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LUMINESCENCE QUENCHING MECHANISMS OF X- AND γ-IRRADIATED BARIUM ALUMINOBORATE GLASSES DOPED WITH Fe

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case, all terms proportional to $1/m$ (including the kinetic energy) in (30) can be dropped, and the resulting equations give the binding energy $V = E - 2m$ for states where the quark and anti-quark are localized a distance r apart (i.e. for wave-functions $S(r), E1(r), E2(r)$ and $B(r)$ all proportional to $\delta(r - r_0)$. It is natural to compare this mass-independent binding energy to phenomenological potentials and those extracted from the expectation values of Wilson loops in numerical lattice simulations.

In this limit the $E2$- and B-components decouple and one has to solve the algebraic equations

$$\begin{aligned}(-V + C - \frac{4}{3}\frac{\alpha_s}{r})S(r) &= -\frac{\pi\phi r}{3\sqrt{2}}E_1(r) \\ (-V + \frac{1}{6}\frac{\alpha_s}{r})E_1(r) &= \frac{\pi\phi r}{3\sqrt{2}}S(r)\,. \end{aligned} \tag{31}$$

The eigenfunctions are obviously localized and the eigenvalue or effective potential, $V(r)$, given by

$$V(r) = -\frac{1}{2}(\frac{7}{6}\frac{\alpha_s}{r} + \sqrt{\frac{9}{4}\frac{\alpha_s^2}{r^2} - \frac{9}{3}\frac{\alpha_s C}{r} - \frac{2\pi^2\phi^2 r^2}{9} + C^2}) \tag{32}$$

Figure 1(a) shows this effective potential and its Coulomb part for parameters $\phi^2 = (360MeV)^4$, $C = -756MeV$ and $\alpha_s = 0.39$, which we found appropriate for charmonium.

Although this potential is certainly no longer valid for $r > 0.9$fm, where the root in (32) becomes purely imaginary it does show a nearly linear behaviour for intermediate distances $0.4\text{fm} < r < 0.8\text{fm}$, with a correspondingly constant force of ~ 840MeV/fm, which compares favorably with a string tension of about $800 - 1000$MeV/fm extracted from recent lattice calculations of the potential[25]. In Fig. 1(b) we compare our effective potential (32) to that extracted from lattice data[25] and to the phenomenological potential used by ref[16].

It is encouraging that our rather crude approximations to the vacuum structure seem to qualitatively reproduce the potential for very heavy quarks at small distances. The analytical results of this section justify the numerical calculation of "octet"-components in heavy quarkonia which we now present.

5 Numerical Results

Going beyond the static approximation by including the kinetic energy of the quarks but still neglecting coupling terms of order $1/m$ in (30), the quarkonium spectrum becomes discrete and the following coupled set of differential equations must be solved numerically

$$\begin{aligned}\Big[2m - E + C - \frac{1}{m}\frac{\partial^2}{\partial r^2} - \frac{4}{3}\frac{\alpha_s}{r}\Big]S(r) &= -\frac{\pi\phi r}{3\sqrt{2}}E_1(r) \\ \Big[2m - E - \frac{1}{m}(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}) + \frac{1}{6}\frac{\alpha_s}{r}\Big]E_1(r) &= \frac{\pi\phi r}{3\sqrt{2}}S(r)\,. \end{aligned} \tag{33}$$

This is essentially Leutwyler's[20] approximation to the problem, who obtained the perturbation of the Coulomb spectrum in 2^{nd}-order of ϕ and found that it is exceedingly large for canonical values of the condensate. Note that the Coulomb force is repulsive in the "octet" channel and 1/8-th in strength compared to the "singlet" channel (this is just the ratio of $T_1^a \bar{T}_2^a$ in the two representations) and the E_2- and B-states still decouple in this approximation. In this approximation, vector- and pseudoscalar- quarkonia are furthermore still degenerate. This is expected, since the spin splitting is of order $1/m$. It is however another nontrivial consistency check of our method, because the basis states for vector mesons are of course quite different. Nevertheless equivalent equations to (33) result, if $1/m$ potential terms are neglected.

The relative minus sign of the two off-diagonal coupling terms in (33) shows that this Hamiltonian is not hermitian and that its eigenvalues will generally not be real. This effect is completely missed if the coupling is treated as a perturbation. To any finite order in perturbation theory the correction to the Coulomb spectrum is real. Perturbation theory does however show that a few (low lying) eigenvalues of (33) are real for a sufficiently small ϕ. For the canonical value $\phi^2 = (300 - 380\text{MeV})^4$ we find numerically $2 - 4$ stable bound quarkonia in the pseudoscalar channel, depending on the heavy quark mass m.

But even for these low states, the deviation of the exact correction to the Coulomb eigenvalue from the 2^{nd}-order estimate is large for the charmonium and bottonium systems as shown in table 1. We conclude with Leutwyler[20] that perturbation theory is not applicable for canonical values of ϕ, but that

the effective non-hermitian Hamiltonian equations (33) do yield reasonable corrections in an exact solution.

The value $C = -756$MeV used by us was ajusted to reproduce the correct splitting between η_c and η_c'[26]. The corrections to the Coulomb spectrum are however not extremely sensitive to the value of C once it is large enough. Using $C = -1400$MeV instead would only reduce the nonperturbative contributions for the ground and first excited bottonium-states in table 1 to 6 and 75MeV respectively.

