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ANYONS IN 1+1 DIMENSIONS ARE ANOMALOUS

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Abstract

We consider the analog in one spatial dimension of the Bose-Fermi transmutation for planar systems. That is, the construction of a purely bosonic effective local theory starting from a system of bosons and fermions upon integration over the fermionic variables. We consider a quantum mechanical system of a spin 1/2 particle coupled to an abelian gauge field, which is classically invariant under gauge transformations and charge conjugation. It is found that, unless the flux enclosed by the particle orbits is quantized, and the spin takes a value $n+1/2$, at least one of the two symmetries would be anomalous. Thus, charge conjugation invariance and the existence of abelian instantons simultaneously avoid the anomaly and force the particles to be either bosons or fermions, but not anyons.

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It is the aim of theoretical physics to provide economical explanations for the fundamental features of nature. Two brilliant examples of this are the quantization of spin in multiples of $\hbar/2$, and the connection between spin and the (anti-) symmetry of the wave function under exchange of identical particles. It has been powerfully stressed however, that both results are crucially dependent on the fact that the rotation group in three spatial dimensions is non-abelian. Since this fails to be so for lower dimensions, spin is not necessarily quantized and the states need not be symmetric or antisymmetric under particle exchange.

In the last ten years intense research on the physical and mathematical properties of planar - *i.e.*, (2+1)-dimensional - systems has taken place. From the mathematical point of view, the discovery of topological quantum field theories [1] and fractional statistics [2] are surely the most important results. On the other hand, the fact that spin and statistics could be fractional and their potential applications is, probably, one the most exciting discoveries in theoretical physics.

However although there are many well established theoretical [3] and experimental results[4] for linear - *i.e.*, (1+1)-dimensional - systems, a deeper understanding of their origin is still lacking.

One of these intriguing results is the bosonization in one-dimensional (1+1) systems. It is generally believed that bosonization occurs naturally in those systems because there is no rotation group in one dimension and, as a consequence, the spin could be considered a matter of convention.

There are two approaches to bosonization: A non-linear, non-local field transformation that maps a fermionic action into a bosonic one [5, 6]; and the more recent two-dimensional construction of Polyakov [7], where the fermions are integrated out and the resulting effective action is written in terms of purely bosonic fields . These two approaches seem to be related, as it can be argued in the context of duality [8].

In all approaches, topology plays a key role. The integer spin excitations are found to exist in a topologically non-trivial sector of the bosonized theory. The resulting (effective) action has a boundary term of topological origin that is responsible for the antisymmetry

of the wave function under particle exchange, thus respecting the fermionic statistics in "bosonized" theory.

In the construction based on the non-local mapping, the appearence of this topolog boundary term is not explicit because the identification is either made at the classical grangian level, or it is perturbative. Its is therefore of interest to investigate a simple sys where the topological features of the mapping can be studied in the quantum theory .

In the approach based on the integration over the fermionic fields can be exempli with a relativitic spinning particle in two spatial dimensions. Upon integration of the degrees of freedom in the action $S[x,\theta]$, the resulting effective action reads [9],

$$S_{eff} = \int d\tau [m\sqrt{\dot{x}^2} + \frac{1}{2}\mathcal{W}],$$

which is just a bosonic way of describing a relativistic particle of spin one-half. The writl number, $\mathcal{W}$, is a topological invariant which classically does not contribute to the equati of motion but quantum mechanically is responsible for the non-trivial character of the B Fermi transmutation. The factor $\frac{1}{2}$ is precisely the spin of the particle.

One could also note that (1) is a particular case of a more general construction. In f when one couples a spinless particle to an abelian Chern-Simons field A_μ in a planar syst the action reads[10]

$$S = \int d\tau [m\sqrt{\dot{x}^2} + A_\mu \dot{x}^\mu] + \frac{1}{2\sigma}\int d^3x \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho .$$

Integrating out the gauge field, one finds the effective Lagrangian

$$L_{eff} = m\sqrt{\dot{x}^2} + \frac{\sigma}{4\pi}\mathcal{W},$$

which is exactly (1) for $\sigma = 2\pi$. The coefficient $\frac{\sigma}{4\pi}$ corresponds to the spin of the effec system and, in this sense, (3) describes a quantum particle with fractional spin and statis [11]. In one spatial dimension the rotation group is discrete; its representations are c dimensional and labeled by a phase. Spin is associated with the symmetry or antisymm of the states under the exchange of identical particles. This phase (spin) in turn can shown to be determined by the class of boundary conditions that render the Hamiltonian self-adjoint [12]. If no further constraints are imposed on this phase, the spin can take any real value, interpolating continuously between bosons and fermions. If one takes seriously this idea, then a quantum field theory in 1+1 dimensions should be anyonizable and not just bosonizable, as is commonly assumed.

