi$ and $<\bar{q}q>$ at $P_F = 0$ to their experimental values and requiring $M_q$ to be roughly one-third of the nucleon mass. The curves for the quark mass and condensate obtained by PMS2 with the renormalized parameters are not shown because their behavior is analogous to $f_\pi$: they go to zero at $P_F = 1.6$ fm$^{-1}$. At $P_F = 0$ we have $M_q = 314$ MeV. These are the usual HF results and the renormalized parameters are the same as used in Ref. [14] for two flavors and a three momentum cutoff.

¿From these figures we see that the results change appreciably when different criteria are used. The main difference is related with the density dependence of the quantities: while with PMS1 the quantities smoothly approach zero at some critical density, they go to zero through a first order phase transition with PMS2 (or Hartree-Fock) [14].

## III. WALECKA MODEL

In this section we consider the Walecka model [15] for nuclear matter. The Lagrangian density of the model is given by

$$\mathcal{L}_{\rm W} = \bar{\psi}\left[\gamma_\mu(i\partial^\mu - g_\omega V^\mu) - (M - g_\sigma\phi)\right]\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_\sigma^2\phi^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2V_\mu V^\mu , \qquad (32)$$

where $\psi$ represents the nucleon field operators, $\phi$ and $V_\mu$ are respectively the field operators of the scalar and vector mesons, and $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$.

The energy-momentum tensor density corresponding to this Lagrangian density is given by:

$$T^{\mu\nu}_{\rm W} = i\bar{\psi}\gamma^\mu\partial^\nu\psi + \partial^\mu\phi\partial^\nu\phi + \partial^\nu V_\lambda F^{\lambda\mu} - g^{\mu\nu}\mathcal{L}_{\rm W} . \qquad (33)$$

Note that we have not used the nucleon equation of motion. Next, we eliminate the meson field operators in favor of the nucleon field operators. The Euler-Lagrange equations yield the meson field equations:

$$\left(\partial_\mu\partial^\mu + m_\sigma^2\right)\phi = g_\sigma\bar{\psi}\psi\,, \tag{34}$$

$$\left(\partial_\mu\partial^\mu + m_\omega^2\right)V^\nu = g_\omega\bar{\psi}\gamma^\nu\psi\,. \tag{35}$$

In obtaining the second equation above we have used baryon current conservation, which implies that $\partial_\mu V^\mu = 0$. These equations can formally be integrated as:

$$\phi(x) = -g_\sigma\int d^4y\Delta_\sigma(x-y)\bar{\psi}(y)\psi(y)\,, \tag{36}$$

$$V_\mu(x) = -g_\omega\int d^4y\Delta_\omega(x-y)\bar{\psi}(y)\gamma_\mu\psi(y)\,, \tag{37}$$

where $\Delta_i(x)$, $i=\sigma,\omega$, is given by:

$$\Delta_i(x) = \int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m_i^2+i\epsilon}e^{-iqx}\,. \tag{38}$$

In Eq. (37) above, because of baryon current conservation, we have neglected the term proportional to $p^\mu p^\nu/m_\omega^2$ in the vector meson propagator.

Using the expressions above for $\phi$ and $V^\mu$ in Eq. (33), taking the expectation value of the resulting expression in the many-nucleon state, and evaluating this with the help of Wick's contraction technique, we obtain:

$$\langle T^{\mu\nu}\rangle = -i\int\frac{d^4q}{(2\pi)^4}\left\{\mathrm{Tr}\left[\gamma^\mu q^\nu - g^{\mu\nu}(\not{q}-M)\right]S(q)\right\} + \langle T^{\mu\nu}\rangle_\sigma + \langle T^{\mu\nu}\rangle_\omega\,, \tag{39}$$

with

$$\begin{aligned}\langle T^{\mu\nu}\rangle_\sigma = {}& \frac{1}{2}\frac{g_\sigma^2}{m_\sigma^2}\left[\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}S(q)\right]^2 g^{\mu\nu} - g_\sigma^2\int\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[S(q+k)S(k)\right]\Delta_\sigma(q^2)\\ &\times\left\{\left[\frac{1}{2}(q^2-m_\sigma^2)\Delta_\sigma(q^2)-1\right]g^{\mu\nu} - q^\mu q^\nu\Delta_\sigma(q^2)\right\},\end{aligned} \tag{40}$$

and

$$\begin{aligned}\langle T^{\mu\nu}\rangle_\omega = {}& -\frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\left[\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\gamma_\mu S(q)\right]\left[\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\gamma^\mu S(q)\right]g^{\mu\nu}\\ &+ g_\omega^2\int\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma_\lambda S(q+k)\gamma^\lambda S(k)\right]\Delta_\omega(q^2)\\ &\times\left\{\left[\frac{1}{2}(q^2-m_\omega^2)\Delta_\omega(q^2)-1\right]g^{\mu\nu} - q^\mu q^\nu\Delta_\omega(q^2)\right\}\,.\end{aligned} \tag{41}$$



In the same way as in the NJL model, the nucleon propagator is obtained by inverting Dyson's equation

$$S^{-1}(p) = S_0^{-1}(p) - \Sigma(p)\,, \tag{42}$$

where $S_0$ is the propagator corresponding to $\mathcal{L}_0$ in Eq. (55) below,

$$S_0^{-1}(p) = \not{p} - M_0\,, \tag{43}$$

with the self-energy $\Sigma(p)$ calculated as a perturbation expansion in powers of $\delta$.

For infinite nuclear matter, because of the translational, rotational, parity and time reversal invariances, $\Sigma(p)$ can be generally written in terms of the ûnit matrix and the Dirac $\gamma_\mu$ matrices as follows [16]:

$$\begin{aligned}\Sigma(p) &= \Sigma^s(p) - \gamma_\mu\Sigma^\mu(p)\\ &= \Sigma^s(p^0,|\mathbf{p}|) - \gamma^0\Sigma^0(p^0,|\mathbf{p}|) + \boldsymbol{\gamma}\cdot\mathbf{p}\Sigma^v(p^0,|\mathbf{p}|)\,.\end{aligned} \tag{44}$$

Defining the following auxiliary quantities [16]:

$$\begin{aligned}M^*(p) &\equiv M_0 + \Sigma^s(p)\,,\\ \mathbf{p}^* &\equiv \mathbf{p}\left[1+\Sigma^v(p)\right]\,,\\ E^*(p) &\equiv \left[\mathbf{p}^{*2} + M^{*2}(p)\right]^{\frac{1}{2}}\,,\\ p^{*\mu} &= p^\mu + \Sigma^\mu(p) = \left[p^0+\Sigma^0(p),\mathbf{p}^*\right]\,,\end{aligned} \tag{45}$$

we can invert Eq. (42) and write the nucleon propagator in the compact form:

$$S(p) = S_F(p) + S_D(p) \tag{46}$$

$$S_F(p) = \left[\gamma^\mu p_\mu^* + M^*(p)\right]\frac{1}{p^{*\mu}p_\mu^* - M^{*2}(p) + i\epsilon} \tag{47}$$

$$S_D(p) = \left[\gamma^\mu p_\mu^* + M^*(p)\right]\frac{i\pi}{E^*(p)}\delta\left(p^0 - E(p)\right)\theta\left(P_F - |\mathbf{p}|\right)\,, \tag{48}$$

where $E(p)$ is the single-particle energy, which is the solution of the transcendental equation:

$$\begin{aligned}E(p) &= \left[E^*(p) - \Sigma^0(p)\right]_{p_0=E(p)}\\ &= \left\{\mathbf{p}^2\left[1+\Sigma^v(|\mathbf{p}|,E(p))\right]^2 + \left[M+\Sigma^s(|\mathbf{p}|,E(p))\right]^2\right\}^{\frac{1}{2}} - \Sigma^0(|\mathbf{p}|,E(p))\,.\end{aligned} \tag{49}$$

Note that we have assumed that the nucleon propagator has simple poles with unit residue. Within the approximation scheme we are working in this paper, this assumption is satisfied, as can be seen below.

