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THE ASYMPTOTIC BEHAVIOR OF SINGLE GRADED RESPONSE NEURON MODEL WITH A DELAYED SELF-CONNECTION

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Research announcement:

The asymptotic behavior of single graded response neuron model with a delayed self-connection

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1 The graded response neuron model

The graded response neuron activation evolves according to the following delay differential equation (DDE):

$$\frac{dx}{dt}(t) = \lambda(-x(t) + \eta\sigma(x(t-1))) \tag{1}$$

Where σ is defined by:

$$\sigma(x) = \tanh(\alpha x) \tag{2}$$

We suppose $\lambda > 0$, $\eta = \pm 1$ and $\alpha > 1$.

Let $S = \mathcal{C}[-1,0]$ be the space of continuous real functions of the interval $[-1,0]$. For ϕ in S, there exists a unique real function $x(t,\phi)$ on the interval $[-1,+\infty)$, such that $x(t,\phi) = \phi(t)$ for $-1 \leq t \leq 0$, and $x(t,\phi)$ satisfies equation (1) for $t \geq 0$. For such a solution of the DDE, we denote by $x_t(\phi)$ the element of S defined by $x_t(\phi)(\theta) = x(t+\theta,\phi)$, for $-1 \leq \theta \leq 0$.

Our research project aims to establish the following description of the global attractor of the DDE (1).

2 Negative feedback

In this section we suppose $\eta = -1$. Let $\lambda_0 = 0$, and λ_k be the value of the parameter λ at the kth Hopf bifurcation occuring at 0. For this model, all Hopf bifurcations are supercritical.

The discrete time Lyapunov function counts the number of zeros of a solution on an interval of unit length of the form $[t-1,t]$ such that $x(t) = 0$ (Mallet-Paret, 1988). Let us denote this function by V.

We define the Morse sets as follows: $S_{2N+1} = \{\phi : V(\phi) = 2N+1\}$ where ϕ is taken in the attractor.

Let N^* be the number of eigenvalues of the characteristic equation at 0 with positive real parts. N^* is necessarily even. We consider only the cases where there are no eigenvalues with zero real part. $S_{N^*} = \{0\}$, for other even values of N, S_N is empty.

Then the family S_n forms a Morse decomposition of the attractor of the oscillating solutions.

For the parameter λ in the range: $\lambda_k < \lambda < \lambda_{k+1}$, we have $N^* = 2k$ and

1. $S_n = \emptyset$ for $n > N^*$
2. S_{2p+1} for $0 \leq p \leq k-1$ is composed of a unique limit periodic solution C_p.

Moreover, the stable and the unstable manifolds of the various limit cycles intersect transversally. These are the connecting sets between the Morse sets. There are connecting orbits between S_m and S_q for all $m > q$ such that the corresponding Morse sets are non empty.

This gives the Morse decomposition of the attractor. To this decomposition, there corresponds a partition of the phase space S.

There exists a strictly ordered sequence of subsets of S such that:

1. $W_k \subset W_{k-1} \subset \cdots \subset W_0 = S$
2. W_p is of codimension $2p$
3. $x_t(\phi)$ tends to C_p if and only if ϕ is in $W_p - W_{p+1}$ (with $0 \le p \le k-1$).
4. $x_t(\phi)$ tends to 0 if and only if ϕ is in W_k.

3 Positive feedback

In this section we suppose $\eta = +1$. Let $\lambda_0 = 0$, and λ_k be the value of the parameter λ at the kth Hopf bifurcation occuring at 0. For this model, all Hopf bifurcations are supercritical.

The discrete time Lyapunov function counts the number of zeros of a solution on an interval of unit length of the form $[t-1, t]$ such that $x(t) = 0$. Let us denote this function by V (Arino, 1993).

We define the Morse sets as follows: $S_{2N} = \{\phi \ : V(\phi) = 2N\}$ where ϕ is taken in the attractor of the oscillating solutions.

Let N^* be the number of eigenvalues of the characteristic equation at 0 with positive real parts. N^* is necessarily odd. We consider only the cases where there are no eigenvalues with zero real part. $S_{N^*} = \{0\}$, for other odd values of N, S_N is empty.

Then the family S_n forms a Morse decomposition of the attractor of the oscillating solutions.

For the parameter λ in the range: $\lambda_k < \lambda < \lambda_{k+1}$, we have $N^* = 2k+1$ and

1. $S_n = \emptyset$ for $n > N^*$
2. S_{2p} for $1 \le p \le k$ is composed of a unique periodic solution C_p.

Moreover, the stable and the unstable manifolds of the various limit cycles intersect transversally. These are the connecting sets between the Morse sets. There are connecting orbits between S_m and S_q for all $m > q$ such that the corresponding Morse sets are non empty.

This gives the Morse decomposition of the attractor. To this decomposition, there corresponds a partition of the manifold W of oscillating solutions.

There exists a strictly ordered sequence of subsets of W such that:

1. $W_k \subset W_{k-1} \subset \cdots \subset W_0 = W$
2. W_p is of codimension $2p+1$
3. $x_t(\phi)$ tends to C_p if and only if ϕ is in $W_{p-1} - W_p$ (with $1 \le p \le k$).
4. $x_t(\phi)$ tends to 0 if and only if ϕ is in W_k.

4 References

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