The purpose of this letter is to show how it is possible to write an expression analogous to (3) for a simple quantum mechanical system in one dimension, and to discuss some subtleties associated with the bosonization procedure. In particular, we will show how, after bosonization, the system remembers that it comes from a fermionic one.

In order to describe our results let us proceed in analogy with analysis of [7] and [9], starting with a non-relativistic spinning particle described by the action

$$S = \int_{t_1}^{t_2} dt\Big[\frac{1}{2}\dot{x}^2 + \frac{1}{2}V^2(x) + \psi^\dagger(i\partial_t + A)\psi\Big], \tag{4}$$

which for $A = V'$ is $N = 1$ supersymmetric quantum mechanics (SSQM), where $V(x)$ is the superpotential.

This action (4) has two classical symmetries *v.i.z.*

i) Invariance under local $U(1)$ gauge transformations

$$\psi'(t) = e^{i\omega(t)}\psi(t), \qquad \psi'^\dagger(t) = e^{-i\omega(t)}\psi^\dagger(t), \tag{5}$$

where the gauge potential A transform as [13]

$$A \to A + \frac{d\omega}{dt}. \tag{6}$$

ii) Invariance under "charge-conjugation" *i.e.*

$$\psi \leftrightarrow \psi^\dagger, \qquad A \to -A. \tag{7}$$

The partition function is defined as

$$Z = \int \mathcal{D}x\mathcal{D}\psi\mathcal{D}\psi^\dagger e^{-S}. \tag{8}$$

It is customary to assume the orbits to be periodic in the bosonic coordinates, w either periodic [14] or antiperiodic [15] in the fermions. Since we want to investigate possibility of having anyons, which should not be expected to obey neither periodic antiperiodic boundary conditions, we will allow for twisted boundary conditions, namel

$$x(t_1) = x(t_2),$$

and

$$\psi(t_2) = e^{2\pi i\alpha}\psi(t_1),$$

where α is an arbitrary real number.

Integrating over the fermionic variables,

$$Z_\alpha = \int \mathcal{D}x \det(i\partial_t + A)_\alpha e^{i\int_{t_1}^{t_2} dt(\frac{1}{2}\dot{x}^2 - \frac{1}{2}V^2)}.$$

As usual, the determinant of an operator Ω is computed as $\det(\Omega)_\alpha = \prod_n \lambda_n^{(\alpha)}$, wl $\lambda_n^{(\alpha)}$ are the eigenvalues. Using (10), the eigenvalues are

$$\lambda_n^{(\alpha)} = \frac{1}{T}\int_{t_1}^{t_2} dt A(x) - \frac{2\pi(\alpha+n)}{T},$$

where $T = t_2 - t_1$, and the fermionic determinant becomes

$$\begin{aligned}\Gamma(A) &= \det(i\partial_t + A)_\alpha \\ &= \prod_{n=-\infty}^{n=\infty}\Big[\frac{1}{T}\int_{t_1}^{t_2} dt A(x) + \frac{2\pi(\alpha+n)}{T}\Big].\end{aligned}$$

In order to compute the infinite product, one can isolate the $n = 0$ eigenvalue, t standard manipulations lead to

$$\Gamma(A) = \frac{1}{T}(y + 2\pi\alpha)\prod_{n=1}^{\infty}\Big(\frac{-2\pi}{T}\Big)^2 n^2 \prod_{n=1}^{\infty}\Big[1 - \frac{(y+2\pi)\alpha^2}{4n^2}\Big],$$

with $y = \int_{t_1}^{t_2} dt A(x)$ and, using well known identities, we arrive at [16]

$$\Gamma_\alpha(A) = \mathcal{N}\sin\Big[\int_{t_1}^{t_2} dt\Big(\frac{1}{2}A + \frac{\pi\alpha}{T}\Big)\Big],$$

where $\mathcal{N}$ is a normalization constant independent of x.

This formula reduces to the known results when $\alpha = 0$ (bosons), and $\alpha = 1/2$ (fermions). This can be checked by direct calculation in SSQM [15].

The determinant (15) can be understood as follows. Assume one starts with some definite boundary condition for the fermions, say periodic boundary conditions. Then eq. (24) can be viewed as the result of a gauge transformation on the gauge potential A of the form (6) with ω given by

$$\omega(t) = \frac{2\pi\alpha}{t_2 - t_1}(t - t_1). \qquad (16)$$

As a consequence, there is a one to one correspondence between gauge transformations of the class (16) and the twist chosen for the fermionic boundary condition (10). Thus, by means of successive gauge transformations one can continuosly interpolate between $\Gamma_{\alpha=0}(A)$ (bosons) and $\Gamma_{\alpha=1/2}(A)$ (fermions).

