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Q-DEFORMED ALGEBRAS AND MANY-BODY PHYSICS*

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A review is presented of some applications of $q-$deformed algebras to many-body systems. The rotational and pairing nuclear problems will be discussed in the context of $q-$deformed algebras, before presenting a more microscopically based application of $q-$deformed concepts to many-fermion systems.

I. INTRODUCTION

In the last decade a great effort has been devoted to the development and understanding of deformed algebras, although their direct physical interpretation is sometimes incomplete or even completely lacking. In some cases like the XXZ-model, where the ferromagnetic/antiferromagnetic nature of a spin $\frac{1}{2}$ chain of length N can be simulated through the introduction of a q-deformed algebra [1], or the rotational bands in deformed nuclei and molecules which can be fitted via a q-rotor Hamiltonian [2–4], instead of using the variable moment of inertia (VMI-model), is the physical meaning of such a deformation established. From the original studies which appeared in connection with problems related to solvable statistical mechanics models [5] and quantum inverse scattering theory [6], a solid development has emerged which encompass nowadays various branches of mathematical problems related to physical applications, such as deformed superalgebras [7], knot theories [8], non-commutative geometries [9] and so on. In this context, the introduction of a q-deformed bosonic harmonic oscillator, derived in such a way to pass from a $su(2)$ symmetry, originally present in the non-deformed case, to a $su_q(2)$ one, gave origin to new commutation relations which have been extensively studied in several papers [10–12] being all these results unambiguously obtained due to the underlying $sl_q(2)$ structure [13].

The nucleus is a finite quantal many-baryon system. It provides a scenario where electromagnetic, weak and strong interactions play their role altogether. The nucleus may be, therefore, a natural place to look for manifestations of new symmetries and/or deviations from old ones.

A plethora of models have been developed to deal with the physics of such a complex system, some of them with more phenomenological foundations, for instance the nuclear collective model, being others based on the underlying fermionic structure of the nuclear many body system, like the mean-field plus residual interaction description of the nuclei.

The many-body problem in all its complexity calls for the use of approximate methods or the development of simple solvable models which should entail most of the relevant physics combined with a technically simple treatment [14]. A long heritage of such models is available in the nuclear physics literature, among which the Lipkin model [15] has been extensively used as a laboratory to test approximate methods and to point out the main features of the many-body systems.

From the point of view of $q-$deformed algebra applications to physical systems it is

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*Dedicated to Prof. Paulo Leal Ferreira on his 70th birthday. His scientific contribution to the development of Theoretical Physics, particularly in our country, will never be acknowledged enough.

important to understand how the basic characteristics and the general behavior of many-body systems are modified when the underlying algebra is deformed. The use of q-deformed algebra in the description of some many-body systems has lead to the appearance of new features when compared to the non-deformed case. In this connection we mention some examples: a) in the q-oscillator many-body problem [12] it was shown that, when promoting the symmetries of the standard oscillator system to q-symmetries, the spectrum of the system is found to exhibit interactions between the levels of the individual oscillators, b) the revivals phenomenon present in the Jaynes-Cummings model [16] disappears when the original $su(2)$ symmetry is deformed, c) an extensive study of a deformed collective Lipkin Hamiltonian was performed and the q-deformed second order phase transition was found to be suppressed [17]. The second order phase transition associated to the spherical symmetry breaking in the quasi-spin space [18] for this deformed model was also discussed in the q-coherent states framework [19]. A recent paper [20] has shown, in the framework of the $q-$deformed Lipkin Hamiltonian, the importance of a careful treatment of the mean field, when dealing with $q-$deformed fermionic systems.

In this talk different aspects of the application of $q-$algebras to many-body physics will be presented. After a short review of the basic concepts of $q-$deformed algebras in section 2, "phenomenological" applications of $q-$ algebras in nuclear physics will be discussed in section 3. Section 4 will deal with $q-$deformed many fermion systems in the context of a q-fermionic extension of the standard Lipkin model (SLM). In this section the discussion will be concentrated on the phase transition behavior of fermionic $q-$deformed systems, both at zero and finite temperatures. Finally some conclusions will be drawn on section 5.

