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Abstract

We show that two-dimensional topological BF theories coupled to particles carrying non-Abelian charge admit a new coupling involving the Lagrange multiplier field. When applied to the gauge theoretic formulation of dilatonic gravity it gives rise to a source term for the gravitational field. We show that the system admits black hole solutions.

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Two-dimensional gravity theories have been studied in the hope to have a better understanding of the quantum properties of its four-dimensional counterpart. Usually others fields are introduced if we require non trivial dynamics for the gravitational field. A class of interesting models introduces just one scalar field known as the dilaton. A particular model where black holes can be formed and the quantization can be performed is the Callan, Giddings, Harvey and Strominger (CGHS) model [1]. It is described by the action

$$S = \int d^2x\,\sqrt{-\bar{g}}[e^{-2\phi}(\bar{R} - 4\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \Lambda) + \frac{1}{2}\sum_i \bar{g}^{\mu\nu}\partial_\mu f_i\partial_\nu f_i], \quad (1)$$

where $\bar{R}$ is the curvature scalar build with the metric $\bar{g}_{\mu\nu}$, ϕ is the dilaton, Λ is the cosmological constant and f_i is a set of scalar matter fields. If the conformal transformation $g_{\mu\nu} = e^{-2\phi}\bar{g}_{\mu\nu}$ is performed the action (1) takes the form [2]

$$S = \int d^2x\,\sqrt{-g}\,(\eta R - \Lambda + \frac{1}{2}\sum_i g^{\mu\nu}\partial_\mu f_i\partial_\nu f_i), \quad (2)$$

where $\eta = e^{-2\phi}$ and R is the scalar curvature built with the metric $g_{\mu\nu}$. Now the field equation for η implies that $R = 0$ so that the two-dimensional space-time is locally flat and there is no black hole solution. This happens because the conformal transformation changes the geometry but since is just a field redefinition the physical content of theory should not be affected by it. Usually the metric $\bar{g}_{\mu\nu}$, which can describe black holes, is called the "physical" metric while $g_{\mu\nu}$, which is locally flat, is called the "stringy" metric and physical interpretations depending on the space-time geometry are usually taken using the "physical" metric. At the quantum level the conformal transformation is more problematic. While the model described by (1) presents Hawking radiation the model described by (2) has no Hawking radiation. It has been argued that with proper care of the conformal transformation no ambiguity exists [3]. Even so the quantization in either form is not free of troubles [4].

An important aspect of the action (2) is that it can be rewritten as a topological gauge theory of the BF type [5] with a gauge group which is a central extension of the two-dimensional Poincaré group [6]. If we couple matter in this formulation it should be coupled in a gauge invariant way. A proposal to do that is to use a Higgs-like mechanism which introduces a new field, the Poincaré coordinate, to describe point-like matter [7]. Another possibility makes use of a formulation of relativistic particles which carry non-Abelian charges [8]. It was applied to the gauge theoretic version of dilatonic gravity and some solutions were shown to be equivalent to those of the Poincaré coordinate formulation [9]. An important feature of both approaches to include matter is that the curvature equation $R = 0$ never acquires a source term. We will show that in the formulation where particles carry non-Abelian charge a new gauge invariant coupling does exist for topological BF theories. This new coupling provides a source term for the curvature equation and black hole solutions can then be found.

Non-Abelian degrees of freedom for point particles were originally introduced in the context of QCD [10, 8]. They are described by the group element $g(\tau)$ and a real constant element of the algebra K, τ being the proper time of the particle. It is useful to introduce the variable

$$Q(\tau) = g(\tau) K g^{-1}(\tau), \tag{3}$$

which is in the adjoint representation. The minimal coupling between the particle and the gauge field can then be performed by introducing a covariant derivative

$$D_\tau = \frac{d}{d\tau} + e\dot{x}^\mu A_\mu(x(\tau)). \tag{4}$$

If we also consider a kinetic term for the relativistic particle then an action which is gauge and reparametrization invariant is [8]

$$S = -m\int d\tau\, \sqrt{\dot{x}^2} + \int d\tau\, Tr(K g^{-1}(\tau) D_\tau g(\tau)). \tag{5}$$

This action is also invariant under the transformation $K \to SKS^{-1}$ where S is τ independent. This shows that the action (5) is independent of the direction in the internal symmetry space given by K. Varying the action (5) with respect to $x^\mu(\tau)$ we get a non-Abelian version of the Lorentz force

$$m\ddot{x}^\mu + \Gamma^\mu_{\nu\rho}\dot{x}^\nu\dot{x}^\rho = -eTr(F^\mu{}_\nu Q)\dot{x}^\nu, \tag{6}$$

while varying with respect to $g(\tau)$ we get a covariant conservation equation for the non-Abelian charge Q

$$\frac{dQ}{d\tau} + [A_\mu(x(\tau)), Q(\tau)]\dot{x}^\mu(\tau) = 0. \tag{7}$$

These equations are known as the Wong equations [10].