The fact that a perturbative evaluation in ϕ is not appropriate in (33) can also not be circumvented by including loop corrections (higher orders in α_s than we have treated so far) to the perturbative Coulomb potential. Titard and Ynduráin[27] recently proposed to modify the perturbative part of the interaction in the following manner

$$\frac{\alpha_s}{r} \to \frac{\alpha_s(\mu^2)\,[1 + (a_1 + \gamma_E\beta_0/2)\alpha_s(\mu^2)/\pi]}{r} + \frac{\beta_0\alpha_s^2(\mu^2)}{2\pi}\frac{\log r\mu}{r}\,, \qquad (34)$$

where the appropriate constants for the SU(3) color group with 4 light quark flavors are $\beta_0 = 8.33$ and $a_1 = 1.47$. The first term of (34) which contains a piece of one-loop radiative corrections was taken by Titard and Ynduráin as an effective Coulomb potential and solved exactly. The effective coupling constant is defined as

$$\tilde{\alpha}_s(\mu^2) = [1 + (a_1 + \gamma_E\beta_0/2)\frac{\alpha_s(\mu^2)}{\pi}]\alpha_s(\mu^2)\,. \qquad (35)$$

The second term in (34) was treated to first order in perturbation theory. A new scale parameter μ was introduced which depends on the quarkonium system under consideration. Taking the effective Coulomb potential we obtain the deviation of the eigenvalues from 2^{nd}-order perturbation theory in ϕ shown in table 2 for two sets of parameters in the bottonium system (still neglecting $1/m$ corrections).

The deviation of the exact correction from the perturbative estimate is reduced somewhat, especially in the second case, but still far from negligible. In assessing the quality of a perturbative evaluation in this case, one should also keep in mind that the scale parameter μ of Ref.[27] was chosen in such a way that the 2^{nd} order correction in ϕ is precisely canceled by the correction terms to the effective Coulomb potential in (34). For the first set

of parameters this cancellation occurs for the ground state energy. The second set was chosen so that the splitting between the first excited state and the ground state is not affected to 2^{nd} order. This is an arbitrary procedure which requires an additional parameter and does not cure the problem that perturbation theory simply does not apply (as table 2 clearly indicates).

We therefore will not include these modifications to the Coulomb force in our discussion of the numerical solution to the full set of coupled equations (30). The eigenvalues we obtained at order $(1/m)^0$ are summarized in table 3 and compared to those obtained by Eichten et al.[16] with the phenomenological funnel potential. Note that we only found 3 or 4 stable solutions in the charmonium and bottonium systems respectively. We used the same values for the quark masses and the coupling constant as Eichten et al.[16].The constant C was chosen to reproduce the experimental splitting (not confirmed[26]) between η_c and η_c'. It was not adjusted in the bottonium system. The gluon condensate value $\phi^2 = (360\text{MeV})^4$ that we used is within QCD-sumrule estimates[28] for this nonperturbative quantity. All eigenvalues were finally shifted by $E_0 = 98$MeV to give the correct η_c mass. (This small shift in the overall energy normalization can be eliminated by a slight change of ~ 50MeV in the quark masses used by Eichten et al.[16] and a corresponding small adjustment of the other parameters. To have a more direct comparison of the wavefunctions and spectra, we refrained from making these adjustments here.)

The inclusion of $1/m$ potential terms lifts the degeneracy of pseudoscalar- and vector- quarkonia and we restrict our discussion here to the pseudoscalar case. The B and E_2-states now couple in, but generally have small norms because the coupling is of order $1/m$. The effects of the $1/m$ corrections in the splittings are shown in table 4.

We also show in table 4 the results obtained using a regularized running coupling constant with the correct two-loop behaviour for small distances[29]:

$$\alpha_s = \alpha_s(r) = 4\pi\frac{1}{b_0 f(r)}[1 + \frac{2\gamma_E + 53/75}{f(r)} - \frac{462}{625}\frac{ln(f(r)}{f(r)}]$$

where

$$f(r) = ln[\frac{1}{(\Lambda r)^2} + b]$$

$$b_0 = 25/3$$

In the above expression $\gamma_E = 0.5772$, and as in [29], we used $\Lambda = 300MeV$ and $b = 19$.

In Figs. 2 and 3 we show the eigenfunctions for the various components of our quarkonium states (7) as well as the corresponding eigenfunction of Eichten et al.[16]. The singlet component of our ground state wave functions are very similar to those of the funnel potential. At small radii this is true also for the excited states, since the coupling to "octet" components is proportional to r in (30). The "octet"-components increase with increasing excitation energy of the quarkonium and lead to the appearance of extra nodes in the higher lying "singlet" wave functions at large radii (since this is a coupled channel problem, the extra nodes do not mean that we missed some bound states). As noted earlier, the E-components of the meson state contribute negatively to its norm. All the stable quarkonia states that we found are however positive norm states. We could not obtain any stable state where the octet components are dominant.

Let us speculate at this point on the fact that only very few stable quarkonium states were found. This is of course quite in line with the experimental observation that only a few heavy quarkonia are stable against decay by strong interactions. For reasons which we had not anticipated, this basic property seems already to be incorporated in the nonhermitian coupling to E-components.

