UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 66318
05389-970 SÃO PAULO - SP
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FLUCTUATION EFFECTS IN INITIAL CONDITIONS FOR HYDRODYNAMICS

Yogiro Hama
Instituto de Física, Universidade de São Paulo

Samya Paiva
Instituto de Física Teórica, UNESP
Rua Pamplona, 145, 01405-901 São Paulo-SP, Brazil

Takeshi Kodama
Instituto de Física, Universidade do Rio de Janeiro
CP. 68528, 21945-970 Rio de Janeiro-RJ, Brazil

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Fluctuation Effects in Initial Conditions for Hydrodynamics

Samya Paiva* and Yogiro Hama
Instituto de Física, Universidade de São Paulo
C.P.66318, 05389-970 São Paulo-SP, Brazil

Takeshi Kodama
Instituto de Física, Universidade do Rio de Janeiro
C.P.68528, 21945-970 Rio de Janeiro-RJ, Brazil

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Abstract

We have studied the fluctuation effects in proton-proton collisions through the analysis of its observables. To investigate the role of fluctuations in initial conditions, we have calculated them using the *Interacting Gluon Model* modified by the inclusion of the impact parameter, and have applied them to the Landau's Hydrodynamical Model in one dimension. The rapidity and pseudo-rapidity distributions were calculated using two distinct procedures. One in which we have considered the fluctuations in initial conditions and the other the usual method considering only one fireball with average initial conditions. The obtained results have shown the importance of considering these fluctuations.

1 Introduction

It is well known that the hydrodynamical models usually describe well the various aspects of the multiple particle production phenomena in high energy nuclear and hadronic collisions. Nevertheless, a simple application of these models may fail when we try to analyze in detail the experimental data that carry information about

*Present Adress: Instituto de Física Teórica, UNESP, Rua Pamplona, 145, 01405-901 São Paulo, S.P., Brazil

or even associated with the impact parameter. Usually, the so-called inclusive data for the final particle distribution is the average over such event-to-event fluctuations for a given experimental initial condition. Let us denote such an averaging process by $\overline{\cdots}$. On the other hand, the hydrodynamic models also deal mainly with the inclusive quantities such as density, mean energy and entropy, which are average values in momentum space over a statistical ensemble. Let us denote this averaging process by $\langle\cdots\rangle$. In the usual application of hydrodynamic models in describing the inclusive data, we presumably expect, by means of a sort of ergodic assumption [1], that the statistical ensemble average $\langle\cdots\rangle$ substitutes the average over event-to-event fluctuations $\overline{\cdots}$, that is,

$$\overline{\cdots} \simeq \langle\cdots\rangle \tag{1}$$

However, not all the average of physical fluctuations can be expressed in terms of the above average over statistical ensemble of the constituent configurations. For example, the quantum-mechanical and impact parameter fluctuations that occur in the initial condition of each event can never be averaged out with the use of the ergodic hypothesis. The main aim of this report is to discuss the effects of such fluctuations on the observed quantities in a hydrodynamical description.

When we want to introduce fluctuations in the initial conditions of a hydrodynamic system, we must go beyond the hydrodynamic degrees of freedom. They should be calculated from some microscopic model. For this purpose, we use here the Interacting Gluon Model [2] (IGM). This is a simple model which permits us to calculate the energy and momentum distribution of fireballs produced in a hadronic collision. In addition to the dynamical fluctuation of the microscopic degrees of freedom, we would also like to include the impact parameter fluctuation. Quantum mechanically, it is in principle impossible to fix the impact parameter. Even if we could theoretically define the trajectories of the incident particles like in heavy-ion collisions where the incident objects are nearly classical, it would equally not be possible in practice to fix the impact parameter due to the actual experimental conditions. We may recall that there exist some experimental techniques to discriminate central from peripheral collisions in such reactions. But they do not eliminate fluctuations. So, in any realistic description of nuclear and hadronic collisions, the impact parameter fluctuation has to be taken into account. Thus, we modify the original IGM to take account of the impact parameter fluctuation. The fluctuations we are mentioning become especially important in hadronic collisions rather than in heavy ion collisions. Thus, in this paper we shall mainly be concerned with pp and $\bar{p}p$ collisions.

In high-energy pp and $\bar{p}p$ collisions, it is probable that the inelasticity, that is, the fraction of the incident energy used to produce the final particles be determined before the hydrodynamic scenario sets in. In other words, the inelasticity is the input for the hydrodynamics. We thus calculate the inelasticity distribution using the impact parameter dependent IGM. To have a first look on the effect of impact

parameter, we compare the inelasticity distribution calculated by the IGM with and without the impact parameter fluctuation. A sizeable change in inelasticity distribution, as well as in the leading-particle spectrum leads us to verify the sensitivity of these observables with impact parameter fluctuation. A better agreement of our results with data suggests that we are in the right way.

After introducing the initial conditions given by our IGM, the next step is to choose some hydrodynamic model and study the fluctuation effects on the final particle spectra. For this, we have chosen one dimensional Landau's Hydrodynamical Model [3],[4]. In this model, an analytical solution can be obtained over the whole kinematical region just in terms of the invariant fireball mass. This enormously simplifies our task of averaging over all the fluctuation considered.

In order to quantitatively analyze the consequences of these fluctuations, we calculate the rapidity distributions in two distinct ways. One is that we take first the average of fireballs over all the fluctuations given by the IGM, then calculate the rapidity distribution applying the hydrodynamics to this unique averaged fireball. This process would correspond to the usual application of a hydrodynamical model. The other is that, we apply the hydrodynamics for each event with fluctuating initial conditions to obtain the event-by-event rapidity distributions and then by summing up these distributions over all the events we calculate the averaged rapidity distribution. The comparison of the above two results reveals us that the rapidity distribution is very sensitive to fluctuations in the initial conditions, pointing out the importance of taking care of them.