Consider now a two-dimensional BF topological field theory

$$S = \int d^2x\, Tr(\eta\, F), \tag{8}$$

where $F = dA + A^2$ is the curvature two-form corresponding to the connection one-form A and η is a zero-form transforming in the co-adjoint representation of the gauge group. The action of particles carrying non-Abelian charge (5) combined with the BF action (8) was studied in [9] while the Abelian case was treated in [11]. When compared with the pure BF case, the main consequence of adding non-Abelian matter is that the field equation for the Lagrange multiplier η acquires a source term $\epsilon^{\mu\nu}D_\nu\eta = J^\mu$ (where J is the non-Abelian current of the particle) while the connection remains flat $F = 0$.

Since the structure of the BF theory requires two fields then, besides the coupling involving the gauge field, we can consider another coupling involving the Lagrange multiplier η. A coupling of the type $Tr(\eta\, Q)$ is gauge and reparametrization invariant so we can consider an extension of (8) plus (5) to

$$\begin{aligned} S &= \int d^2x\, Tr(\eta\, F) + \int d\tau\, Tr(g^{-1}K\dot{g}) + \\ &+ e\int d^2x \int d\tau\, Tr(Q(\tau)A_\mu(x))\, \delta^2(x - x(\tau))\, \dot{x}^\mu(\tau) + \\ &+ g\int d^2x \int d\tau\, Tr(Q(\tau)\eta)\, \delta^2(x - x(\tau)), \end{aligned} \tag{9}$$

where e and g are independent coupling constants. The main consequence of the new coupling when compared to the former case is that the connection is no longer flat and has as source the non-Abelian charge Q

$$\epsilon^{\mu\nu}F_{\mu\nu} = g\int d\tau\, Q(\tau)\, \delta^2(x - x(\tau)). \tag{10}$$

As we shall see in the context of dilatonic gravity theory this change allows us to find black hole solutions without making reference to any conformal transformation to a "physical" metric. The field equations for the Lagrange multiplier is now

$$\epsilon^{\mu\nu}D_\nu\eta = e\int d\tau\, Q(\tau)\, \delta^2(x - x(\tau))\, \dot{x}^\mu(\tau), \tag{11}$$

while the equation for non-Abelian charge is modified to

$$\frac{dQ}{d\tau} + e[A_\mu(x(\tau)), Q]\dot{x}^\mu + g[\eta(x(\tau)), Q] = 0. \quad (12)$$

generalizing the conservation equation (7) to the BF theory. Notice that the field equations obtained by varying $x^\mu(\tau)$ vanish identically. The reason is that by varying the action with respect to $x^\mu(\tau)$ we get a contribution which is proportional to the previous field equations (10-12) so that it vanishes on-shell. This shows the topological character of the the non-Abelian particle in the sense that its local motion (described by $x^\mu(\tau)$) is completely arbitrary not being determined by any field equation. There are only global restrictions to the motion of the particle as was shown in [9, 11] in the case $g = 0$.

To consider the two dimensional dilatonic gravity theory we now choose the gauge group as the central extension of the Poincaré group [6]

$$\begin{aligned} [P_a, P_b] &= \epsilon_{ab} Z, \\ [J, P_a] &= \epsilon_a{}^b P_b, \\ [P_a, Z] &= [J, Z] = 0, \end{aligned} \quad (13)$$

where P_a is the translation generator, J is the Lorentz transformation generator and Z is a central charge. The supersymmetric extension of (13) was performed in [12]. The flat Minkowski metric is $h_{ab} = diag(-1, +1)$ and $\epsilon^{01} = 1$. When we consider the algebra (13) we can expand the one form gauge potential in terms of the generators of the algebra

$$A = e^a P_a + wJ + AZ. \quad (14)$$

The fields e^a, w and A are going to be identified with the zweibein, the spin connection and an Abelian gauge field, respectively. The Lagrange multiplier η can be expanded as

$$\eta = \eta^a P_a + \eta_3 J + \eta_2 Z, \quad (15)$$

with components η^a, η_2 and η_3 with η_2 being proportional to the dilaton in (2). Then the curvature two-form F has components

$$\begin{aligned} F^a(P) &= de^a + we^b \epsilon_b{}^a, \\ F(J) &= d\omega, \\ F(Z) &= dA + \frac{1}{2} e^a e^b \epsilon_{ab}. \end{aligned} \quad (16)$$

