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Δ WIDTH FROM QCD SUM RULES

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Δ width from QCD sum rules

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Abstract

The decay width of the Δ isobar is calculated using the QCD sum rule method. To include the effect of finite hadronic widths, we replace the δ-function in the spectral density by a normalized Breit-Wigner distribution. The resulting width is in reasonable agreement with the experimental value.

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The QCD sum rule approach is an attempt to understand hadronic parameters in the low energy region in terms of QCD perturbation theory and non-vanishing condensates, which characterizes the non-perturbative QCD vacuum. It has been widely applied to calculate hadron properties [1–4] and its fundamental assumption is the principle of duality: specifically one assumes that there exists an energy interval over which a hadron may be equivalently described at both quark and hadronic level.

The underlying procedure of the QCD sum rule technique is to look at the correlation function between QCD hadronic currents and to study its dispersion relation. The real part of the correlation function is calculated in QCD using the *operator product expansion* (OPE) and the imaginary part is modeled with phenomenological parameters, such as mass and particle-current coupling.

In almost all QCD sum rule calculations, the phenomenological side is parametrized by writing the spectral density as a single sharp pole, representing the lowest-energy hadron state, plus a smooth continuum representing higher states. This approximation is justified by saying that the lowest-energy hadron state is usually fairly narrow when compared to higher-mass states. Nevertheless, one knows that the Δ-particle is unstable, and therefore it does have a finite width. In this note we will evaluate the Δ width in the framework of the QCD sum rules, and show that the result obtained is compatible with experimental data.

Also, besides the intrinsic interest of the problem, there is the fact that the Δ width is expected to change in the nuclear medium [5,6]. Therefore, once one is able to extract information about the Δ width from the QCD sum rules in vacuum, the approach can be extended to nuclear matter in order to evaluate the effect of the medium in the Δ width from the QCD sum rule point of view [7]. The QCD sum rules for the Δ in nuclear matter with the sharp pole approximation were recently analyzed in Refs. [8,9].

We must stress here that our main goal in introducing a width is not to try to improve the agreement between the two Δ sum rules, as was done in Ref. [10] by the inclusion of instanton effects, but rather to explore the QCD sum rules prediction for the width. We have knowledge of only one work which has considered the effect of finite hadronic widths in a QCD sum rule calculation [11]. The authors of Ref. [11] have shown that the large difference between the ρ and ω widths can have a dramatic effect on the value of the off-shell ρ-ω mixing matrix element. In this work we follow an opposite direction than that used by these authors and extract the value of the width from the QCD sum rule analysis, instead of using it as an input in the calculation.

The correlation function for the Δ isobar is defined by

$$\Pi_{\mu\nu}(q) \equiv i \int d^4x \, e^{iqx} \langle 0| T \{\eta_\mu(x)\bar{\eta}_\nu(0)\} |0\rangle, \tag{1}$$

where

$$\eta_\mu(x) = \varepsilon_{abc} \left[u_a^{\mathrm{T}}(x) C \gamma_\mu u_b(x)\right] u_c(x) \tag{2}$$

is the current with the Δ isobar quantum numbers [2]. In Eq. (2) $u(x)$ is a quark field operator, T denotes a transpose in Dirac space, C is the charge conjugation matrix and a, b, c are color indices.

The Δ correlator, Eq. (1), has many tensorial structures. However, in this work we will consider only the sum rules obtained from the structures $g_{\mu\nu}$ and $g_{\mu\nu}\not{q}$, which receive contributions just from spin $\frac{3}{2}$ states [2]:

$$\Pi_{\mu\nu}(q) = g_{\mu\nu}\left[\Pi_1(q^2) + \not{q}\Pi_{\not{q}}(q^2)\right] + \cdots \tag{3}$$

For each of these two structures, we can write a dispersion relation:

$$\Pi_{1,\not{q}}(q^2) = \frac{1}{\pi}\int_0^\infty ds \, \frac{\mathrm{Im}\left[\Pi_{1,\not{q}}(s)\right]}{s-q^2} + \cdots, \tag{4}$$

where "$\cdots$" in the last equation stands for subtraction terms (polynomials in q^2).

In previous studies the spectral density, $\mathrm{Im}\left[\Pi_{1,\not{q}}(s)\right]$, was always represented by a sharp pole at the mass of the Δ, plus a continuum starting at an effective threshold s_0 [2–4,10]:

$$\mathrm{Im}\left[\Pi_{1,\not{q}}(s)\right] = \pi\lambda_\Delta^2 c_{1,\not{q}}\delta(s-M_\Delta^2) + \mathrm{Im}\left[\Pi_{1,\not{q}}^{\mathrm{OPE}}(s)\right]\theta(s-s_0), \tag{5}$$

where $c_1 = M_\Delta$, $c_{\not{q}} = 1$ and λ_Δ gives the strength of the coupling between the current and the Δ-particle (M_Δ is the mass of the Δ isobar).

