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DIRECTING A CHAOTIC SYSTEM TO AN AIMED STATE OR TARGET

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Abstract

In this work, we use a set of constant perturbations to rapidly direct a chaotic system to a desired state or target. This procedure was applied to the one-dimensional Logistic map, and to the two-dimensional Hénon map. Using the Logistic map, we show numerically that the resulting trajectory (from the starting point to the target) goes along the stable manifold of the target. Also, using this map, we show the procedure robustness when noise is introduced. The targeting procedure is usually applied for experimental systems only if this system is one dimensional or for those approximately modeled by a one dimensional equation. However, we apply the method for a three dimension dynamical system, the Double Scroll circuit, without using any one dimensional model.

1 Introduction

Since chaotic systems have sensitivity on initial conditions, small perturbations lead to enormous changes in the system trajectories. In reference [1] the authors applied small perturbations on a parameter to control an unstable periodic orbit. Since then, many other methods to control chaos have been proposed [2], [3], [4], [5], [6].

One of these several ways chaos can be controlled is by applying a resonant perturbation that can originate a stable phase locked trajectory [7], [8], [9]. In some cases, the resonant amplitudes to eliminate chaos are large and, therefore, the perturbation modifies the original dynamics. Although this modification would not be always tolerable, in some cases it may be still convenient. Thus, new orbits introduced by the perturbations may be used to drive the system.

A system with a periodic behaviour can not reach some essential regions not located along the periodic orbit. Also, it may be necessary a large amount of time requisited for a chaotic system to reach some point. In addition, to apply some methos of chaos control the trajectory of the system to be controlled needs to be on some desired point, namely, in the vicinity of the unstable periodic orbit one wants to control. So, targeting methods have been proposed to rapidly direct a chaotic system to some specific location.

The idea of targeting a chaotic system is owned to Shimbrot and others [10], who used the Hénon map to demonstrate their method. This method considers one determined initial perturbation, which will direct a starting point to a desired target. If the dynamical equations are unkown, the method can only be used if the system can be one dimensionally modeled [11], [12].

For the Lorenz system, a three dimensional flow with one positive Lyapunov coefficient, Shinbrot applied his method to direct flows to stationary states [13]. However, this method is not useful if the system is high dimensional, that is, the system has more than one positive Lyapunov exponent [14] and [15].

So, in [16], it is presented a method that can be applied to high dimensional systems with known equations, considering one determined peturbation, for each positive Lyapunov coefficient.

These previously mentioned methods do not focus the question about how to target most efficiently a chaotic system. In the reference [17] the authors use optimal control theory to target the Hénon map, in the fastest possible way by applying n definite perturbations.

Other works about targeting chaotic systems can be seen in [18],[19].

The method we show in this paper does not need to use prescribed amplitude perturbations, but rather a sequence of constant perturbations.

In section 2 we present a general idea of this method applied to maps.

Using the Logistic map, in section 2.1, we explain geometrically how the perturbation targets this chaotic system.

In section 3 we use our method to create a new orbit that was not present in the original (non perturbed) chaotic attractor and, then, this orbit is controled using the OGY method [1].

In section 4 we show how to target a three dimensional flow (the Double Scroll circuit [20]) without the necessity of modelating the system or using the previous knowledge of the dynamical equations.

In section 5 we use our method to optimize the OGY method of control [1].

2 Targeting Maps

Suppose that you are dealing with a one-parameter map, represented by the equation $X_{n+1} = F(b, X_n)$ whose parameter b can be changed by $\pm\delta$. So, the parameter b can assume three values, $b+\delta$, b, and $b - \delta$. We want to show that with these three possible parameter values we are able to direct a starting point X_0 to a target located at the vicinity of X_f, by applying N times these perturbations on the b parameter.

To understand the idea of directing a starting point X_0 to the vicinity of X_f, we perform the following steps. Initially, we apply the map F to the point X_0, using the three possible values of the parameter b. So, we get from the first iterate three new points: $X_1 = F(b + \delta, X_0)$, $X_2 = F(b, X_0)$, $X_3 = F(b - \delta, X_0)$.

We keep performing this procedure, by applying the map F, to each of the three points obtained from the first iterate (X_1, X_2, X_3), for the three possible parameter values. Thus, at the second iteration we get nine points (X_1, ..., X_9). Furthermore, for the N^{th} iterate we can get 3^N points. If one of these points reaches the vicinity of the target, that is, the interval given by $[X_f - \epsilon, X_f + \epsilon]$, then we stop iterating the map.

The set of perturbations that direct the point X_0 to the target is indicated in figure 1, which indicates a path to reach the target at X_3, after N=2 iterations.

