UNIVERSIDADE DE SÃO PAULO

PUBLICAÇÕES

INSTITUTO DE FÍSICA
CAIXA POSTAL 66318
05389-970 SÃO PAULO - SP
BRASIL

IFUSP/P-1163

PION-NUCLEON SCATTERING AND THE TAIL OF TWO-PION EXCHANGE NUCLEON-NUCLEON POTENTIAL

M.R. Robilotta
Instituto de Física, Universidade de São Paulo

Junho/1995

PION-NUCLEON SCATTERING AND THE TAIL OF TWO-PION EXCHANGE NUCLEON-NUCLEON POTENTIAL

M.R. Robilotta

Instituto de Física, Universidade de São Paulo
C.P. 66318, 05389-970 São Paulo, SP, Brazil

An amplitude for pion-nucleon scattering based on a chiral background supplemented by the Höhler, Jacob and Strauss coefficients is used to determine the long range part of the two-pion exchange nucleon-nucleon potential.

I. INTRODUCTION

The only very well known component of the nucleon-nucleon interaction is due to one pion exchange, has long range and determines many deuteron and high angular momentum scattering observables. The next layer of the potential has intermediate range and is associated with various processes, involving pions, meson resonances and deltas. At short distances, quark dynamics is supposed to dominate.

The spatial features of a given process are determined by the mass exchanged in the t-channel. In the intermediate part of the potential, the lightest system that can be exchanged involves just two pions and has a mass of about 300 MeV. Other important effects, associated with resonances such as the rho and the omega, are shorter ranged, since these states have masses around 750 MeV. Therefore, intermediate range interactions also exhibit a marked spatial hierarchy.

The long range part of the two-pion exchange nucleon-nucleon potential ($\pi\pi E$-NNP) is closely related to the pion-nucleon scattering amplitude, as pointed out long ago by Brown and Durso [1]. This relationship underlies the construction of the Paris potential [2], where empirical information about the πN process, treated by means of dispersion relations, is used to obtain the intermediate part of the force. It has also motivated a detailed study based on dispersion relations by Chemtob, Durso and Riska [3].

Field theory provides an alternative framework for the evaluation of the $\pi\pi E$-NNP. In this case, the basic idea is to write down a lagrangian involving the degrees of freedom one considers to be relevant and then calculate a certain number of Feynman diagrams. This leads to an amplitude which may afterwards be transformed into a potential. An important step along this line was given by Partovi and Lomon [4], who considered the box and crossed box diagrams that determine the long range part of the force, using a lagrangian containing just pions and nucleons with pseudoscalar (PS) coupling. A study of the same diagrams using a pseudovector (PV) coupling was performed later on by Zuilhof and Tjon [5].

In recent times, several authors considered the problem of constructing the $\pi\pi E$-NNP using field theory, motivated by the realization that chiral symmetry provides a suitable theoretical framework for the calculation of strong processes [6]–[10]. A system containing just pions and nucleons yields a minimal chiral potential, that can be obtained by means of a non-linear lagrangian based on either PS or PV πN couplings. The theoretical foundations of this part of the interaction are rather well established and it is reasonable to expect that it should become a standard ingredient of any modern NN potential. A shortcoming of the minimal chiral model is that it fails to reproduce experimental information in the case of the intermediate πN amplitude that enters the $\pi\pi E$-NNP. In order to overcome this difficulty, one may extend the approach, so as to encompass other degrees of freedom. This possibility was recently considered by Ordóñez, Ray and Van Kolck [11], who have shown that the inclusion of deltas in the model improves considerably its predictive power. Alternatively, one may choose to introduce the empirical information that is missing in the intermediate πN amplitude in a model independent way, with the help of the Höhler, Jacob and Strauss [12]–[14] subthreshold coefficients. The main purpose of the present work is to show that the detailed knowledge of the πN amplitude provided by these coefficients allows a precise determination of the outer part of the intermediate range nucleon-nucleon potential, which is due to the exchange of two pions.

This work is organized as follows. In section II we display the main results for the πN amplitude, in section III these results are used to construct the $\pi\pi E$-NNP and in section IV we discuss the main features of its leading terms. Finally, in section IV we present our conclusions.

II. PION-NUCLEON AMPLITUDE

The amplitude for the process $\pi^a(k)N(p) \rightarrow \pi^b(k')N(p')$, for on-shell nucleons, is written as

$$F = F^+ \delta_{ab} + F^- i\, \epsilon_{bac}\, \tau_c \ , \tag{1}$$

where

$$F^\pm = \overline{u}\left(A^\pm + \frac{\not{k} + \not{k}'}{2} B^\pm\right) u \ . \tag{2}$$

The functions $A^\pm$ and $B^\pm$ depend on two of the following four variables:

$$s \equiv (p+k)^2 = (p'+k')^2 \ , \tag{3}$$

$$u \equiv (p-k')^2 = (p'-k)^2 \ , \tag{4}$$

$$t \equiv (k-k')^2 = (p-p')^2 \ , \tag{5}$$

$$\nu \equiv \frac{(s-u)}{4m} = \frac{(p+p')\cdot(k+k')}{4m} \ . \tag{6}$$

If the pions are off-shell, $A^\pm$ and $B^\pm$ may also depend on k^2 and k'^2.

Various dynamical processes, involving the interactions of pions, nucleons, rhos and deltas, as well as higher baryon resonances, may contribute to the amplitude. Chiral models that include these degrees of freedom are successful in reproducing experiment data, both below threshold and for energies up to 350 MeV [15]–[17]. Nevertheless, in order to keep model dependence to a minimum, in this work we consider the dynamics of a system containing just pions and nucleons and then supplement it with the coefficients of Höhler, Jacob and Strauss (HJS) [12]–[14]. For the πN system we adopt a non-linear lagrangian with the interaction term given by

$$\mathcal{L} = \cdots - g\,\overline{N}\left[\sqrt{f_\pi^2 - \phi^2} + i\boldsymbol{\tau}\cdot\boldsymbol{\phi}\gamma_5\right] N + \cdots \quad , \tag{7}$$

where N and ϕ represent the nucleon and pion fields, whereas f_π and g are the pion decay and πN coupling constants respectively. This lagrangian gives rise to three diagrams at tree level, represented within brackets in fig. 1. Their evaluation produces the following contributions to $A^\pm$ and $B^\pm$:

$$A_N^+ = \frac{g^2}{m} \quad , \tag{8}$$

$$B_N^+ = -\frac{g^2}{s-m^2} + \frac{g^2}{u-m^2} \quad , \tag{9}$$

$$A_N^- = 0 \quad , \tag{10}$$

$$B_N^- = -\frac{g^2}{s-m^2} - \frac{g^2}{u-m^2} \quad . \tag{11}$$

It is worth pointing out that the use of a PV lagrangian would yield exactly the same results.

The HJS coefficients, summarizing all the other dynamical effects, are used to construct the remainder (R) of the sub-amplitudes, as the following series on ν and t:

$$A_R^+ = \sum a_{mn}^+\,\nu^{2m}\,t^n \quad , \tag{12}$$

$$B_R^+ = \sum b_{mn}^+\,\nu^{(2m+1)}\,t^n \quad , \tag{13}$$

$$A_R^- = \sum a_{mn}^-\,\nu^{(2m+1)}\,t^n \quad , \tag{14}$$

$$B_R^- = \sum b_{mn}^-\,\nu^{2m}\,t^n \quad . \tag{15}$$

The numerical values of the coefficients of these series are reproduced in table 1. They are compatible with the following values for the other constants: $g = 13.40$, $\mu = 139.57$ MeV, $m = 6.7227\,\mu$.

