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PUBLICAÇÕES

IFUSP/P-1187

STUDY OF EXCHANGE NONLOCAL EFFECTS ON FUSION BARRIER DISTRIBUTIONS

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Novembro/1995

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The barrier distribution obtained from an unfolding of the nuclear fusion cross section can reveal basic characteristics of the possible channels involved in the fusion process. In this paper we discuss how the nonlocal effects of exchange origin can affect the description of the nuclear fusion tunnelling process and consequently the results for the barrier distribution also. The analysis is performed in a simple schematic model and an application to a nuclear system is also presented.

I. INTRODUCTION

Nuclear fusion reactions are known to constitute a convenient way of investigating the nucleus-nucleus potential barrier and the peculiar nuclear effects in the fusion mechanism. The theoretical description of the physical process of nuclear fusion involves classical degrees of freedom, as represented for example by the relative motion coordinate, as well as the

subtleties of quantum effects, as the transmission function of the nucleus-nucleus barrier. In the past years it was clearly demonstrated that besides those effects just mentioned it was necessary to introduce some additional nuclear structure effects in order to bring the theoretical results in agreement with the experimental data. In this sense, the coupling of channels, representing some internal degrees of freedom associated to the nuclear structure, to the relative motion can be viewed as an efficient way of enhancing the nuclear fusion cross section. Furthermore, it has been shown that, to a certain extent, it is possible to treat the heavy ion fusion cross section, σ, as been produced by a set of eigenchannels which can give rise to a barrier distribution that can distinguish those channels participating in the fusion process. It has been also established in this approach that the quantity $d^2(E\sigma)/dE^2$ has, in the classical limit ($\hbar \to 0$), discontinuities in the values of the barrier height in the channel α, B_α, which allow to obtain B_α as well as the weight of the corresponding channel, w_α, in the distribution directly from σ [1]. Quantum tunnelling effects are known to smear the effect of the presence of B_α in a tunnelling width of $\sim 0.56\ \hbar\omega$, where ω is the harmonic oscillator frequency associated to the curvature of the top of the barrier. Even though, one can still obtain a distribution with discernible structure if the channel coupling is strong [2].

In recent papers [3,4], it was shown that kinematical nonlocal effects of exchange nature – as opposed to the nonlocal contributions coming from the coupled channels– that must also be present in the nucleus-nucleus potential, modify the quantum tunnelling process as compared with a local potential calculation. In that approach those nonlocal effects were taken into account through an effective mass of the system which is then a function of the relative motion coordinate. Moreover, if one uses a contact model between the colliding nuclei in order to describe the separation dependence of the effective mass, it is possible to straightforwardly calculate the fusion cross section which embodies those nonlocal effects. The main result obtained from the new fusion cross section analytical expression, obtained through the effective reduced mass model, is that one can recognize the origin of the modification introduced by the kinematical nonlocal effects, namely, they redefine the barrier curvature $\hbar\omega$ in such a form to enhance the transmission, contrary to what is verified in the schematic model by Dasso [5] in which a family of barriers of different heights is generated by the coupling of the relative motion to some internal degrees of freedom.

In the present paper we consider again the same approach used before to study the kinematical nonlocal effects in the fusion cross section in order to analyze the consequences of their presence in barrier distributions through the quantity $d^2(E\sigma)/dE^2$.

This paper is organized as follows. In section 2 we present the essentials of the distribution of barrier formalism. The discussion of the nonlocal effects on the tunnelling process and consequently on the fusion cross section is described in section 3. Finally in section 4 we discuss how the nonlocal effects manifest themselves in the analysis of the barrier distribution and an example is presented to illustrate our considerations.