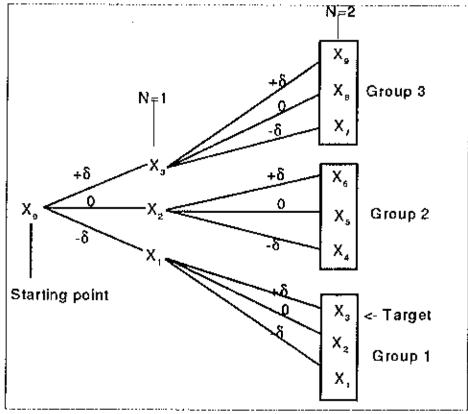

Figure 1: The path to reach the target, and the value of the constant perturbations ($+\delta$,0, $-\delta$) at each iteration. The starting point is indicated by X_0, and the target that can be achieved by a set of two constant perturbations ($-\delta$, $+\delta$).

When N is high the amount of memory needed to keep all the points that form the 3^N paths is large. So, we developed a numerical procedure to avoid this problem. In fact, the only information we need to know is the 3^N points obtained at the N^{th} iteration.

In figure 1 we see the paths from the starting point X_0. We put together the three points (X_1, X_2, X_3) obtained from the point X_1 of the first iteration and call this set **group 1**. So, **group 2** is formed by the three points (X_4, X_5, X_6) obtained from the point X_2 of the first iteration and **group 3** is formed by the other three points. The point that has the highest index inside a group was obtained from the iteration of the former point applying a positive parameter perturbation ($+\delta$), and the point that has the lowest index was obtained applying a negative parameter perturbation ($-\delta$). Thus, the index of this points represents the position of the point in a vector we use to store the 3^N points.

This figure shows paths only up to the iteration N=2. However, for N higher than two, the set of applied perturbations S (to reach the target) is easily obtained by knowing only the index H of the point that reach the target $X_H \cong X_f$ and the number N.

Next, we show how to determine the set S. Thus, imagine that the target was reached by the point X_H at the N^{th} iteration and this point belongs to **group M**.

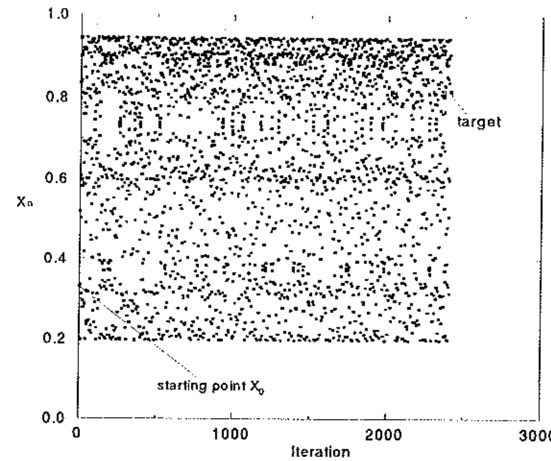

Figure 2: The time (n = 2446) that the Logistic map expends to be directed from the point X_0 = 0.33 to the target (X_f=0.816).

The **group M** (formed at the N^{th} iteration) is obtained by the iteration of the point X_M, obtained at the $N-1^{th}$ iteration. This point, depending on the perturbation ($+\delta$, 0, $-\delta$), generates the points (X_{3M-0},X_{3M-1},X_{3M-2}) that form **group M**. So, depending on the position of the point X_H in **group M** we know the value of the perturbation. For exemple, if X_H=X_{3M-2}, we know that the point X_M of the iteration $N-1$ was iterated using the perturbation $-\delta$.

To know how the point X_M at the $N-1^{th}$ iteration was obtained, we have to find out the group L in which this point belongs and its position inside this group.

So, in figure 1 the set of perturbation is S={ $-\delta$, $+\delta$ }.

2.1 Targeting the Logistic Map

The equation of the Logistic map is

$$X_{n+1} = bX_n(1 - X_n) \tag{1}$$

where b is the control parameter.

Following the procedure introduced in the previous section, we choose to change the parameter $b = 3.78$ by an amount $\delta = 0.005$, with $\epsilon = 0.0001$.

The necessity of the targeting procedure can be seen in figure 2, where a map (1) trajectory takes 2406 iterates until the trajectory goes from $X_0 = 0.33$ to $X_f = 0.816$. However, using our method, after only eight iterations through the set of perturbation S={0,$+\delta$, $+\delta$, 0,+δ, 0, 0, -δ} the trajectory reaches the same target located on the point X_f=0.816, as shown in figure 3. We should emphasize that the perturbed trajectories reach points that are never visited by the attractor of the map (1) with constant b, as the point $X_f = 0.750$, which can be reached from the point $X_0 = 0.5$ in 10 iterations, for b=3.78.