II. A BRIEF REVIEW OF $SU_Q(2)$ DEFORMED ALGEBRAS

The concept of deformed algebra emerged in the eighties and is still object of continuous developments in mathematics and physics. It was introduced in the context of exactly soluble statistical models, integrable systems in field theory, non-commutative geometry and other

fields. Of particular interest are the developments by Biedenharn [10] and Macfarlane [11] on the $q-$analogues of the quantum harmonic oscillator. If the importance of the harmonic oscillator in many branches in physics will be reproduced in the context of $q-$algebras, it can be expected that its study may provide some guidance in this new field.

The quantum algebra $su_q(2)$ is a deformation of the Lie algebra of the $SU(2)$ group. Deformation in this context does not mean exactly what we may be acquainted with, for instance in a non spherical nucleus or a molecule, but rather deformation here is to be understood as modifications in the commutation relations among the generators of the algebra according to given, although not unique, prescriptions. A possible realization of the $su_q(2)$ quantum algebra in terms of three hermitian operators J_+, J_- and J_0 is shown below along with the usual $su(2)$ algebra:

$$\boxed{\begin{array}{ccc} su(2) & & su_q(2) \\ [J_0, J_\pm] = \pm J_\pm & \rightarrow & [J_0, J_\pm] = \pm J_\pm \\ & & \\ [J_+, J_-] = 2J_0 & \rightarrow & [J_+, J_-] = [2J_0]_q \end{array}} \tag{2.1}$$

The new quantity $[x]_q$ appearing above is a $q-$number and it can be defined as,

$$[x]_q = \frac{q^x - q^{-x}}{q - q^{-1}} = \frac{\sinh \gamma x}{\sinh \gamma}, \tag{2.2}$$

where $q = e^\gamma$ may be a complex number. $Q-$numbers go to the usual numbers as $q \rightarrow 1$ (or $\gamma \rightarrow 0$), meaning that the well known commutation relations of the $su(2)$ algebra are recovered in this limit.

The quadratic Casimir operator of $su_q(2)$ given by,

$$C_q = J_+J_- + [J_0]_q[J_0 - 1]_q, \tag{2.3}$$

still commutes with the generators of the algebra. Jimbo [21] has shown that, given Eqs. (2.1), exists a representation, for each j ($j = 0, 1/2, 1, ...,$) with basis $|jm\rangle$ $(-j \leq m \leq j)$, such that,

$$J_0 \left| jm \right\rangle = m \left| jm \right\rangle \tag{2.4}$$

$$J_\pm \left| jm \right\rangle = \sqrt{[j \mp m]_q [j \pm m + 1]_q} \left| jm \pm 1 \right\rangle . \tag{2.5}$$

The $su_q(2)$ irreducible representations D^j are obtained from the maximum weight states. The basis states $\left| jm \right\rangle$ are connected to the maximum weight states $\left| jj \right\rangle$ in the following way,

$$\left| jm \right\rangle = \sqrt{\frac{[j+m]_q!}{[2j]_q![j-m]_q!}} (J_-)^{j-m} \left| jj \right\rangle , \tag{2.6}$$

where, $[n]_q! = [1]_q[2]_q....[n]_q$ and,

$$C_q \left| jm \right\rangle = [j]_q [j+1]_q \left| jm \right\rangle . \tag{2.7}$$

Analogously to the standard construction of irreducible representations of $su(2)$ due to Schwinger [22], Macfarlane proposed a way to write $\mathbf{J}$ in terms of the creation and destruction operators of a pair of independent $q-$deformed harmonic oscillator degrees of freedom [11]. $Q-$bosons can be defined through the commutation relations,

$$a_i a_i^\dagger - q a_i^\dagger a_i = q^{-N_i} \tag{2.8}$$

$$[N_i, a_i^\dagger] = a_i^\dagger \text{ and } [N_i, a_i] = -a_i, \tag{2.9}$$

where $a_i^\dagger$ (a_i) are creation (annihilation) operators and N_i is the corresponding number operator. The whole $su_q(2)$ spectrum can now be described by two commuting oscillators,

$$\left| jm \right\rangle = \frac{(a_1^\dagger)^{j-m}}{\sqrt{[j-m]_q!}} \frac{(a_2^\dagger)^{j+m}}{\sqrt{[j+m]_q!}} \left| 0 \right\rangle . \tag{2.10}$$

In terms of $q-$bosons the generators of $su(2)$ can be written as,

$$J_+ = a_1^\dagger a_2, \; J_- = a_2^\dagger a_1, \; 2J_0 = N_1 - N_2. \tag{2.11}$$

We would like to call the reader's attention to the fact that the above way of $q-$deform the $su(2)$ algebra is by no means unique [23]. In fact, the lack of uniqueness in deriving $q-$deformed objects is a source of skepticism on the reliability of the application of $q-$algebras to physical systems. As a matter of fact, the unclear (if any) physical meaning of the $q-$deformation is another source of such a skeptical attitude. In the next section we will briefly review two examples where the physical meaning of the $q-$deformation seems to be established.