II. THE DISTRIBUTION OF BARRIERS METHOD

An schematic and simple way of taking into account the structure properties of the colliding nuclei in a fusion reaction is given by the coupled channels calculation [5,6], where the internal degrees of freedom of the nuclei are coupled to the variable associated to the relative motion. The fusion cross section is then given by

$$\sigma_f(E) = \sum_\alpha w_\alpha \sigma_\alpha(B_\alpha) \ , \tag{1}$$

where $\sum_\alpha w_\alpha = 1$ and $\sigma_\alpha(B_\alpha)$ are fusion cross sections calculated using the simple barrier penetration formulation, for instance, Wong's expression [7]

$$\sigma_W = \frac{\hbar\omega}{2E}R^2 \ln\left\{1+\exp\left[\frac{2\pi}{\hbar\omega}(E-B)\right]\right\} \ , \tag{2}$$

where $\hbar\omega$, R and B are the curvature, radial position and height of the fusion barrier respectively. The direct interpretation of Eq. (1) is that fusion process occurs through certain given intermediate states distinguished by the label α. The intermediate state α faces a barrier with an effective height given by B_α, which is given by the maximum of the function $V(r)+\lambda_\alpha F(r)$, being λ_α the eigenvalues of the coupling matrix, $F(r)$ is the strength of the coupling and w_α is the probability of the system attain the state α.

Let us consider now the fusion cross section written as a continuum weighted average over the ensemble of barriers [2]

$$\sigma_f(E) = \int_0^\infty \sigma(E,B)\, D(B)\, dB \tag{3}$$

with the normalization condition

$$\int_0^\infty D(B)\, dB = 1 \ . \tag{4}$$

By comparing Eqs. (3) and (1) we are lead to a possible form for $D(B)$, namely,

$$D(B) = \sum_\alpha w_\alpha \delta(B - B_\alpha) \ . \tag{5}$$

In a more general way, a classically parametrized distribution $D_c(B)$ was found [1] by the use of the classical expression for the barrier radius R in Eq. (3),

$$\begin{aligned}\sigma(E,B) &= \sigma_c(E,B) = \pi R^2\left(1-\frac{B}{E}\right) \text{ for } E > B \\ &= 0 \text{ for } E < b\end{aligned} \tag{6}$$

which leads to the following relation concerning the single barrier problem

$$d^2(E\sigma(E,B))/dE^2 = \pi R^2 \delta(E-B) \ . \tag{7}$$

Substituting Eq. (7) in Eq. (3) we arrive at an expression which corresponds to the total cross section

$$\frac{1}{\pi R^2}\frac{d^2(E\sigma_f(E))}{dE^2} = D(E) \tag{8}$$

such that, in principle at least, could be directly used to extract the barrier distribution from the fusion experimental data.

It is important to stress, however, that the classical formula (6) does not take into account quantum tunneling effects. A convenient way of introducing these effects is to use Wong's formula, Eq. (2), derived for a parabolic barrier. In this case, one finds the analogue of Eq. (7) which is now written as

$$\frac{1}{\pi R^2}\frac{d^2\left(E\sigma_W\right)}{dE^2}=G\left(x\right) \tag{9}$$

where

$$G\left(x\right)=\left[\left(\frac{2\pi}{\hbar\omega}\right)\frac{e^x}{\left(1+e^x\right)^2}\right] \tag{10}$$

and

$$x=\left(\frac{2\pi}{\hbar\omega}\right)\left(E-B\right) \tag{11}$$

Since $\int G\left(x\right)dE=1$ and $G\left(x\right)\rightarrow\delta\left(E-B\right)$ for $\hbar\omega\rightarrow 0$, the function $G\left(x\right)$ is a generalization of the delta function appearing in Eq. (7), where the width now is a manifestation of the quantum tunneling effect, being its energy width of 0.56 $\hbar\omega$. Thus, in order to obtain a conspicuous barrier distribution it is necessary that $D(B)$ present a width larger than around 2 MeV (since $\hbar\omega\approx 4$ MeV) and that is expected to occur in the strong coupling regime [2].