Figure 3 shows the trajectory obtained (solid black line) by applying the determined sequency of perturbations . This trajectory is very close to the stable manifold (dotted gray line) of the target X_f. This means that the driven trajectory from the starting point to the target is along the stable manifold of the target. Therefore, the stable manifold of the target and the targeting trajectory can hardly be distinguished.

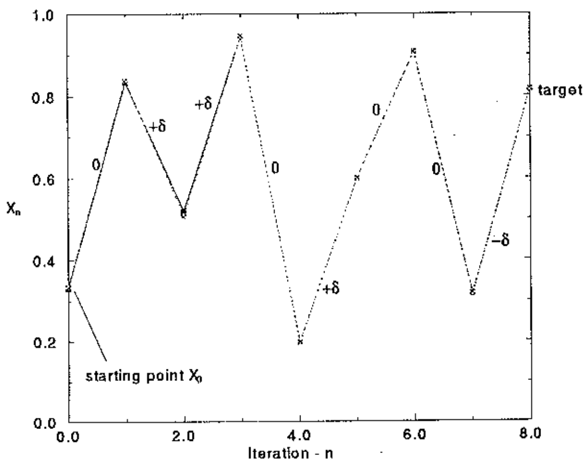

Figure 3: Using the targeting method we can direct the point X_0=0.33 to the target X_f=0.816 in only 8 iterations, applying a set of 8 perturbation to the Logistic control parameter b=3.78. The amplitude perturbation is δ=0.005. The trajectory (solid black line) is very close to the stable manifold (dotted gray line) of the target.

2.2 "Experimental" Application

To simulate the application of our method to an experimental situation we add a noise term in the right side of equation (1), and we use the this new system as a generator of experimental data. As previously introduced, we change the parameter b by making it assume three values, b_1=b+δ, b_2=b, and b_3=b-δ. Thus, we collect three sets of data from equation (1) randomly perturbed.

Each of these three trajectories is fitted by a polynomium in the form G(X, i, j, k)=iX^2 + jX + k. So, we refer to the map (1) randomly perturbed (with a noise amplitude 0.005) by the letter F and then using data from the map F(X,b_1) we obtain the fitted polynomium

G1, from the map F(X,b_2) we obtain G2, and from the map F(X,b_3) we obtain G3. The fitting polynomiums are

$$\begin{aligned} G1 &= -3.79060980X^2 + 3.79090945X - 0.00029370 \\ G2 &= -3.78018933X^2 + 3.78024169X - 8.11875400\ 10^{-5} \\ G3 &= -3.77170930X^2 + 3.77199330X - .00049564. \end{aligned} \tag{2}$$

and, in this case, b=3.78, δ=0.01 and X_0=0.3300000.

So, for calculating the set of perturbations, we calculate the set which directs the trajectory to the target, considering the fitting maps G1, G2 and G3. Thus, the starting point X_0 is direct to the target X_f=0.816, by applying the set of perturbations (-δ,0,+δ,0,+δ,-δ,0,-δ). The trajectory reaches the point X_8=0.8171709 which is very close to the desired point.

3 Targeting the Hénon Map

The equations of the Henon map are

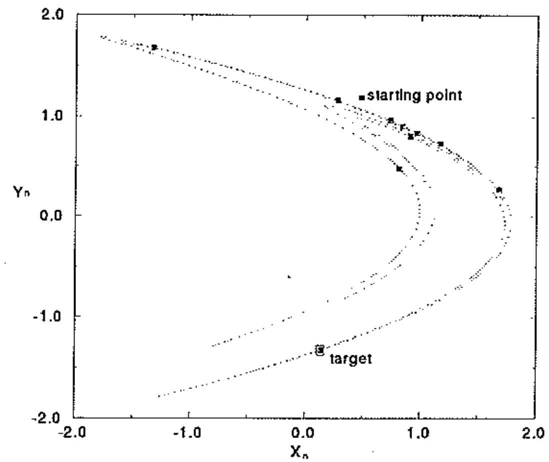

Figure 4: The targeting method applied to the Hénon map. Without using the targeting 6062 iterations are necessary to direct the starting point to the target. Using the targeting method only 10 iterations are enough.

$$\begin{aligned} X_{n+1} &= a + 0.3Y_n - X_n \\ Y_{n+1} &= X_n. \end{aligned} \tag{3}$$

In this case, we consider that the control parameter a=1.40 can be changed by an amount of δ=0.01. So, the parameter **a** can assume the values a=1.39, a=1.40, and a=1.41. Then, one example of the use of our method is considering the starting point P_i={0.4772, 1.1880} and the target at the point P_f={0.1371 , -1.3280}; 6062 iterations are necessary for the trajectory to go from the point P_i to the vicinity of P_f. However, with our method, the trajectory reaches the target in only 10 iterations, by applying the map (3) with the following set of control perturbations: ($-\delta,+\delta,+\delta,-\delta,+\delta,+\delta,+\delta, 0,-\delta,+\delta$). This example can be seen in figure 4.