III. NUCLEON-NUCLEON INTERACTION

The amplitude T, for the process $N(p_1)N(p_2) \to N(p_1')N(p_2')$ and due to the exchange of two pions, is represented in fig. 2, where the vertices with four legs include the diagrams shown in fig. 1. We call k and k' the momenta of the exchanged pions and formulate the problem in terms of the variables:

$$W \equiv p_1 + p_2 = p_1' + p_2' \;, \tag{16}$$

$$Q \equiv \frac{1}{2}(k + k') \;, \tag{17}$$

$$\Delta \equiv k' - k = p_1' - p_1 = p_2 - p_2' \;, \tag{18}$$

$$z \equiv \frac{1}{2}\left[(p_1 + p_1') - (p_2 + p_2')\right] \;. \tag{19}$$

The inversion of these expressions leads to the following forms for the momenta of the various particles

$$p_1 = \frac{1}{2}(W + z - \Delta) \;, \tag{20}$$

$$p_2 = \frac{1}{2}(W - z + \Delta) \;, \tag{21}$$

$$p_1' = \frac{1}{2}(W + z + \Delta) \;, \tag{22}$$

$$p_2' = \frac{1}{2}(W - z - \Delta) \;, \tag{23}$$

$$k = Q - \frac{1}{2}\Delta \;, \tag{24}$$

$$k' = Q + \frac{1}{2}\Delta \;. \tag{25}$$

The nucleons are assumed to be on shell, and hence the following constraints hold

$$W \cdot z = W \cdot \Delta = z \cdot \Delta = 0\ . \tag{26}$$

With these definitions, the Mandelstam variables for nucleon 1 are given by

$$s_1 - m^2 = Q^2 + Q \cdot (W + z) - \frac{1}{4}\,\Delta^2\ , \tag{27}$$

$$u_1 - m^2 = Q^2 - Q \cdot (W + z) - \frac{1}{4}\,\Delta^2\ , \tag{28}$$

$$\nu_1 = \frac{Q \cdot (W + z)}{2m}\ . \tag{29}$$

The variables for nucleon 2 are obtained by replacing $z \to -z$.

In the centre of mass system one has

$$W = (2E\,;\ 0)\ , \tag{30}$$

$$\Delta = (0\,;\ \mathbf{p}' - \mathbf{p})\ , \tag{31}$$

$$z = (0\,;\ \mathbf{p}' + \mathbf{p})\ , \tag{32}$$

where $\mathbf{p}$ and $\mathbf{p}'$ are the initial and final trimomenta of nucleon 1 and E is the common energy.

The diagram of fig. 2 yields the following form for T

$$i\,T = \frac{1}{2} \int \frac{d^4Q}{(2\pi)^4}\, F^{(1)}\, F^{(2)}\, \frac{1}{k^2 - \mu^2}\, \frac{1}{k'^2 - \mu^2}\ , \tag{33}$$

where $F^{(i)}$ is the πN amplitude for nucleon i, whose general form is given in eqs. (1–2). The factor $\frac{1}{2}$ was introduced because fig. 2 is symmetric under the exchange of the intermediate pions. The use of eq. (1) allows one to write

$$i\,T = \frac{1}{2} \int \frac{d^4Q}{(2\pi)^4} \left[3F^{+(1)}\, F^{+(2)} + 2\boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)}\, F^{-(1)}\, F^{-(2)}\right] \frac{1}{k^2 - \mu^2}\, \frac{1}{k'^2 - \mu^2}\ . \tag{34}$$

Using eq. (2) and eqs. (8–11) one obtains

$$
\begin{aligned}
iT &= (2m)^2 \frac{1}{2} \int \frac{d^4Q}{(2\pi)^4} \frac{1}{k^2-\mu^2} \frac{1}{k'^2-\mu^2} \\
&\times \left\{ 3 \left[\left(\frac{g^2}{m} + A_R^+ \right) I + \left(-\frac{g^2}{s-m^2} + \frac{g^2}{u-m^2} + B_R^+ \right) \not{Q} \right]^{(1)} \left[\left(\frac{g^2}{m} + A_R^+ \right) I \right.\right. \\
&+ \left. \left(-\frac{g^2}{s-m^2} + \frac{g^2}{u-m^2} + B_R^+ \right) \not{Q} \right]^{(2)} + 2\boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} \left[A_R^- I + \left(-\frac{g^2}{s-m^2} \right.\right. \\
&- \left.\left.\left. \frac{g^2}{u-m^2} + B_R^- \right) \not{Q} \right]^{(1)} \left[A_R^- I + \left(-\frac{g^2}{s-m^2} - \frac{g^2}{u-m^2} + B_R^- \right) \not{Q} \right]^{(2)} \right\} \qquad (35)
\end{aligned}
$$

where I and $\not{Q}$ are given by

$$
I = \frac{1}{2m} \, \overline{u} u \ , \qquad (36)
$$

$$
\not{Q} = \frac{1}{2m} \, \overline{u} \, Q_\mu \gamma^\mu u \ . \qquad (37)
$$

At this point it is interesting to decompose T into two pieces:

$$
T = T_N + T_R \qquad (38)
$$

where T_N originates in the pure pion-nucleon sector and is proportional to g^4; T_R, on the other hand, encompasses all the other dynamical effects.

The amplitude T_N is the part of the interaction considered recently in several works [6]–[10] and corresponds to the minimal chiral background to the $\pi\pi N$-NNP. In order to make explicit the correspondence with earlier calculations, we write

$$
\begin{aligned}
iT_N &= g^4 \int \frac{d^4Q}{(2\pi)^4} \frac{1}{k^2-\mu^2} \frac{1}{k'^2-\mu^2} \left\{ 6\, I^{(1)} I^{(2)} + 3 \left(-\frac{2m}{s_1-m^2} + \frac{2m}{u_1-m^2} \right) \not{Q}^{(1)} I^{(2)} \right. \\
&+ 3 \left(-\frac{2m}{s_2-m^2} + \frac{2m}{u_2-m^2} \right) I^{(1)} \not{Q}^{(2)} + \frac{3}{2} \left(-\frac{2m}{s_1-m^2} + \frac{2m}{u_1-m^2} \right) \\
&\times \left(-\frac{2m}{s_2-m^2} + \frac{2m}{u_2-m^2} \right) \not{Q}^{(1)} \not{Q}^{(2)} + \boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} \\
&\times \left. \left(-\frac{2m}{s_1-m^2} - \frac{2m}{u_1-m^2} \right) \left(-\frac{2m}{s_2-m^2} - \frac{2m}{u_2-m^2} \right) \not{Q}^{(1)} \not{Q}^{(2)} \right\} . \qquad (39)
\end{aligned}
$$

The integration is symmetric under the operation $Q \to -Q$, which also corresponds to $s_1 \leftrightarrow u_1$ and $s_2 \leftrightarrow u_2$. Therefore we have

$$
\begin{aligned}
i\,T_N \;=\;& g^4 \int \frac{d^4Q}{(2\pi)^4}\,\frac{1}{k^2-\mu^2}\,\frac{1}{k'^2-\mu^2}\,\Big\{ 6I^{(1)}\,I^{(2)} + 6\,\frac{2m}{u_1-m^2}\,\not{Q}^{(1)}\,I^{(2)} \\
&- 6\,\frac{2m}{s_2-m^2}\,I^{(1)}\,\not{Q}^{(2)} + 3\,\frac{2m}{u_1-m^2}\left(-\frac{2m}{s_2-m^2}+\frac{2m}{u_2-m^2}\right)\not{Q}^{(1)}\,\not{Q}^{(2)} \\
&+ 2\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}(2)\,\frac{2m}{u_1-m^2}\left(\frac{2m}{s_2-m^2}+\frac{2m}{u_2-m^2}\right)\not{Q}^{(1)}\,\not{Q}^{(2)}\Big\}\;. \qquad (40)
\end{aligned}
$$

This expression is identical to eq. (30) of ref. [9], where only the pion-nucleon sector was considered. It is worth pointing out that this equivalence has a diagramatic counterpart, displayed in fig. 3. The configuration space potential corresponding to eq. (40) was evaluated in ref. [9] and parametrized in ref. [18].