The strength of this coupling in our model is however determined by the gluon condensate ϕ^2, which is not expected to vanish even in the purely gluonic theory. From table 3 we see that the instability sets in at an excitation energy of between $1 - 1.3$GeV in this model. Since we cannot account for the decay into light $q\bar{q}$-mesons with a parameter which is essentially independent of the number of (light) flavours, we speculate that the nonhermitian coupling proportional to ϕ effectively accounts for the possible decay channel

$$\text{Quarkonium}^* \longrightarrow \text{Quarkonium} + \text{Gluonium}\,, \qquad (36)$$

in this model. From the fact that we do not seem to find any stable quarkonium state more than 1.3GeV above the ground state, we would estimate this to be the threshold for the production of the lightest gluonium. This estimate of the lightest gluonium mass $m_G \gtrsim 1.3$GeV, is in almost perfect agreement with our previous interpretation of the energy shift $-C = m_g \sim m_G/2$.

Since the production of light $q\bar{q}$-pairs has a much lower threshold, it is this process which limits the stability of physical quarkonia. We therefore

expect this model, which does not (not even effectively) incorporate this decay channel, to still predict more stable states than are experimentally observed.

The influence of a running constant is more visible in the wave functions at the origin, which are important for production and annihilation decays. We compare the wave functions (not multiplied by r or r^2) with and without running coupling constants in figures 4 and 5.

6 Conclusion

We developed a hamiltonian formalism, which enabled us to estimate the effect of a nonperturbative gluonic ground state on quarkonia in a systematic short distance, weak coupling and $1/m$ expansion of the effective hamiltonian. The gauge invariant basis[8] was extended to include color octet quark-antiquark pairs coupled to vacuum fluctuations. Hamiltonian matrix elements in this basis are gauge invariant to the order considered in the short distance expansion.

After separating hard and soft gluons in the gauge (13), we obtained the effective hamiltonian neglecting radiative corrections to the Coulomb interaction from hard gluons. We thus neglected the logarithmic corrections to the effective coupling strength at very short distances. The correct behaviour of the potential for $r < 0.2fm$ can in principle be included by "hand" in a modification of the Coulomb part of the interaction[19]. Although the quarkonium spectrum is not very sensitive to this correction at small distances, it could become important for the evaluation of decay widths, which depend on the wave function at the origin.

In the limit of very heavy quark masses, where all terms of order $1/m$ can be neglected, an energy independent effective potential was obtained, which shows an approximately linear behaviour at intermediate distances $0.4fm < r < 0.8fm$ with an effective string tension of $\sim 840MeV$. In our approach this behaviour of the potential arises due to the nonperturbative structure of the gluonic vacuum parametrized by its gluon condensate and was not assumed from the outset as in most of phenomenological quark models. This potential compares favourably with recent lattice results[25], the discrepancies at very small $r < 0.2fm$ being due to our neglect of radiative corrections. Our short distance expansion for the effective potential however

is only valid for $r < 0.9fm$, beyond which the potential acquires an imaginary part. An extension of the model to larger distances would require a more detailed knowledge of the vacuum structure in the form of higher dimensional condensates, or some other effective parametrization of this structure. The approach in this case would become increasingly phenomenological and also more complicated in this case. Its predictive power is therefore probably limited to heavy quarkonia, where a detailed knowledge of the potential for very large radii does not seem to be necessary.

We showed that the numerical solution of the coupled channel problem for vector- and pseudoscalar- quarkonia (they are degenerate to order $1/m$) is feasible and an exact diagonalization of the hamiltonian in the extended basis therefore possible. The resulting exact spectrum does not show the far too rapid increase of the eigenvalues with the principal quantum number of the perturbative approach to the vacuum effects[9][20]. In addition to the usual "singlet" wave-functions describing the quark and anti-quark of the quarkonium when they are coupled in a colour singlet, we also obtain the "octet" components of quarkonium states describing the quarks in the octet configuration when an additional (soft) gluon is around. Since hadronic decays may be very sensitive to this "octet" configuration with the creation of an additional octet $q\bar{q}$-pair from a hard gluon, this approach opens the possibility of estimating nonperturbative contributions to hadronic decays.

We compare our results for the spectrum and wavefunctions with those of the Cornell potential[16] for pseudoscalar quarkonia. With the standard value for the gluon condensate and quark masses and coupling constant used by the Cornell group we obtain the correct splitting between η_c and η_c' and make predictions for the η_b's. Our main concern was however a better theoretical understanding and justification of the phenomenological ingredients common to most nonrelativistic models for heavy quarkonia and we refrained from adjusting the few parameters of this approach to optimally reproduce the experimental data. A better description of the potential at short distances with the inclusion of radiative corrections to the Coulomb force and the consideration of hadronic decay channels is clearly desirable before a detailed comparison with phenomenology is attempted.

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Figure 1: (a) Effective potential (solid curve) from (32) and Coulomb potential (dotted curve) with $\alpha_s = 0.39$, $C = -756$ MeV and $\phi^2 = (360\ \text{MeV})^4$. (b) Effective potential (solid curve) with the same parameters as in (a). The potential extracted from lattice data[25] with $\sqrt{\sigma} = 365$ MeV (dot-dashed curve) and $\sqrt{\sigma} = 505$ MeV (dashed curve). Cornell potential[16] (dotted curve) with $\alpha_s = 0.39$ and $a = 2.34\ \text{GeV}^{-1}$.