The same initial and final points were used in [10]. In that work 12 iteration were necessary to direct the initial point to the target.

It is also possible to use the targeting method to create unstable periodic orbits. To do so we choose the starting point as the target. Next, two examples are presented.

For the first application we choose the starting point shown in figure 4. As this point is outside the region of the original attractor, any periodic orbit that passes through this point is a new orbit. Thus, we want to direct the orbit from the starting point X_i={ 0.4772 , 1.1880} to the same point, with a precision $\epsilon = 0.005$ and amplitude $\delta = 0.02$. The set of perturbations to target this point is S={$+\delta$, 0, $+\delta$, $+\delta$, $+\delta$, 0, $+\delta$, $+\delta$, 0}. So, we create a new unstable periodic orbit with period p=9.

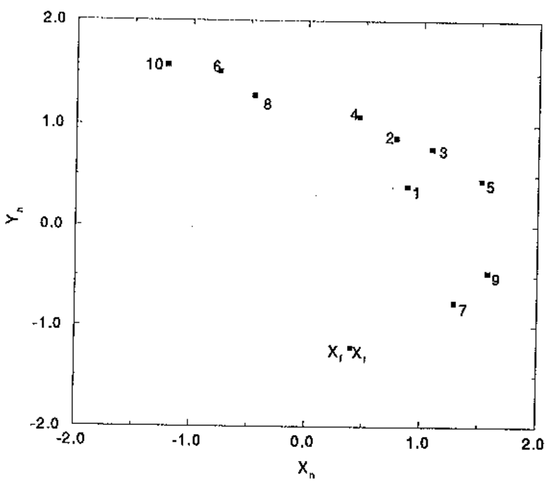

Figure 5: The new period eleven unstable periodic orbit created by applying our targeting method to the system (3).

As the second application, we create an unstable periodic orbit that passes close to a point present in the non-perturbed Hénon's attractor. Thus, we apply our target method to direct the trajectory from the point X_i={0.391806 , -1.216145} to the point $X_f = X_i$ with a precision $\epsilon = 0.0005$ and an amplitude $\delta = 0.01$. To target this point we need to apply

eleven perturbations on the parameter a. The set S={$-\delta$, $+\delta$, $+\delta$, $-\delta$, $-\delta$, $-\delta$, $+\delta$, $+\delta$, 0, $-\delta$, 0}. This period-11 orbit can be seen in figure 5.

However, since this orbit is unstable, we would have to apply a new set of perturbations for each cycle. A more convenient way for stabilizing this orbit is to apply the OGY method [1], adapted to the control of this large-periodic orbit (see [21]).

The control of this orbit is shown in figure 6 where we plot the evolution of the variable X_n. In figure 7 we see the value of the parameter a, for each iteration. During the targeting, a can only assume the values $a+\delta$, a or $a-\delta$. When we start applying the OGY method, a small correction is done, at each iteration, to the value of the parameter a.

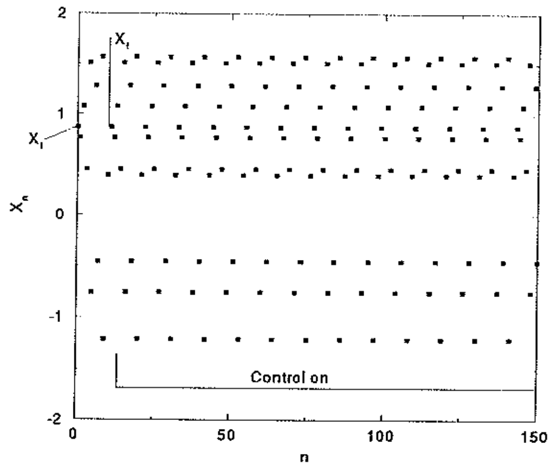

Figure 6: Controlling the orbit of figure 5 by applying the OGY method. In this figure is shown the variable X_n of the system (3).