Recently NN observables were calculated by means of a OPEP supplemented with the $\pi\pi E$-NNP derived from T_N [19][20]. In these works the structure of the phase shifts was investigated by means of the variable phase method and the conclusion was reached that the calculated phase shifts were the outcome of large cancellations due to chiral symmetry. The equivalence between the present calculation and that of ref. [5] demonstrates that the cancellations noticed in NN scattering are due to those occurring in the intermediate πN amplitude.

We now turn to the problem of evaluating T_R, the remainder of the amplitude, that is associated with the diagrams of fig. 4 and given by

$$
\begin{aligned}
i\,T_R \;=\;& \int \frac{d^4Q}{(2\pi)^4}\,\frac{1}{k^2-\mu^2}\,\frac{1}{k'^2-\mu^2} \\
&\times \Big\{ 6\Big[\left(g^2\,m\,A_R^{+(1)} + g^2\,m\,A_R^{+(2)} + m^2\,A_R^{+(1)}\,A_R^{+(2)}\right) I^{(1)}\,I^{(2)} \\
&+ \left(g^2\,m\,B_R^{+(2)} + m^2\,A_R^{+(1)}\,B_R^{+(2)} - m\,A_R^{+(1)}\,\frac{mg^2}{s_2-m^2} + m\,A_R^{+(1)}\,\frac{mg^2}{u_2-m^2}\right) I^{(1)}\,\not{Q}^{(2)}
\end{aligned}
$$

$$+ \left(g^2\, m\, B_R^{+(1)} + m^2\, B_R^{+(1)}\, A_R^{+(2)} - \frac{mg^2}{s_1 - m^2}\, m\, A_R^{+(2)} + \frac{mg^2}{u_1 - m^2}\, m\, A_R^{+(2)}\right) \not{Q}^{(1)}\, I^{(2)}$$

$$+ \left(m^2\, B_R^{+(1)}\, B_R^{+(2)} - \frac{mg^2}{s_1 - m^2}\, m\, B_R^{+(2)} + \frac{mg^2}{u_1 - m^2}\, m\, B_R^{+(2)}\right.$$

$$\left. - \; m\, B_R^{+(1)} \frac{mg^2}{s_2 - m^2} + m\, B_R^{+(1)}\, \frac{mg^2}{u_2 - m^2}\right) \not{Q}^{(1)}\, \not{Q}^{(2)}\Bigg]$$

$$+ \; 4\boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} \Bigg[m^2\, A_R^{-(1)}\, A_R^{-(2)}\, I^{(1)}\, I^{(2)} + \left(m^2\, A_R^{-(1)}\, B_R^{-(2)}\right.$$

$$\left. - \; m\, A_R^{-(1)} \frac{mg^2}{s_2 - m^2} - m\, A_R^{-(1)}\, \frac{mg^2}{u_2 - m^2}\right) I^{(1)}\, \not{Q}^{(2)}$$

$$+ \left(m^2\, B_R^{-(1)}\, A_R^{-(2)} - \frac{mg^2}{s_1 - m^2}\, m\, A_R^{-(2)} - \frac{mg^2}{u_1 - m^2}\, m\, A_R^{-(2)}\right) \not{Q}^{(1)}\, I^{(2)}$$

$$+ \left(m^2\, B_R^{-(1)}\, B_R^{-(2)} - \frac{mg^2}{s_1 - m^2}\, m\, B_R^{-(2)} - \frac{mg^2}{u_1 - m^2}\, m\, B_R^{-(2)} - m\, B_R^{-(1)}\right.$$

$$\left. \times \; \frac{mg^2}{s_2 - m^2} - m\, B_R^{-(1)}\, \frac{mg^2}{u_2 - m_2}\right) \not{Q}^{(1)}\, \not{Q}^{(2)}\Bigg]\Bigg\} \; . \tag{41}$$

Using the symmetry of the integral under the operation $Q \to -Q$, we obtain

$$i\, T_R = \int \frac{d^4Q}{(2\pi)^4}\, \frac{1}{k^2 - \mu^2}\, \frac{1}{k'^2 - \mu^2}$$

$$\times \Bigg\{ 6 \Bigg[\left(g^2\, m\, A_R^{+(1)} + g^2\, m\, A_R^{+(2)} + m^2\, A_R^{+(1)}\, A_R^{+(2)}\right) I^{(1)}\, I^{(2)}$$

$$+ \left(g^2\, m\, B_R^{+(2)} + m^2\, A_R^{+(1)}\, B_R^{+(2)} - m\, A_R^{+(1)}\, \frac{2m\, g^2}{s_2 - m^2}\right) I^{(1)}\, \not{Q}^{(2)}$$

$$+ \left(g^2\, m\, B_R^{+(1)} + m^2\, B_R^{+(1)}\, A_R^{+(2)} + \frac{2m\, g^2}{u_1 - m^2}\, A_R^{+(2)}\right) \not{Q}^{(1)}\, I^{(2)}$$

$$+ \left(m^2\, B_R^{+(1)}\, B_R^{+(2)} + \frac{2m\, g^2}{u_1 - m^2}\, m\, B_R^{+(2)} - m\, B_R^{+(1)}\, \frac{2m\, g^2}{s_2 - m}\right) \not{Q}^{(1)}\, \not{Q}^{(2)}\Bigg]$$

$$+ 4\boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} \Bigg[m^2\, A_R^{-(1)}\, A_R^{-(2)}\, I^{(1)}\, I^{(2)} + \Bigg(+ \; m^2\, A_R^{-(1)}\, B_R^{-(2)} - m\, A_R^{-(1)}$$

$$\times \left. \frac{2m\, g^2}{s_2 - m^2} \right) I^{(1)}\, \not{Q}^{(2)} + \left(m^2\, B_R^{-(1)}\, A_R^{-(2)} - \frac{2mg^2}{u_1 - m^2}\, m\, A_R^{-(2)} \right) \not{Q}^{(1)}\, I^{(2)}$$

$$+ \left. \left. \left(m^2\, B_R^{-(1)}\, B_R^{-(2)} - \frac{2m\, g^2}{u_1 - m^2}\, m\, B_R^{-(2)} - m\, B_R^{-(1)}\, \frac{2m\, g^2}{s_2 - m^2} \right) \not{Q}^{(1)}\, \not{Q}^{(2)} \right] \right\} . \quad (42)$$

In these expressions, the amplitudes $A_R^{\pm}$ and $B_R^{\pm}$ describe the interactions of pions which are off-shell and should be expanded in powers of t, ν, $(k^2 - \mu^2)$ and $(k'^2 - \mu^2)$. Thus, in principle, each of the coefficients of eqs. (12–15) should be reexpanded in order to account for the fact that the pions are off-shell. However, the use of terms proportional to $(k^2 - \mu^2)$ or $(k'^2 - \mu^2)$ in eq. (42) produces contributions that cancel at least one of the pion propagators and hence correspond to very short distance effects in configuration space. In this work we are interested only in the outer part of the $\pi\pi E$-NNP and hence we neglect pion off-shell effects in $A_R^{\pm}$ and $B_R^{\pm}$. This amounts to using directly eqs. (12–15) in the calculation of the amplitude.