In what follows, we present in section II the basic ideas of the Interacting Gluon Model modified by the inclusion of the impact parameter and the calculation of the initial conditions. The results of the inelasticity distribution and the leading particle spectrum are also shown. In section III we show the implementation of the initial conditions in Landau's hydrodynamical model and the rapidity distributions are calculated. Our conclusions are given in section IV.

2 Fluctuations in the Initial Conditions

One of the present problems which high energy nuclear and hadronic collisions are faced with is the determination of the energy deposited in the reaction or, equivalently, the fraction k of the total incident energy $\sqrt{s}$ consumed to produce particles. As mentioned in the Introduction, this fraction, or inelasticity, is an essential ingredient for statistical models of high energy hadronic collisions. We apply the Interacting Gluon Model to calculate this quantity. The main reason for this is that, in terms of few parameters, it allows us to obtain analytically the inelasticity distribution as functions of the incident energy. This, in its turn, is immediately related to the leading-particle spectrum.

The Interacting Gluon Model is based on an idea[5] that in high energy hadronic collisions valence quarks weakly interact so that they almost pass thorough, whereas gluons interact strongly. To be more specific[2], the valence quarks are supposed to be responsible for the fragmentation regions, while the interacting gluons produce an indefinite number of mini fireballs through gluon fusion, which eventually form a unique large fireball in the central region. A further simplification in this model can be made by assuming that there is no fragmentation of valence quarks. That is, all the remaining energy not deposited in the central fireball is to be found in the leading particles. In [2], no reference is made about the impact parameter. We here reformulate the model to introduce this new kind of fluctuation for the description of proton-proton and nucleus-nucleus collisions.

2.1 Impact Parameter

The role of the impact parameter in providing the initial condition for a hydrodynamical model is twofold. First, a given value of the impact parameter $\vec{b}$ defines the probability density of occurrence of a reaction. Second, once a reaction takes place, it determines how do the mass and momentum fluctuate in the initial conditions of the hydrodynamics. To account for the first point, the best way to introduce the impact parameter in quantum mechanical reaction process is the use of the eikonal formalism[7]. In the impact parameter representation, the total inelastic cross section of proton-proton reaction can be written as

$$\int d\vec{b}\, F(\vec{b}) = \sigma_{pp}^{inel}(\sqrt{s})\,, \tag{2}$$

where the incident energy dependence of the inelastic cross section is well expressed as[6]

$$\sigma_{pp}^{inel} = 56\,(\sqrt{s})^{-1.12} + 18.16\,(\sqrt{s})^{0.16}.$$

The function $F(\vec{b})$ is nothing but the partial cross section with respect to the impact parameter $\vec{b}$. We also write

$$F(\vec{b}) = 1 - |S(\vec{b})|^2. \tag{3}$$

In the IGM where we assume that the inelastic processes occur due to the gluon-gluon fusion, we write the eikonal function as the convolution of the projectile and target gluon thickness function. Thus we write

$$|S(\vec{b})|^2 = \exp\{-C \int d\vec{b}' \int d\vec{b}''\, D(\vec{b}')\, D(\vec{b}'') f(\vec{b}+\vec{b}'-\vec{b}'')\}, \tag{4}$$

where $D(\vec{b})$ is the proton thickness function and C is a constant which should be determined by the normalization condition (2). For $D(\vec{b})$ we take here a Gaussian

distribution with the range equal to the proton radius. The function $f(\vec{b})$ in (4) gives account of the finite interaction range of the gluons and is subject to the constraint

$$\int f(\vec{b})d\vec{b} = 1\,. \tag{5}$$

The simplest choice of $f(\vec{b})$ would be $\delta(\vec{b})$, which represents a point interaction, but it is not consistent (especially in the case of pp collision) with the finite range of the strong interaction. We preferred to parametrize it as a Gaussian with a range $\approx$ 0.8 fm, which gives also a better agreement with the data. Thus we have eventually

$$D(\vec{b}) = f(\vec{b}) = \frac{a}{\pi}\, e^{-ab^2}, \tag{6}$$

with $a = 3/2\ R_p^2$, where $R_P \approx 0.8\ fm$ is the proton radius. Thus we get

$$F(\vec{b}) = 1 - \exp\left\{-\left[\frac{aC}{3\pi}\right]\exp(-\frac{a\vec{b}^2}{3})\right\}\,. \tag{7}$$

2.2 IGM with Impact Parameter Fluctuation

Now, to implement the second point mentioned above, we apply the basic idea of the IGM for each impact parameter $\vec{b}$. That is, having occurred a reaction for a given $\vec{b}$, we assume that the colliding protons form, by gluon exchange, a central fireball depositing in it fractions respectively $x(\vec{b})$ and $y(\vec{b})$ of their momenta. Let n_i be the number of pairs of gluons that carry fractions x_i and y_i. The fractions $x(\vec{b})$ and $y(\vec{b})$ are thus a sum over all such gluon pairs

$$\sum_i n_i x_i = x(\vec{b}) \quad \text{and} \quad \sum_i n_i y_i = y(\vec{b})\,. \tag{8}$$

From now on, we omit the explicit dependence of x and y on $\vec{b}$ in order not to overload the notation, if otherwise necessary. The energy and momentum of the central fire ball in the center-of-mass frame of protons are given by

$$\begin{aligned} E(x,y) &= \frac{\sqrt{s}}{2}(x+y)\,, \\ P(x,y) &= \frac{\sqrt{s}}{2}(x-y) \end{aligned} \tag{9}$$

and its invariant mass M and rapidity Y are

$$M = \sqrt{sxy} \equiv k\sqrt{s} \quad \text{and} \quad Y = \frac{1}{2}\ln\frac{x}{y}\,. \tag{10}$$