Figure 2: The wavefunctions of the ground and 1^{st} excited pseudoscalar states of charmonium are shown in figures (a) and (b) respectively. The solid curve is the wavefunction for the funnel potential[16] for comparison. The long dashed curve is the singlet component $S(r)$ of the quarkonium state in our calculation. The dashed, dotted and dot-dashed curves are the "octet" components $E_1(r)$, $E_2(r)$ and $B(r)$ of equation (30) respectively. The solution was obtained with the parameters $m_c = 1840$ MeV, $\alpha_s = 0.39$, $\phi^2 = (360\ \text{MeV})^4$ and $C = -756$ MeV.

Figure 3: The wavefunctions of the ground, 1^{st}- and 2^{nd} excited pseudoscalar states of bottonium are shown in figures (a), (b) and (c) respectively. The solid curve is the wavefunction for the funnel potential[16] for comparison. The long dashed curve is the singlet component $S(r)$ of the quarkonium state in our calculation. The dashed, dotted and dot-dahsed curves are the "octet" components $E_1(r)$, $E_2(r)$ and $B(r)$ of equation (30) respectively. The solution was obtained with the parameters $m_b = 5170$ MeV, $\alpha_s = 0.39$, $\phi^2 = (360\ \text{MeV})^4$ and $C = -756$ MeV.

Figure 4: The wavefunctions (not multiplied by r or r^2) of the ground and 1^{st} excited pseudoscalar states of charmonium are shown in figures (a) and (b) respectively. The solid curves are for the results without a running coupling constant. The dashed curves are obtained with a running coupling constant. The solutions were obtained with the parameters $m_c = 1840$ MeV, $\alpha_s = 0.39$ (without running), $\Lambda = 300$ MeV and $b = 19$ (with running), $\phi^2 = (360\ \text{MeV})^4$ and $C = -756$ MeV.

Figure 5: The wavefunctions of the ground, 1^{st}- and 2^{nd} excited pseudoscalar states of bottonium are shown in figures (a), (b) and (c) respectively. The solid curves are for the results without a running coupling constant. The dashed curves are obtained with a running coupling constant. The solutions were obtained with the parameters $m_b = 5170$ MeV, $\alpha_s = 0.39$ (without running), $\Lambda = 300$ MeV and $b = 19$ (with running), $\phi^2 = (360\ \text{MeV})^4$ and $C = -756$ MeV.

Table 1: The Coulomb energy in MeV is presented in the first column for the ground state and first excitation of $c\bar{c}$ and $b\bar{b}$ (pseudoscalar or vector). Second and third columns contain the nonperturbative contributions in MeV calculated respectively within perturbation theory and with our method. We used $m_c = 1840$ MeV, $m_b = 5170$ MeV, $\alpha_s = 0.39$, $\phi^2 = 0.012\ \text{GeV}^4$ and $C = -756$ MeV.

	Coulomb	Pert. theory	Non-pert.
η_c, J/ψ	-124.4	311	66
η_c', ψ'	-31.1	21525	424
η_b, Υ	-349.5	14	8
η_b', Υ'	-87.4	970	111

Table 2: The Coulomb energy in MeV is presented in the first column for the ground state and first excitation of $b\bar{b}$ (pseudoscalar or vector) at two different scales μ. Second and third columns contain the nonperturbative contributions in MeV calculated respectively within perturbation theory and with our method. We used $C = -756$ MeV, $\phi^2 = 0.042\ \text{GeV}^4$ and for $\mu = 1.44$ GeV: $m_b = 4866$ MeV, $\tilde{\alpha}_s = 0.38$. For $\mu = 0.99$ GeV: $m_b = 5010$ MeV, $\tilde{\alpha}_s = 0.54$.

