UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 66318
05389-970 SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-1237

PATH INTEGRAL FOR SPINNING PARTICLES IN ODD DIMENSIONS

Dmitri M. Gitman
Instituto de Física, Universidade de São Paulo

Agosto/1996

Path Integral for Spinning Particles in Odd Dimensions

Dmitri M. Gitman*

Instituto de Física, Universidade de São Paulo

Caixa Postal 66318-CEP 05389-970-São Paulo, S.P., Brasil

(August 28, 1996)

Abstract

The propagator of a spinning particle in external Abelian field is presented by means of a path integral in odd dimensions. For the first time the problem is solved and its solution differs essentially from the one in even dimensions (the latter has been known before). The path integral representation derived provides one a new pseudoclassical action to describe spinning particles in odd dimensions.

03.70, 11.10.Kk, 11.15.Kc, 03.65.Ca

Typeset using REVTEX

*e-mail: gitman@snfma1.if.usp.br; fax: 55-11-8186833; tel: 55-11-8134257

Path integral representations for relativistic particles propagators were discussed in numerous papers. Spinning particles propagator (Dirac propagator) in four dimensions was presented via path integral both over even (bosonic) and odd (Grassmann) trajectories [1]. The latter trajectories are connected with spinning degrees of freedom of the particles. The representation can be easily generalized to arbitrary even-dimensional case. The absence of an analog of γ^5 matrix in odd dimensions was an obstacle to write the path integral similar to that one in even dimensions. In the present paper we have succeeded to construct a path integral representation for spinning particle propagator in arbitrary odd-dimensional cases. Remember, that, in fact, the path integral in four dimensions was constructed [2], using a super-generalization of the Schwinger proper-time representation for the inverse operator, in which the proper time is presented by a pair of even and odd variables. In this case the effective action, which one can extract from the path integral, coincides with Berezin-Marinov one [3]. In odd dimensions one has to use a more complicated super-generalization of the Schwinger representation where the proper-time has already one even and two Grassmann components. Extracting a gauge-invariant part of the effective action from the path integral in odd dimensions we get a new pseudoclassical action to describe spinning particles in these dimensions. Recently the problem of such an action construction was discussed intensively, especially, in connection with the $2+1$ field theory [4–6,10]. We present both classical analysis and quantization of the new model and discuss its peculiarities.

A path integral representation for Dirac propagator in arbitrary even $D = 2d$ dimensions can be constructed completely similar to the four-dimensional case [1]. Such a representation (in the notations of the paper [2]) has the form

$$S^c(x_{out}, x_{in}) = \exp\left(i\Gamma^n \frac{\partial_\ell}{\partial\theta^n}\right) \int_0^\infty de_0 \int d\chi_0 \int_{e_0} M(e)De \int_{\chi_0} D\chi \int_{x_{in}}^{x_{out}} Dx \int D\pi \int D\nu$$
$$\times \int_{\psi(0)+\psi(1)=\theta} \mathcal{D}\psi \, \exp\left\{ i\int_0^1 \left[-\frac{\dot{x}^2}{2e} - \frac{e}{2}m^2 - g\dot{x}A + iegF_{\mu\nu}\psi^\mu\psi^\nu \right.\right.$$
$$\left.\left. + i\left(\frac{\dot{x}_\mu\psi^\mu}{e} - m\psi^D\right)\chi - i\psi_n\dot{\psi}^n + \pi\dot{e} + \nu\dot{\chi}\right] d\tau + \psi_n(1)\psi^n(0)\right\}\Bigg|_{\theta=0} . \quad (1)$$

The propagator (causal Green function) $S^c(x, y)$ obeys the Dirac equation transformed by