As in [2], we can then express the probability of forming a fireball with the specific energy and momentum as a sum over all the set of gluon pairs $\{n_i\}$ which satisfies the relations (8).

$$\Gamma(x,y;\vec{b}) = \sum_{\{n_i\}} \delta\left[x(\vec{b}) - \sum_i n_i x_i\right]\,\delta\left[y(\vec{b}) - \sum_i n_i y_i\right]\,\prod_i P(n_i)\,, \tag{11}$$

where $P(n_i)$ is the probability of occurring fusions of n_i gluon pairs $\{x_i, y_i\}$. If these fusions are independent, we may take $P(n_i)$ as a Poisson distribution

$$P(n_i) = \frac{\overline{n}_i^{n_i} e^{-\overline{n}_i}}{n_i!}\,. \tag{12}$$

Note that Eq.(11) is normalized,

$$\int dx \int dy \Gamma(x,y;\vec{b}) = 1\,.$$

Now, expressing the delta functions by Fourier integrals, one can perform all summations in Eq.11 and arrive at

$$\Gamma(x,y;\vec{b}) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} ds\ e^{i(ux+sy)}\ \exp\{\sum_j \overline{n}_j[e^{-i(ux_j+sy_j)} - 1]\}\,. \tag{13}$$

At this stage, we shift to the continuous limit rather than the discrete version considered above. Then the summation in (13) must be replaced by an integral,

$$\sum_j \overline{n}_j \to \int_0^1 dx' \int_0^1 dy'\ w(x',y';\vec{b})\,, \tag{14}$$

where $w(x,y;\vec{b})$ is the density of gluon pairs that fuse contributing to the final fireball, with the fraction x of the projectile and y of the target proton momenta, respectively. This quantity is the central ingredient in the present version of the model. Within the eikonal model, we may express it as

$$w(x,y;\vec{b}) \equiv \int\ G(x,\vec{b}')\ G(y,\vec{b}'')\frac{\sigma_{gg}(x,y)}{\sigma_{pp}^{inel}} f(\vec{b}+\vec{b}'-\vec{b}'')\ d\vec{b}'\ d\vec{b}''\,, \tag{15}$$

where $G(x,\vec{b})$ and $G(y,\vec{b})$ are the momentum distributions of gluons in the projectile and target protons at a given impact parameter $\vec{b}$ and $\sigma_{gg}(x,y,s)$ is the gluon-gluon interaction cross section. The function f is the same one as in 4. In this work, we assume that the dependences of G on x and $\vec{b}$ factorize, that is, we take

$$G(x,\vec{b}) = \frac{1}{x}D(\vec{b})\,, \quad \text{e} \quad G(y,\vec{b}) = \frac{1}{y}D(\vec{b})\,, \tag{16}$$

where $D(\vec{b})$ is the proton thickness function appeared before, and the gluon-gluon cross section is parametrized[2] as

$$\sigma_{gg}(x,y) = \frac{c}{xys} \tag{17}$$

where c is a parameter of the model.

Because of the form of $G(x,\vec{b})$ (gluons carrying small fraction of proton momentum) and of σ_{gg}, the spectral function is sharply peaked at small (x,y) which justifies the approximation

$$e^{-i(ux'+sy')} - 1 \simeq -i(ux'+sy') - \frac{1}{2}(ux'+sy')^2. \tag{18}$$

Replacing the sum in equation (13) by the integral given by (14) and using the approximation above, one can obtain an analytical form for $\Gamma(x,y;\vec{b})$ given by

$$\Gamma(x,y;\vec{b}) \simeq \frac{1}{\pi\sqrt{\det(\mathbf{G})}} \exp\{-\vec{\mathbf{X}}^T\mathbf{G}^{-1}\vec{\mathbf{X}}\}, \tag{19}$$

where

$$\mathbf{X} = \begin{pmatrix} x-\langle x\rangle \\ y-\langle y\rangle \end{pmatrix}, \qquad \mathbf{G} = 2\begin{pmatrix} \langle x^2\rangle & \langle xy\rangle \\ \langle xy\rangle & \langle y^2\rangle \end{pmatrix}, \tag{20}$$

with the notation,

$$\langle x^m y^n\rangle = \langle x^m y^n(\vec{b})\rangle = \int_0^1 dx' \int_0^1 dy'\, x'^m\, y'^n\, w(x',y';\vec{b}). \tag{21}$$

In terms of the total energy E and the momentum P the probability density is calculated as

$$\Gamma(E,P;\vec{b}) \simeq \frac{2\sqrt{a_1 a_2}}{\pi} \exp\{-a_1(E-\langle E\rangle)^2 - a_2P^2\}, \tag{22}$$

where

$$a_1 = [s(\langle x^2\rangle + \langle xy\rangle)]^{-1}, \quad \text{and} \quad a_2 = [s(\langle x^2\rangle - \langle xy\rangle)]^{-1} \tag{23}$$

and

$$\langle E\rangle = \frac{\sqrt{s}}{2}(\langle x\rangle + \langle y\rangle). \tag{24}$$

Note that (22) is still normalized,

$$\int dE \int dP\, \Gamma(E,P;\vec{b}) = 1. \tag{25}$$

The expression (19) or (22) describes the relative probability of formation, at a given impact parameter, of a central fireball with energy E and momentum P. In

these expressions, however, no restriction has imposed neither on the fireball mass M nor on its momentum P. Nevertheless, there exist natural upper limits on these variables determined by the total collision energy $\sqrt{s}$. There is also a minimum allowed fireball mass $m_0 \sim m_\pi$, which is a parameter of the model. So, in order to recover the correct normalization, we have to put some additional factor in (23). Combining this with the probability of occurrence of a reaction with a given impact parameter as discussed in 2.1, we finally write the probability $\chi(E,P;\vec{b})$ of having a fireball with energy E and momentum P at an impact parameter $\vec{b}$ as

$$\chi(E,P;\vec{b}) = \chi_0(\vec{b})\Gamma(E,P;\vec{b}), \tag{26}$$

where $\chi_0(\vec{b})$ should be determined by the condition

$$\int dE \int dP\, \chi(E,P;\vec{b})\, \theta(\sqrt{(E^2-P^2)/s} - k_{min}) = \frac{1}{\sigma_{pp}^{inel}} F(\vec{b}). \tag{27}$$