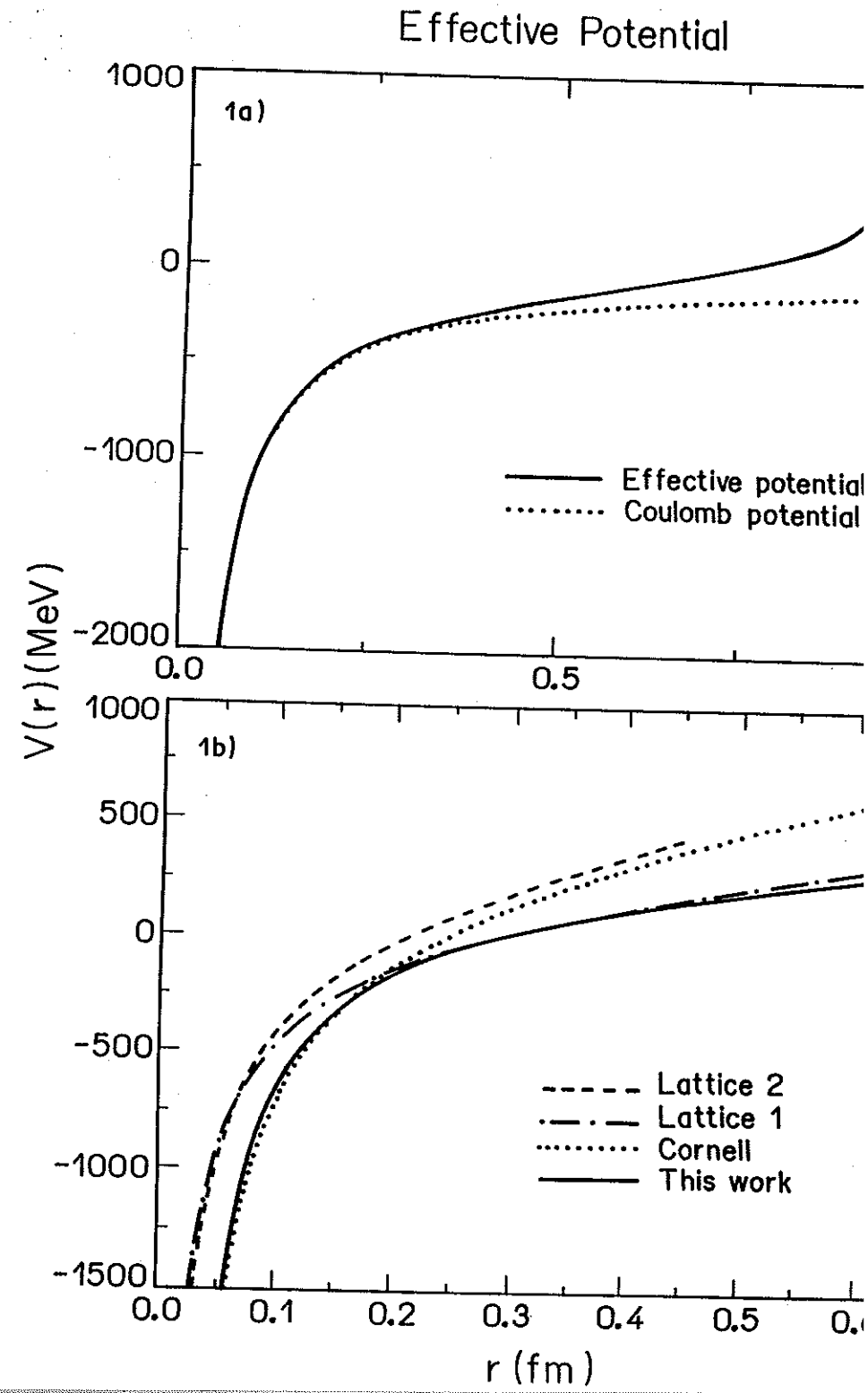
	Coulomb	Pert. theory	Non-pert.
η_b, Υ ($\mu = 1.44$ GeV)	-312	25	11
η_b', Υ' ($\mu = 1.44$ GeV)	-78	1762	129
η_b, Υ ($\mu = 0.99$ GeV)	-649	6	3
η_b', Υ' ($\mu = 0.99$ GeV)	-162	396	72

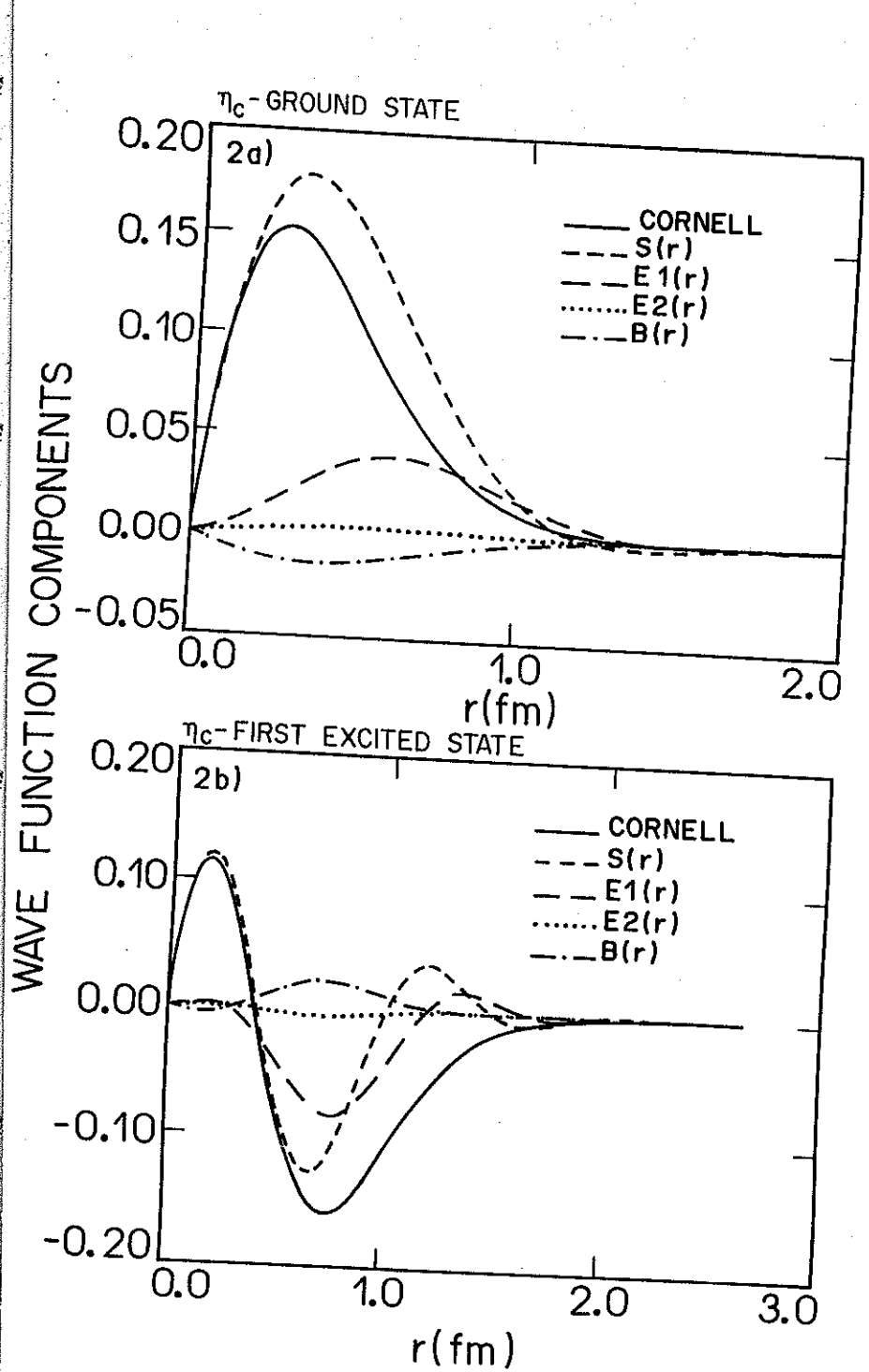
Table 3: Masses in MeV of pseudoscalar quarkonia with the funnel potential and with our effective nonperturbative hamiltonian. Parameters used: $m_c = 1840$ MeV, $m_b = 5170$ MeV, $\alpha_s = 0.39$, $\phi^2 = (360\ \text{MeV})^4$, $C = -756$ MeV.