Now, introducing Eq. (2) in Eq. (3) one gets another expression which substitutes relation (8), namely

$$\begin{aligned}\frac{1}{\pi R^2}\frac{d^2\left(E\sigma_f\left(E\right)\right)}{dE^2}&=\int_0^\infty D\left(B\right)G\left(x\right)dB\\&\equiv\bar{D}\left(E\right)\end{aligned} \tag{12}$$

which, upon using expression (5) for $D\left(B\right)$, leads to

$$\frac{1}{\pi R^2}\frac{d^2\left(E\sigma_f\left(E\right)\right)}{dE^2}=\sum_\alpha w_\alpha G\left(x_\alpha\right)\ , \tag{13}$$

with

$$x_\alpha=\left(2\pi/\hbar\omega\right)\left(E-B_\alpha\right)\ . \tag{14}$$

In this form, the experimental quantity appearing on the left hand side of Eq. (12) is directly related to the distribution (5) which is now modified by the quantum tunnelling factor $G\left(x\right)$. Therefore, since $D(b)$ is extracted through a second order derivative calculation, it is necessary to have very precise experimental data in order to obtain realistic results. Unfortunately this leads to a function that is weakly defined in the region where σ is large, i.e., at energies above the Coulomb barrier.

III. NONLOCAL EFFECTS IN THE BARRIER DISTRIBUTION

An important consequence of the fermionic character of colliding heavy ion many-body systems is that the interaction between them is nonlocal [8]. The kinematical nonlocal character of the nucleus-nucleus potential reflects the correlations that will appear in the total wave function of the system and will produce important effects in the reaction involving those nuclei when their density functions begin to overlap. These correlations can be understood as exchange effects in the total wave function that will be a function of the separation between the colliding nuclei. These kinematical nonlocal effects, not initially taken into account in the real parametrized nucleus-nucleus potential, can be shown to introduce important modifications in the total barrier transmission coefficient calculation for a problem which also includes coupled channels [3,4]. For simplicity, assuming a simple geometrical model in which the effective reduced mass of the system changes with the density distribution profile of the colliding nuclei only at the radius of the barrier, R, defined by the total potential, we are led to

$$T = \sum_{\alpha} \mid \langle 0 \mid \alpha \rangle \mid^2 T\left(E, V\left(x\right) + \lambda_\alpha F(R); b\right) \ , \tag{15}$$

where $\mid 0 \rangle$ denotes the ground state of the nuclei, $\mid \langle 0 \mid \alpha \rangle \mid^2 \equiv w_\alpha$, and b is the nonlocality range. These coefficients have already been obtained through the use of the Weyl-Wigner quantum phase space formalism in the Feynman path integral approach [4] for a Perey and Buck-like nonlocal potential and are written for the present case as

$$T\left(E, V\left(x\right) + \lambda_\alpha F(R); b\right) = \left\{ 1 + \exp\left[2 \int_{x_1}^{x_2} \sqrt{\frac{2\mu\left(x; b\right)}{\hbar^2} \left(V\left(x\right) + \lambda_\alpha F(R) - E\right) dx} \right] \right\}^{-1} \ , \tag{16}$$

where x_1 and x_2 are the classical turning points of the potential barrier $V\left(x\right)$ and $\mu\left(x; b\right)$ is the reduced effective mass which embodies the kinematical nonlocal effects and is written, in the contact model [3], as

$$\mu(x;b) = \begin{cases} \mu & , \quad x > R \\ \mu/\left(1 + \frac{\mu b^2}{2\hbar^2} \mid V_N(R) \mid\right) & , \quad 0 \le x \le R \end{cases}, \tag{17}$$