Here, k_{min} is related to the minimum fireball mass m_0 through

$$k_{min} = \frac{m_0}{\sqrt{s}}.$$

The expression (26) shows that, when the matter overlap is small (corresponding to a large impact parameter), not only the average fireball mass or the inelasticity is small as implied by (22) and (15), but also the probability of such fireball formation is small. This second effect, which we have discussed in 2.1, is a reflection of quantum effect in impact parameter and has shown to be crucial in our description.

2.3 Inelasticity Distribution

Now we have all the necessary ingredients for the calculation of $\chi(E,P;\vec{b})$ function, that is our generator function of the initial conditions which includes the energy, momentum and impact parameter fluctuations. Note that, in comparison with the original model, we have introduced a new kind of fluctuation without including any additional free parameter, except for the geometrical radius of the proton R_p which appear in Eq.(7).

Once $\chi(E,P;\vec{b})$ has been obtained, the inelasticity distribution $\chi(k)$ can easily be calculated by integrating it over E, P and $\vec{b}$ with the inelasticity $k = \sqrt{(E^2-P^2)/s}$ fixed, namely

$$\chi(k) = \int d\vec{b} \int dE \int dP\, \chi(E,P;\vec{b})\, \delta(\sqrt{(E^2-P^2)/s} - k)\, \theta(\sqrt{(E^2-P^2)/s} - k_{min}). \tag{28}$$

As for the leading-particle spectrum, assuming an approximate factorization of $x_L(=2p_L/\sqrt{s})$ and p_T dependences, we have

$$E_L \frac{d^3\sigma}{dp^3} \approx f(x_L)h(p_T)\,, \tag{29}$$

where

$$f(p_L) = \int d\vec{b} \int dP \int dE\ \chi(E,P;\vec{b})\ \theta(\sqrt{(E^2-P^2)/s} - k_{min})\ \delta(\frac{\sqrt{s}-(E+P)}{2} - p_L) \tag{30}$$

and we have parametrized [8]

$$h(p_T) = \frac{\beta^2}{2\pi}\, e^{-\beta p_T}\,; \quad \beta \approx 4.0\ GeV^{-1}, \tag{31}$$

with $x_L = 2p_L/\sqrt{s}$.

The only experimental information available on $\chi(k)$ at the present moment is the one extracted from [9] corresponding to $\sqrt{s} = 16.5\ GeV$. In this reference, what is presented is the unnormalized cross section measured in the range $0.08 \leq k \leq 0.72$. However, both the original IGM result [2] and the ours are normalized curves in whole the k range ($k_{\min} \leq k \leq 1$). In order to compare correctly our result with the data, we gave the latter the same normalization as our curve in the range where they have been measured. We show in Fig. 1 the comparison of our result with the experimental data together with the curve obtained in the original IGM [2], i.e. without impact parameter fluctuations. It is clearly seen that the inclusion of the impact-parameter appreciably changes the inelasticity distribution. Our curve is much more uniform compared to the original one. We understand that this is due to the increase in the probability of the small fireball formation at larger impact parameters. The increase of $\chi(k)$ close to $k = 1$ is due to the fact that our c in (17) is larger than in the original version. It is also seen that our result is better.

The result of the leading particle spectrum is compared in Fig. 2 with the experimental data. Again, we see a better agreement of our result with the data as compared with the one obtained in the original version. The effect of enhancement of the formation of smaller fireballs is clearly seen there.

3 Particle Spectra

Having calculated the distribution of fluctuations in the initial conditions, expressed by $\chi(E,P;\vec{b})$, now we proceed to study their effects on the final particle spectra. For this purpose, we adopt one-dimensional Landau's hydrodynamical model for an ideal gas. Despite all the simplifications, this model is known to reproduce the main features of the measured momentum (or rapidity) distributions and has advantage

of having an analytical solution over the whole rapidity range. The only input of the model is the total energy and the geometrical size of the initial fireball. We will apply this model to the fireballs produced by the gluon fusions as discussed in the previous section. Note that these fireballs do not contain the leading particles anymore. In other words, the hydrodynamics introduced here will not affect the inelasticity calculated in the previous section.

The invariant momentum distribution of produced particles in a hydrodynamical model is usually given by Cooper-Frye formula [10]

$$E\frac{dN}{d\vec{p}} = \int_{\sigma(T_d)} f(p^\mu u_\mu)\, p^\mu\, d\sigma_\mu\,, \tag{32}$$

where $\sigma(T_d)$ is a constant temperature freeze-out hypersurface, p^μ is the 4-momentum of the emitted particle and u^μ is the 4-velocity of the fluid. Although it is possible to use more realistic freeze-out criteria [11]-[15], here we limit ourselves to the simplest choice (32) without sophistication. This will be enough for our present purpose to study how the initial condition fluctuations affect the final particle spectra.