Using the same notation employed in ref. [8], we write:

$$\begin{aligned} i\, T_R &= 6(K_1^+ + K_2^+ + K_3^+ + K_4^+ + K_5^+ - K_6^+ + K_7^+ - K_8^+) \\ &+ 4\boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} (K_1^- + K_2^- + K_3^- + K_4^- - K_5^- - K_6^- - K_7^- - K_8^-) \, , \end{aligned} \quad (43)$$

where the $K_i^{\pm}$ are integrals of the form

$$K_i^{\pm} = \int \frac{d^4Q}{(2\pi)^4} \frac{1}{\left[\left(Q - \frac{1}{2}\Delta\right)^2 - \mu^2 \right] \left[\left(Q + \frac{1}{2}\Delta\right)^2 - \mu^2 \right]}\, g_i^{\pm} \, , \quad (44)$$

with $g_i^{\pm}$ given by

$$g_1^+ = \left[g^2 m \left(A_R^{+(1)} + A_R^{+(2)} \right) + m^2\, A_R^{+(1)}\, A_R^{+(2)} \right] I^{(1)}\, I^{(2)} \, , \quad (45)$$

$$g_2^+ = \left[g^2 m\, B_R^{+(2)} + m^2\, A_R^{+(1)}\, B_R^{+(2)} \right] I^{(1)}\, \not{Q}^{(2)} \, , \quad (46)$$

$$g_3^+ = \left[g^2 m\, B_R^{+(1)} + m^2\, B_R^{+(1)}\, A_R^{+(2)} \right] \not{Q}^{(1)}\, I^{(2)} \, , \quad (47)$$

$$g_4^+ = \left[m^2\, B_R^{+(1)}\, B_R^{+(2)} \right] \not{Q}^{(1)}\, \not{Q}^{(2)} \, , \quad (48)$$

$$g_5^+ = \frac{2m\,g^2}{u_1 - m^2}\, m\, A_R^{+(2)}\, \not{Q}^{(1)}\, I^{(2)} , \tag{49}$$

$$g_6^+ = m\, A_R^{+(1)}\, \frac{2m\,g^2}{s_2 - m^2}\, I^{(1)}\, \not{Q}^{(2)} , \tag{50}$$

$$g_7^+ = \frac{2m\,g^2}{u_1 - m^2}\, m\, B_R^{+(2)}\, \not{Q}^{(1)}\, \not{Q}^{(2)} , \tag{51}$$

$$g_8^+ = m\, B_R^{+(1)}\, \frac{2m\,g^2}{s_2 - m^2}\, \not{Q}^{(1)}\, \not{Q}^{(2)} , \tag{52}$$

$$g_1^- = m^2\, A_R^{-(1)}\, A_R^{-(2)}\, I^{(1)}\, I^{(2)} , \tag{53}$$

$$g_2^- = m^2\, A_R^{-(1)}\, B_R^{-(2)}\, I^{(1)}\, \not{Q}^{(2)} , \tag{54}$$

$$g_3^- = m^2\, B_R^{-(1)}\, A_R^{-(2)}\, \not{Q}^{(1)}\, I^{(2)} , \tag{55}$$

$$g_4^- = m^2\, B_R^{-(1)}\, B_R^{-(2)}\, \not{Q}^{(1)}\, \not{Q}^{(2)} , \tag{56}$$

$$g_5^- = \frac{2m\,g^2}{u_1 - m^2}\, m\, A_R^{-(2)}\, \not{Q}^{(1)}\, I^{(2)} , \tag{57}$$

$$g_6^- = m\, A_R^{-(1)}\, \frac{2m\,g^2}{s_2 - m^2}\, I^{(1)}\, \not{Q}^{(2)} , \tag{58}$$

$$g_7^- = \frac{2m\,g^2}{u_1 - m^2}\, m\, B_R^{-(2)}\, \not{Q}^{(1)}\, \not{Q}^{(2)} , \tag{59}$$

$$g_8^- = m\, B_R^{-(1)}\, \frac{2m\,g^2}{s_2 - m^2}\, \not{Q}^{(1)}\, \not{Q}^{(2)} . \tag{60}$$

The integrals entering the functions $K_i^\pm$ are evaluated in the appendix. The next step in the construction of $V(r)$, the potential in configuration space, is to note that in the centre of mass of the two-nucleon system one has $\Delta^2 = -\boldsymbol{\Delta}^2$, and then use the definition

$$V(r) = -\frac{1}{4m^2} \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}}\, T(\boldsymbol{\Delta}) . \tag{61}$$

In the appendix we have shown that the functions $K_i^\pm$ can be expressed as integrals over auxiliary Feynman parameters of functions of the form $1/(\boldsymbol{\Delta}^2 + M^2)$, where M is a running mass. Thus, contributions to the configuration space potential are obtained by means of the replacement

$$\frac{1}{\boldsymbol{\Delta}^2 + M^2} \longrightarrow \frac{1}{4\pi}\frac{e^{-\mu r}}{r} \ . \tag{62}$$

The spin-dependence of the potential is obtained by means of the results [4]

$$I^{(1)}\,I^{(2)} \cong 1 - \frac{\Omega_{SO}}{2m^2} \ , \tag{63}$$

$$\gamma^{0(1)}\,\gamma^{0(2)} \cong 1 + \frac{\Omega_{SO}}{2m^2} \ , \tag{64}$$

$$\frac{1}{2}\left(\gamma^{0(1)}\,I^{(2)} + I^{(1)}\,\gamma^{0(2)}\right) \cong 1 \ , \tag{65}$$

$$\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)} = \frac{\Omega_{SS}}{6m^2} - \frac{\Omega_{SO}}{m^2} = \frac{\Omega_T}{12m^2} \ , \tag{66}$$

where

$$\Omega_{SS} = -\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right) \ , \tag{67}$$

$$\Omega_{SO} = \mathbf{L}\cdot\mathbf{S}\left(\frac{1}{r}\frac{\partial}{\partial r}\right) \ , \tag{68}$$

$$\Omega_T = \left(3\boldsymbol{\sigma}^{(1)}\cdot\hat{\mathbf{r}}\,\boldsymbol{\sigma}^{(2)}\cdot\hat{\mathbf{r}} - \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\right)\left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}\right) \ . \tag{69}$$

The details of the various profile functions originating from these expressions will be presented elsewhere. In this work we consider just the leading terms, which provide some insight concerning the dynamics of the $\pi\pi E$-NNP.

IV. LEADING CONTRIBUTIONS

In this section we use as much as possible the notation developed in ref. [9]. The leading contributions to the tail of the $\pi\pi E$-NNP arise from the terms proportional to K_1^+, K_5^+ and K_6^+. Using the results of the appendix, we have

$$K_1^+ = \frac{2i}{(4\pi)^2}\left[2m\,g^2\,a_{0n}^+\,t^n + m^2\,a_{0\ell}^+\,a_{0n}^+\,t^{(\ell+n)}\right]\Sigma_1\,I^{(1)}\,I^{(2)} \ ; \tag{70}$$

the function Σ_1 is

$$\Sigma_1 = 2(\Lambda^2 - \mu^2) \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \, \frac{F_1}{\boldsymbol{\Delta}^2 + M_1^2} \quad , \tag{71}$$

where Λ is a monopole regularizer,

$$F_1 = \left[(\alpha+\beta) - (\alpha-\beta)^2\right]^{-1} \ , \tag{A24}$$

$$M_1 = 2\left\{F_1\left[(\alpha+\beta)\mu^2 + (1-\alpha-\beta)\Lambda^2\right]\right\}^{1/2} \ . \tag{A25}$$