III. $Q-$DEFORMED ALGEBRAS APPLIED TO NUCLEAR PHYSICS

This section will deal with some applications of $q-$deformation to nuclear systems. Two problems will be discussed: a) pairing in a single j shell , and b) the spectra of rotational nuclei. Both cases have a $su(2)$ structure and the $q-$deformation will be performed at the level of the generators of the algebra. We have called these two situations "phenomenological" because the $q-$deformation of the algebra is performed irrespectively of the underlying fermionic structure.

A. Pairing in a single j shell

Pairing plays an important role in the structure of nuclei. It is by far the most important part of the residual interaction around magic nuclei. Among the very many applications of pairing model to nuclei, the ^{40}Ca isotopes represent a particularly simple situation, since the $1f_{7/2}$ level is fairly isolated from the others and can therefore be considered as a single j shell.

In this case, the pairing Hamiltonian can be written,

$$H_{pairing} = -\frac{G}{4} \sum_{jm,j'm'} (-)^{j-m} (-)^{j'-m'} c_{j'm'}^\dagger c_{j'-m'}^\dagger c_{j-m} c_{jm} , \tag{3.1}$$

where $c_{jm}^\dagger (c_{jm})$ is the creation (annihilation) operator in the single j shell orbit. G is the pairing strength.

Due to the underlying $su(2)$ structure of the single j shell pairing Hamiltonian, it can be written in terms of the quasi-spin operators [14],

$$H_{pairing} = -GS_+S_- = -GS^2 - GS_0(S_0 - 1)), \tag{3.2}$$

where $S_+ = \sum_{m>0}(-)^{j+m}c^\dagger_{jm}c^\dagger_{j-m}$, $S_- = \sum_{m>0}(-)^{j+m}c_{j-m}c_{jm}$ and $S_0 = \frac{1}{2}(\sum_m c^\dagger_{jm}c_{jm} - \Omega)$ satisfy quasi-spin (angular momentum) commutation relations. $\Omega = \frac{1}{2}(2j+1)$ is the pair degeneracy. The eigenstates are labeled by the number of particles n and by the number of unpaired particles ν (seniority quantum number) giving rise to the eigenvalues,

$$E(n,\nu) = -\frac{G}{4}(n-\nu)(2\Omega - \nu - n + 2).$$

The $q-$deformation of the quasi-spin pairing Hamiltonian in Eq. (3.2) can be easily performed by using Eq. (2.3) and rewriting it in terms of the Casimir operators of $su_q(2)$,

$$H^q_{pairing} = -GC_q - G[S_0]_q[S_0 - 1]_q. \tag{3.3}$$

Like in the non-deformed case, n and ν label the eigenstates of $H^q_{pairing}$ and their eigenvalues are given by [24],

$$E_q(n,\nu) = -G\frac{(q^{\frac{1}{2}(n-\nu)} - q^{-\frac{1}{2}(n-\nu)})(q^{(\Omega-\frac{1}{2}(\nu+n)+1)} - q^{-(\Omega-\frac{1}{2}(\nu+n)+1)})}{(q-q^{-1})^2}. \tag{3.4}$$

Defining a state dependent parametrization for the deformation of the algebra, $q = \exp(\frac{1}{2}(n-\nu)\gamma')$, we obtain,

$$E_q(n,\nu) = -G\frac{\sinh[\frac{1}{2}(n-\nu)^2\gamma']\sinh\{[\Omega - \frac{1}{2}(\nu+n)+1]\frac{1}{2}(n-\nu)\gamma'\}}{\sinh^2(\frac{1}{2}(n-\nu)\gamma')}. \tag{3.5}$$

Figure 1 shows the corresponding results for the ground state energies of the even-A Ca isotopes.

The results are rather good, even considering the, non usual, state dependent parametrization used. This can be understood if we Taylor expand a typical term appearing in $H^q_{pairing}$, namely,

$$\frac{q^{S_0} - q^{-S_0}}{q - q^{-1}} \approx S_0 + [(S_0-1)^2 + (S_0-3)^2 + \ldots](\delta q)^2. \tag{3.6}$$

Therefore, it seems that, by $q-$ deforming the $su(2)$ algebra, higher order terms of the residual interaction are taken into account in an effective way.