3.1 Landau's Model

In computing the isothermal σ and the velocity (or rapidity) distribution of the fluid in Landau's model, one has to distinguish two different regions. In the so-called non-trivial region ($\alpha \leq -\xi/\sqrt{3}$, $\xi = \ln(T/T_0)$, T_0 =initial temperature, α =fluid rapidity), the temperature $T(x,t)$ and the rapidity $\alpha(x,t)$ are given in terms of the potential [16]

$$\Psi(\alpha,\xi) = -T_0\, l\, \sqrt{3}\, e^{\xi} \int_{\alpha/\sqrt{3}}^{-\xi} e^{2\xi'}\, I_0(\sqrt{\xi'^2 - \alpha^2/3}\,)\, d\xi'\,, \tag{33}$$

through the relations

$$t = \frac{\partial\Psi}{\partial T}\cosh\alpha - \frac{1}{T}\frac{\partial\Psi}{\partial\alpha}\sinh\alpha\,, \tag{34}$$

$$x = \frac{\partial\Psi}{\partial T}\sinh\alpha - \frac{1}{T}\frac{\partial\Psi}{\partial\alpha}\cosh\alpha\,. \tag{35}$$

Here, we are considering only one hemisphere $\alpha \geq 0$. The solution for $\alpha \leq 0$ can be obtained by making a reflection. In the simple-wave region,

$$\alpha = -\frac{\xi}{1/\sqrt{3}} \tag{36}$$

and the fluid velocity is related to (x,t) through

$$x = \frac{v - 1/\sqrt{3}}{1 - v/\sqrt{3}}t\,. \tag{37}$$

Remind that we are supposing that the fluid is an ideal gas of zero mass particles.

The invariant momentum distribution (33), with this solution becomes

$$E\frac{dN}{d\vec{p}} = \frac{g}{(2\pi)^3}\int_{\sqrt{3}\xi_d}^{-\sqrt{3}\xi_d}\frac{m_T[\cosh(y-\alpha)\phi(\alpha)+\sinh(y-\alpha)\psi(\alpha)]}{\exp\{m_T\,\cosh(y-\alpha)/T_c\}-1}\,d\alpha + \frac{V_0\,e^{-2\xi_d}}{2}\frac{g}{2\pi^2}\frac{m_T[\cosh(y-\alpha_d)+\sqrt{3}\,\sinh(y-\alpha_d)]}{\exp\{m_T\cosh(y-\alpha_d)/T_d\}-1}\,, \tag{38}$$

where V_0 is the initial volume and

$$\phi(\alpha) = A\,\frac{e^{-\xi_d}}{T_0}\left(\frac{\partial\Psi}{\partial\xi}-\frac{\partial^2\Psi}{\partial\alpha^2}\right)\,, \tag{39}$$

$$\psi(\alpha) = A\,\frac{e^{-\xi_d}}{T_0}\left(\frac{\partial\Psi}{\partial\alpha}-\frac{\partial^2\Psi}{\partial\alpha\partial\xi}\right) \tag{40}$$

and the suffix "d" stands for dissociation (or freeze-out). In (38), the first term represents the contribution from the non-trivial region and the second one that from the simple wave.

The formula (38) gives us the invariant momentum distributions of the decay products (pions in the majority) of a fireball in terms of V_0 and T_0, once the dissociation temperature T_c is fixed. In the original work of Landau, it was assumed that the total energy $\sqrt{s}$ is liberated as thermal energy in a small Lorentz contracted interaction volume

$$V_0 = \frac{V}{\gamma}\,, \tag{41}$$

where V is the proper volume of proton and $\gamma = 2m_p/\sqrt{s}$. The initial temperature T_0 is then computed assuming the fluid is an ideal gas, i.e., $p = \varepsilon/3$. Nowadays we know that neither the hypothesis of instantaneous thermalization nor the appearance of extremely high values of the initial temperature are physically reasonable so that many people are reluctant in accepting the model itself. However, in spite of these rather *non conventional* initial conditions, many of the qualitative and the quantitative results (average multiplicity, particle ratios, momentum distributions, $\cdots$) are surprisingly good when compared with data. In our point of view, perhaps the equilibrium is attained at a later time when the system has already suffered some expansion, but then the temperature and the rapidity distributions at the onset of the hydrodynamical regime are approximately those of Landau's model whose initial conditions correspond to high temperature and energy density if extrapolated back in time. So, for any practical purpose, we can use Landau's solution to describe the system.

Now, as mentioned before, we are going to apply Landau's model to each fireball characterized by its mass M and momentum $\vec{P}$. Then, the total energy is replaced

by M and everything is computed in the fireball's rest frame (which is boosted with respect to the center of mass of the collision). But, then which is the initial volume V_0 in this case? It has been shown [17] that, in the case of the incident-particle fragmentation,

$$V_0 = \frac{V}{\gamma} = \frac{2m_p}{M}\,V\,. \tag{42}$$

In the case of the central fireball, we do not have such a simple expression. However, phenomenological analysis [19] of the M dependence of average multiplicity data [18] has shown that in terms of Landau's model those data may well be reproduced if one assumes (42). So, in the present work we shall also assume (42) and T_0 will be computed by putting M into this volume.

3.2 Rapidity and Pseudorapidity Distributions

We now compute the rapidity distributions in two distinct ways. In the first case, with the help of our generator function $\chi(E,P;\vec{b})$, we calculate the average initial conditions, i.e., the average mass with $\langle P\rangle = 0$ because of the symmetry. Then the rapidity distribution is computed as done usually by using the formula (38). In the other case, fluctuations are taken into account and the rapidity distribution given by (38) is computed for each event and summed over M, P (or E, P) and $\vec{b}$ according to

$$\frac{\overline{dN}}{dy} = \int d\vec{b}\int dP\int dE\,\frac{dN}{dy}(E,P;\vec{b})\,\chi(E,P;\vec{b})\,\theta(\sqrt{(E^2-P^2)/s}-k_{min})\,. \tag{43}$$

The results obtained with these two prescriptions at the incident energy $\sqrt{s} = 24.0\ GeV$ are given in Fig. 3. Observe that, since we have included also the simple wave solution, the rapidity distribution for one fireball with the average initial conditions presents a large peak at this energy. When the fluctuations are taken into account, such a peak is completely smoothed away.