The other contribution is given by

$$K_5^+ - K_6^+ \ = \ -\frac{4i}{(4\pi)^2}\left[m\, g^2\, a_{0n}^+\, t^n\right] \Sigma_2\, I^{(1)}\, I^{(2)} \ , \tag{72}$$

$$\Sigma_2 = 2m^2 \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \, \frac{F_2(1-\alpha-\beta)}{\boldsymbol{\Delta}^2 + M_2^2} \quad ,$$

where

$$F_2 = (4\alpha\beta)^{-1} \ , \tag{A44}$$

$$M_2 = 2\left\{F_2\left[(\alpha+\beta)\mu^2 + (1-\alpha-\beta)^2 m^2\right]\right\}^{1/2} \ . \tag{A45}$$

In order to obtain the potential in coordinate space, we note that t becomes a laplacian and write

$$V_R(r) \ = \ -\frac{6}{(4\pi)^2}\left[\frac{g^2}{m}\, a_{0n}^+ (\boldsymbol{\nabla}^2)^n\, (U_1 - 2U_2) + \frac{1}{2}\, a_{0\ell}^+\, a_{0n}^+ (\boldsymbol{\nabla}^2)^{(\ell+n)}\, U_1\right]\left(1 - \frac{\Omega_{s0}}{2m^2}\right) \ , \tag{73}$$

where

$$U_1 = \frac{2}{4\pi}\,(\Lambda^2 - \mu^2) \int_0^1 d\alpha \int_0^{1-\beta} d\beta\, F_1\, \frac{e^{-M_1 r}}{r} \quad , \tag{74}$$

$$U_2 = \frac{2}{4\pi}\, m^2 \int_0^1 d\alpha \int_0^{1-\beta} d\beta\, F_2(1-\alpha-\beta)\, \frac{e^{-M_2 r}}{r} \quad . \tag{75}$$

The functions U_1 and U_2 were also present in our previous calculation of the chiral background to the $\pi\pi E$-NNP [9]. For large distances, the function U_1 can be calculated analytically [18] and is given by

$$U_1 = \frac{\mu^3}{4\sqrt{\pi}}\ \frac{e^{-2x}}{x^2\sqrt{x}}\left[1 + \mathcal{O}\left(\frac{1}{x}\right)\right] \ , \tag{76}$$

where $x \equiv 2\mu r$.

For U_2 one has the numerical identity [18] $U_2 = \frac{5}{16} U_1$. On the other hand, the action of the laplacian over U_1 yields

$$\nabla^2 U_1 = 4\mu^2 U_1 \left[1 + \mathcal{O}\left(\frac{1}{x}\right)\right] . \tag{77}$$

Thus, the very long range part of $V_R(r)$ is given by

$$V_R(r) = -\frac{6}{(4\pi)^2} \left\{ \frac{3}{8} \frac{g^2}{m} \left[a_{0n}^{+}(4\mu^2)^n\right] + \frac{1}{2} \left[a_{0n}^{+}(4\mu^2)^n\right]^2 \right\} U_1 . \tag{78}$$

The full long range scalar-isoscalar potential due to the $\pi\pi E$-NNP is obtained by adding the contributions from the pure nucleon sector, given by eq. (59) of ref. [9] to $V_R(r)$. However, the studies made in refs. [18], [19] and [20] indicate that the contribution from the nucleon sector is repulsive and very small, because of chiral cancellations. Thus the behaviour of this component of the force is due to V_R.

In ref. [14] we find $g^2/m = 26.70\,\mu^{-1}$ whereas the results from table 1 yield $[a_{0n}(4\mu^2)^n] = 3.68\,\mu^{-1}$. Thus, as expected, one finds an attractive long range scalar-isoscalar interaction associated with the $\pi\pi E$-NNP. The structure of the potential given in eq. (78) sheds some light in the dynamical content of the large distance behaviour of this component of the force. The first point to be noted is that the leading contribution to the potential comes from the term proportional to g^2 in eq. (78), indicating that it is due to the diagrams involving nucleons in one of the sides and other effects in the other. The functions U_1 and U_2 that determine the spatial dependence of the leading term reflect the contact and pole contributions to the intermediate πN amplitude and hence the partial cancellation noticed in $V_R(r)$ is a consequence of chiral symmetry. The scale of the final result is determined by the coefficients a_{00}^{+} and a_{01}^{+}. The value of the former, $a_{00}^{+} = -1.46\,\mu^{-1}$, comes almost entirely from the delta pole and non-pole delta intermediate states, whose individual contributions are respectively $-23.7\,\mu^{-1}$ and $25.2\,\mu^{-1}$. On the other hand, the coefficient $a_{01}^{+} = 1.14\,\mu^{-3}$, which dominates V_R, receives contributions from the sigma term ($\sim 0.40\,\mu^{-3}$) and the delta pole ($\sim 0.74\,\mu^{-3}$) [15]–[17].

V. CONCLUSIONS

The relationship between the elastic πN process and the NN amplitude was stressed long ago [1],[3] and already successfully employed in the construction of the Paris potential [2], by means of dispersion relations. On the other hand, the NN interaction has been considered recently by several authors, in the framework chiral symmetric field theory [6]–[11]. In this work we have made a sort of bridge between the approaches based on chiral symmetry and on dispersion relations. We have used the knowledge about pion-nucleon scattering in order to determine the tail of the two-pion exchange nucleon-nucleon potential. We used the fact that the intermediate πN scattering amplitude at low energies is well represented by means of a minimal chiral background, involving just pions and nucleons, supplemented by the Höhler, Jacob and Strauss [12]–[14] subthreshold coefficients. Our main conclusions are the following:

1) As far as the minimal background is concerned, we have shown that the diagrams considered in recent calculations of the $\pi\pi E$-NNP [6]–[10] can be obtained from those describing the intermediate πN amplitude. On the other hand, it is well known that the latter encompasses a large cancellation due to chiral symmetry. So, large cancellations must also occur in the framework of the NN interaction, confirming the results presented in refs. [9] and [18]–[20].

2) The inclusion of the part of the amplitude represented by the HJS coefficients, denoted globally as "remainder" (R), allows a simple and model independent evaluation of the outer part of the $\pi\pi E$-NNP. This establishes a very direct relationship between the precision of the coefficients and that of the potential.

3) The study of the leading contribution to the $\pi\pi E$-$NNP(R)$ has shown that

it happens, as expected, in the scalar-isoscalar channel, is attractive and due to HJS coefficients associated with the deltas and with the πN sigma term. Its final shape is determined by two rather strong cancellations. The first one occurs between pole and contact terms within the subsystem containing just pions and nucleons, which is the signature of chiral symmetry. The second one happens between the delta pole and non-pole contributions. This result conveys an important message, namely that non-pole or contact terms play a very important role in the interaction, and hence must be taken into account in any model aiming at being realistic.

APPENDIX A

In this appendix we evaluate the integrals given in eqs. (44–60). It is useful to define the four vectors V_1 and V_2 by

$$V_1^\mu = \frac{1}{2m}\,(W+z)^\mu\ , \tag{A1}$$

$$V_2^\mu = \frac{1}{2m}\,(W-z)^\mu\ . \tag{A2}$$

With these definitions we have

$$\nu_i = Q \cdot V_i \tag{A3}$$

and

$$\not{V}_i^{(i)} = I^{(i)}\ . \tag{A4}$$

The pion propagators are represented with the help of the shorthands

$$d\ =\ \left[\left(Q-\frac{\Delta}{2}\right)^2 - \mu^2\right]\ , \tag{A5}$$

$$d'\ =\ \left[\left(Q+\frac{\Delta}{2}\right)^2 - \mu^2\right]\ . \tag{A6}$$

We also make use of the result $\Delta^2 = t = -\boldsymbol{\Delta}^2$.