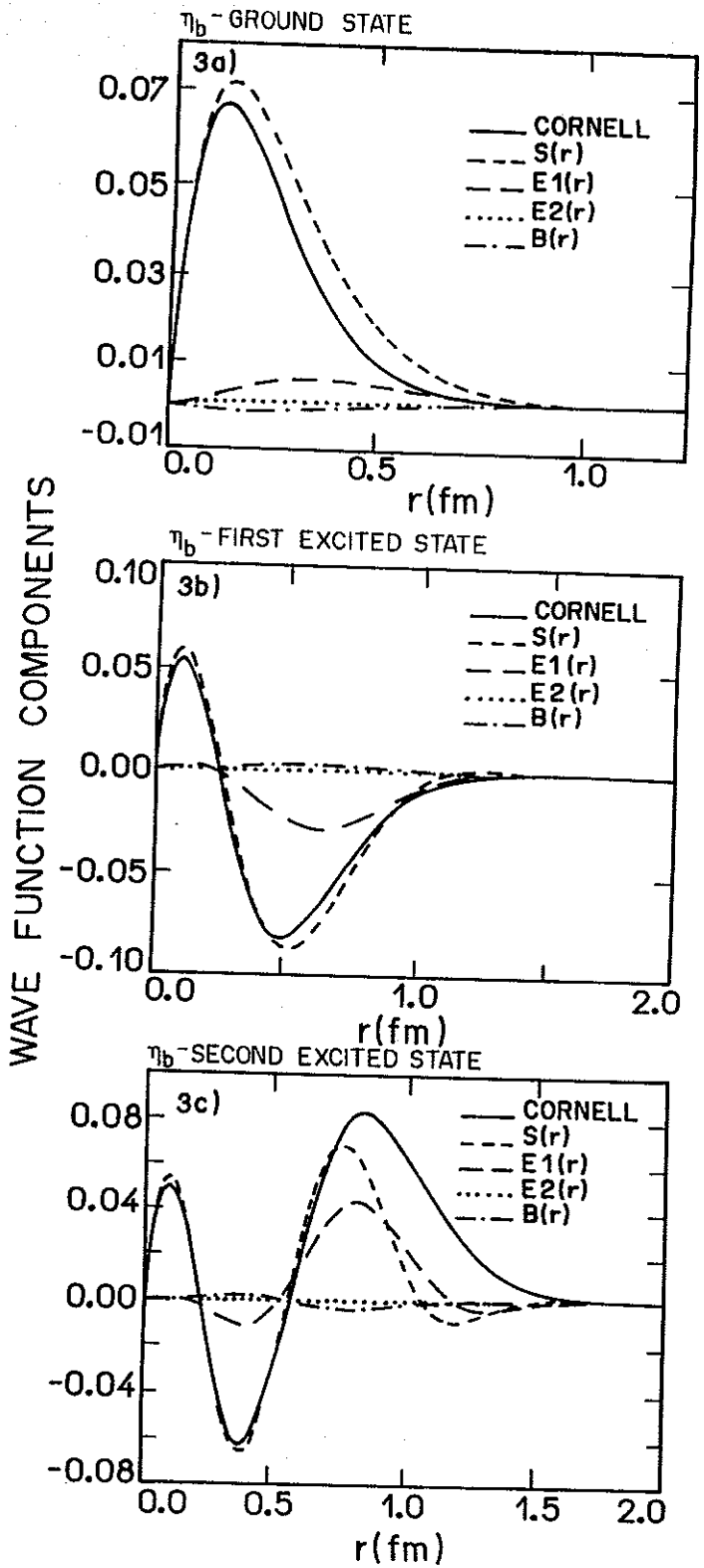
	funnel	nonpert.
η_c	2980	2980
η_c'	3571	3594
η_c''	3994	3993
η_b	9213	9344
η_b'	9805	9739
η_b''	10150	10084
η_b'''	10427	10610