$V_N(R)$ is the nuclear potential calculated at the contact radius R. Furthermore, if we use a parabolic approximation for $V(x)$ and consider that the nonlocal effects only produces a small change in the reduced mass at the barrier, then the coefficients $T(E, V(x) + \lambda_\alpha F(R); b)$ satisfy the following form [3]

$$T(E, V(x) + \lambda_\alpha F(R); b) = \left\{1 + \exp\left[\frac{2\pi(B + \lambda_\alpha F(R) - E)}{\hbar\omega_B} S(b, R)\right]\right\}^{-1}, \tag{18}$$

where

$$S(b, R) = 1 - b^2 \frac{f(R)}{4}, \tag{19}$$

and

$$f(R) = \frac{\mu}{2\hbar^2} |V_N(R)| . \tag{20}$$

The total fusion cross section can then be calculated by adopting the approximations proposed by Wong [7], and is written as

$$\sigma_f^{NL}(E; b) = \sum_\alpha w_\alpha \frac{R^2 \hbar\omega}{2ES(b, R)} \ln\left\{1 + \exp\left[\frac{2\pi}{\hbar\omega}(E - (B + \lambda_\alpha F(R))) S(b, R)\right]\right\} . \tag{21}$$

At this point it is important to note that the factor $S(b, R)$, coming from the effective mass, redefine the potential curvature $\hbar\omega$ while keeping the barrier height $(B + \lambda_\alpha F(R))$ and the weights w_α unaltered.

The nonlocal effects embodied in the present effective mass approach are essentially of kinematical nature, i.e., it comes mainly from the exchange character of the Perey and Buck [9] ansatz as discussed by Rawitscher [10] and do not substantially overlap with the nonlocal effects which will also come in from possible relevant channels to the fusion. However, during the fusion processes, their contributions will produce, in an entangled way, a strong modification in the nuclear barrier whose height and width will change simultaneously. In spite of this, as a consequence of the models used here, expression (21) shows how the kinematical nonlocal effects introduced here mix together the coupled channels contributions to the total fusion cross section and it also displays how the separated contributions manifest themselves on the barrier distribution $D(B)$. It is immediate to see that the substitution of expression (21) in Eq. (8) leads to

$$\frac{1}{\pi R^2}\frac{d^2\left(E\sigma_f^{NL}\right)}{dE^2}=\sum_\alpha w_\alpha G\left(\tilde{x}_\alpha\right) \tag{22}$$

with

$$\tilde{x}_\alpha=\frac{2\pi}{\hbar\tilde{\omega}}\left(E-B_\alpha\right) \tag{23}$$

where

$$\hbar\tilde{\omega}=\frac{\hbar\omega}{S\left(b,R\right)}, \tag{24}$$

thus giving $\tilde{x}_\alpha = S\left(b,R\right)x_\alpha$. The main modification introduced in the barrier distribution equation is that the function $G\left(\tilde{x}_\alpha\right)$ now presents a diffuseness induced by the nonlocal effects on the barrier tunnelling whose energy width is 0.56 $\hbar\tilde{\omega}$.

IV. RESULTS AND DISCUSSION

Exchange nonlocal effects in nucleus-nucleus colliding systems were taken into account in a simple model through a contact hypothesis for the effective reduced mass and it was shown how these effects can modify the barrier transmission coefficient and, consequently, the fusion cross section also. In this model, the hallmark of these nonlocal effects in the nuclear barrier is the redefinition of its curvature which becomes narrower than the standard local corresponding function. At the same time, the introduction of the relevant coupled channels, as described by the simple model proposed by Dasso [5], is known to generate a family of barriers of different heights. The combination of these two effects are then responsible for a strong modification of the nuclear fusion barrier during the fusion process.

In this connection, we can discuss, at least within the models realm, how the combined effect of these two contributions alter the barrier distribution predictions, as discussed previously.