The integrands of $K_1^\pm$, $K_2^\pm$, $K_3^\pm$ and $K_4^\pm$ are even polynomials in V_1 and V_2, which have the following explicit forms:

$$\begin{aligned} g_1^+ \ &=\ \Big\{g^2\, m\, a_{mn}^+ \left[(Q\cdot V_1)^{2m} + (Q\cdot V_2)^{2m}\right] t^n \\ &\qquad m^2\, a_{k\ell}^+\, a_{mn}^+ (Q\cdot V_1)^{2k}\,(Q\cdot V_2)^{2m}\, t^{(\ell+n)}\Big\}\, I^{(1)}\, I^{(2)}\ , \end{aligned} \tag{A7}$$

$$g_1^- \ =\ \Big\{m^2\, a_{k\ell}^-\, a_{mn}^- (Q\cdot V_1)^{(2k+1)}\,(Q\cdot V_2)^{(2m+1)}\, t^{(\ell+n)}\Big\}\, I^{(1)}\, I^{(2)}\ ; \tag{A8}$$

$$\begin{aligned} g_2^+ \ &=\ \Big\{g^2\, m\, b_{mn}^+ (Q\cdot V_2)^{(2m+1)}\, t^n \\ &\qquad +\ m^2\, a_{k\ell}^+\, b_{mn}^+ (Q\cdot V_1)^{2k}\,(Q\cdot V_2)^{(2m+1)}\, t^{(\ell+n)}\Big\}\, I^{(1)}\, \not{Q}^{(2)}\ , \end{aligned} \tag{A9}$$

$$g_2^- \ =\ \Big\{m^2\, a_{k\ell}^-\, b_{mn}^- (Q\cdot V_1)^{(2k+1)}\,(Q\cdot V_2)^{2m}\, t^{(\ell+n)}\Big\}\, I^{(1)}\, \not{Q}^{(2)}\ ; \tag{A10}$$

the expressions for $g_3^\pm$ are obtained from $g_2^\pm$ by exchanging V_1 and V_2;

$$g_4^+ = \left\{m^2\, b_{k\ell}^+\, b_{mn}^+ (Q\cdot V_1)^{(2k+1)}\,(Q\cdot V_2)^{(2m+1)}\, t^{(\ell+n)}\right\} \not{Q}^{(1)}\,\not{Q}^{(2)}\ , \tag{A11}$$

$$g_4^- = \left\{m^2\, b_{k\ell}^-\, b_{mn}^- (Q\cdot V_1)^{2k}\,(Q\cdot V_2)^{2m}\, t^{(\ell+m)}\right\} \not{Q}^{(1)}\,\not{Q}^{(2)}\ . \tag{A12}$$

These results show that we need to evaluate integrals of the type

$$I^{\mu\ldots\sigma} = \int \frac{d^4Q}{(2\pi)^4}\,\frac{Q^\mu\ldots Q^\sigma}{dd'}\ . \tag{A13}$$

The simplest case corresponds to the function

$$I = \int \frac{d^4Q}{(2\pi)^4}\,\frac{1}{dd'}\ . \tag{A14}$$

This integral is divergent and requires regularization. This goal may be achieved either by using dimensional regularization techniques or with the help of a monopole regularizer. The former procedure has the advantage of producing a compact closed expression in momentum space whereas the latter is useful when going to configuration space. Therefore we discuss both methods in the sequence.

In the case of dimensional regularization we use Feynman integration parameters in order to write

$$I_D = \int_0^1 d\alpha \int \frac{d^4Q}{(2\pi)^4}\,\frac{1}{[Q^2+2p\cdot Q-M^2]^2} \tag{A15}$$

where

$$p = \left(\alpha-\frac{1}{2}\right)\Delta\ , \tag{A16}$$

$$M^2 = \mu^2-\frac{1}{4}\Delta^2\ . \tag{A17}$$

Going to Euclidean coordinates, denoted by a bar, and performing the integration in Q, we obtain

$$I_D = \frac{i}{(4\pi)^2}\int_0^1 d\alpha\,\frac{\Gamma(2-\omega)}{\left[\bar{M}^2-\bar{p}^2\right]^{(2-\omega)}}\ , \tag{A18}$$

where ω is half of the number of dimensions. Making the limit $\omega \to 2$ and neglecting divergent or constant terms, we have

$$I_D = -\frac{i}{(4\pi)^2} \int_0^1 d\alpha \ln \left[1 - \alpha(1-\alpha) \frac{\Delta^2}{\mu^2} \right] . \tag{A19}$$

The integration over α may be performed, yielding the closed result

$$I_D = \frac{i}{(4\pi)^2} \left\{ 2 - \sqrt{1 - \frac{4\mu^2}{\Delta^2}} \ln \left[\frac{\sqrt{1 - \frac{4\mu^2}{\Delta^2}} + 1}{\sqrt{1 - \frac{4\mu^2}{\Delta^2}} - 1} \right] \right\} . \tag{A20}$$

If one is interested in going to configuration space, it is useful to rewrite eq. (A19) as

$$I_D = \frac{i}{(4\pi)^2} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \, \frac{\alpha \Delta^2}{-\alpha\beta\Delta^2 + \mu^2} . \tag{A21}$$

The use of the monopole regularizer, on the other hand, leads to the expression

$$I_\Lambda = 2\left(\mu^2 - \Lambda^2\right) \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int \frac{d^4Q}{(2\pi)^4} \frac{1}{[Q^2 + 2p \cdot Q - M^2]^3} . \tag{A22}$$

Changing to Euclidean coordinates and integrating over Q, we get

$$I_\Lambda = \frac{i}{(4\pi)^2} \left(\Lambda^2 - \mu^2\right) 4 \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \, \frac{F_1}{-\Delta^2 + M_1^2} , \tag{A23}$$

where

$$F_1 = \left[(\alpha + \beta) - (\alpha - \beta)^2 \right]^{-1} , \tag{A24}$$

$$M_1 = 2 \left\{ F_1 \left[(\alpha+\beta)\mu^2 + (1 - \alpha - \beta)\Lambda^2 \right] \right\}^{1/2} . \tag{A25}$$

For large values of Λ the integrals I_D and I_Λ are related by

$$I_\Lambda = \frac{i}{(4\pi)^2} \left[\frac{\mu^2}{\Lambda^2 - \mu^2} \ln \frac{\Lambda^2}{\mu^2} - 1 \right] + I_D . \tag{A26}$$

In order to evaluate $I^{\mu\nu}$, we write

$$I_D^{\mu\nu} = \int_0^1 d\alpha \int \frac{d^4Q}{(2\pi)^4} \frac{Q^\mu Q^\nu}{[Q^2 + 2Q \cdot p - M^2]^2} , \tag{A27}$$

where P and M^2 are given in eqs. (20) and (21). Using dimensional regularization and neglecting divergent terms, we obtain, after some manipulations

$$I_D^{\mu\nu} = -\frac{i}{(4\pi)^2}\int_0^1 d\alpha\, p^\mu\, p^\nu\, \ln\left(M^2+p^2\right) + \frac{1}{2}g^{\mu\nu}\int_0^1 d\alpha \int \frac{d^4Q}{(2\pi)^4}\,\frac{1}{(Q^2+2Q\cdot p-M^2)}\ . \tag{A28}$$

The variable p is proportional to Δ. Using eq. (26) and the Dirac equation, one has

$$V_1\cdot\Delta = V_2\cdot\Delta = \not{\Delta}^{(i)} = 0\ , \tag{A29}$$

and the first term in eq. (A28) does not contribute. Moreover, the second term in this equation is much more singular than a double pion exchange and hence can contribute only to very short distance effects. Therefore, in this work we do not consider contributions from $I^{\mu\nu}$ or integrals of the form (A13) containing higher powers of Q^μ.