Following the scheme of neglecting the Feynman part of the nucleon propagator, Eq. (47), we obtain for the energy density of nuclear matter the following expression:

$$\begin{aligned}\mathcal{E}_{\rm W} &= \frac{1}{V}\int d^3x T^{00}_{\rm W} - \text{V.E.V.} \\ &= \gamma\int_0^{P_F}\frac{d^3q}{(2\pi)^3}\frac{\mathbf{q}\cdot\mathbf{q}^* + MM^*(q)}{E^*(q)} + \mathcal{E}^D_{\rm W} + \mathcal{E}^E_{\rm W}\,,\end{aligned} \tag{50}$$

where V.E.V. means the vacuum expectation value of $T^{00}_{\rm W}$, and $\mathcal{E}^D_{\rm W}$ and $\mathcal{E}^E_{\rm W}$ are the direct and exchange contributions, given by:

$$\mathcal{E}^D_{\rm W} = \left(\frac{1}{2}-1\right)\frac{g_\sigma^2}{m_\sigma^2}\left[\gamma\int_0^{P_F}\frac{d^3q}{(2\pi)^3}\frac{M^*(q)}{E^*(q)}\right]^2 - \left(\frac{1}{2}-1\right)\frac{g_\omega^2}{m_\omega^2}\left[\gamma\int_0^{P_F}\frac{d^3q}{(2\pi)^3}\right]^2\,, \tag{51}$$

$$\begin{aligned}\mathcal{E}^E_{\rm W} = \frac{1}{2}\gamma\int_0^{P_F}\frac{d^3q}{(2\pi)^3E^*(q)}\int_0^{P_F}\frac{d^3k}{(2\pi)^3E^*(k)}\Big\{&g_\sigma^2\Delta_\sigma(q-k) \\ &\times\left[\left(\frac{1}{2}-1\right) - [E(q)-E(k)]^2\Delta_\sigma(q-k)\right]\left[q^{*\mu}k^*_\mu + M^*(q)M^*(k)\right] \\ &+2g_\omega^2\Delta_\omega(q-k)\left[\left(\frac{1}{2}-1\right) - [E(q)-E(k)]^2\Delta_\omega(q-k)\right] \\ &\times\left[q^{*\mu}k^*_\mu - 2M^*(q)M^*(k)\right]\Big\}\,.\end{aligned} \tag{52}$$

These expressions are very similar to the ones obtained in the Hartree-Fock approximation. Differences are contained in the fermion kinetic energy, the first term in Eq. (50), and in the factors $\left(\frac{1}{2}-1\right)$ in Eqs. (51) and (52). These differences arise because we are not using the nucleon field equation of motion.

To implement the $\delta$ expansion, we need to specify $\mathcal{L}_0$. We choose

$$\mathcal{L}_0 = \bar\psi\left(i\gamma_\mu\partial^\mu - M_0\right)\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_\sigma^2\phi^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2V_\mu V^\mu\,, \tag{53}$$

where

$$M_0 \equiv M + \mu\,. \tag{54}$$

The interpolated Walecka model is then given by

$$\mathcal{L}_{\rm W}(\delta) = \mathcal{L}_0 + \delta\left(-g_\omega\bar\psi\gamma_\mu V^\mu\psi + g_\sigma\bar\psi\phi\psi + \mu\bar\psi\psi\right)\,. \tag{55}$$



Notice that the $\delta$ expansion technique could have also been applied to the meson fields explicitly. However, we have chosen to eliminate the meson fields in favor of the nucleon fields by means of Eqs. (36) and (37), therefore meson effects enter via the nucleon fields.

Next we obtain the self-energy in perturbation theory, always neglecting vertex corrections and the Feynman part of the nucleon propagator. In zeroth order in $\delta$, the nucleon self-energy, corresponding to the interpolated Lagrangian Eq. (55), is obviously zero:

$$\Sigma^{(0)} = 0\,. \tag{56}$$

Therefore, the auxiliary quantities to be used in Eq. (46) become:

$$\begin{aligned}M^*(p) &\equiv M_0 = M + \mu\,, \\ \mathbf{p}^* &\equiv \mathbf{p}\,, \\ E^*(p) &\equiv \left[\mathbf{p}^2 + M_0^2\right]^{\frac{1}{2}}\,, \\ p^{*\mu} &= p^\mu = (p^0, \mathbf{p})\,.\end{aligned} \tag{57}$$

The single-particle energy is simply given by:

$$E(p) = E^*(p) = E_0(p) = \left[\mathbf{p}^2 + M_0^2\right]^{\frac{1}{2}}\,. \tag{58}$$

Using these in Eqs. (51) and (52), we obtain for the zeroth order energy density the following expression:

$$\mathcal{E}^{(0)}_{\rm W} = \gamma\int_0^{P_F}\frac{d^3q}{(2\pi)^3}\frac{\mathbf{q}^2 + MM_0}{E_0(q)} + \mathcal{E}^{(0)D}_{\rm W} + \mathcal{E}^{(0)E}_{\rm W}\,, \tag{59}$$

with the direct and exchange contributions given by:

$$\mathcal{E}^{(0)D}_{\rm W} = \left(\frac{1}{2}-1\right)\frac{g_\sigma^2}{m_\sigma^2}\left[\gamma\int_0^{P_F}\frac{d^3q}{(2\pi)^3}\frac{M_0}{E_0(q)}\right]^2 - \left(\frac{1}{2}-1\right)\frac{g_\omega^2}{m_\omega^2}\left[\gamma\int_0^{P_F}\frac{d^3q}{(2\pi)^3}\right]^2\,, \tag{60}$$

$$\begin{aligned}\mathcal{E}^{(0)E}_{\rm W} = \frac{1}{2}\gamma\int_0^{P_F}\frac{d^3q}{(2\pi)^3E_0(q)}\int_0^{P_F}\frac{d^3k}{(2\pi)^3E_0(k)}\Big\{&g_\sigma^2\Delta_\sigma([E_0(q)-E_0(k)]^2 - (\mathbf{q}-\mathbf{k})^2) \\ &\times\left[\left(\frac{1}{2}-1\right) - [E_0(q)-E_0(k)]^2\Delta_\sigma([E_0(q)-E_0(k)]^2 - (\mathbf{q}-\mathbf{k})^2)\right][E_0(q)E_0(k) \\ &- \mathbf{q}\cdot\mathbf{k} + M_0^2\Big] + 2g_\omega^2\Delta_\omega([E_0(q)-E_0(k)]^2 - (\mathbf{q}-\mathbf{k})^2)\left[\left(\frac{1}{2}-1\right) - [E_0(q)-E_0(k)]^2\right. \\ &\times\left.\Delta_\omega([E_0(q)-E_0(k)]^2 - (\mathbf{q}-\mathbf{k})^2)\right]\left[E_0(q)E_0(k) - \mathbf{q}\cdot\mathbf{k} - 2M_0^2\right]\Big\}\,.\end{aligned} \tag{61}$$

Let us consider the direct term first. Application of the PMS to it:

$$\frac{d\mathcal{E}_{\rm W}^{(0)}}{d\mu} = \frac{d\mathcal{E}_{\rm W}^{(0)}}{dM_0}\frac{dM_0}{d\mu} = \frac{d\mathcal{E}_{\rm W}^{(0)}}{dM_0} = 0\ , \tag{62}$$

yields the following self-consistency condition for $M_0$:

$$M_0 = M - \frac{g_\sigma^2}{m_\sigma^2}\gamma\int_0^{P_F}\frac{d^3q}{(2\pi)^3}\frac{M_0}{E_0(q)}\ . \tag{63}$$

This is exactly the same self-consistency condition for the effective nucleon mass obtained by means of the Hartree, or mean-field, approximation.