Thus, the complete partition function is

$$Z_\alpha = \int \mathcal{D}x \; \Gamma_\alpha(A) e^{i\int_{t_1}^{t_2} dt(\frac{1}{2}\dot{x}^2 - \frac{1}{2}V^2)}. \qquad (17)$$

At this point one can ask whether the classical symmetries of the model are respected in the quantum theory. In order to address this question, one notes that gauge invariance requires $\Gamma_\alpha(A) = \Gamma_0(A)$, and therefore either

$$a) \qquad \alpha = 2n, \qquad (18)$$

or

$$b) \qquad \Phi = (2n + 1 - \alpha)\pi, \qquad (19)$$

where $\Phi \equiv \int_0^T dtA$ is the flux enclosed by the particle orbit in Euclidean space (instanton). On the other hand, invariance under charge conjugation implies $\Gamma_\alpha(A) = \Gamma_\alpha(-A)$ and hence,

$$c) \qquad \alpha = n + 1/2, \qquad (20)$$

or

$$d) \qquad \Phi \equiv \int_0^T dtA = 2n\pi. \qquad ($$

It is clear from this that if the flux <u>were not</u> quantized, it would be impossibl to resp both symmetries simultaneously. Furthermore, if $\alpha \epsilon \mathbf{Z}$, then

$$\Gamma_\alpha(A) = (-1)^n sin[\int_0^T dtA], \qquad ($$

is an odd functional of A, which combined with charge conjugation invariance i iplies t $\Gamma_\alpha(A) = 0$, and the theory would be inconsistent. Thus, the only combination of the ab conditions that ensures consistency and absence of anomalies is b and c, namely,

$$\Phi = (m + 1/2)\pi, \quad \alpha = n + 1/2, \quad n - m = \text{odd}. \qquad ($$

Thus, we see that

$$\Gamma_\alpha(A) = \mathcal{N}'(\alpha) e^{i\int_{t_1}^{t_2} dtA[x(t)]}, \qquad ($$

where $\mathcal{N}'(\alpha)$ is a normalization constant, and the effective action (17) is

$$S = \int_{t_1}^{t_2} dt[\frac{1}{2}\dot{x}^2 - \frac{1}{2}V^2 + \frac{1}{2}A]. \qquad ($$

In conclusion, (25) is the two-dimensional analog of (3) with the flux $\frac{1}{2}\int_{t_1}^{t_2} dtA =$ plays the role of $\frac{\pi}{4\pi}W$ in 2+1 dimensions. However, this analogy is not correct unless flux is quantized, which occurs for the cases 3 and 4 as shown in the the followi g table

	Φ	α	Gauge Inv.	Ch. Conj.	Anomalous
1	arbitrary	$2n$	yes	no	yes
2	arbitrary	$n + 1/2$	no	yes	yes
3	$2n\pi$	noninteger	no	yes	yes
4	$(m + 1/2)\pi$	$n + 1/2$ $(n - m)$ odd	yes	yes	no

Possibility 3, however does not preserve gauge invariance and the theory is anomalous. Case 4 is more interesting because it shows that the two classical symmetries are preserved and α takes on half-integer values only. This means that the particles described by the bosonized action (25) are not bosons –as naively expected–, nor anyons –as the elementary group-theoretical analysis suggests–, but in fact *fermions*.

In the path integral (or partition function) it is customary to integrate over periodic (antiperiodic) orbits for bosonic (fermionic) variables. The reason for this is essentially classical: it is under these conditions that the action has an extremum on the classical orbits [18]. Nevertheless, it is not obvious that this is necessarily so at the quantum level. What one learns from the preceding analysis is that unless fermions are antiperiodic, the theory would not respect gauge and charge conjugation invariance.

This result can also be reached through a geometrical analysis. The boundary conditions (9) and (10) correspond to superimposing a gauge transformation on the orbit so that the spinor comes back to itself, modulo a finite gauge transformation, when x completes a full turn. This implies that the family of boundary conditions considered splits into the homotopy classes in $\Pi_1(SO(2)/U(1)) \simeq \Pi_1(U(1)) = \mathbf{Z}$. These classes are labeled by a value of $\alpha \in \mathbf{Z}$, or $\alpha \in \mathbf{Z}+1/2$ [19]. The additional requirement of charge conjugation invariance, rules out the integer values for α.

The flux quantization results from the compactification of the time direction as a consequence of the (anti-) periodicity of the fields. This is analogous to the quantization of the abelian Chern-Simons coefficient in 2+1 dimensions when the theory is defined on a multiply connected manifold [20]. The fact that the flux enclosed by the orbits is quantized can also be interpreted as a condition for quantization of the orbits, similar to the Bohr-Sommerfeld rule.

The extension of these results to higher dimensions and their connection with non abelian anomalies will discussed elsewhere [17].

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