γ^{D+1},

$$\left(\hat{\mathcal{P}}_\mu \gamma^\mu - m\gamma^{D+1}\right) S^c(x,y) = \delta^D(x-y), \tag{2}$$

where $\hat{\mathcal{P}}_\mu = i\partial_\mu - gA_\mu(x)$, $\mu = 0,\ldots,D-1$, $A_\mu(x)$ are potentials of external Abelian field, γ^μ are γ-matrices in D dimensions with dimensionality $\dim\gamma^\mu = 2^{D/2} = 2^d$, $[\gamma^\mu,\gamma^\nu]_+ = 2\eta^{\mu\nu}$, $\eta_{\mu\nu} = \mathrm{diag}(\underbrace{1,-1,\ldots,-1}_{D})$, and γ^{D+1} is analog of γ^5 in four dimensions, $\gamma^{D+1} = r\gamma^0\gamma^1\ldots\gamma^{D-1}$, $[\gamma^{D+1},\gamma^\mu]_+ = 0$, $\left(\gamma^{D+1}\right)^2 = -1$, $r=1$ if d is even, $r=i$ if d is odd. The set of $D+1$ γ-matrices γ^ν and γ^{D+1} form a representation of the Clifford algebra in odd $2d+1$ dimensions. We denote here such matrices via Γ^n, $\Gamma^\mu = \gamma^\mu$, $\Gamma^D = \gamma^{D+1}$, $[\Gamma^k,\Gamma^n]_+ = 2\eta^{kn}$, $\eta_{kn} = \mathrm{diag}(\underbrace{1,-1,\ldots,-1}_{D+1})$, $k,n = 0,\ldots,D$. In the representation (1) $x(\tau)$, $p(\tau)$, $e(\tau)$, $\pi(\tau)$, are even and $\chi(\tau)$, $\nu(\tau)$ are odd trajectories, obeying the boundary conditions $x(0) = x_{\rm in}$, $x(1) = x_{\rm out}$, $e(0) = e_0$, $\chi(0) = \chi_0$, $\psi(0)+\psi(1) = \theta$, where θ are odd variables and $\frac{\partial_\ell}{\partial\theta^n}$ stands for the left derivative. The measure $M(e)$ has the form

$$M(e) = \int Dp \exp\left\{\frac{i}{2}\int_0^1 ep^2 d\tau\right\} . \tag{3}$$

A discussion of the role of the measure (3) can be found in [2].

In odd dimensions a possibility to construct the matrix γ^{D+1} does not exist. Hence, here a possibility does not exists to work with the transformed by γ^{D+1} Dirac equation (2). Namely that allowed [1,2] to write the path integral representation (1). Nevertheless, the problem in odd dimensions can be solved in a different way.

As it is known, in odd dimensions $D = 2d+1$ there exist two exact non-equivalent irreducible representations of the Clifford algebra with the dimensionality $2^{[D/2]} = 2^d$. Let us mark these representations by the index $s = \pm$. Thus, we have two non-equivalent sets of γ-matrices which we are going to denote as $\Gamma^n_{(s)}$, $n = 0,1,\ldots,2d$ (remark that now we use Latin indices n,k as Lorentz ones). Such matrices can be constructed, e.g. from the corresponding matrices in $D = 2d$ dimensions as follows:

$$\Gamma^n_{(s)} = \begin{cases} \gamma^\mu, & n = \mu = 0,\ldots,D-1 \\ s\gamma^{D+1}, & n = D \end{cases} , \tag{4}$$

In odd dimensions there exists also a duality relation which is important for our purposes

$$\Gamma^n_{(s)} = \frac{sr}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\Gamma_{(s)k_1}\ldots\Gamma_{(s)k_{2d}} , \tag{5}$$

where $r=1$ if d is even, $r=i$ if d is odd. The propagator $S^c(x,y)$ obeys the Dirac equation in the dimensions under consideration

$$\left(\hat{\mathcal{P}}_n\Gamma^n_{(s)} - m\right) S^c(x,y) = -\delta^D(x-y), \tag{6}$$

where $\hat{\mathcal{P}}_n = i\partial_n - gA_n(x)$. Following Schwinger [7], one can present $S^c(x,y)$ as a matrix element of an operator $\hat{S}^c$, $S^c(x,y) = <x|\hat{S}^c|y>$, where $|x>$ are eigenvectors for some self-conjugated operators of coordinates X^n; the corresponding canonical-conjugated operators of momenta are P_n, so that:

$$\begin{gathered} X^n|x> = x^n|x> , \quad <x|y> = \delta^D(x-y) , \quad \int |x><x|dx = I , \\ \left[P_n, X^k\right]_- = -i\delta^k_n , \quad P_n|p> = p_n|p> , \quad <p|p'> = \delta^D(p-p') , \\ \int |p><p|dp = I , \quad <x|P_n|y> = -i\partial_n\delta^D(x-y) , \quad <x|p> = \frac{1}{(2\pi)^{D/2}}e^{ipx} , \\ [\Pi_n,\Pi_k]_- = -igF_{nk}(X) , \quad \Pi_n = -P_n - gA_n(X) . \end{gathered} \tag{7}$$

Equation (6) implies $\hat{S}^c = -\hat{F}^{-1}$, $\hat{F} = \Pi_n\Gamma^n_{(s)} - m$. In the case under consideration it is convenient to present the inverse operator in the following form

$$\begin{gathered} \hat{S}^c = \frac{\hat{F}_{(+)}}{-\hat{F}_{(+)}\hat{F}} = s\,\frac{\hat{A}}{\hat{B}} , \quad \hat{F}_{(+)} = \Pi_n\Gamma^n_{(s)} + m , \\ \hat{A} = \frac{r}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\Pi_n\Gamma_{(s)k_1}\ldots\Gamma_{(s)k_{2d}} + sm , \; \hat{B} = m^2 - \Pi^2 + \frac{ig}{2}F_{kn}\Gamma^k_{(s)}\Gamma^n_{(s)} . \end{gathered} \tag{8}$$

The form of the operator $\hat{A}$ is obtained from the operator $\hat{F}_{(+)}$ by means of the duality relation (5). Now both operators $\hat{A}$ and $\hat{B}$ are even in γ-matrices, so we can treat them as Bose-type operators. For their ratio we are going to use a new kind of integral representation which is a combination of the Schwinger proper-time representation for $\hat{B}^{-1}$ and additional representation of the operator $\hat{A}$ by means of a Gaussian integral over two Grassmann variables χ_1 and χ_2 with the involution property $(\chi_1)^+ = \chi_2$. Namely, one can write

$$\hat{S}^c = s\int_0^\infty d\lambda \int e^{-i[\lambda\hat{B}+\chi\hat{A}]}d\chi\,,\quad \chi=\chi_1\chi_2\,,\quad d\chi = d\chi_1\,d\chi_2\,. \tag{9}$$

Thus, we get for the propagator

$$S^c(x_{out},x_{in}) = s\int_0^\infty d\lambda \int \langle x_{\rm out}|e^{-i\hat{\mathcal{H}}(\lambda,\chi)}|x_{\rm in}\rangle d\chi\,, \tag{10}$$
$$\hat{\mathcal{H}}(\lambda,\chi) = \lambda\left(m^2-\Pi^2+\frac{ig}{2}F_{kn}\Gamma^k_{(s)}\Gamma^n_{(s)}\right)+\chi\left(\frac{r}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\Pi_n\Gamma_{(s)k_1}\ldots\Gamma_{(s)k_{2d}}+sm\right)\,.$$

Starting from this point one can proceed similarly to the four-dimensional case [2] to construct the Hamiltonian path integral for the right side of (10),

$$S^c(x_{out},x_{in}) = s\,\exp\left(i\Gamma^n_{(s)}\frac{\partial_l}{\partial\theta^n}\right)\int_0^\infty d\lambda_0\int d\chi_0\int_{\lambda_0}D\lambda\int_{\chi_0}D\chi\int_{x_{in}}^{x_{out}}Dx\int Dp\int D\pi\int D\nu$$
$$\times\int_{\psi(0)+\psi(1)=\theta}\mathcal{D}\psi\,\exp\left\{i\int_0^1\left[\lambda\left(\mathcal{P}^2-m^2+2igF_{kn}\psi^k\psi^n\right)-\chi\left(sm\right.\right.\right.$$
$$\left.\left.\left.+r\frac{(2i)^{2d}}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\mathcal{P}_n\psi_{k_1}\ldots\psi_{k_{2d}}\right)-i\psi_n\dot{\psi}^n+p\dot{x}+\pi\dot{\lambda}+\nu\dot{\chi}\right]d\tau+\psi_n(1)\psi^n(0)\right\}\Big|_{\theta=0}\,, \tag{11}$$