Now, application of the PMS to the full energy density, which includes both direct and exchange contributions, leads to a nonlinear equation for $\mu$, or equivalently for $M_0$, which is more complicated than the one of Eq. (63). We do not present the expression here because it is rather lengthy and not very instructive. To investigate the size of the exchange corrections we carry out two sets of comparisons. In Figure 4 we compare the nucleon binding energy, obtained by using only the first and direct terms in Eq. (59) (solid line) and coupling constants fixed by fitting the binding energy and density of equilibrium nuclear matter, with the full binding energy, keeping the same coupling constants (dotted line). The value of the coupling constants are $g_s^2 = 91.64$ and $g_v^2 = 136.2$. The masses used in all calculations are $M = 939$ MeV, $m_v = 783$ MeV and $m_\sigma = 550$ MeV. We find that the exchange corrections coincide with those obtained in a relativistic Hartree-Fock calculation [15,16] which we also show for comparison (long-dashed line). The dashed line shows the results obtained for the total binding energy (including both direct and exchange contributions) after renormalizing the model parameters to reproduce the bulk saturation properties of nuclear matter: $g_s^2 = 83.11$ and $g_v^2 = 108.05$. These coupling constants are the same used when renormalizing the relativistic Hartree-Fock calculation of Ref. [16]. Therefore, the PMS condition on the zeroth order energy density of the Walecka model is also equivalent to the usual Hartree-Fock solution.

In Figure 5 we show the results for $\mu$ as a function of the Fermi momentum $P_F$ obtained with the application of the PMS to the zeroth order energy density. The solid line corresponds to the first and direct terms only and the dashed one (almost unnoticeable) corresponds to the full energy density with the renormalized constants. In Figure 6 we compare the results for the effective nucleon mass in nuclear matter as a function of $P_F$



obtained from $\mu$. In both figures, it is clear that the results with the exchange terms and renormalized constants coincide with the results obtained by using the direct terms only.

We now consider the second-order contribution to the self-energy. The self-energy to second-order in delta is is given by:

$$\begin{aligned}\Sigma^{(2)}(p) = &-\mu\delta + i\frac{g_\sigma^2\delta^2}{m_\sigma^2}\int\frac{d^4q}{(2\pi)^4}{\rm Tr}\left[S^{(0)}(q)\right] - i\frac{g_\omega^2\delta^2}{m_\omega^2}\int\frac{d^4q}{(2\pi)^4}\gamma_\mu{\rm Tr}\left[\gamma^\mu S^{(0)}(q)\right]\\ &+ ig_\sigma^2\delta^2\int\frac{d^4q}{(2\pi)^4}S^{(0)}(q)\Delta_\sigma(p-q) - ig_\omega^2\delta^2\int\frac{d^4q}{(2\pi)^4}\gamma_\mu S^{(0)}(q)\Delta_\omega(p-q)\gamma_\mu\ .\end{aligned} \tag{64}$$

where $\Delta_\sigma$ and $\Delta_\omega$ are given in Eq. (38), and again we have made use of the baryon current conservation. We evaluate this expression neglecting the Feynman part of the nucleon propagator, the term $S_F(p)$ given by Eq. (47). Because of this, all integrals in Eq. (64) are finite and can easily be evaluated; there is no need for renormalization. The first term in Eq. (64) comes from the first order contribution in $\delta$ and must be kept at second order. The results are very similar to the ones obtained with the Hartree-Fock approximation [16]. Since there are subtle differences, we write them explicitly below. Each component of the self-energy, $\Sigma^s$, $\Sigma^0$, and $\Sigma^v$, can be decomposed in a direct and an exchange part. The direct components are given by:

$$\Sigma_D^{s(2)} = -\delta\mu - \gamma\frac{g_\sigma^2\delta^2}{m_\sigma^2}\int_0^{P_F}\frac{d^3q}{(2\pi)^3}\frac{M_0}{E_0(q)}\ , \tag{65}$$

$$\Sigma_D^{0(2)} = -\gamma\frac{g_\omega^2\delta^2}{m_\omega^2}\int_0^{P_F}\frac{d^3q}{(2\pi)^3}\ , \tag{66}$$

$$\Sigma_D^{v(2)} = 0\ . \tag{67}$$

Notice that they are independent of energy and momentum. The exchange terms are given by:

$$\Sigma_E^{s(2)}(p) = \frac{1}{4\pi^2 p}\int_0^{P_F}dq\,q\,\frac{M_0}{E_0(q)}\left[\frac{1}{4}g_\sigma^2\delta^2\Theta_\sigma(p,q) - g_\omega^2\delta^2\Theta_\omega(p,q)\right]\ , \tag{68}$$

$$\Sigma_E^{0(2)}(p) = -\frac{1}{4\pi^2 p}\int_0^{P_F}dq\,q\,\left[\frac{1}{4}g_\sigma^2\delta^2\Theta_\sigma(p,q) + \frac{1}{2}g_\omega^2\delta^2\Theta_\omega(p,q)\right]\ , \tag{69}$$

$$\Sigma_E^{v(2)}(p) = -\frac{1}{4\pi^2 p^2}\int_0^{P_F}dq\,q\,\frac{q}{E_0(q)}\left[\frac{1}{2}g_\sigma^2\delta^2\Phi_\sigma(p,q) + g_\omega^2\delta^2\Phi_\omega(p,q)\right]\ , \tag{70}$$

where the functions $\Theta_i(p,q), \Phi_i(p,q), i = \sigma,\omega$, are defined by:

$$\Theta_i(p,q) = \ln\left|\frac{A_i(p,q)+2pq}{A_i(p,q)-2pq}\right|\ , \tag{71}$$

$$\Phi_i(p,q) = \frac{1}{4pq}A_i(p,q)\Theta_i(p,q) - 1\ , \tag{72}$$

where

$$A_i(p,q) = \mathbf{p}^2 + \mathbf{q}^2 + m_i^2 - [E(p) - E_0(q)]^2 \ . \tag{73}$$

The auxiliary quantities to be substituted into Eq. (46) are then given by:

$$\begin{aligned} M^*(p) &\equiv M_0 + \left[\Sigma_D^{s(2)} + \Sigma_E^{s(2)}(p)\right] \ , \\ \mathbf{p}^* &\equiv \mathbf{p}\left[1 + \Sigma_E^{v(2)}(p)\right] \ , \\ E^*(p) &\equiv \left[\mathbf{p}^{*2} + M^{*2}\right]^{\frac{1}{2}} \ , \\ p^{*\mu} &= p^\mu + \Sigma^\mu(p) = \left[p^0 + \Sigma_D^{0(2)} + \Sigma_E^{0(2)}(p), \mathbf{p}^*\right] \ , \end{aligned} \tag{74}$$

and the single-particle energy is the solution of:

$$\begin{aligned} E(p) &= \left[E^*(p) - \Sigma^{0(2)}(p)\right]_{p_0=E(p)} \\ &= \left\{\mathbf{p}^2\left[1 + \Sigma^{v(2)}(|\mathbf{p}|, E(p))\right]^2 + \left[M + \Sigma^{s(2)}(|\mathbf{p}|, E(p))\right]^2\right\}^{\frac{1}{2}} - \Sigma^{0(2)}(|\mathbf{p}|, E(p)) \ . \end{aligned} \tag{75}$$