Before we verify the consequences of the exchange nonlocal effects on the barrier distribution $D(B)$ in multi-channel cases, we are going to investigate those effects on the quantity $d^2(E\sigma)/dE^2$ for the fusion cross section present in an one-channel calculation through the comparison between the results coming from Eq. (9) and those from Eq. (22) with $w_\alpha = 1$ and $\lambda_\alpha = 0$ for the $^{16}O +^{154} Sm$ system [2]. The potential parameters used in these calculations are $B_0 = 58.5$ MeV, $R = 10.5$ fm and $\hbar\omega = 4.3$ MeV. The nonlocal range is assumed to be $b = 0.85$ fm, that is, the value usually associated to the nonlocality coming from the exchange effects only [3,10]. Figure 1 shows the results from local calculations, Eq. (9), (full curve) and those from nonlocal ones, Eq. (22), in the no-coupling limit (dashed curve). We see that when we take into account the exchange effects on the fusion reaction, introduced in a phenomenological way through a Perey-Buck-like potential, the width of the distribution for a single channel is enhanced with respect to the local width by a factor $1/S(b, R)$, as it is immediate to verify from Eq. (24) too.

Now, in order to investigate the exchange nonlocal effects on the barrier distribution in a multi-channel case, we analyze the fusion of the same system with local and nonlocal coupled channel calculations. Here we use a three-channel model in which the relative motion is coupled to rotational states $(0^+ \rightarrow 2^+, 4^+)$ of the deformed target, as already discussed in [11]. The couplings give rise to three barriers $B_\alpha = B_0 + \lambda_\alpha F(R)$, where $F(R) = 2.15$ MeV is the coupling strength at the barrier radius, being the barriers identified by the following set of parameters $(w_\alpha, \lambda_\alpha) = (0.17, -1.80); (0.36, -0.35)$ and $(0.47, 0.93)$ respectively.

Figure 2 shows the results of the calculations performed with expressions (13) and (22), corresponding to local and nonlocal models respectively. The behavior of $d^2(E\sigma)/dE^2$ with E, for the local model (full curve) when three channels are taken into account, displays structures which clearly identify the different barrier heights through the positions of the peaks in energy present in the distribution. The relative weight of the corresponding channel in the composition of the cross section is proportional to the height of the peak. Now, as one increases the number of the coupled channels in the calculation $\bar{D}(E)$ looses these structures until the manifestations of the presence of the individual barriers vanish; however $\bar{D}(E)$ will retain an well defined overall structure which, in the limit of coupling to all members of the rotational band, will be equivalent to that one obtained when all the orientations of the deformed target are considered [2].

The results of the nonlocal model (dashed curve) calculation include the same channels as those considered in the local case. It is remarkable that the individual structures of the local model do not show up here and only the overall behavior of the distribution is kept. However, $\bar{D}(E)$ lost its structures not because more barriers were included in the calculations, but because the individual distributions were widened due to the redefinition of the curvature $\hbar\omega$. In spite of this spreading of the individual distributions in the nonlocal results, the overall width of this function is approximately the same as the local one. Thus, a few coupled channels plus exchange nonlocal effects produce a similar barrier distribution to that obtained from a coupled channels calculation with a large number of eigenchannels.

We considered here a particular model where only couplings to rotational states are taken into account because of the available local results [11] and also because we have adopted from the very beginning an schematic model which is able to exhibit the main features of the interplay between the exchange nonlocality and coupled channels [4]. However, even without considering other kinds of coupling, for instance to transfer channels and to excited phonon states(although it was shown that these couplings produce a distribution with well defined peaks [12]) we are able to describe, at least in the model validity range, the exchange nonlocal effects on barrier distributions in a simple way.

Acknowledgement The authors want to thank Prof. D. Pereira from Universidade de São Paulo, Brazil, for valuable suggestions. M.A.C.R. was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo, FAPESP, and D.G. was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico, CNPq, Brazil.