4 Targeting fluxes - the Double Scroll Circuit

The Double Scroll circuit (see figure 8) is an autonomous non-linear electronic circuit composed by two capacitors, C_1 and C_2, one indutor, L, two linear resistor, R, r, and a nonlinear resistor R_{NL}. The circuit dynamic is described by

$$\begin{aligned} C_1 d_t V_{C1} &= (V_{C2} - V_{C1})/R - i_{NR}(V_{c1}) \\ C_2 d_t V_{C2} &= (V_{C1} - V_{C2})/R + i_L \\ L d_t i_L &= -V_{C2} - q(t) \end{aligned} \tag{4}$$

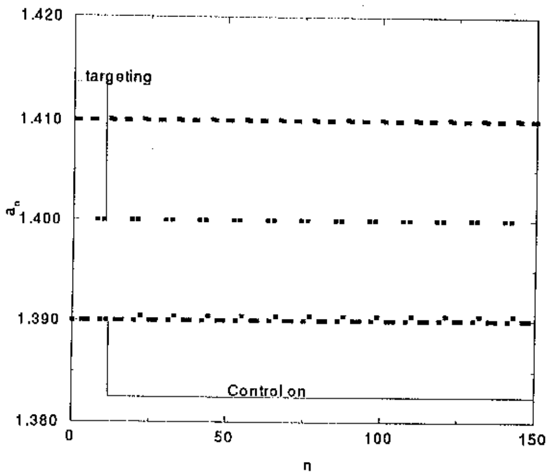

Figure 7: Using the OGY method, small adjustments are done on the amplitude value δ to control the orbit of figure 5.

where V_{C1}, V_{C2}, and i_L are the dynamical variables and represent the voltage across C_1, the voltage across C_2, and the current through L, respectively, and q is tension across r, which is the external perturbation we consider to apply our targeting method.

The term i_{NR} is the characteristic curve of the non-linear resistor R_{NL} and is the piecewise-linear function represented by the equation

$$i_{NR} = m_0 V_{C1} + \frac{1}{2}(m_1 - m_0)(\mid V_{C1} + B_p \mid - \mid V_{C1} - B_p \mid) \tag{5}$$

The equations (4) were integrated using the following parameters,

$$\frac{1}{C_1} = 9.0 \qquad \frac{1}{C_2} = 1.0 \qquad \frac{1}{L} = 7.0$$
$$\frac{1}{R} = 0.7 \qquad m_0 = -0.5 \qquad m_1 = -0.8 \qquad B_p = 1.0 \tag{6}$$

Figure 9 shows the Double Scroll chaotic attractor of (4) for q = 0.

In the chosen example, we want to direct the trajectory of the system (4) to the target located at the point

$$V_{C1}^f = -1.500 \qquad V_{C2}^f = 0.238 \qquad i_L^f = 1.723. \tag{7}$$

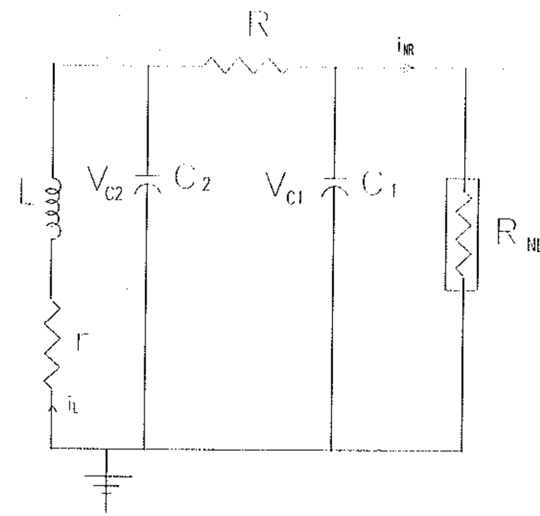

Figure 8: The Double Scroll circuit composed by two capacitors, C_1 e C_2, one inductor, L, two linear resistors, R e r, and one non-linear resistor R_{NL} .

with a precision of ϵ_f=0.001.

To apply our method to a 3-D flow, some adjustments to the original method are necessary. For a map, the perturbation is introduced at each iteration, yet for a flow we should consider a time interval T, for which the perturbing parameter, q, is not zero.

We consider that both, the starting point and the target, are on a surface α (a Poincaré section [14]), positioned at the plane determined by V_{C1}=-1.5 (the line in figure 9)).

The number of times the trajectory crosses the section α, is J. As done in maps, we should consider a number of times N we want to apply the perturbing parameter q, which as before can assume three possible values: +δ, 0, -δ.

Intending to simulate a real experiment, we do not set up the system initial condition as it can be done for a system of known equations. So, before starting to apply our method, we must force the system (4) to oscilate in a periodic way. The advantage of this procedure is the facility of determining the starting point, since any periodic orbit crossing the Poincaré section can be chosen as our starting point.