Although the main purpose of this work is just to show the influences of the fluctuations in the initial conditions, we may proceed to a comparison with some data [21]. These are given in terms of pseudo-rapidity. So, we calculate the pseudo-rapidity distributions for the energies $\sqrt{s} = 53$ and $546\ GeV$, by taking the average value of $m_T = 0.41$ and $0.49\ GeV$, respectively. In this approximation the pseudo-rapidity distribution is given by

$$\frac{dN}{d\eta} = \frac{dN}{dy}\times\frac{\sqrt{(\langle p_T\rangle/\langle m_T\rangle)^2+\sinh^2 y}}{\cosh y}\,. \tag{44}$$

The results are shown in the Figs. 5 and 6. The comparison of both results leads us to conclude that the rapidity or equivalently pseudo-rapidity distributions are very

sensitive to the fluctuations in the initial conditions. These fluctuations cause a widening, a smoothing and a lowering of the distributions. Moreover, if we compare the distributions with and without fluctuations with experimental data, we see that the behavior of the first ones is more similar to the data than the other ones and the presence of the simple-wave peaks in each event does not invalidate the overall agreement with data.

4 Concluding Remarks

We have studied in details the effects produced by the fluctuations in the initial conditions on the final observables of emitted particles. As a mechanism of fluctuations, we have used a modified version of the Interacting Gluon Model, by including also the impact-parameter fluctuations. The inclusion of the latter has shown to be significant, since a spreading in impact parameter causes a corresponding widening in the fireball mass distribution or, equivalently, in the inelasticity distribution. This widening causes, in its turn, flattening of the leading-particle spectrum. A better agreement of our results with data on these quantities, as compared with the predictions of the original version, may indicate that our version is indeed an improvement.

The modified IGM allowed us to introduce fluctuations in the fireball energy-momentum as well as in the impact parameter, through the distribution function $\chi(E, P; \vec{b})$. We have then studied the effects of these fluctuations on the rapidity and pseudo-rapidity distributions, using Landau's hydrodynamical model. This has been done by computing the rapidity (or pseudo-rapidity) distributions in two distinct ways. First, by the usual procedure in which only one fireball is assumed with the average characteristics (mass, momentum and impact parameter). Second, by taking the fluctuations into account, by generating each event according to the probability distribution $\chi(E, P; \vec{b})$ and by summing up over all the events. The difference between them are found to be quite appreciable. As expected, the rapidity (or pseudo-rapidity) distributions become smoother, wider and lower when fluctuations are considered. The version of hydrodynamical model we used shows peaks in the rapidity distribution(originated from the simple waves) of each event. Nevertheless, in the over-all distribution they are entirely smoothed away, showing that even analyses of such a simple quantity as the inclusive one particle distribution may lead to a complete wrong conclusions if fluctuations are totally neglected.

It is well known that there are several observables which cannot be understood if fluctuations are not properly taken into account. For instance, KNO distribution [20], forward-backward correlation [21], [22], semi-inclusive distributions [21] and so on. However, as is easily be convinced the fluctuations we considered in the present work are not enough to account for these quantities. One of the fluctuations we

did not includ here and which seemingly plays an important role is the multiplicity fluctuation in the fireball decay, given its mass M. Investigation in this direction is in progress.

In conclusion, despite all the simplifications made in our description, our results do show that the fluctuations are indeed a very important feature in the hadronic collisions and must be considered in any realistic description of these collisions [23].

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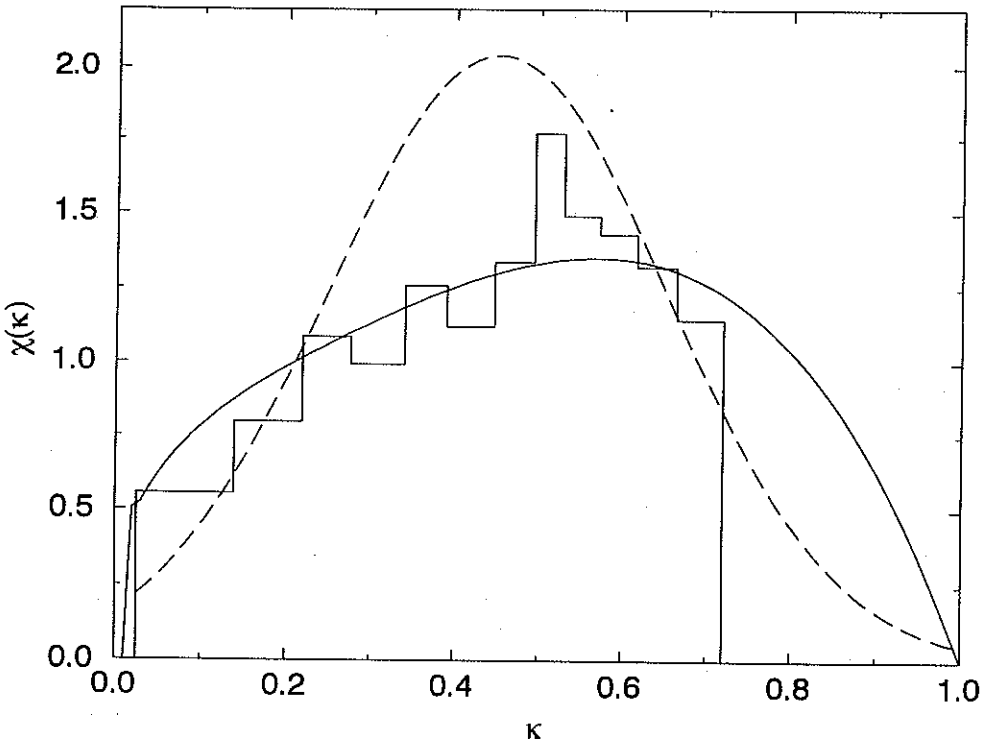

Figure 1: $\chi(k)$ for $s^{1/2} = 16.5\ GeV$. The data are extracted from ref. [7]. The dashed curve corresponding the original IGM, without impact parameter fluctuation ($c = 4$), and the solid one represents our result ($c = 6.4$).