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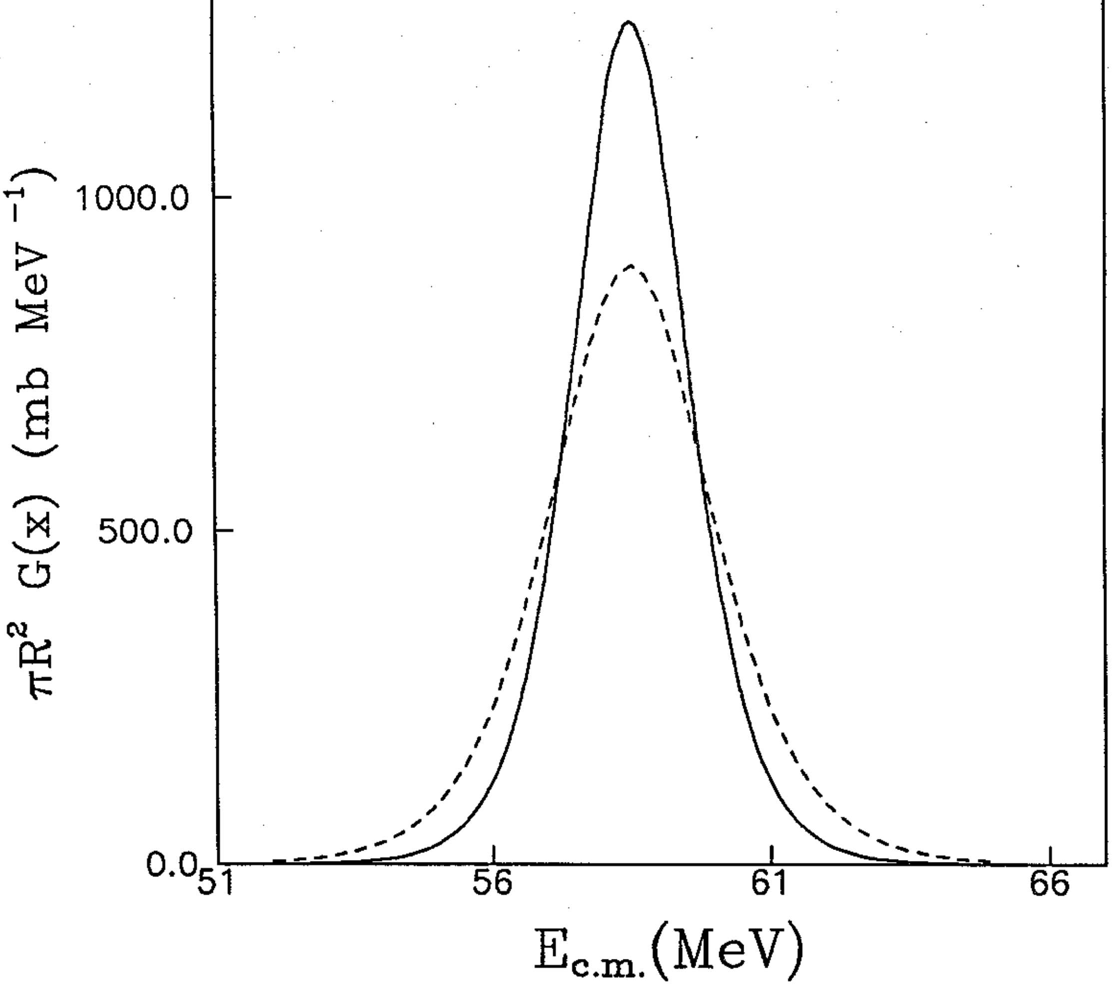
VI. FIGURE CAPTION

Figure 1

The expression for $\pi R^2 G(x)$ as obtained from equations (9) (local - full curve) and (22) with $w_\alpha = 1$ and $\lambda_\alpha = 0$ (nonlocal - dashed curve) for ${}^{16}O + {}^{154}Sm$.