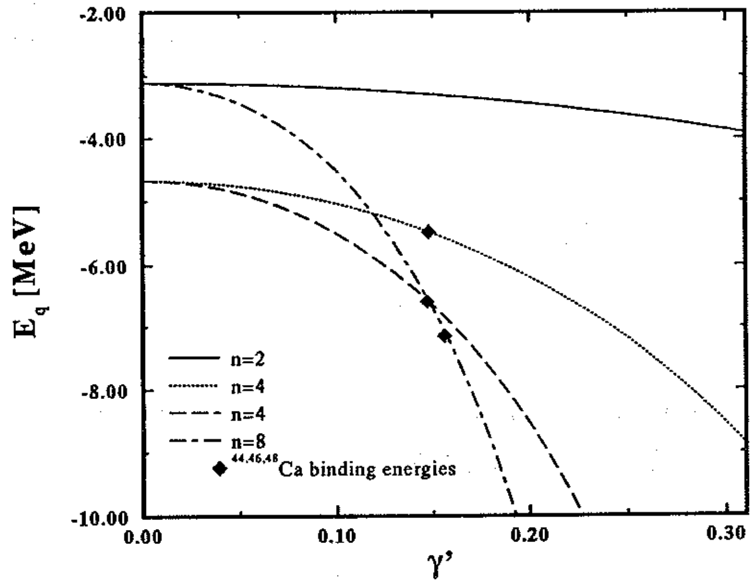

FIG. 1. This figure shows the binding energies of even A Ca isotopes as a function of the deformation parameter γ'. n=2,4,6,8 correspond to $^{42,44,46,48}Ca$ respectively. The diamonds are the experimental data for $^{44,46,48}Ca$. The pairing strength was adjusted to the ^{42}Ca binding energy.

B. The spectra of rotational nuclei

This case has some similarities with the pairing one. The $q-$deformed Hamiltonian for rotational nuclei can be written in terms of the usual one simply substituting the $su(2)$ Casimir operator by its deformed version:

$$\boxed{H_{rotor} = E_0 + \frac{1}{2I}J(J+1) \rightarrow H^q_{rotor} = E_0 + \frac{1}{2I}C_q,} \tag{3.7}$$

where E_0 is the band-head energy and I is the moment of inertia.

There are (at least) two different approaches:

1) q is a phase ($q = e^{i\gamma}$) and $\mathcal{C}_q = J_+J_- + [J_0]_q[J_0 - 1]_q$. Rather good fits to the rotational spectra of nuclei and molecules are obtained.

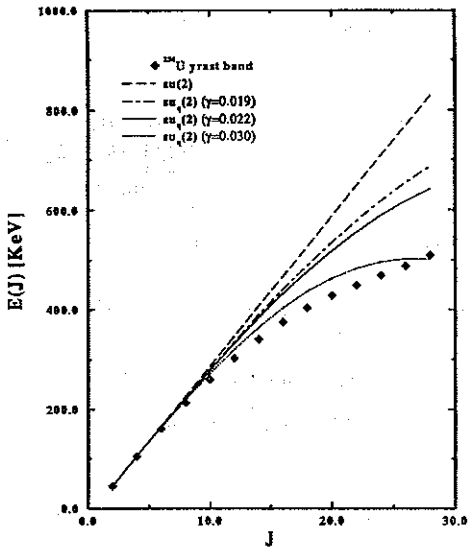

FIG. 2. Experimental transition energies $\Delta E(J) = E(J) - E(J-2)$ for the yrast states of ^{236}U are shown along with the $su(2)$ and $su_q(2)$ predictions calculated with three different values of the deformation parameter γ.

Figure 2 presents the ^{236}U experimental transition energies along with fittings using the non-deformed and deformed rotor Hamiltonians. Three different values of the deformation parameter γ are presented and for $\gamma = 0.030$ the agreement is quite good. The reason for this success is simple: $\mathcal{C}_q$ is equivalent to an expansion in powers of $j(j+1)$:

$$E_j = E_0 + a(q)j(j+1) + b(q)[j(j+1)]^2 + ...$$

Moreover, the coefficients $a(q), b(q), ...$ have alternating signs ($a > 0$) and magnitudes in agreement with phenomenology [3]. On the other side, looking at the corresponding transi-

tion probabilities (Fig.3) [25], this success looks less spectacular: good agreement is with $\gamma = 0.019$ whereas $\gamma = 0.030$ is completely off. This clearly should not be considered a difficulty of the $q-$deformation scheme but rather it is an inherent dificulty in any attempt to get good descriptions of both spectra and wave functions.