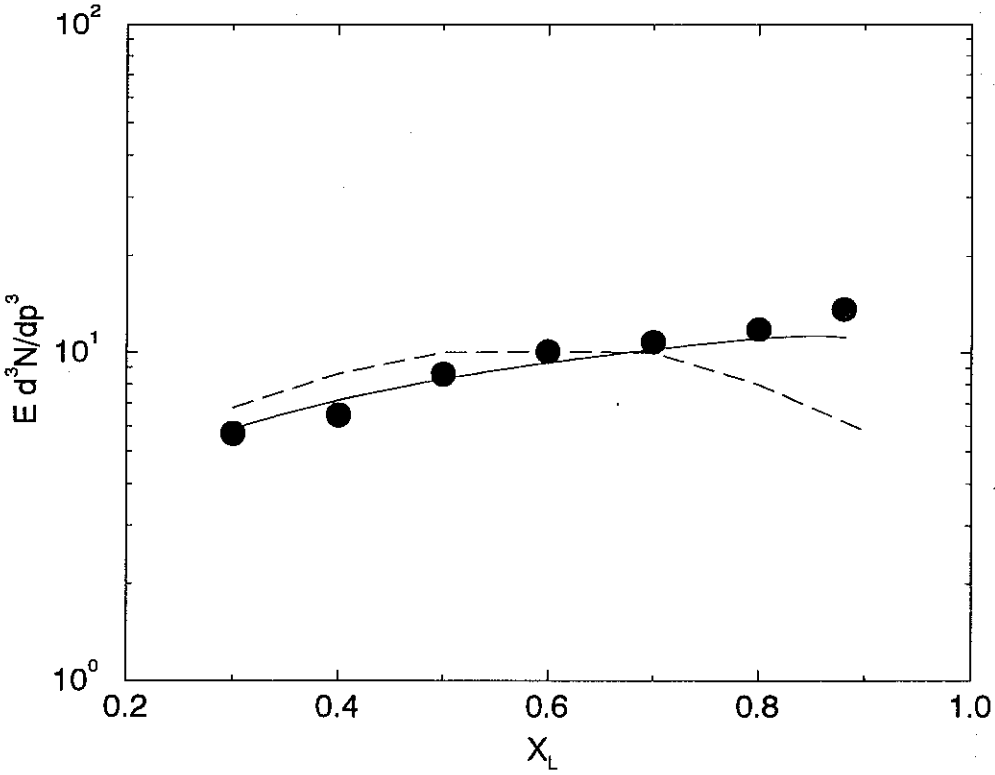

Figure 2: Leading particle spectrum for $s^{1/2} = 14\ GeV$, the data are extracted from ref. [8] with $p_T = 0.3\ Gev/c$, the dashed curve corresponding the IGM without impact parameter fluctuation and the solid one represents our result.

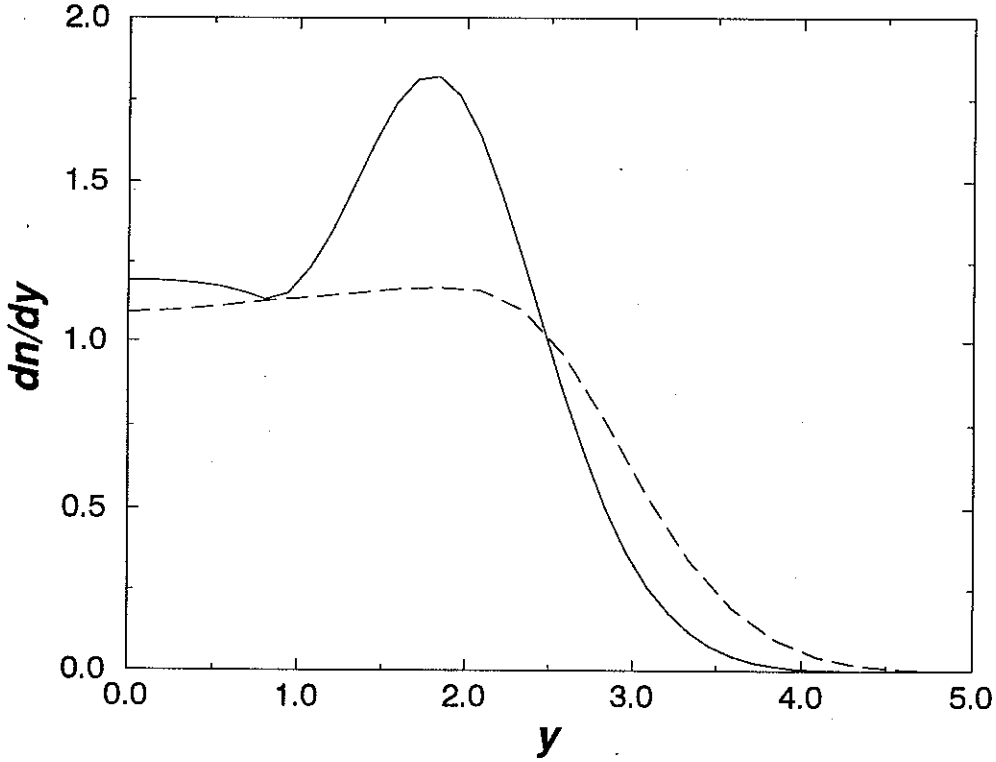

Figure 3: The rapidity distributions calculated in usual procedure and with fluctuations for $s^{1/2} = 24\ GeV$. The solid line represents the one with average initial conditions and the dashed is the obtained with fluctuations.