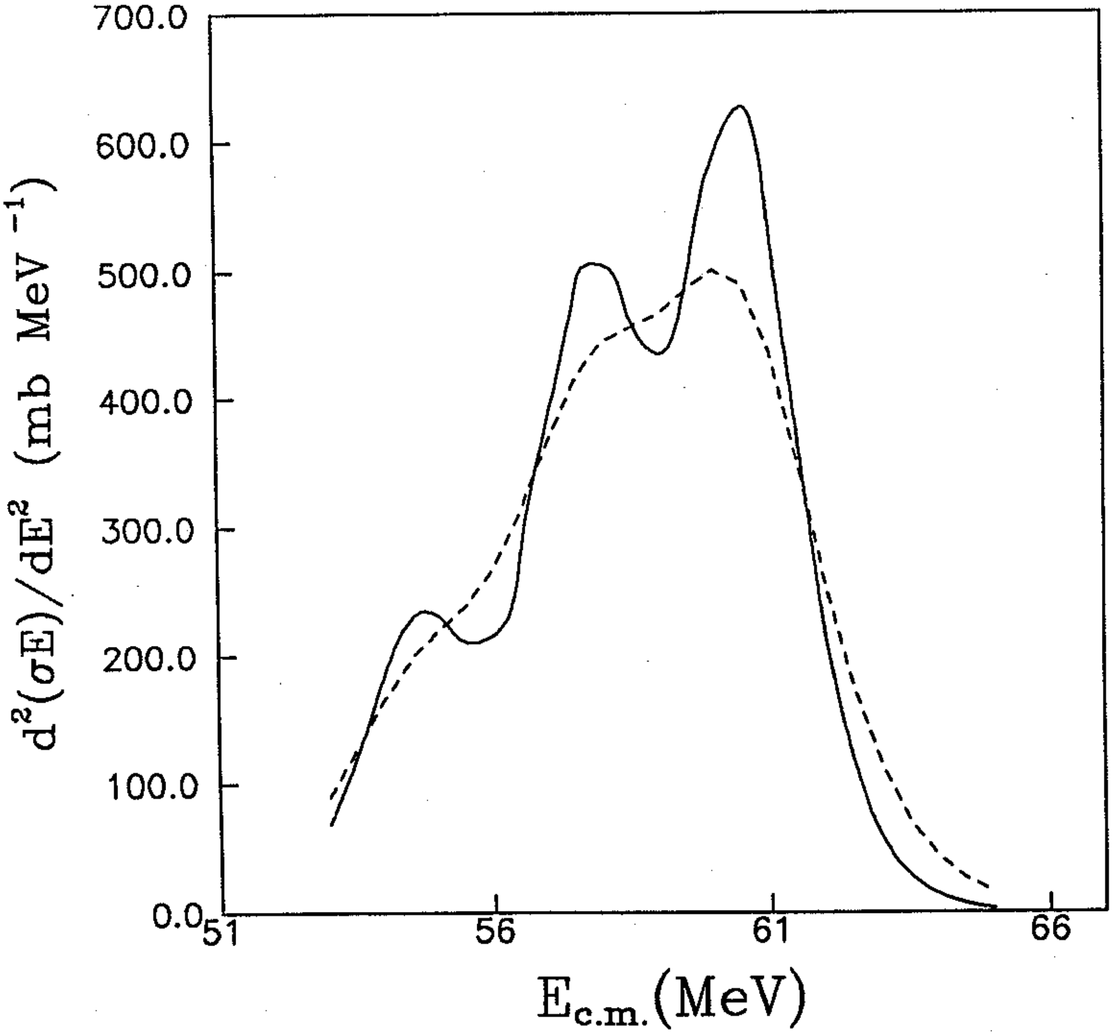
Figure 2

Theoretical values for $d^2\left(E\sigma\left(E,B\right)\right)/dE^2$ for the ${}^{16}O + {}^{154}Sm$ system which includes the coupling to the 2^+ and 4^+ states. The full curve represents the local potential calculation and the dashed curve the nonlocal one with the nonlocal parameter $b = 0.85$ fm.



1000.0
500.0
0.0
51
56
61
66
$\pi R^2\ G(x)$ (mb MeV^{-1})
$E_{c.m.}$(MeV)

Fig. 1



700.0
600.0
500.0
400.0
300.0
200.0
100.0
0.0
51
56
61
66
$d^2(\sigma E)/dE^2$ (mb MeV^{-1})
$E_{c.m.}$(MeV)

Fig. 2