We are in the position to calculate the energy density. Initially we consider the direct terms only. The energy density is given by:

$$\mathcal{E}_W^{(2)D} = \gamma \int_0^{P_F} \frac{d^3q}{(2\pi)^3} \frac{\mathbf{q}\cdot\mathbf{q}^* + MM^*}{E^*(q)} + \frac{g_\omega^2}{2m_\omega}\left[\frac{2}{3\pi^2}P_F^3\right]^2 - \frac{g_\sigma^2}{2m_\sigma^2}\left[\gamma\int_0^{P_F}\frac{d^3q}{(2\pi)^3}\frac{M^*}{E^*(q)}\right]^2 \ . \tag{76}$$

Application of the PMS to this yields again the familiar Hartree result, Eq. (63), with $M_0$ and $E_0(q)$ replaced respectively by $M^*$ and $E^*(q)$. The exchange terms are shown in Eq. (52) and we believe it is not necessary to rewrite them here. Also in this case, despite numerical imprecisions, the exchange corrections coincide with the usual Hartree-Fock solution, as can be seen in Figure 7. The behavior of $M^*$ as a function of the Fermi momentum at this order does not show any noticeable difference as compared with the zeroth order results. However, as can be seen in Figure 8 the behavior of $\mu$ as a function of $P_F$ obtained with the application of the PMS to the full second order energy density (dashed line) is rather different from the one obtained when only the direct term is taken into account (solid line). It is very interesting to notice that this different behavior does not manifest itself either in the values of $M^*$ or of the binding energy. This is because the energy density $\mathcal{E}$ is a very flat function of $\mu$, as can be seen in Figure 9, where the energy density is shown for $P_F = 1.19$ fm$^{-1}$. Recall that if one had an exact solution, the energy density would be independent of



$\mu$. The solid line is obtained without the inclusion of the exchange term (the PMS solution in this case is given by $\mu/M = -0.275$) and the dashed line gives the full second order density energy (the PMS solution is $\mu/M = -0.35$). This stability in the value of the energy density as a function of $\mu$ is very desirable and guarantees that even big changes in the value of $\mu$ will not affect physical quantities, as the binding energy for instance.

## IV. CONCLUSIONS AND FUTURE PERSPECTIVES

In this paper we have utilized the optimized $\delta$ expansion to study medium effects in two commonly used models in hadron and nuclear physics: the NJL model and the Walecka model. We have investigated an alternative way of fixing the arbitrary parameters introduced by the $\delta$ expansion, by applying the PMS to the energy density of the system.

The most important and concrete conclusion we can draw from this work is that when applying the PMS to the energy density of the NJL model we reproduce, at any order, the familiar Hartree-Fock solution for the dynamically generated mass, in the approximation of neglecting vertex corrections. In the case of Walecka model, we obtained results quantitatively similar to the ones of the usual Hartree-Fock approximation, although the analytical expressions are not evidently equivalent. If one neglects the exchange term in the energy density then clearly the mean-field solution is reproduced at any order. It is also worth mentioning that, in the Walecka model, the energy density is a very flat function of $\mu$ and this guarantees that the PMS solution is indeed very stable.

On the basis of our results, we believe that the optimized $\delta$ expansion is a very robust nonperturbative approximation scheme. Compared with the Hartree-Fock approximation, the $\delta$ expansion is very economical because of its perturbative nature. Once the reliability of the scheme has been established, one is ready to proceed to other interesting applications. These include vertex and, obviously, vacuum effects. The study of the vacuum in the Walecka model is an important issue since one needs to know the limits of applicability of such model to high densities and/or temperatures before quark and gluon degrees of freedom have to be invoked. Of course, one has to face renormalization problems when including the vacuum. Renormalization in a Hartree-Fock scheme is very complicated [17] and one expects that this will be facilitated within the $\delta$ expansion method.

## V. ACKNOWLEDGMENTS

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## FIGURES

FIG. 1. $P_F$ dependence of $\mu$ in the NJL model obtained with PMS on $f_\pi$.

FIG. 2. Constituent quark mass (solid and dotted lines) and $- < \bar{q}q >^{1/3}$ (dashed and dot-dashed lines) as a function of $P_F$. The solid and dashed lines are the PMS1 solution and the dotted and dot-dashed lines are the PMS2 solution.

FIG. 3. $P_F$ dependence of $f_\pi$ for the NJL model. The solid and dotted lines give respectively the PMS1 and PMS2 solutions with the same parameters. The dashed line gives the PMS2 solution with renormalized parameters.

FIG. 4. $P_F$ dependence of the binding energy of the Walecka model at zeroth order in $\delta$. The solid line represents the first and direct terms of Eq. (59) only. The dotted and long-dashed lines give the full binding energy and the Hartree-Fock solution respectively, both determined with the same coupling constants used in the solid line solution. Finally the dashed line gives the full binding energy with the renormalized coupling constants.

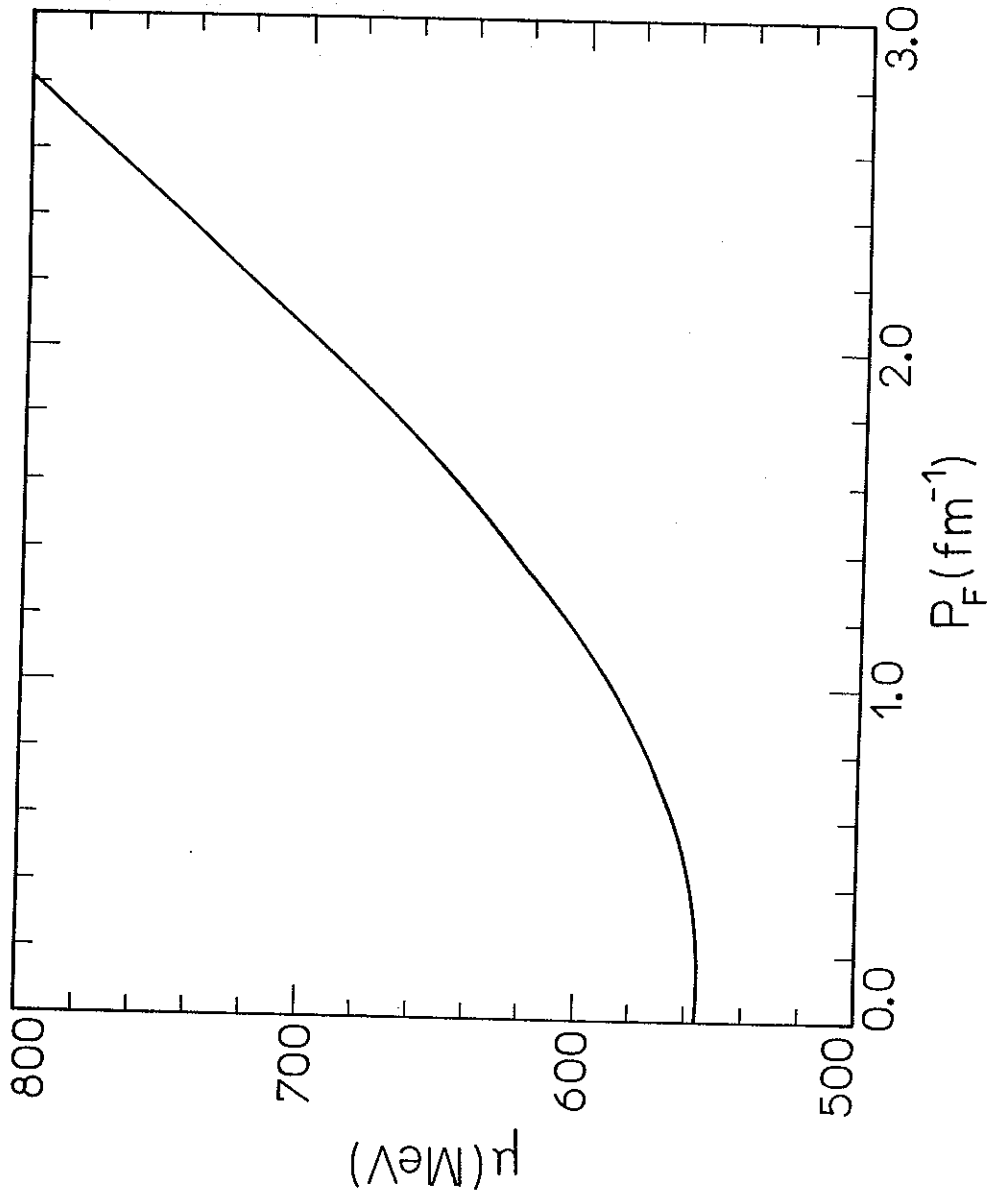
FIG. 5. $P_F$ dependence of $\mu$ for the Walecka model at zeroth order in $\delta$. The dashed line represents $\mu$ determined with the full self-energy, which is the sum of direct and exchange terms. The solid line represents $\mu$ determined without the exchange term.