In order to include a finite width for the Δ, we follow Ref. [11] and replace the δ-function in Eq. (5) by a normalized Breit-Wigner distribution:

$$\delta(s-M_\Delta^2) \longrightarrow \left[\pi/2 + \arctan(M_\Delta/\Gamma_\Delta)\right]^{-1} \frac{M_\Delta\Gamma_\Delta}{(s-M_\Delta^2)^2 + M_\Delta^2\Gamma_\Delta^2}, \tag{6}$$

where Γ_Δ denotes the width of the Δ.

For the calculation of the correlator using OPE, we have considered quark and quark-gluon condensates up to dimension 5, pure gluon condensates of dimension 4 and the four-quark condensate of dimension 6. The corresponding diagrams are shown in Fig. 1.

As usual, we proceed by equating the correlator evaluated using OPE [called the left-hand-side (LHS)] to the correlator obtained using the dispersion relation [called the right-hand-side (RHS)], which is a function of the physical quantities. After applying a Borel transform [1], we obtain the following sum rule from the structure $g_{\mu\nu}$:

$$\frac{1}{M_\Delta}\left[\frac{4}{3}aE_1L^{16/27}M^4 - \frac{2}{3}m_0^2 aE_0L^{4/27}M^2\right] = R(M^2) \tag{7a}$$

and this one bellow from the structure $g_{\mu\nu}\not{q}$:

$$\frac{1}{5}E_2L^{4/27}M^6 + \frac{4}{3}a^2L^{28/27} - \frac{5}{72}bE_0L^{4/27}M^2 = R(M^2), \tag{7b}$$

where we have defined

$$R(M^2) = \tilde{\lambda}_\Delta^2 \int_0^\infty ds \, \left[\pi/2 + \arctan(M_\Delta/\Gamma_\Delta)\right]^{-1} \frac{M_\Delta\Gamma_\Delta}{(s-M_\Delta^2)^2 + M_\Delta^2\Gamma_\Delta^2} e^{-s/M^2}, \tag{8}$$

and

$$\begin{aligned} a &= -(2\pi)^2\langle\bar{q}q\rangle, \\ b &= \left\langle g_s^2G^2\right\rangle, \\ m_0^2 &= \langle\bar{q}g_s\sigma\cdot Gq\rangle / \langle\bar{q}q\rangle, \\ \tilde{\lambda}_\Delta^2 &= (2\pi)^4\lambda_\Delta^2, \\ L &= \log(M/\Lambda_{\mathrm{QCD}})/\log(\mu/\Lambda_{\mathrm{QCD}}). \end{aligned} \tag{9}$$

In the above equations μ is the QCD normalization point and Λ_{QCD} is the QCD scale parameter. The powers of L take care of the anomalous dimension, while the factors E_i are responsible for the continuum contribution to the sum rules and are given by [2]

$$\begin{aligned} E_0 &= 1 - e^{-s_0/M^2} \\ E_1 &= 1 - e^{-s_0/M^2}\left(\frac{s_0}{M^2}+1\right) \\ E_2 &= 1 - e^{-s_0/M^2}\left(\frac{s_0^2}{2M^4}+\frac{s_0}{M^2}+1\right) \end{aligned} \tag{10}$$

The hadronic parameters are then determined by the requirement that the Eqs. (7a) and (7b) are simultaneously satisfied in a certain Borel window, that in our case is $1.2\,\mathrm{GeV}^2 \leq M^2 \leq 1.8\,\mathrm{GeV}^2$. In doing that, we have chosen to hold M_Δ **fixed** at its experimental value, $M_\Delta = 1.23\,\mathrm{GeV}$. The values for the QCD parameters used in our calculations are shown in Table I.

The values obtained for the hadronic parameters, using the same numerical optimization procedure described in Ref. [4], are shown bellow. The resulting fit can be seen from Fig. 2.

$$\begin{aligned} M_\Delta &= 1.23\,\mathrm{GeV} \\ \tilde{\lambda}_\Delta^2 &\approx 2.3\,\mathrm{GeV}^6 \\ s_0 &\approx 3.7\,\mathrm{GeV}^2 \\ \Gamma_\Delta &\approx 160\,\mathrm{MeV} \end{aligned}$$

The values for λ_Δ and s_0, as well as the quality of the fit, are roughly the same as that obtained from the Δ sum rules with the sharp pole approximation. This attests for the stability of our calculations.

It is gratifying to note that the value obtained for Γ_Δ is in good agreement with the experimental value $\Gamma_\Delta \approx 110\,\mathrm{MeV}$, obtained recently from analysis of $\pi^+ p$ scattering data [12].