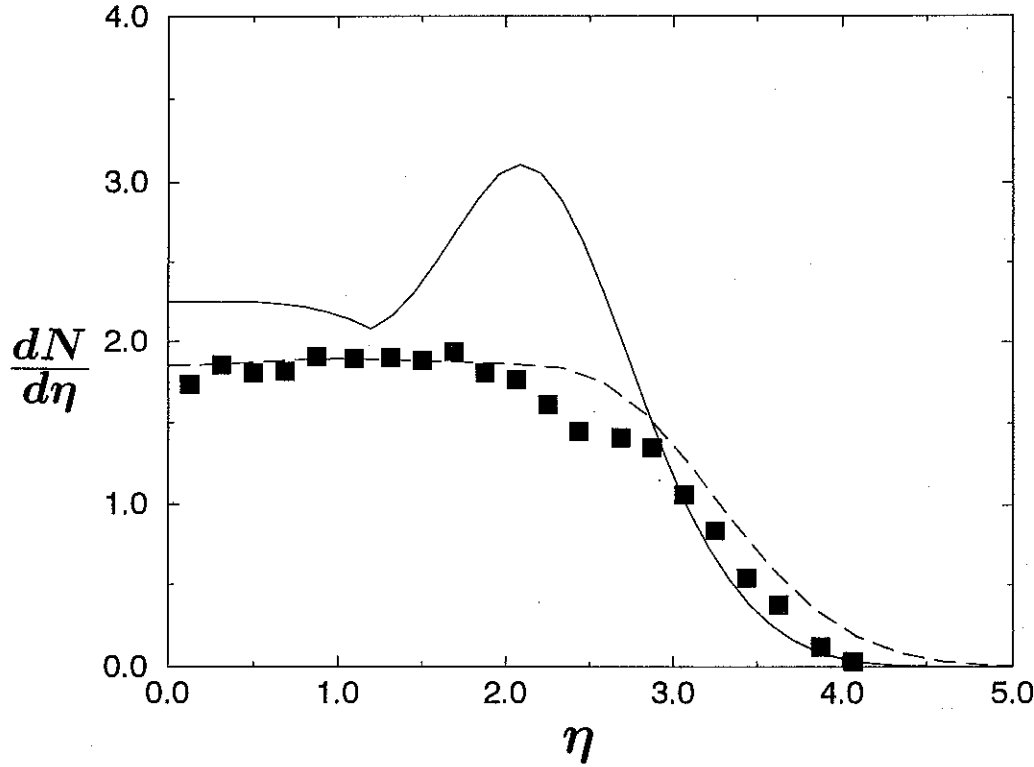

Figure 4: The pseudo-rapidity distributions calculated in usual procedure and with fluctuations for $s^{1/2} = 53\ GeV$. The solid line represents the one with average initial conditions and the dashed is the obtained with fluctuations. Experimental data [24] are shown for comparison.

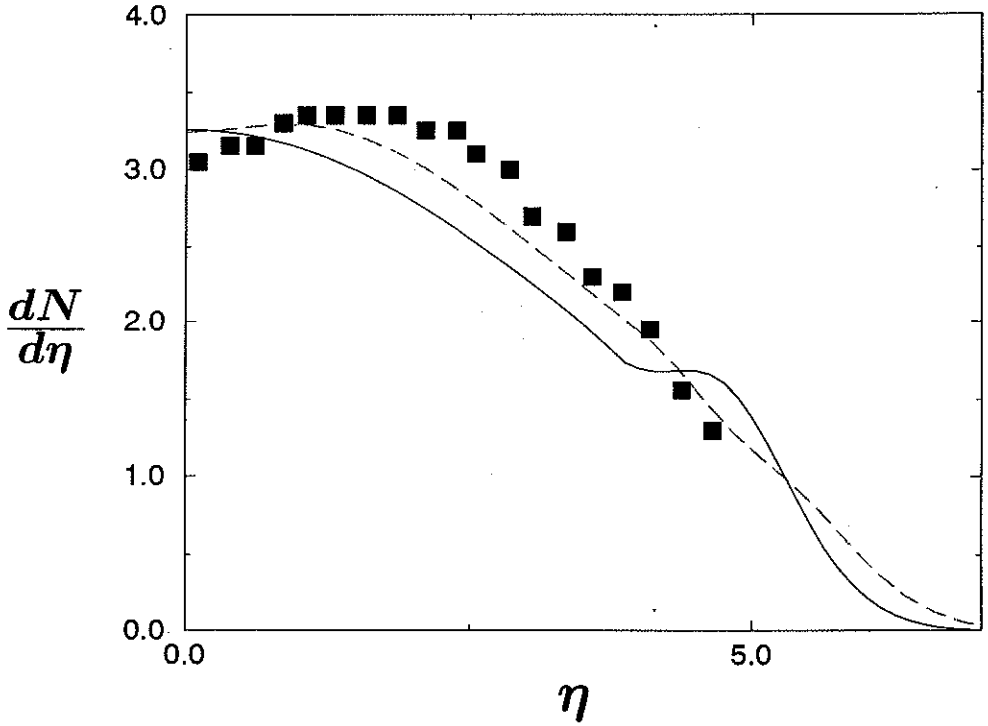

Figure 5: The pseudo-rapidity distributions calculated in usual procedure and with fluctuations for $s^{1/2} = 546\ GeV$. The solid line represents the one with average initial conditions and the dashed is the obtained with fluctuations. Experimental data [21] are shown for comparison.