FIG. 6. Zeroth order nucleon effective mass $M_0$ as a function of $P_F$. The solid curve is the result obtained without the exchange term and the dashed curve is the result using the full energy density.

FIG. 7. $P_F$ dependence of the binding energy of the Walecka model at second order in $\delta$. The solid, dashed, dotted and long-dashed lines are the same as in fig. 4

FIG. 8. $P_F$ dependence of $\mu$ for the Walecka model at second order in $\delta$. The solid and dashed lines are the same as in Figure 5.

FIG. 9. $\mu$ dependence of the energy density for the Walecka model at second order in $\delta$, calculated at $P_F = 1.19$ fm$^{-1}$. The solid line gives the solution when the exchange term is not included. The dashed line gives the full solution.



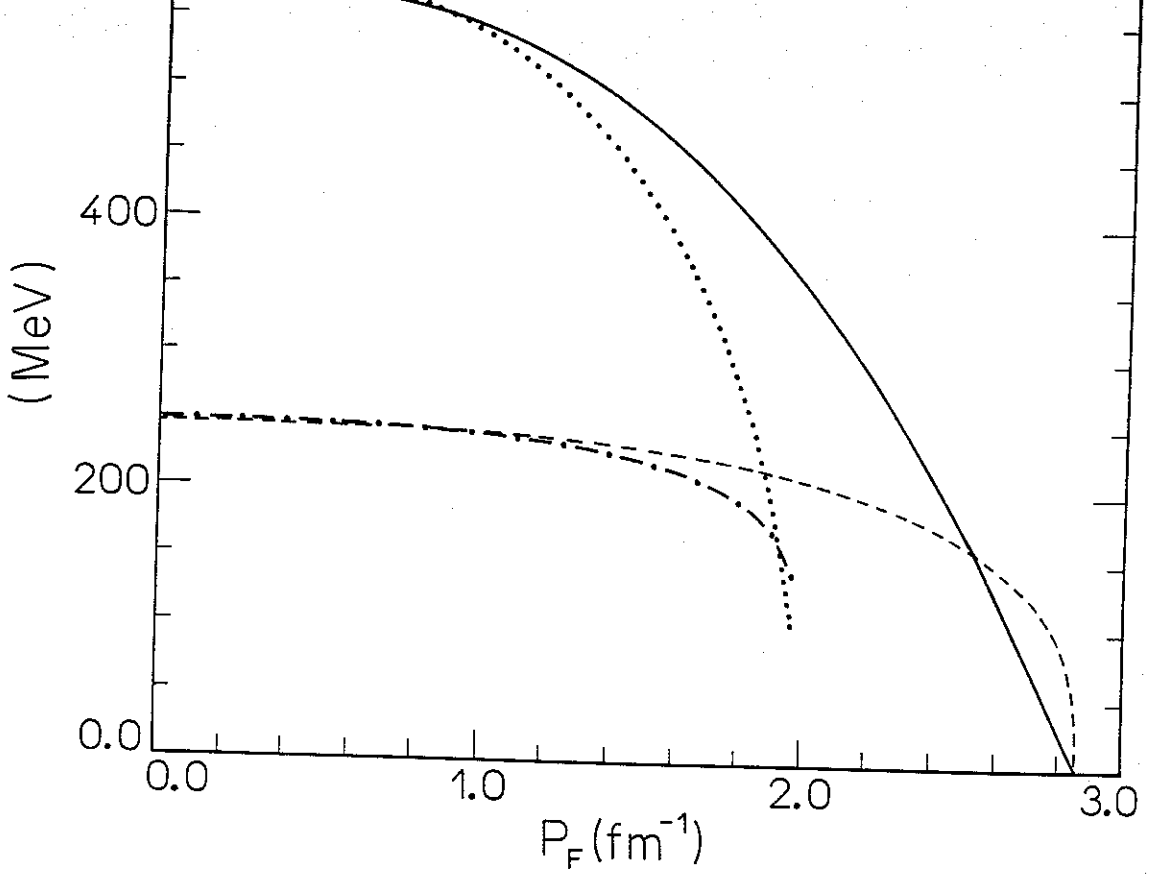



Fig.2-Studying medium effects...-by G.Krein et al

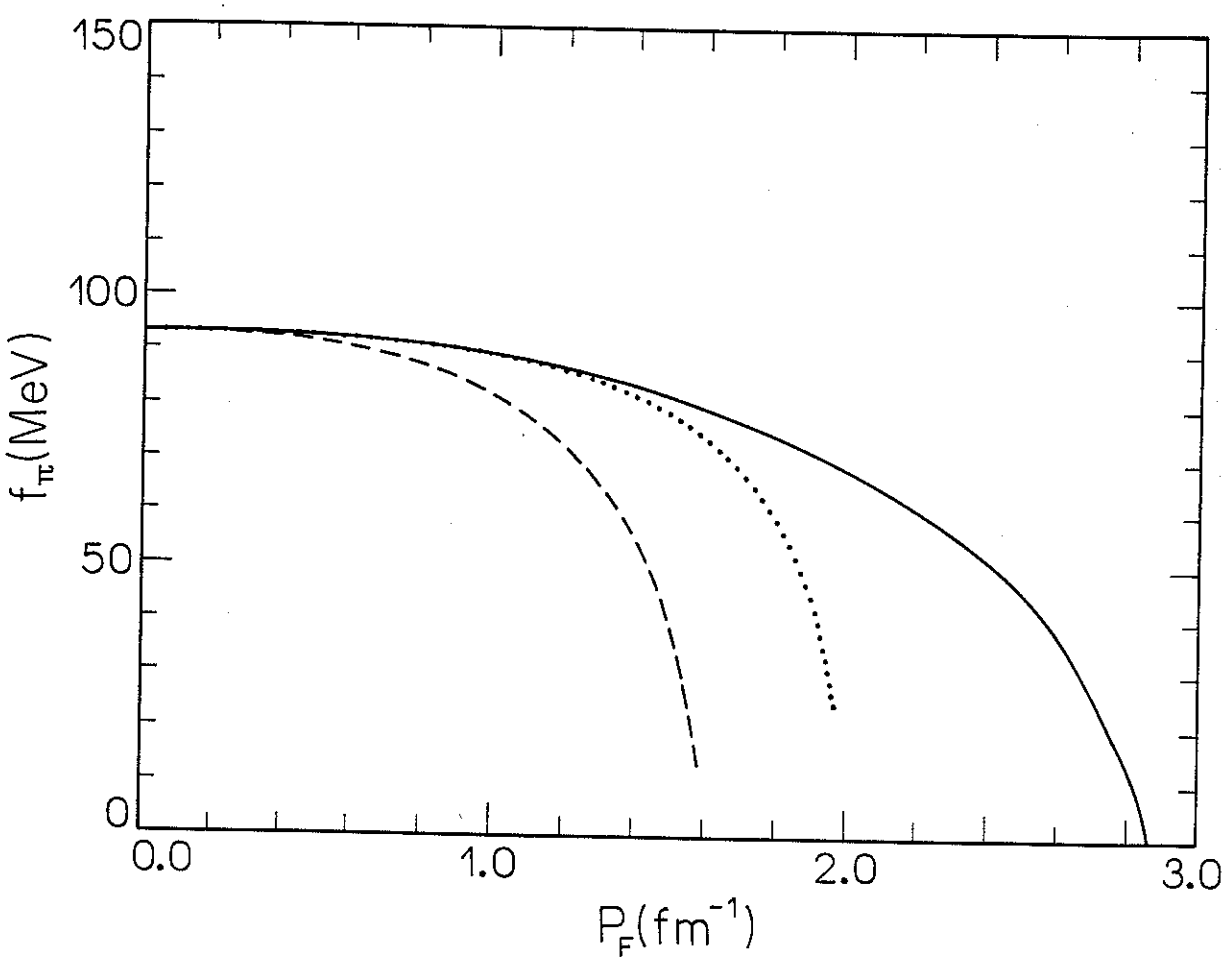



Fig.3-Studying medium effects...-by G.Krein et al

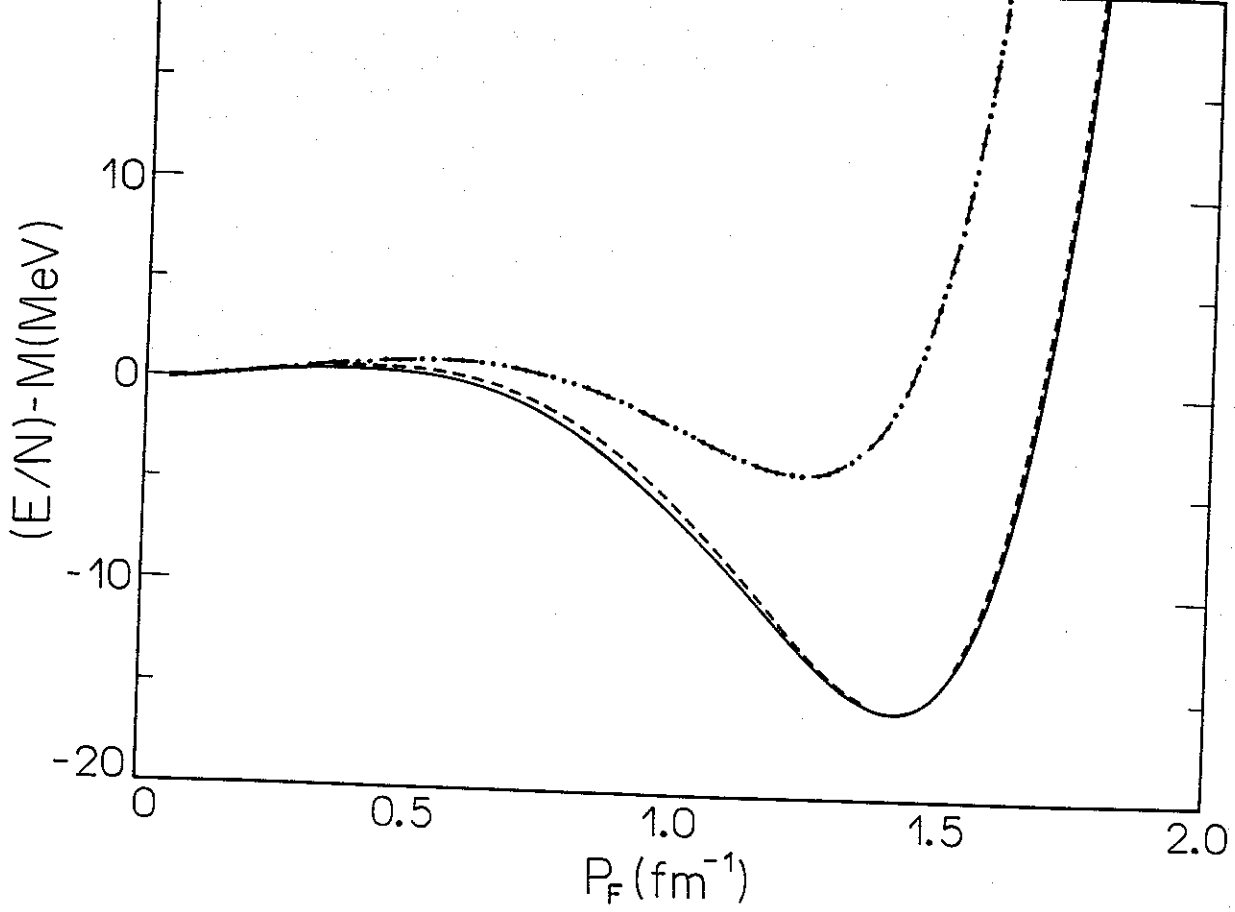



Fig.4-Studying medium effects...-by G.Krein et al

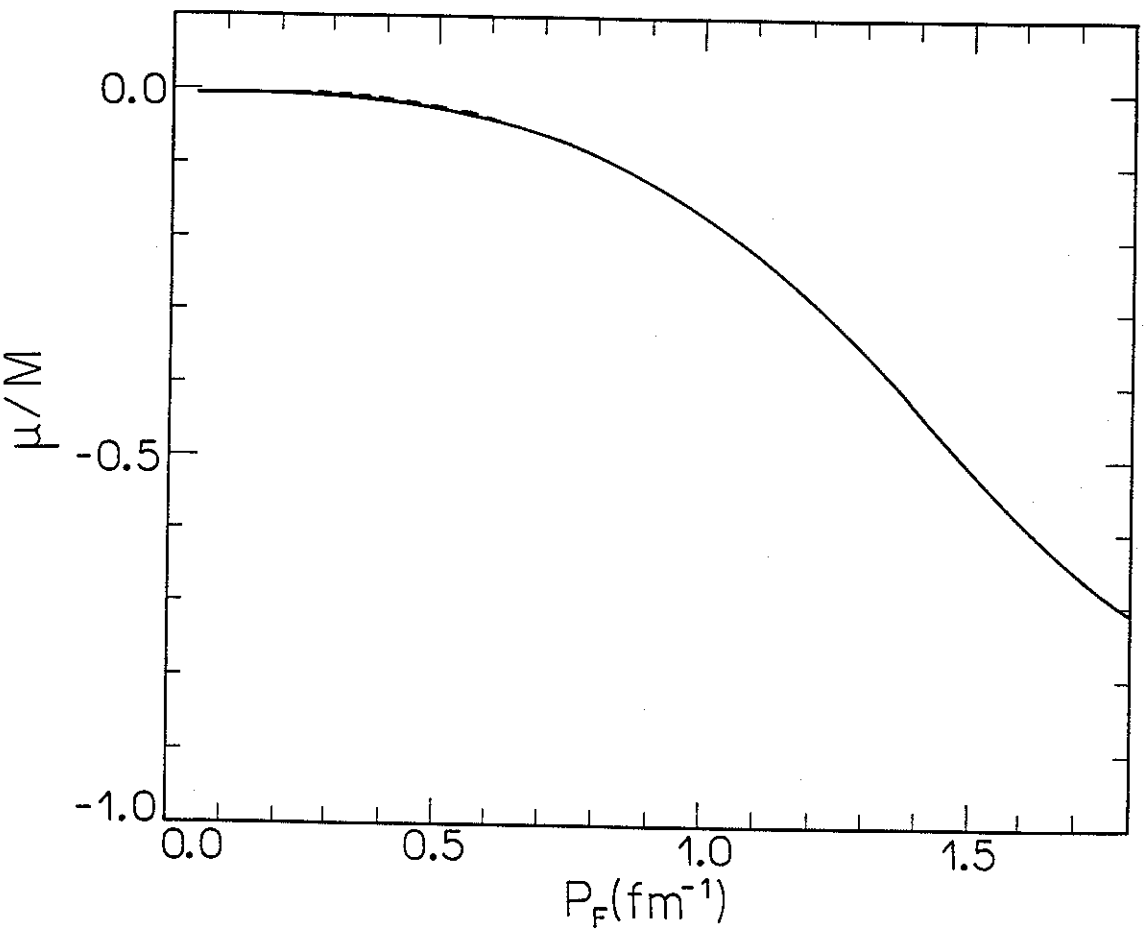



Fig.5-Studying medium effects...-by G.Krein et al