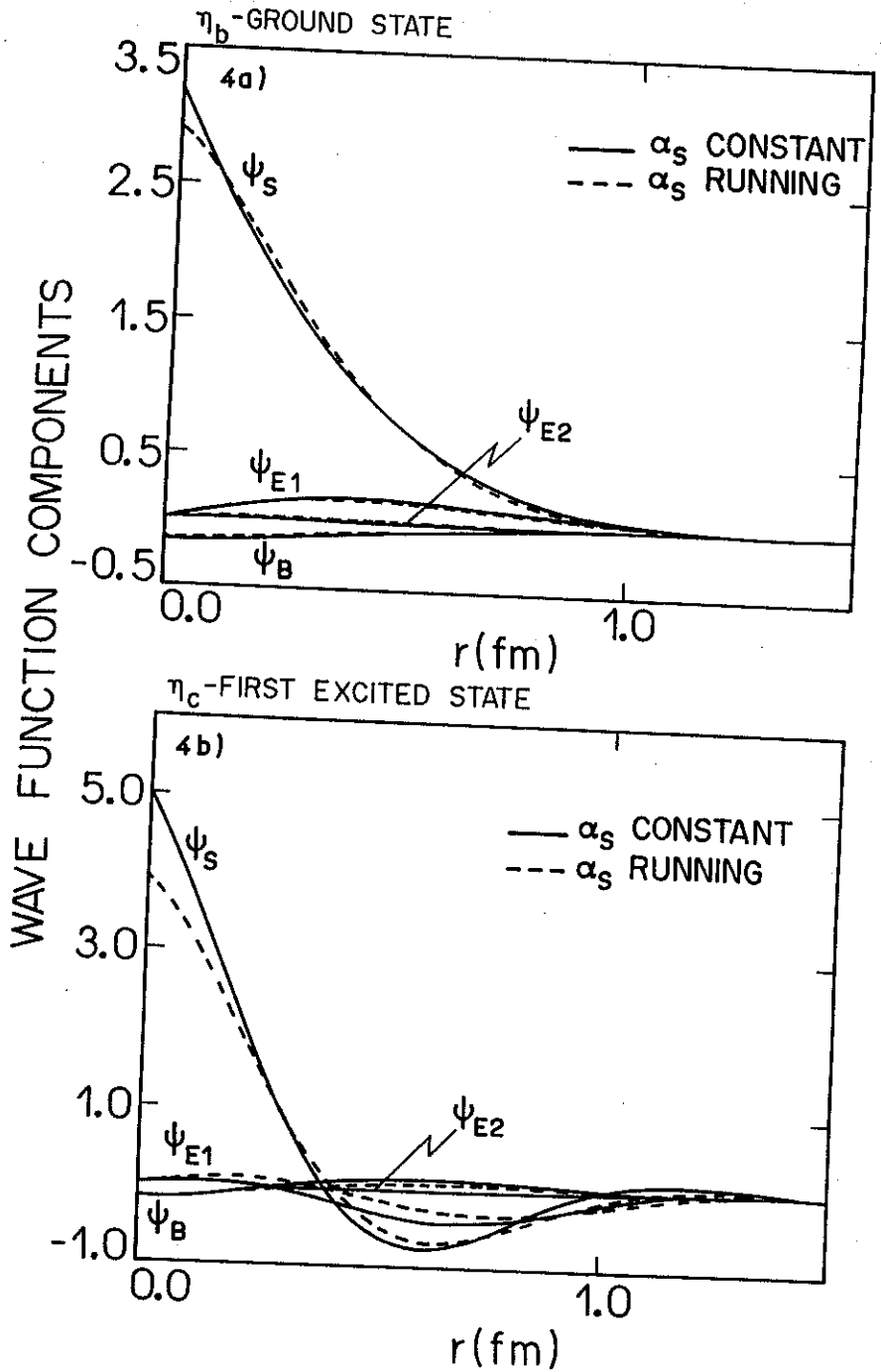
Table 4: Energy splittings among radial excitations of pseudoscalar charmonium and bottonium. The parameters are the same as in table 3. For the running coupling constant (see text) we use $\Lambda = 300$ MeV and $b = 19$. First column: no $1/m$ terms. Second column: $1/m$ corrections included. Third column: $1/m$ terms and running coupling constant.