In conclusion, we have calculated the width of the Δ isobar using the QCD sum rule approach. A finite hadronic width is introduced in the phenomenological side of the sum

rule through the replacement of the δ-function by a normalized Breit-Wigner distribution. The inclusion of the width has little effect on the values of the coupling constant and the continuum threshold, indicating the stability of the result. The width obtained is in good agreement with the experimental value.

It will be very interesting to analyze the effect of the nuclear medium in the Δ width using the QCD sum rule technique. Work in this direction is in progress [7].

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FIGURES

FIG. 1. Diagrams accounted for in the calculation of the correlator [Eq. (1)] using OPE.

FIG. 2. Fit of the phenomenological and QCD sides of the Δ sum rules. $R(M^2)$ is given by Eq. (8), while LHS_1 and LHS_2 denote the left-hand-size of Eqs. (7a) and (7b) respectively.

TABLE I. Values for the QCD parameters used in evaluating the left-hand-size of the sum rules.

a	b	m_0^2	μ	Λ_{QCD}
0.5 GeV^3	0.474 GeV^4	0.8 GeV^2	0.5 GeV	0.1 GeV

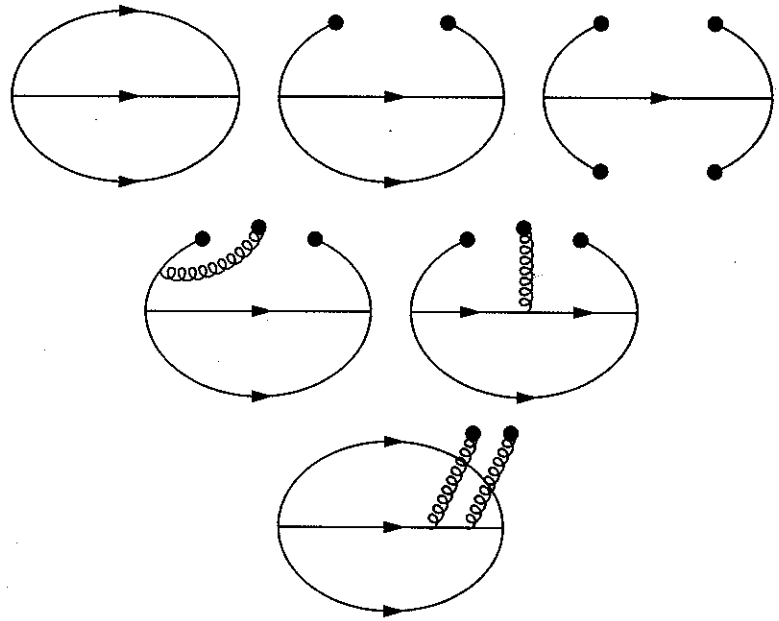

FIGURE 1 – Δ width from QCD sum rules, E. C. Lopes and M. Nielsen.

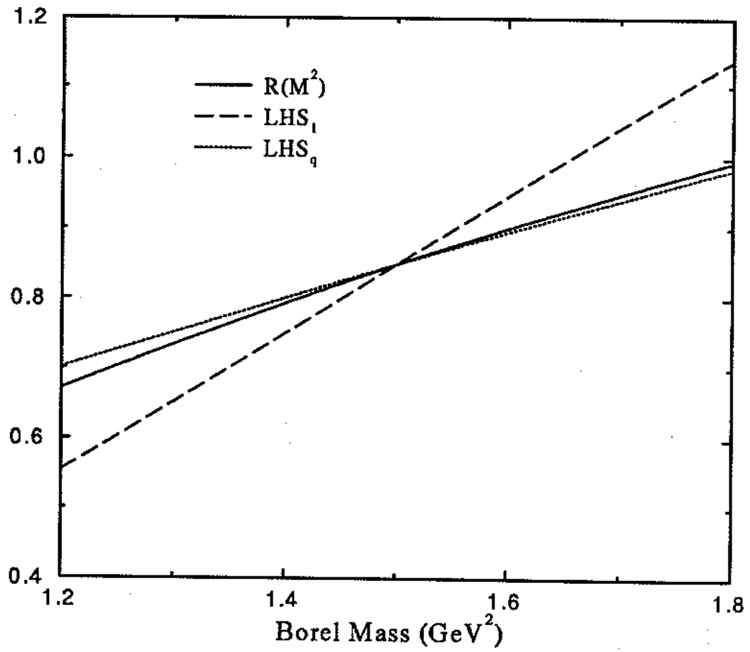

FIGURE 2 – Δ width from QCD sum rules, E. C. Lopes and M. Nielsen.