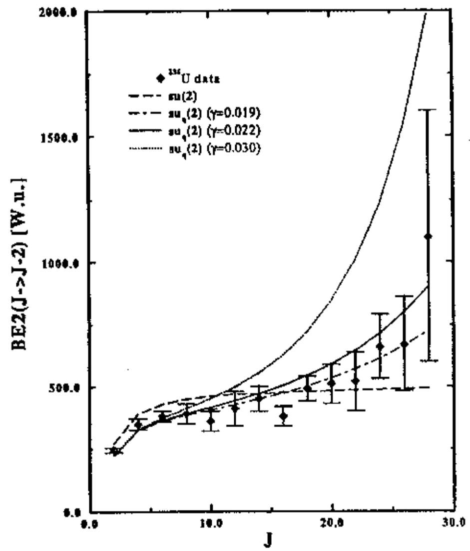

FIG. 3. The same as Fig 2, but for the transition probabilities $BE2(J \to J-2)$.

We would like to call the readers' attention to a difficulty of this approach: if q is a phase, it is not clear how to calculate the norm of the $q-$deformed bosonic state. This can be considered as a minor problem since we are just fitting the data, however this may bring some doubts on the correctness of this deformation scheme, as applied to rotational nuclei.

2) q is real and the $q-$deformed Casimir is taken to be [26],

$$\mathcal{C}_q = \frac{[4]_p}{2[2]_p} \frac{[2J+1]_p^2[2J]_p[2J+2]_p}{[4J+2]_p^2},$$

($p = \sqrt{q}$). In this case, also impressive good fits to rotational spectra of nuclei and molecules can be obtained [2]. We would like to emphasize that this way of $q-$deforming the algebra is free from the normalization disease pointed out previously.

IV. PHASE TRANSITION IN A $Q-$DEFORMED LIPKIN MODEL

Along the years, the Lipkin Hamiltonian [15] has been a theoretical laboratory to test models in many-body physics. It is exactly soluble and is a "quasi-spin like" model. Its structural simplicity will again be useful in an attempt to understand the influence of the $q-$deformation on the behaviour of physical systems. In the standard Lipkin model N fermions occupy two N-fold degenerate levels. The Hamiltonian of the SLM can be written as,

$$H_{Lipkin} = \frac{\epsilon}{2}\sum_{p,\sigma} \sigma a^{\dagger}_{p,\sigma} a_{p,\sigma} + \frac{V}{2}\sum_{p,p',\sigma} a^{\dagger}_{p,\sigma} a^{\dagger}_{p',\sigma} a_{p',-\sigma}\, a_{p,-\sigma}. \tag{4.1}$$

In the above expression p ranges from 1 to N and $\sigma = \pm 1$.

Galetti and Pimentel [17] studied the SLM in the deformed quasi-spin formalism to investigate the influence of the $q-$deformation in the phase transition. As a result they found the suppression of the phase transition with the increasing of q.

Along with this phenomenological approach, a question as emerged, namely, to understand how the general behavior of the many-body system is modified when the fermionic algebra is deformed.

In order to have some guidance we have looked at the bosonic case [12]:

In a system of M bosonic harmonic oscillators, the full Hamiltonian is not just the sum of the individual oscillators, but rather it is a sum of terms involving coproducts if we want to preserve the symmetry of the algebra when deformed, namely, $su(M) \to su_q(M)$.

For example, in the two oscillators case: $N_i = a_i^{\dagger} a_i$, $h_i = N_i + \frac{1}{2}$; $H_i = \frac{1}{2}([h_i + \frac{1}{2}]_q + [h_i - \frac{1}{2}]_q)$, $i = 1, 2$, we have,

normal	$q-$deformed
$[a_i, a_i^{\dagger}] = 1$	$a_i a_i^{\dagger} - q a_i^{\dagger} a_i = q^{-N_i}$
$H = h_1 + h_2$	$H^q = H_1 + H_2$
	$\hookrightarrow H^q = \dfrac{\sinh(\frac{1}{2}\gamma(h_1 + h_2))}{2\sinh(\gamma/2)}$

A similar prescription applied to the $q-$deformed Lipkin Hamiltonian gives rise to:

$$H^q_{Lipkin} = \frac{\epsilon}{4\sinh\frac{\gamma}{2}}\sinh(2\gamma S_0) + \frac{V}{2}(S_+^2 + S_-^2), \tag{4.2}$$

where S_0, S_+ and S_- are the quasi-spin operators for the $q-$deformed Lipkin Hamiltonian. As a consequence, the mean field is also $q-$deformed [20].