We now turn to the evaluation of integrals $K_5^\pm$, $K_6^\pm$, $K_7^\pm$ and $K_8^\pm$. Their integrands are given by

$$g_5^+ = \frac{1}{Q^2-2m\,V_1\cdot Q-\frac{1}{4}\Delta^2}\,\left[2m^2\,g^2\,a_{mn}^+(Q\cdot V_2)^{2m}\,t^n\right]\not{Q}^{(1)}\,I^{(2)}\ , \tag{A30}$$

$$g_7^+ = \frac{1}{Q^2-2m\,V_1\cdot Q-\frac{1}{4}\Delta^2}\,\left[2m^2\,g^2\,b_{mn}^+(Q\cdot V_2)^{(2m+1)}\,t^n\right]\not{Q}^{(1)}\,\not{Q}^{(2)}\ ; \tag{A31}$$

$$g_6^+ = \frac{1}{Q^2+2m\,V_2\cdot Q-\frac{1}{4}\Delta^2}\,\left[2m^2\,g^2\,a_{mn}^+(Q\cdot V_1)^{2m}\right]I^{(1)}\,\not{Q}^{(2)}\ , \tag{A32}$$

$$g_8^+ = \frac{1}{Q^2+2m\,V_2\cdot Q-\frac{1}{4}\Delta^2}\,\left[2m^2\,g^2\,b_{mn}^+(Q\cdot V_1)^{(2m+1)}\right]\not{Q}^{(1)}\,\not{Q}^{(2)}\ ; \tag{A33}$$

$$g_5^- = \frac{1}{Q^2-2m\,V_1\cdot Q-\frac{1}{4}\Delta^2}\,\left[2m^2\,g^2\,a_{mn}^-(Q\cdot V_2)^{(2m+1)}\right]\not{Q}^{(1)}\,I^{(2)}\ , \tag{A34}$$

$$g_7^- = \frac{1}{Q^2-2m\,V_1\cdot Q-\frac{1}{4}\Delta^2}\,\left[2m^2\,g^2\,b_{mn}^-(Q\cdot V_2)^{2m}\right]\not{Q}^{(1)}\,\not{Q}^{(2)}\ ; \tag{A35}$$

$$g_6^- = \frac{1}{Q^2+2m\,V_2\cdot Q-\frac{1}{4}\Delta^2}\,\left[2m^2\,g^2\,a_{mn}^-(Q\cdot V_1)^{(2m+1)}\right]I^{(1)}\,\not{Q}^{(2)}\ , \tag{A36}$$

$$g_8^- = \frac{1}{Q^2+2m\,V_2\cdot Q-\frac{1}{4}\Delta^2}\,\left[2m^2\,g^2\,b_{mn}^-(Q\cdot V_1)^{2m}\right]\not{Q}^{(1)}\,\not{Q}^{(2)}\ . \tag{A37}$$

It is worth noting that the isospin symmetric and antisymmetric integrals involve numerators with odd and even powers of Q respectively.

We consider first the integrals with denominators depending on V_1, that have the general form

$$I_1^{\mu\ldots\sigma} = \int \frac{d^4Q}{(2\pi)^4} \frac{Q^\mu \ldots Q^\sigma}{dd' \left(Q^2 - 2m\, V_1 \cdot Q - \frac{1}{4}\Delta^2\right)} \quad . \tag{A38}$$

The simplest of these integrals is given by

$$I_1^\mu = \int \frac{d^4Q}{(2\pi)^4} \frac{Q^\mu}{dd' \left(Q^2 - 2m\, V_1 \cdot Q - \frac{1}{4}\Delta^2\right)} \quad . \tag{A39}$$

Using Feynman parameters, we obtain

$$I_1^\mu = 2\int_0^1 d\alpha \int_0^{1-\beta} d\beta \int \frac{d^4Q}{(2\pi)^4} \frac{Q^\mu}{[Q^2 + 2p_1 \cdot Q - M^2]^3} \quad . \tag{A40}$$

where

$$p_1 = \frac{1}{2}\left[(\beta - \alpha)\Delta - (1 - \alpha - \beta)(W + z)\right] , \tag{A41}$$

$$M^2 = (\alpha + \beta)\mu^2 + (1 - 2\alpha - 2\beta)\frac{\Delta^2}{4} \quad . \tag{A42}$$

Changing to Euclidean coordinates and integrating over Q, we obtain

$$I_1^\mu = \frac{i}{(4\pi)^2}\, 4\int_0^1 d\alpha \int_0^{1-\beta} d\beta\, \frac{F_2\, p_1^\mu}{-\Delta^2 + M_2^2} \quad , \tag{A43}$$

where

$$F_2 = (4\alpha\beta)^{-1} , \tag{A44}$$

$$M_2 = 2\left\{F_2\left[(\alpha + \beta)\mu^2 + (1 - \alpha - \beta)^2 m^2\right]\right\}^{1/2} \quad . \tag{A45}$$

Using eq. (A28), we rewrite I_1^μ as

$$I_1^\mu = -\frac{i}{(4\pi)^2}\, 4\int_0^1 d\alpha \int_0^{1-\beta} d\beta\, \frac{F_2(1 - \alpha - \beta)m\, V_1^\mu}{-\Delta^2 + M_2^2} \quad . \tag{A46}$$

The same procedure may be applied to the other integrals of this type. After neglecting divergent terms, we get

$$I_1^{\mu\nu} = -\frac{i}{(4\pi)^2}\, 4\int_0^1 d\alpha \int_0^{1-\alpha} d\beta\, F_2 \left\{ \frac{(1-\alpha-\beta)^2\, m^2\, V_1^\mu\, V_1^\nu}{-\Delta^2 + M_2^2} \right.$$

$$\left. + \;\frac{1}{2}\, g^{\mu\nu} \ln\left[\frac{(-\Delta^2 + M_2^2)}{\mu^2}\right]\right\} \qquad \text{(A47)}$$

$$I_1^{\mu\nu\lambda} = \frac{i}{(4\pi)^2}\, 4\int_0^1 d\alpha \int_0^{1-\alpha} d\beta \left\{ F_2\, \frac{(1-\alpha-\beta)^3\,(2m)^3\, V_1^\mu\, V_1^\nu\, V_1^\lambda}{-\Delta^2 + M_2^2} \right.$$

$$\left. + \;\frac{1}{2}(1-\alpha-\beta)\, 2m \left(g^{\mu\nu}\, V_1^\lambda + g^{\lambda\mu}\, V_1^\nu + g^{\nu\lambda}\, V_1^\mu\right) \ln\left[\frac{(-\Delta^2 + M_2^2)}{\mu^2}\right]\right\} . \qquad \text{(A48)}$$

The integrals of the form

$$I_2^{\mu\ldots\sigma} = \int \frac{d^4Q}{(2\pi)^4}\, \frac{Q^\mu \ldots Q^\sigma}{dd'\left(Q^2 + 2m\, V_2 \cdot Q - \frac{1}{4}\Delta^2\right)} \qquad \text{(A49)}$$

may be obtained from the results for $I_1^{\mu\ldots\lambda}$ by means of the operation $W \to -W$.

References

[1] G.E. Brown and J.W. Durso, *Phys. Lett.* **35B**, 120 (1971).

[2] M. Lacombe, B. Loiseau, J.M. Richard, R. Vinh Mau, J. Coté, P. Pires and R. de Tourreil, *Phys. Rev.* **C21**, 861 (1980).

[3] M. Chemtob, J.W. Durso and D.O. Riska, *Nucl. Phys.* **B38**, 141 (1972).

[4] M.H. Partovi and E. Lomon, *Phys. Rev.* **D2**, 1999 (1970).