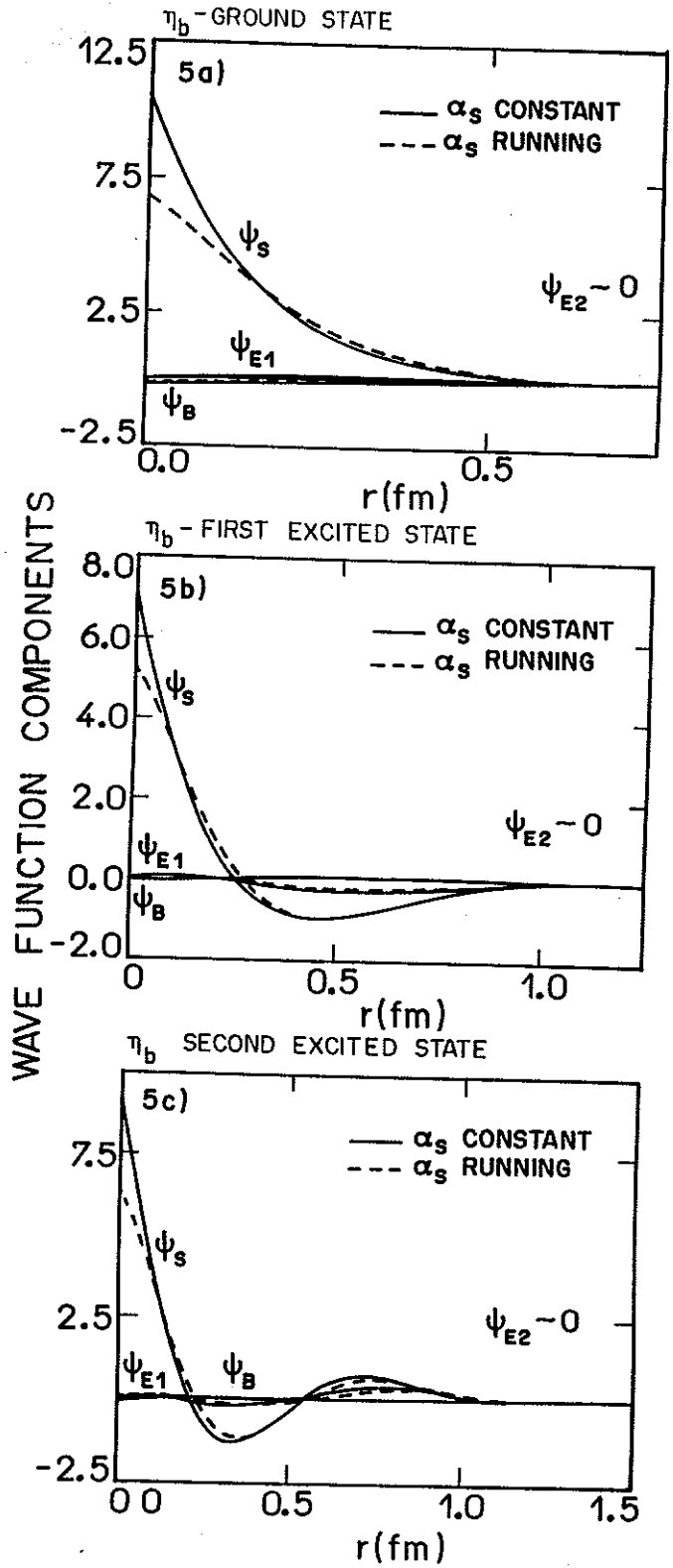
	no 1/m	1/m included	1/m and running
$c\bar{c}$ - 2S-1S	614	666	593
$b\bar{b}$ - 2S-1S	395	397	413
$b\bar{b}$ - 3S-1S	740	753	738




ηc - GROUND STATE
2a)
CORNELL
S(r)
E1(r)
E2(r)
B(r)
0.20
0.15
0.10
0.05
0.00
-0.05
0.0
1.0
2.0
r(fm)
ηc - FIRST EXCITED STATE
2b)
CORNELL
S(r)
E1(r)
E2(r)
B(r)
0.20
0.10
0.00
-0.10
-0.20
0.0
1.0
2.0
3.0
r(fm)
WAVE FUNCTION COMPONENTS


ηb - GROUND STATE
3a)
CORNELL
S(r)
E1(r)
E2(r)
B(r)
0.07
0.05
0.03
0.01
-0.01
0.0
0.5
1.0
r(fm)
ηb - FIRST EXCITED STATE
3b)
CORNELL
S(r)
E1(r)
E2(r)
B(r)
0.10
0.05
0.00
-0.05
-0.10
0.0
1.0
2.0
r(fm)
ηb - SECOND EXCITED STATE
3c)
CORNELL
S(r)
E1(r)
E2(r)
B(r)
0.08
0.04
0.00
-0.04
-0.08
0.0
0.5
1.0
1.5
2.0
r(fm)
WAVE FUNCTION COMPONENTS


η_b-GROUND STATE
4a)
α_s CONSTANT
α_s RUNNING
ψ_s
ψ_{E2}
ψ_{E1}
ψ_B
WAVE FUNCTION COMPONENTS
r(fm)
η_c-FIRST EXCITED STATE
4b)
α_s CONSTANT
α_s RUNNING
ψ_s
ψ_{E2}
ψ_{E1}
ψ_B
r(fm)


η_b-GROUND STATE
5a)
α_s CONSTANT
α_s RUNNING
ψ_s
$\psi_{E2}\sim 0$
ψ_{E1}
ψ_B
r(fm)
η_b-FIRST EXCITED STATE
5b)
α_s CONSTANT
α_s RUNNING
ψ_s
$\psi_{E2}\sim 0$
ψ_{E1}
ψ_B
WAVE FUNCTION COMPONENTS
r(fm)
η_b SECOND EXCITED STATE
5c)
α_s CONSTANT
α_s RUNNING
ψ_s
$\psi_{E2}\sim 0$
ψ_{E1}
ψ_B
r(fm)