Similarly the non-Abelian charge Q can be expanded as

$$Q = Q^a P_a + Q_3 J + Q_2 Z. \quad (17)$$

Then the field equations for the gauge fields (10) are

$$\epsilon^{\mu\nu}(\partial_\mu e^a_\nu + \omega_\mu e^b_\nu \epsilon_b{}^a) + g \int d\tau\, Q^a(\tau)\, \delta^2(x - x(\tau)) = 0, \quad (18)$$

$$\epsilon^{\mu\nu}\partial_\mu \omega_\nu + g \int d\tau\, Q_3(\tau)\, \delta^2(x - x(\tau)) = 0, \quad (19)$$

$$\epsilon^{\mu\nu}(\partial_\mu a_\nu + \frac{1}{2} e^a_\mu e^b_\nu \epsilon_{ab}) + g \int d\tau\, Q_2(\tau)\, \delta^2(x - x(\tau)) = 0, \quad (20)$$

while the field equations for the Lagrange multipliers (11) are

$$\epsilon^{\mu\nu}(\partial_\nu \eta_a - \omega_\nu \epsilon_a{}^b \eta_b + \eta_3 \epsilon_{ab} e^b_\nu) + e \int d\tau Q_a(\tau) \delta^2(x - x(\tau)) \dot{x}^\mu(\tau) = 0 \quad (21)$$

$$\epsilon^{\mu\nu}(\partial_\nu \eta_2 + e^a_\nu \epsilon_a{}^b \eta_b) + e \int d\tau\, Q_2(\tau)\, \delta^2(x - x(\tau)) \dot{x}^\mu(\tau) = 0, \quad (22)$$

$$\epsilon^{\mu\nu}\partial_\nu \eta_3 + e \int d\tau\, Q_3(\tau)\, \delta^2(x - x(\tau)) \dot{x}^\mu(\tau) = 0. \quad (23)$$

The equations of motion for the non-Abelian charge (12) are

$$\frac{dQ^a}{d\tau} + e\epsilon_b{}^a(e^b_\mu Q_3 - \omega_\mu Q^b)\dot{x}^\mu + g\epsilon_b{}^a(\eta^b Q_3 - \eta_3 Q^b) = 0, \quad (24)$$

$$\frac{dQ_2}{d\tau} - \epsilon_{ab}(e e^a_\mu Q^b \dot{x}^\mu - g\eta^a Q^b) = 0, \quad (25)$$

$$\frac{dQ_3}{d\tau} = 0. \quad (26)$$

Due to the relation $\epsilon^{\mu\nu}\partial_\mu \omega_\nu = \sqrt{-g}R$ it is now clear from (19) that in order to have a non vanishing curvature R we need to have $g \neq 0$. Notice also that torsion $T_{\mu\nu} = \epsilon^{\mu\nu}(\partial_\mu e^a_\nu + \omega_\mu e^b_\nu \epsilon_b{}^a)$ may also be present due to (18).

We will now look for some solutions of the above equations. We will find the general solution in the absence of matter and in the presence of a non-Abelian point particle at rest. A more general analysis will be done elsewhere. In order to solve (18-26) we have to perform several gauge fixings and to choose a space-time trajectory for the particle since, as remarked before, there is no field equation for $x^\mu(\tau)$. We will look for static solutions so we use Rindler like coordinates (x,t). For simplicity let us consider the particle at rest at the origin

$$x=0, \qquad t=\tau. \tag{27}$$

Let us consider first the gauge field sector. In the gravitational sector we will choose a diagonal zweibein $e_0^1 = e_1^0 = 0$ with the non vanishing components satisfying $e_0^0 = (e_1^1)^{-1}$, and a vanishing space component of the connection $\omega_1 = 0$. For the Abelian gauge field we will choose the axial gauge $A_1 = 0$. Then eqs.(18-20) reduce to

$$\begin{aligned} \partial_1 e_0^0 + \omega_0 e_1^1 &= gQ^0\,\delta(x), && (28)\\ 0 &= gQ^1\delta(x), && (29)\\ \partial_1\omega_0 &= gQ_3\,\delta(x), && (30)\\ \partial_1 A_0 + e_0^0 e_1^1 &= gQ_2\,\delta(x). && (31)\end{aligned}$$

If no matter is present ($e=g=0$) then we find flat space-time as the only solution. Explicitly we find

$$\begin{aligned} \omega_0 &= -a, \quad e_0^0 = (b+2ax)^{\frac{1}{2}},\\ A_0 &= -x + A, \end{aligned} \tag{32}$$

where a, b and A are integration constants. Notice that the line element $ds^2 = -(b+2ax)dt^2 + (b+2ax)^{-1}dx^2$ has a singularity at $x=-b/2a$. This coordinate singularity can be removed by the coordinate transformation

$$\begin{aligned} \sigma &= \frac{1}{a}\sqrt{b+2ax}\ \cosh(at),\\ \tau &= \frac{1}{a}\sqrt{b+2ax}\ \sinh(at) \end{aligned} \tag{33}$$

for the patch $x > -b/2a, \sigma > 0$ and similar transformations for the three remaining patches. This coordinate transformation brings the metric in (32) to its Minkowski form. The curvature scalar vanishes dues to (30).