where $x(\tau)$, $p(\tau)$, $\lambda(\tau)$, $\pi(\tau)$ are even and $\psi(\tau)$, $\chi_1(\tau)$, $\chi_2(\tau)$, $\nu_1(\tau),\nu_2(\tau)$ are odd trajectories, obeying the boundary conditions $x(0)=x_{\rm in}$, $x(1)=x_{\rm out}$, $\lambda(0)=\lambda_0$, $\chi(0)=\chi_0$, $\psi(0)+\psi(1)=\theta$. Following notations are used: $\chi=\chi_1\chi_2$, $\nu\dot{\chi}=\nu_1\dot{\chi}_1+\nu_2\dot{\chi}_2$, $d\chi=d\chi_1\,d\chi_2$, $D\chi=D\chi_1\,D\chi_2$, $D\nu=D\nu_1\,D\nu_2$. Integrating over momenta we get a path integral in the Lagrangian form,

$$S^c(x_{out},x_{in}) = \frac{s}{2}\,\exp\left(i\Gamma^n_{(s)}\frac{\partial_\ell}{\partial\theta^n}\right)\int_0^\infty de_0\int d\chi_0\int_{e_0}M(e)De\int_{\chi_0}D\chi\int_{x_{in}}^{x_{out}}Dx\int D\pi\int D\nu$$
$$\times\int_{\psi(0)+\psi(1)=\theta}\mathcal{D}\psi\,\exp\left\{i\int_0^1\left[-\frac{\dot{x}^2}{2e}-\frac{e}{2}m^2-g\dot{x}_nA^n+iegF_{kn}\psi^k\psi^n\right.\right. \tag{12}$$
$$\left.\left.-\chi\left(sm+\frac{r}{e}\frac{(2i)^{2d}}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\dot{x}_n\psi_{k_1}\ldots\psi_{k_{2d}}\right)-i\psi_n\dot{\psi}^n+\pi\dot{e}+\nu\dot{\chi}\right]d\tau+\psi_n(1)\psi^n(0)\right\}\Big|_{\theta=0}\,,$$

where the measure $M(e)$ is defined by the eq. (3) and $e(0)=e_0$.

One can also get a different form of the path integral for the Dirac propagator in odd dimensions. To this end, instead of (8), one has to write

$$\hat{S}^c=\frac{\hat{F}}{-\hat{F}\hat{F}}=s\frac{\hat{A}}{\hat{B}},\quad \hat{A}=\frac{r}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\Pi_n\Gamma_{(s)k_1}\ldots\Gamma_{(s)k_{2d}}-sm\,, \tag{13}$$
$$\hat{B}=m^2-\Pi^2+\frac{ig}{2}F_{kn}\Gamma^k_{(s)}\Gamma^n_{(s)}-\frac{2srm}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\Pi_n\Gamma_{(s)k_1}\ldots\Gamma_{(s)k_{2d}}\,,$$

and then proceed as before. Thus, we get one more form of the Lagrangian path integral