With this new collective deformed Hamiltonian we study the only phase transitions in this $q-$deformed model, which are of second order, *à la* Holzwarth [18], i.e. the spherical symmetry breaking in the quasi-spin space.

Q-analogues [27] of the $su(2)$ Perelomov coherent states [28] are used to define θ and φ as collective variables. The phase transition is analyzed through the behavior of the variationally obtained ground state energy functional,

$$E(\theta, \phi, \gamma, N) = \frac{\langle z|\, H^q_{Lipkin}\, |z\rangle}{\epsilon_q \langle z|z\rangle} = \frac{[N]_q}{2}\left\{\frac{\cos\theta}{\mathcal{D}(\gamma,\theta)} + \frac{\chi}{2}\frac{\sin^2\theta\cos 2\phi}{\mathcal{D}(\gamma,\theta)}\right\} \tag{4.3}$$

where

$$\mathcal{D}(\gamma,\theta) = 1 + \sinh^2\left[\frac{\gamma}{2}(N-1)\right]\sin^2\theta. \tag{4.4}$$

In the above expressions, $\epsilon_q = \frac{\epsilon}{2[1/2]_q}$ is the q-deformed energy spacing and $\chi = \frac{V[N-1]_q}{\epsilon_q}$ is an effective coupling strength.

From $E(\theta, \phi, \gamma, N)$ we extract the main information about the Lipkin model ground state, as described by the $q-$deformed coherent state. The energy depends on the deformation of the algebra and is proportional to $[N]_q$. The terms enclosed by the curly brackets in Eq.[4.3] are function of N and γ through the product $\gamma(N-1)$ and of the effective coupling strength χ.

In order to study the ground state energy we must require the conditions

$$\frac{\partial E(\theta,\phi,\gamma,N)}{\partial\phi}=0 \tag{4.5}$$

$$\frac{\partial E(\theta,\phi,\gamma,N)}{\partial\theta}=0 \tag{4.6}$$

to be satisfied.

From the above equations we can calculate the critical value of the coupling constant χ, which characterizes the phase transition. The frame below presents the differences between the non deformed case and the deformed one.

Standard case	q–deformed
$\chi_c = 1$	$\chi_c = 1 + 2\sinh^2\left[\frac{\gamma}{2}(N-1)\right]$

In the same fashion as discussed by Holzwarth [18], we would expect here the second order phase transition, characterizing the spherical symmetry breaking in the quasi-spin space, to show up as the appearance of two symmetrical minima shifted from the origin and a maximum at the position of the old minimum. In this q–deformed case, however, the phase transition depends **not only** on the strength of the interaction but **also on the deformation** of the algebra and **on the number of particles** through the product $\gamma(N-1)$ [20].