Now if $Q_3 = 0$ and $Q^a \neq 0$ then the space-time has torsion but no curvature since (28) is proportional to the torsion. If $Q_3 \neq 0$ and $Q^a = 0$ then the space-time has curvature but no torsion. Let us consider the last

case. Take Q_2 and Q_3 as constants (as we shall see below Q_2 and Q_3 constants and $Q^a = 0$ is a solution of (24-26)). Then we find as solution of (28-31)

$$\begin{aligned} \omega_0 &= gQ_3\epsilon(x), \quad e_0^0 = (\tilde{b} - 2gQ_3|x|)^{\frac{1}{2}},\\ A_0 &= -x + gQ_2\epsilon(x) + \tilde{A}, \end{aligned} \tag{34}$$

where $\tilde{b}$ and $\tilde{A}$ are integration constants. The space-time described by (34) has a black hole [13] and the curvature scalar is given by (30) $R = gQ_3\delta(x)$. Notice that gQ_3 can now be understood as the black hole mass and it is essential to have $g \neq 0$.

In the Lagrange multiplier sector the gauge choices reduce eqs.(21-23) to

$$\begin{aligned} &\partial_1\eta_0 - \eta_3 e_1^1 = -eQ_0\delta(x),\\ &\partial_1\eta_1 = -eQ_1\delta(x),\\ &\omega_0\eta_1 = \omega_0\eta_0 + \eta_3 e_0^0 = 0,\\ &\partial_1\eta_2 - e_1^1\eta_0 = -eQ_2\delta(x),\\ &e_0^0\eta_1 = 0,\\ &\partial_1\eta_3 = -eQ_3\delta(x). \end{aligned} \tag{35}$$

In the absence of matter, using (32), we find the solution

$$\eta_0 = \frac{\Lambda}{a}(b+2ax)^{\frac{1}{2}}, \quad \eta_1 = 0, \tag{36}$$

$$\eta_2 = \frac{\Lambda}{a}x + c, \quad \eta_3 = \Lambda, \tag{37}$$

where Λ and c are integration constants. Notice that we have identified one of the integration constants, the one coming from integrating η_3 in (35), as the cosmological constant Λ. The reason is that this solution in the absence of matter [6] reproduces the results obtained in the original CGHS formulation after performing the conformal transformation. The dilaton (37) has an unusual form in the coordinates (x,t). By performing the coordinate transformation (33) it becomes proportional to $\sigma\tau$ assuming its usual form [1].

In the presence of matter with Q_2 and Q_3 constants and $Q^a = 0$ we find, using (34)

$$\begin{aligned} \eta_0 &= \frac{e}{g}(\tilde{b} - 2gQ_3|x|)^{\frac{1}{2}}, \quad \eta_1 = 0,\\ \eta_2 &= \frac{e}{g}x - eQ_2\epsilon(x) + \tilde{c}, \quad \eta_3 = -eQ_3\epsilon(x), \end{aligned} \tag{38}$$

where $\tilde{c}$ is another integration constant and $\epsilon(x)$ is the step function. The appearance of the step function in the solution for the Lagrange multiplier fields signals that there are topological restrictions to the motion of particles [9, 11]. It is remarkable that the would be cosmological constant η_3 is now a step function changing sign at the position of the particle. No constant term is allowed in the solution for η_3 so if we set $e = 0$ then the cosmological constant vanishes. The dilaton η_2 still has its linear term but has also acquired a step function. We can however set $Q_2 = 0$ and still have a linear dilaton and the black hole (34), which is independent of Q_2.

Finally the equations for the non-Abelian charge (24-26), after the gauge choice, reduce to

$$\begin{aligned}
&\dot{Q}^0 - eQ^1\omega_0 - g(Q^1\eta_3 - Q_3\eta^1) = 0,\\
&\dot{Q}^1 - e(Q^0\omega_0 - Q_2e_0^0) - g(Q^0\eta_3 - Q_3\eta^0) = 0,\\
&\dot{Q}_2 - eQ^1e_0^0 + g(Q^0\eta^1 - Q^1\eta^0) = 0,\\
&\dot{Q}_3 = 0.
\end{aligned} \tag{39}$$

In the absence of matter these equations are trivially satisfied. In the presence of matter with $Q^a = 0$ we find using (34) and (38) that Q_2 and Q_3 are constants as we have anticipated.

We have presented some local solutions for the gauge theoretic version of dilatonic gravity theories with non-Abelian sources. We are still investigating more general solutions and classifying them. It is also needed to perform a global analysis of the solutions. When $g = 0$ this can be performed essentially because the gauge connection is flat [11]. In the present case this task becomes more difficult since the gauge connection is no longer flat due to (10). Progress on these lines will be reported elsewhere.

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