$$S^c(x_{out},x_{in}) = \frac{s}{2}\,\exp\left(i\Gamma^n_{(s)}\frac{\partial_\ell}{\partial\theta^n}\right)\int_0^\infty de_0\int d\chi_0\int_{e_0}M(e)De\int_{\chi_0}D\chi\int_{x_{in}}^{x_{out}}Dx\int D\pi\int D\nu$$
$$\times\int_{\psi(0)+\psi(1)=\theta}\mathcal{D}\psi\,\exp\left\{i\int_0^1\left[-\frac{\dot{x}^2}{2e}-\frac{e}{2}m^2-g\dot{x}_nA^n+iegF_{kn}\psi^k\psi^n+sm\chi\right.\right. \tag{14}$$
$$\left.\left.-\left(\frac{\chi}{e}-sm\right)\frac{r(2i)^{2d}}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\dot{x}_n\psi_{k_1}\ldots\psi_{k_{2d}}-i\psi_n\dot{\psi}^n+\pi\dot{e}+\nu\dot{\chi}\right]d\tau+\psi_n(1)\psi^n(0)\right\}\Big|_{\theta=0}\,.$$

The path integral representations for particles propagators have also an important heuristic value. They give a possibility to guess the form of actions to describe the particles classically or pseudoclassically. Really, the exponent in the integrand of the right side of (1) can be treated as a pseudoclassical action of the spinning particle in even dimensions. Separating the gauge-fixing terms and boundary terms, we get a trivial generalization of the Berezin-Marinov action [3] to D dimensions. In odd dimensions, the path integral (12) prompt us the following pseudoclassical action to describe spinning particles in such dimensions:

$$S=\int_0^1\left[-\frac{z^2}{2e}-\frac{e}{2}m^2-g\dot{x}_nA^n+iegF_{kn}\psi^k\psi^n-\kappa m\chi-i\psi_n\dot{\psi}^n\right]d\tau=\int_0^1 Ld\tau\,,$$
$$z^n=\dot{x}^n+r\frac{(2i)^{2d}}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\psi_{k_1}\ldots\psi_{k_{2d}}\chi\,. \tag{15}$$

We suppose that κ is an even constant, which is discussed below and $\chi=\chi_1\chi_2$, with χ_1 and χ_2 being Grassmann variables, obeying the involution properties $\chi_1^+=\chi_2$. Interpreting the variable χ in such a way we can discover that the action (15) is gauge-invariant (reparametrization- and supergauge-invariant). The corresponding gauge transformations have the form: reparametrizations

$$\delta x^n=\dot{x}^n\xi,\ \delta e=\frac{d}{d\tau}(e\xi),\ \delta\psi^n=\dot{\psi}^n\xi,\ \delta\chi_1=\dot{\chi}_1\xi+\frac{1}{2}\chi_1\dot{\xi},\ \delta\chi_2=\dot{\chi}_2\xi+\frac{1}{2}\chi_2\dot{\xi}\,, \tag{16}$$

and two sets of nonlocal (in time) supertransformations,

$$\delta x^n=i\epsilon^{nk_1\ldots k_{2d}}\psi_{k_1}\ldots\psi_{k_{2d}}U\,,\quad \delta\psi^n=-\frac{d}{e}\epsilon^{nk_1k_2\ldots k_{2d}}z_{k_1}\psi_{k_2}\ldots\psi_{k_{2d}}U\,,$$
$$\delta e=0\,,\ \delta\chi_1=\theta_1,\ \ \delta\chi_2=\theta_2,\ \ U=\frac{ir(2i)^{2d}}{(2d)!}\int_0^\tau[\chi_1\theta_2-\chi_2\theta_1]d\tau\,, \tag{17}$$

where ξ is even and $\theta_{1,2}$ are odd τ-dependent parameters.

Recently, there were already proposed three different types of pseudoclassical actions to describe the massive spinning particles in odd-dimensional space-time, two in [4,5] respectively and the third one in [6]. The first one is classically equivalent to Berezin-Marinov action, extended to odd dimensions. It is P- and T- invariant on the classical level and the violation of the symmetry takes place only on quantum level, so that an anomaly is presented. Another action [5] is already P- and T- noninvariant and reproduces the adequate quantum theory in course of quantization. The action is close enough to the action (15). However, it contains additional dynamical Grassmann variables, and χ is not interpreted as a composite bifermionic-type variable that is why it is not supersymmetric. In the papers [6] a different action was proposed, which is a natural dimensional reduction from the even-dimensional massless (Weyl particles [8,9] in even dimensions) case. The action is supersymmetric, P- and T- noninvariant and can be extended to describe higher spins in odd dimensions [10]. However, as in two previous cases no path integral quantization of the model was given. We already have proved that our new action (15) allows to write the corresponding path integral and now we are going to check that the direct (operator) quantization leads to the corresponding quantum theory. To this end, as usual, we need to analyse the Hamiltonian structure of the theory with the action (15). Introducing the canonical momenta