So, initially, we choose a perturbation $q(t)$ that forces the system (4) to oscillate periodically. This is done by making the perturbing parameter q assume the form of a sinoidal wave of amplitude V_g and frequency f:

$$q(t) = c \sin(2\pi f t). \tag{8}$$

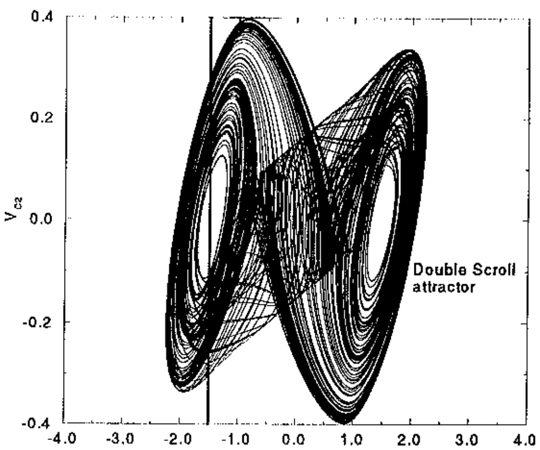

Figure 9: The non perturbed Double Scroll attractor of the system (4) projected on the variables V_{C1} and V_{C2}. The line represents the chosen Poincaré section we consider to apply our targeting method.

It is possible to supress chaotic motion of the system (4) by perturbing it with q given by (8) (chaos is suppressed by phase-locking) [7], [8], and [9]. Then, we choose the frequency f = 0.3 and amplitude c=0.022 to make the system (4) to behave periodically.

This point chosen as the starting point X_i is

$$V_{C1}^i = -1.5000 \qquad V_{C2}^i = 0.3361 \qquad i_L^i = 2.0289. \tag{9}$$

So, we perturb the system (4) using (8) until we verify the trajectory is in the vicinity of the starting point. This happens when the trajectory crosses a three-dimensional sphere with center in the point X_i and radius $e_i = 0.0005$. When that happens, we start applying our method for a set of parameters (T, N, δ) previously chosen, and the perturbation is not anymore given by equation (8), but is rather a series of constants δ as introduced before.

Now, we estimate the number N of perturbations we apply into the system. First, we analyse the area the attractor occupies on the Poincaré section. We find this area (the maximum length plus the maximum width of the attractor on the section) to be $A \approx 0.0091$. Thus, the minimum number N is found by

$$\frac{A}{3^N} < \epsilon_f^2, \tag{10}$$

leading us to N=8.

Now, we determine the time interval T. For that , we verify that the approximated time interval the trajectory spend to return to the point X_i (on the Poincaré section) is $\tau = 16$. Then, the time interval T is obtained from

$$T = \frac{\tau}{N} \tag{11}$$

what give us T=2.

In this work we choose δ=c=0.022, that is, the sinoidal wave (in the phase locking) and the targeting perturbation have the same amplitude. As a matter of fact, the amplitude δ does not determine the success of the targeting.

Summarizing the application of the targeting method to a flux we first stabilize the system until the trajectory crosses the point X_i, by applying a sinoidal wave (the phase-locking targeting phase). Then we apply N perturbations, each one during a time T. After that, we still keep integrating until the trajectory crosses the Poincaré section and reaches the target. If so, we consider J as the number of times the system crosses the section after the perturbations are applied.

For a targeting time T=2, N=8, and δ=0.022, we found that the target can be reached by applying the following sets of perturbations: $\{-\delta,-\delta, 0,0,-\delta, 0, 0, \delta\}$ with J=5 and the set $\{+\delta,-\delta, +\delta,+\delta,0,-\delta,-\delta, -\delta\}$ with J=6. For higher number of perturbations, we find also many more ways to reach the target.

However, we can use other parameters instead of the ones we have estimated. Thus if we consider the same N=8, δ=0.0022 but T=1.2, we find the following perturbations that direct the system to the target:

$$\{-\delta, 0, -\delta, -\delta, -\delta, -\delta, +\delta, -\delta\} \tag{12}$$

with J=2. In figure 10 we show the resulting trajectory for this set.

5 Application of the Targeting Procedure

The system (4) was controlled by applying a perturbation q given by (8). But, there are other ways of doing that, as is the case of the OGY [1] method. With this method it is possible to control a chosen unstable periodic orbit, as the one that can be seen in figure 11. However, this method requires the system trajectory to get closer to the periodic orbit to be controlled. So, we can use our method to rapidly direct the system (4) to a point near the chosen unstable periodic orbit. After we reach the vicinity of this orbit, we apply the OGY method to stabilize this orbit. The control of the unstable periodic orbit (figure 11) by the OGY method can be seen in figure 12.