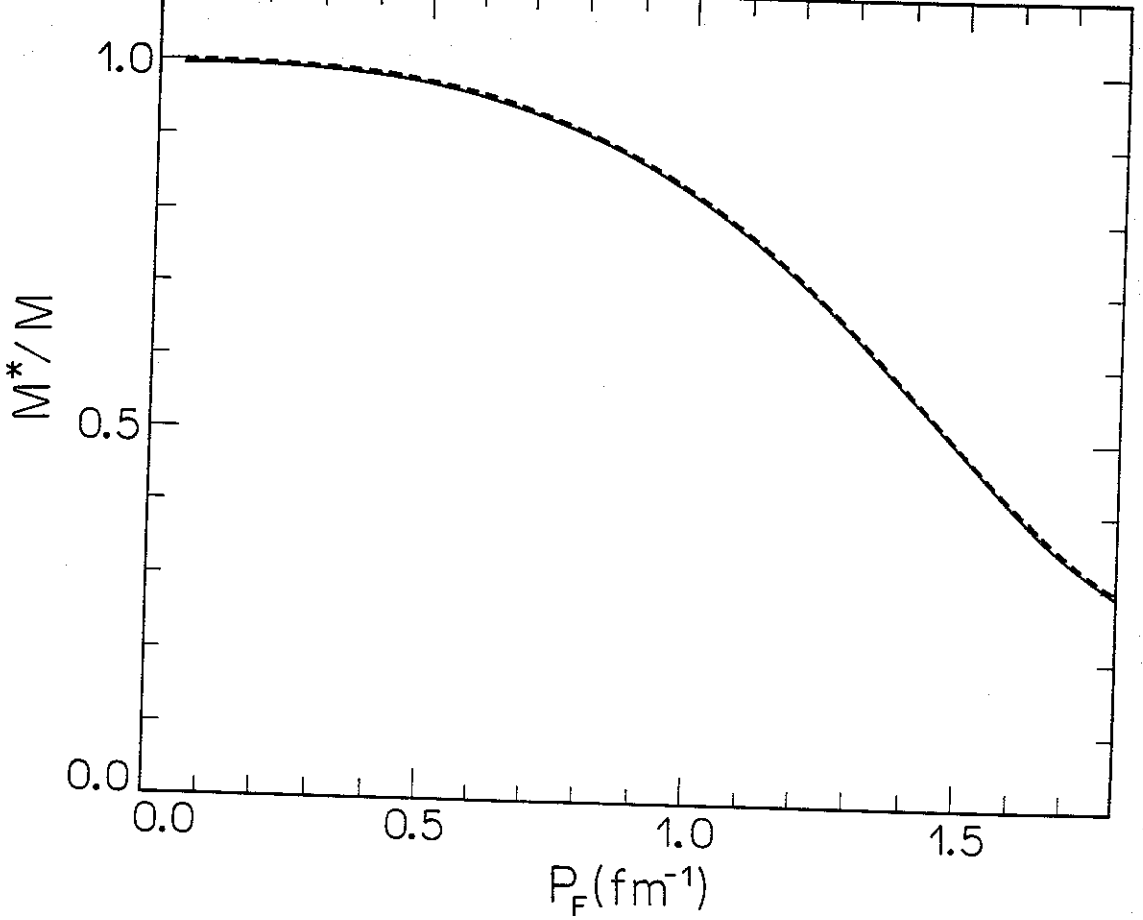



Fig.6-Studying medium effects...-by G.Krein et al

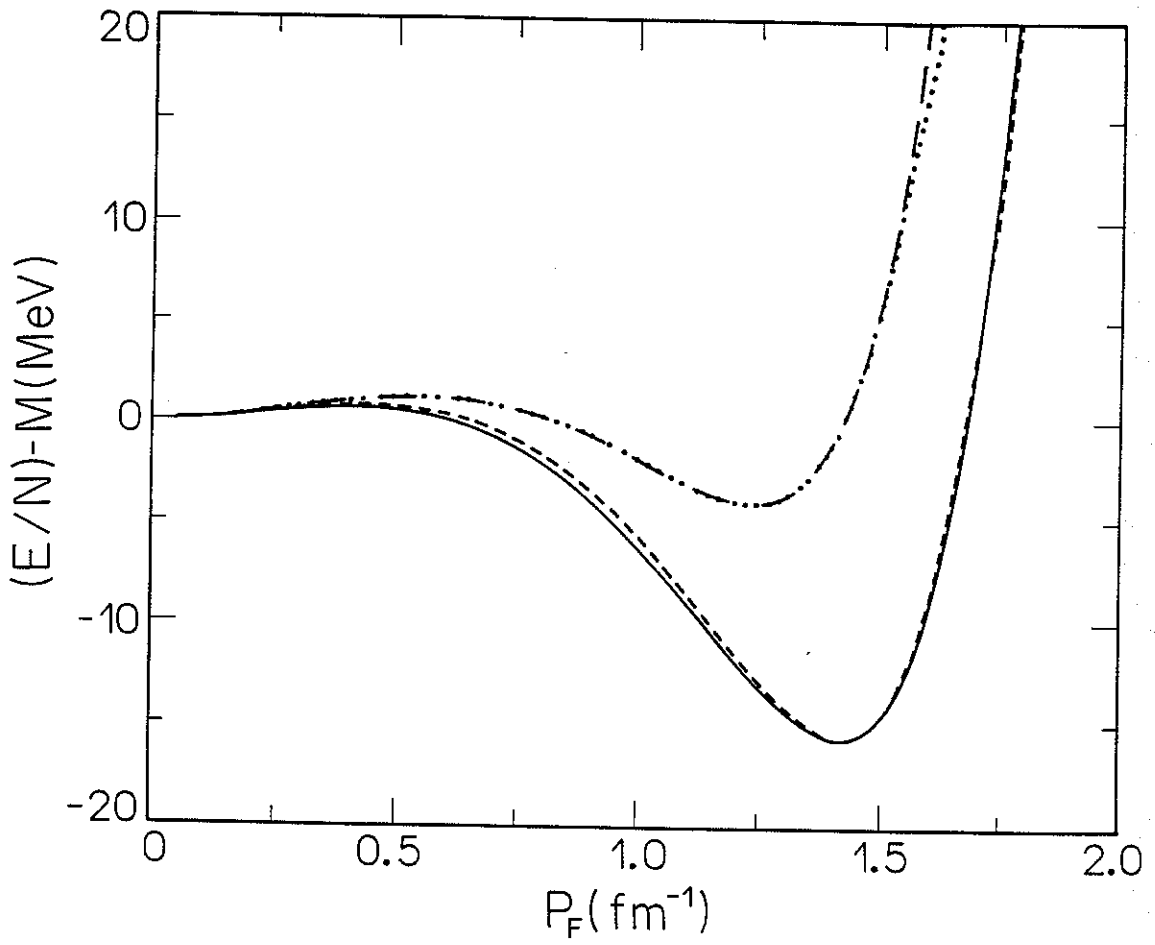



Fig.7-Studying medium effects...-by G.krein et al

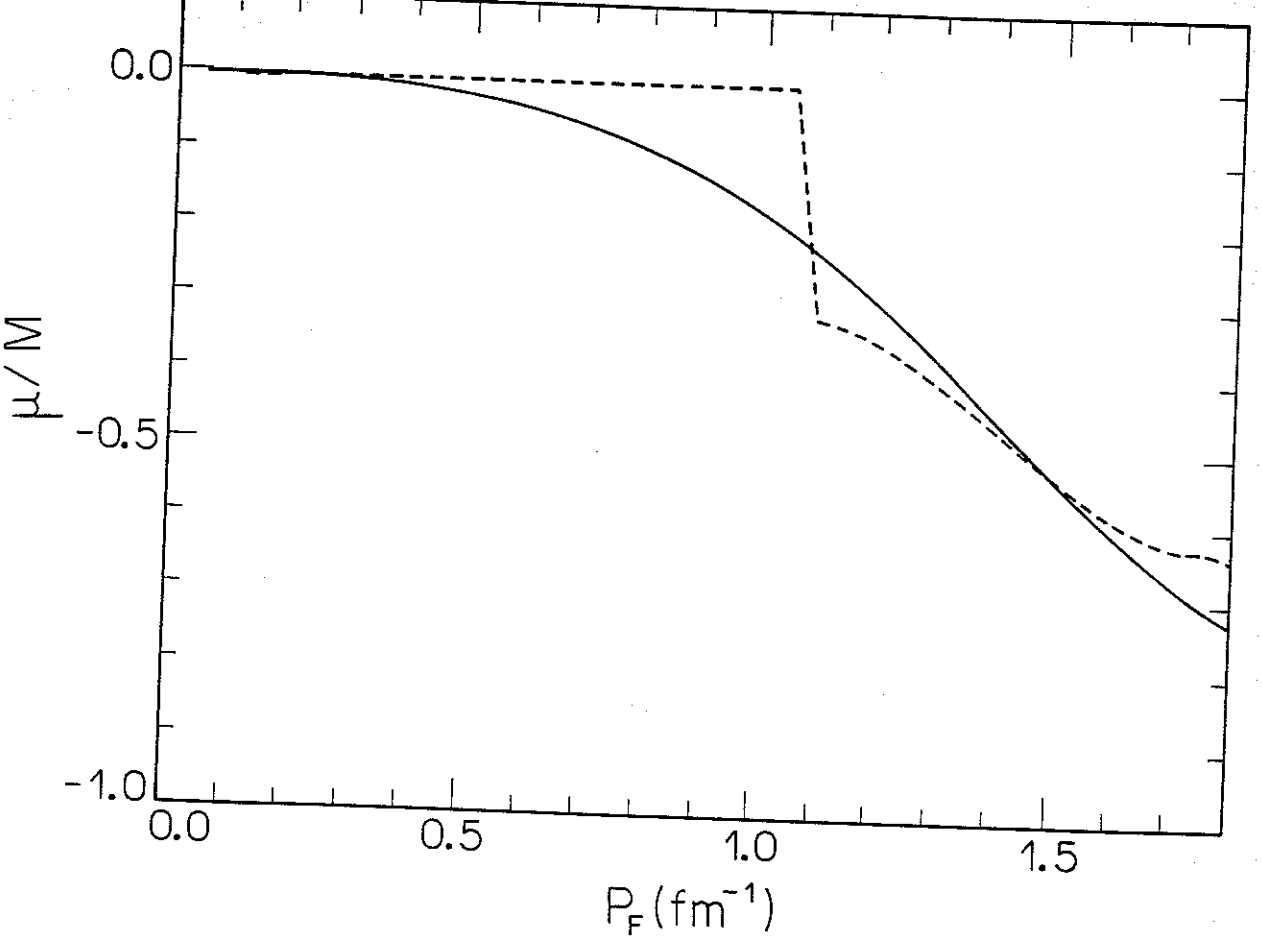



Fig.8-Studying medium effects...-by G.Krein et al

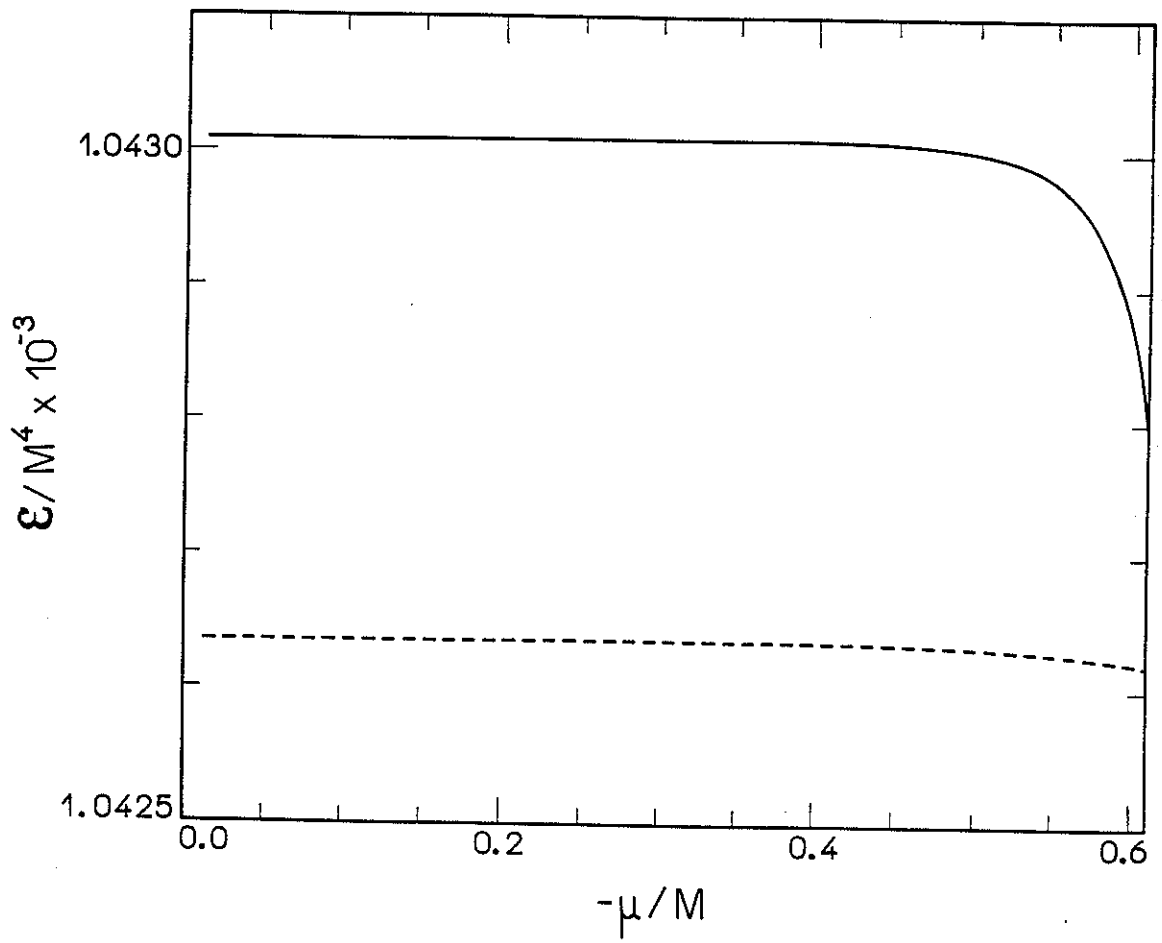



Fig.9-Studying medium effects...-by G.Krein et al