[5] M.J. Zuilhof and J.A. Tjon, *Phys. Rev.* **C24**, 736 (1981); **C26**, 1277 (1982).

[6] C. Ordóñez and U. Van Kolck, *Phys. Lett.* **B291**, 459 (1992).

[7] L.S. Celenza, A. Pantziris and C.M. Shakin, *Phys. Rev.* **C46**, 2213 (1992).

[8] J.L. Friar and S.A. Coon, *Phys. Rev.* **C49**, 1272 (1994).

[9] C.A. da Rocha and M.R. Robilotta, *Phys. Rev.* **C49**, 1818 (1994).

[10] M.C. Birse, *Phys. Rev.* **C49**, 2212 (1994).

[11] C. Ordóñez, L. Ray and U. Van Kolck, *Phys. Rev. Lett.* **72**, 1982 (1994).

[12] G. Höhler, H.P. Jacob and R. Strauss, *Nucl. Phys.* **B39**, 237 (1972).

[13] R. Koch and E. Pietarinen, *Nucl. Phys.* **A336**, 331 (1980).

[14] G. Höhler, group I, vol. 9, subvol. b, part 2 of Landolt-Börnstein Numerical Data and Functional Relationships in Science and Technology, ed. H. Schopper.

[15] M.G. Olson and E.T. Osypowski, *Nucl. Phys.* **B101**, 13 (1975).

[16] M.D. Scadron and L.R. Thebaud, *Phys. Rev.* **D9**, 11544 (1974).

[17] A.M.M. Menezes, M.Sc. Thesis, University of São Paulo, 1985.

[18] C.A. da Rocha and M.R. Robilotta, to be published in *Phys. Rev.* C.

[19] J.L. Ballot, M.R. Robilotta and C.A. da Rocha, submitted for publication.

[20] J.L. Ballot and M.R. Robilotta, submitted for publication.

[21] C.A. da Rocha, Ph.D. Thesis, University of São Paulo, 1993.

FIGURE CAPTIONS

Fig. 1. Diagrams representing the πN amplitude. The process within brackets are associated with minimal chiral symmetry at tree level, whereas the last one corresponds to the net effect of the HJS coefficients and is denoted with R, for "rest".

Fig. 2. Diagram that determines the two-pion exchange NN amplitude; the intermediate πN amplitudes are given by fig. 1.

Fig. 3. Dynamical content of the minimal chiral background to the $\pi\pi E$-NNP.

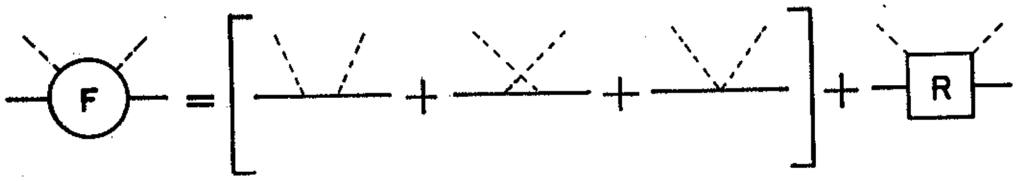
Fig. 4. Dynamical content of the amplitude T_R; conventions are the same as in fig. 1.

TABLE CAPTION

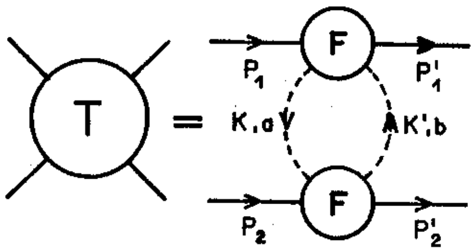
Table 1. Values for the coefficients of eqs. (12–15) in units of μ^{-1}, taken from ref. [14].

Table 1

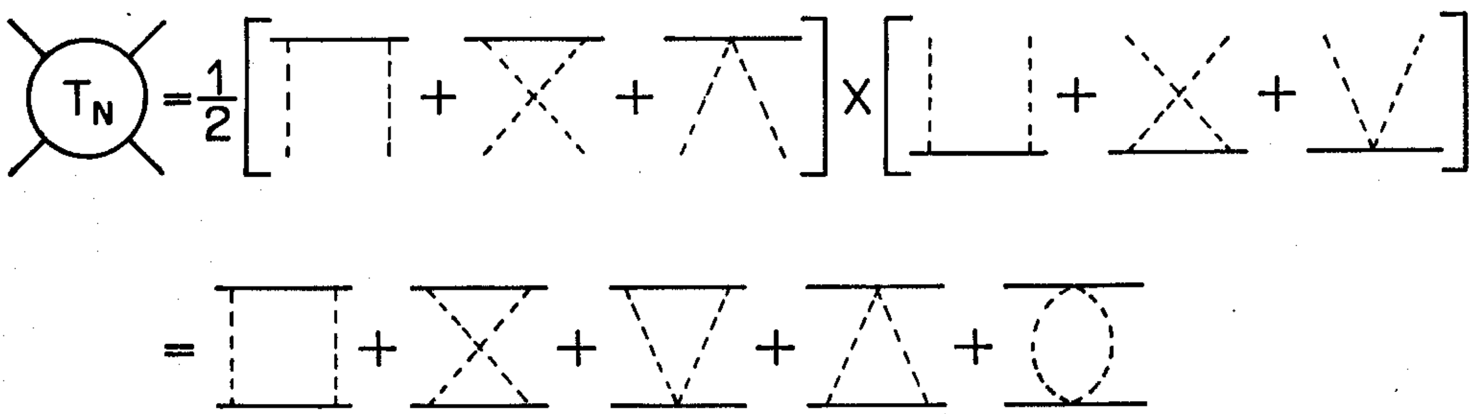
(m,n)	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(2,0)$
a^{+}_{mn}	-1.46 ± 0.10	1.14 ± 0.02	0.036 ± 0.003	4.66	-0.01	1.20 ± 0.02
b^{+}_{mn}	-3.54 ± 0.06	0.18 ± 0.01	-0.01	-1.00 ± 0.02	0.08 ± 0.01	-0.31 ± 0.02
a^{-}_{mn}	-8.83 ± 0.10	-0.374 ± 0.02	-0.015 ± 0.002	-1.247 ± 0.05	0.013 ± 0.006	-0.33 ± 0.02
b^{-}_{mn}	8.37 ± 0.10	0.24 ± 0.01	0.025 ± 0.002	1.08 ± 0.05	-0.055 ± 0.005	0.29 ± 0.02



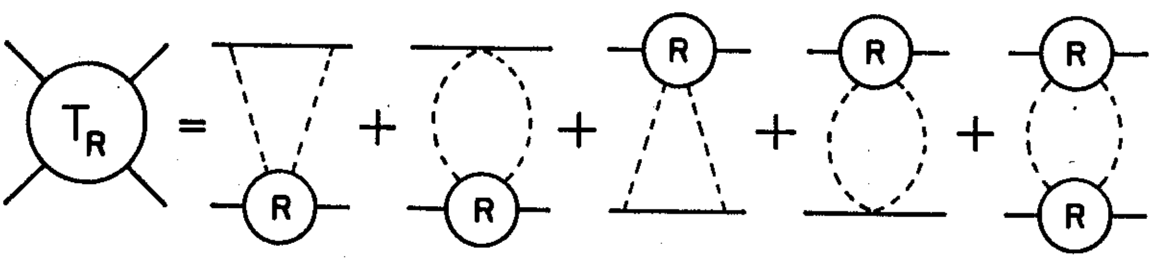
ROBILOTTA, FIG 1.



ROBILOTTA, FIG.2



OBILOTTA, FIG.3



ROBILOTTA, FIG. 4