The orbit can be controlled by applying small perturbations in a previously chosen parameter. To apply the OGY method, we choose the parameter q to be varied. Thus, after the system (4) reached the vicinity of the unstable periodic orbit on the Poincaré section V_{C1}=-1.5, we vary q by δq, calculated by the equation

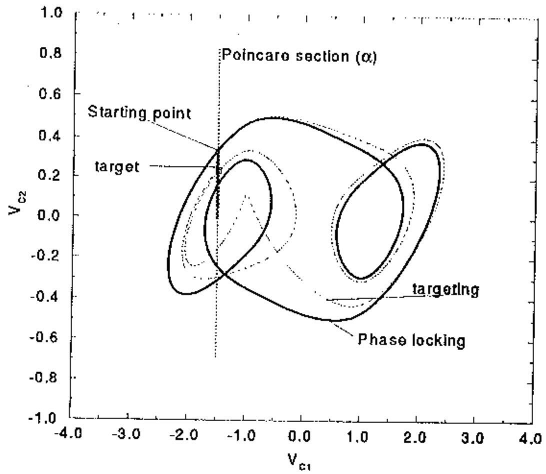

Figure 10: Application of the targeting method to a 3D flow (the Double Scroll circuit). To apply the method to an experimental system we should set up the starting point. That is done by phase locking the circuit with an external sinoidal perturbation (trajectory represented by the large black line). After that, we apply the targeting method to rapidly direct the system to the target applying 8 perturbations (trajectory represented by the thin gray line). On the surface α is represented the region we can reach from the starting point applying different sets of 8 perturbations.

$$\delta q = (0.1649,\ 0.1818)(\xi_n - X_f) \tag{13}$$

where ξ represents the trajectory position (a vector representing the variables V_{C2} and i_L) when it crosses the Poincaré section, and X_f represents the V_{C2} and i_L coordinates of the target. So, each time the trajectory crosses the section we change the value of the parameter q by using equation (13). A detailed manner of obtaining the formula (13) can be found in reference [1].

The point given by equation (7) was chosen as the target because it is in the vicinity of the crossing between the unstable periodic orbit (shown in figure 11) and the section α. So, we can use our method to direct the system (4) rapidly to the vicinity of the this orbit, and then apply the OGY method. The result is shown in figure 13

In this figure we first apply the sinoidal perturbation to induce the phase-locking. Then, when the trajectories reaches the starting point we apply our method to direct it to the target (7) (large black line). When the target is reached, we apply the OGY method to control the unstable periodic orbit (gray line)

In figure 13 we show this control using the time evolution of the variable $V_{C1}(t)$, indicating

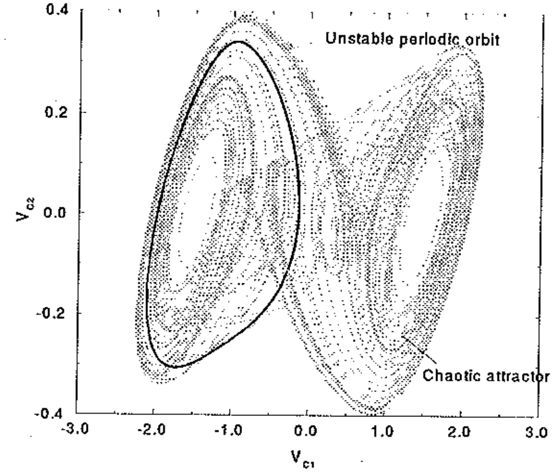

Figure 11: The non perturbed Double Scroll circuit and one of the infinite unstable periodic orbits embbeded in the attractor.

all the phases, the phase-locking, the targeting procedure, and the control of the unstable periodic orbit.

6 Conclusions

We showed that the Logistic and Hénon map, and the Matsumoto's system can have their trajectories rapidly directed from a starting point X_0 to a chosen target by applying a sequency of N constant value parameter perturbations. In fact, the generic parameter p is allowed to assume only constant values with the advantage that the amplitude perturbation δ can have any, but not very small, value. In this article only three possible values $p+\delta$, p, and $p-\delta$ are considered. However, the higher is the number of the constant perturbations the faster the target is reached.

To determine the sequency of N perturbations that we must apply to the parameter, to directs the starting point to the target, a large amount of memory is needed. To avoid this problem, we developed a numerical that requires only the 3^N points that are generated at the N^{th} iteration.

The chosen sequency (responsible to make the target to be reached) among the other 3^N possible ones, may not be unique; however, any of these non unique sequencies makes the system trajectory to evolve along the stable manifold of the target.