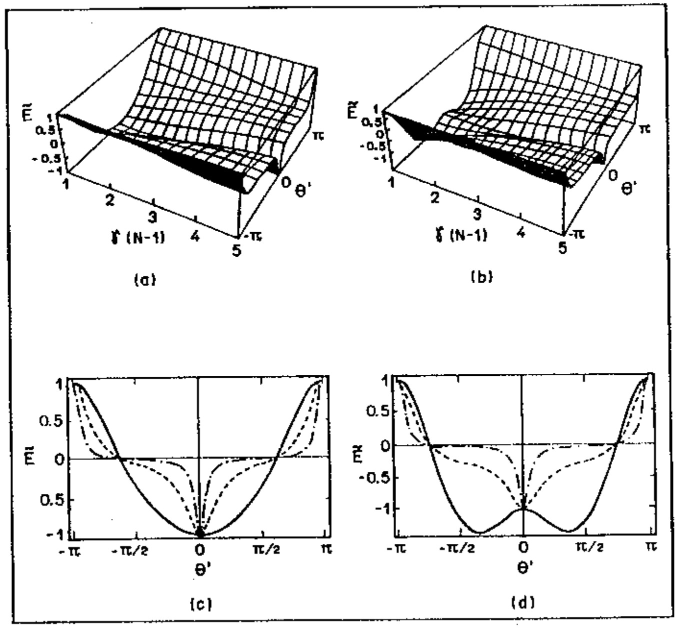

FIG. 4. Figs. 4a and 4b show 3D views of the scaled energy surfaces ($\tilde{E} = 2E/[N]_q$) for $\chi = 1$ and 3 respectively, as function of $\gamma(N-1)$ and of the order parameter $\theta' = \pi - \theta$. Figs. 4c and 4d show sections of the energy surfaces at $\gamma(N-1) = 1$ (full line), 3 (dashed line) and 5 (dot-dashed line). The behavior of $\tilde{E}$ for both global minima at $\varphi = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ is shown together by extending the domain of θ' from -π to π.

Figures 4.a and 4.b show scaled energy surfaces for different values of χ as a function of $\gamma(N-1)$ and the order parameter $\theta' = \pi - \theta$ [29], whereas figures 4.c and 4.d depict sections of the corresponding 3D-pictures for different values of $\gamma(N-1)$. There is a striking difference between the pictures on l.h.s. and r.h.s. of Fig. 4, namely the number of minima. The reason for that behavior in the first case is that χ_c is always greater than one for any value of $\gamma > 0$, as can be seen in Fig. 5. This in turn means that there will be no phase transition when one increases the deformation of the algebra for a fixed $\chi \leq 1$.

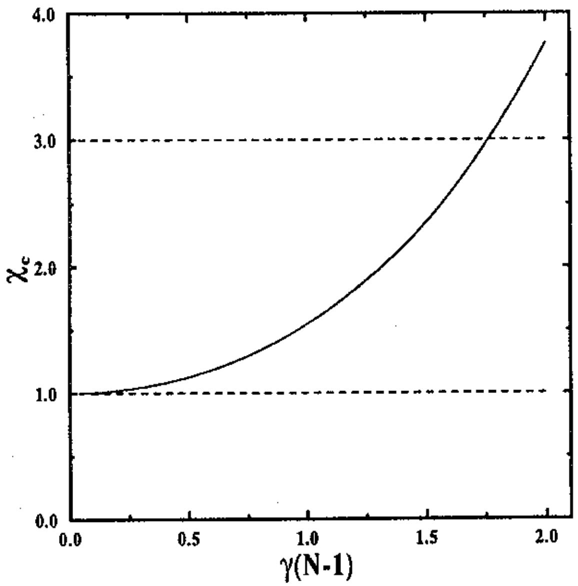

FIG. 5. The critical value of χ as function of $\gamma(N-1)$. The dashed lines indicate the region of existence of phase transition

Figures 4.b and 4.d, however, present a gradual collapse of the two minima, characterizing the phase transition, in a new one at $\theta' = 0$ as γ increases. For low values of $\gamma(N-1)$, $\chi = 3$ is greater than the value of χ_c, as can be seen from Fig. 4. In this range of $\gamma(N-1)$ we clearly identify the phase transition. However for values of $\gamma(N-1)$ for which $\chi_c > 3$, no phase transition is allowed.

Recently, temperature was included in the above framework [30]. For each T a behavior similar to the one at $T = 0$ is obtained, at least before the value of q where the system collapses. Q−deformation has the effect of lowering the critical temperature.

V. CONCLUSIONS

The conclusions will be divided in two parts. In the first one, some general characteristics of the q−deformed algebras, as seen from some naïve applications to nuclear and molecular physics, will be pointed out. In the so called specific conclusions, the focus will be on characteristics of fermionic q−deformed many-body systems.

- **General**

  1. q−deformation perhaps may bring new physics.
  2. In some simple systems it is possible to attribute a physical meaning to the deformation parameter.
  3. However, it seems impossible to attribute a universal significance to it, particularly due to the different deformation schemes.
  4. q−deformation has one interesting aspect: it seems to incorporate in an effective way and in an elegant framework the interaction among the constituents of the system.

- **Specific**

  1. It is important a careful treatment of the q-deformation, already at the fermionic level, in order to take into account its effects correctly in a many-body system.
  2. q−deform a fermionic many-body system gives rise to a q-dependent mean field.
  3. The critical value of χ is a function of $\gamma(N-1)$. This means that no universal character can be anymore assigned to χ as a system independent indicator of the phase transition in a q-deformed system.
  4. Inclusion of temperature does not change the general behavior.

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