The method may be experimentally applied as showed using the noisely perturbed Logistic map.

With the targeting method we can create new periodic orbits that are not present in the

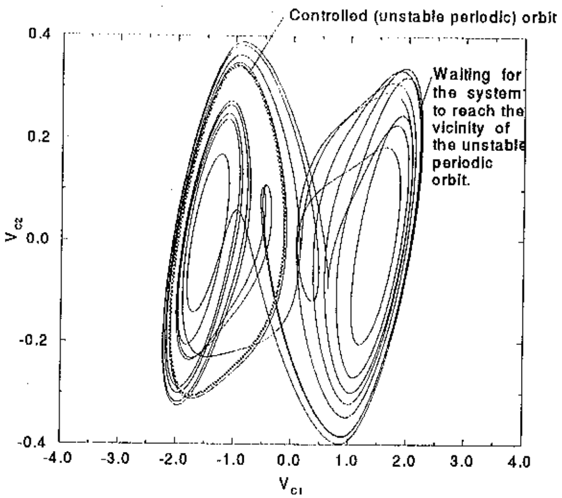

Figure 12: The stabilization of the unstable periodic orbit by applying the OGY method. Before applying the OGY method we should wait until the circuit reaches the vicinity of the unstable periodic orbit.

attractor of the non perturbed system. Even though these new periodic orbits are unstable, their stabilization is possible by applying the method OGY for controlling chaotic behaviour.

The targeting method is applied to the Matsumoto's system that is a three dimension system of equations. Some adaptations to the original aproach, applied to maps, are introduced as the number N of perturbations that must be previously estimate.

Intending to simulate a real experiment, we do not set up the system initial conditions as it can be done for a system of known equations. However, before starting to apply our target method, the system must be placed at some starting point. Thus, to do this, a resonant sinoidal perturbation is applied to the system, in a manner that its trajectory becomes periodic by phase-locking, and then, we consider the starting point as the cross between the periodic trajectory and some chosen Poincaré section.

To increase the performance of the OGY method of control of chaos we can apply our targeting method. Since the OGY method requires the system trajectory to get closer to the periodic orbit to be controlled (fact that sometimes takes a long time), we use our method to rapidily direct the system to a point located at the vicinity of the chosen unstable periodic orbit.

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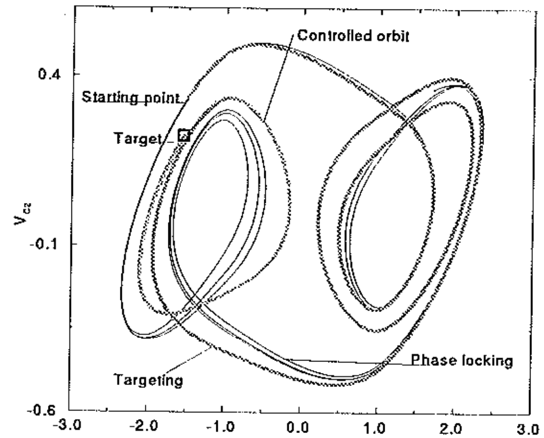

Figure 13: The targeting method is used to rapidly direct the circuit to the vicinity of the unstable periodic orbit. So, we set up the initial condition by phase locking the circuit (trajectory in a thin black line). After, we apply the targeting method to direct the circuit to the target (trajectory in a large black line). Then, we apply the OGY method, stabilizing the unstable periodic orbit (trajectory in a gray line).

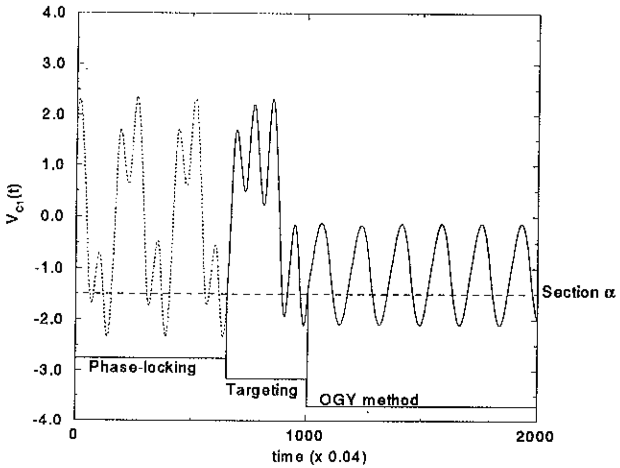

Figure 14: Targeting the system (4) and then controlling the unstable periodic orbit (shown in figure 11) by applying the OGY method.

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