$$p_n = \frac{\partial L}{\partial \dot{x}^n} = -\frac{z_n}{e} - gA_n(x)\,, \quad P_n = \frac{\partial_r L}{\partial \dot{\psi}^n} = -i\psi_n\,, \quad P_e = \frac{\partial L}{\partial \dot{e}} P_{\chi_{1,2}} = \frac{\partial_r L}{\partial \dot{\chi}_{1,2}} = 0\,, \tag{18}$$

one can see that there exist primary constraints $\Phi^{(1)} = 0$, $\Phi^{(1)}_{1,2} = P_{\chi_{1,2}}$, $\Phi^{(1)}_3 = P_e$, $\Phi^{(1)}_{4n} = P_n + i\psi_n$. We construct the total Hamiltonian according to the standard procedure [11,12] (we use the notations of the book [12]), and get $H^{(1)} = H + \lambda\Phi^{(1)}$, with

$$H = -\frac{e}{2}\left(\mathcal{P}^2 - m^2 + 2igeF_{kn}\psi^k\psi^n\right) + \chi\left(r\frac{(2i)^{2d}}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\mathcal{P}_n\psi_{k_1}\ldots\psi_{k_{2d}} + \kappa m\right),$$

where $\mathcal{P} = -p_n - gA_n(x)$. From the consistency conditions (Dirac procedure) we find a set of independent secondary constraints $\Phi^{(2)} = 0$,

$$\Phi^{(2)}_1 = \mathcal{P}^2 - m^2 + 2igeF_{kn}\psi^k\psi^n\,, \quad \Phi^{(2)}_2 = r\frac{(2i)^{2d}}{(2d)!}\epsilon^{nk_1\ldots k_{2d}}\mathcal{P}_n\psi_{k_1}\ldots\psi_{k_{2d}} + \kappa m\,. \tag{19}$$

One can go over from the initial set of constraints $\left(\Phi^{(1)}, \Phi^{(2)}\right)$ to the equivalent one $\left(\Phi^{(1)}, \tilde{\Phi}^{(2)}\right)$, where $\tilde{\Phi}^{(2)} = \Phi^{(2)}(\psi \to \psi + \frac{i}{2}\Phi^{(1)}_4)$. The new set of constraints can be explicitly divided in a set of first-class constraints, which is $\left(\Phi^{(1)}_{1,2,3},\ \tilde{\Phi}^{(2)}\right)$ and in a set of second-class constraints, which is $\Phi^{(1)}_{4n}$. Let us consider the Dirac quantization, where the second-class constraints define the Dirac brackets and therefore the commutation relations. The first-class constraints, being applied to the state vectors, define physical states. Thus, we get for essential operators and nonzero commutation relations:

$$[\hat{x}^\mu, \hat{p}_\nu] = i\{x^\mu, p_\nu\}_{D(\Phi^{(1)}_4)} = i\delta^\mu_\nu\,, \quad [\hat{\psi}^k, \hat{\psi}^n]_+ = i\{\psi^k, \psi^n\}_{D(\Phi^{(1)}_4)} = -\frac{1}{2}\eta^{kn}\,. \tag{20}$$

According to the scheme of quantization selected, operators of the second-class constraints are identically zero, whereas the operators of the first-class constraints have to annul physical state vectors. Taking that into account, one may construct a realization of the commutation relations (20) in a Hilbert space $\mathcal{R}$ whose elements $\mathbf{f} \in \mathcal{R}$ are 2^d-component columns dependent only on x, such that $\hat{x}^n = x^n\mathbf{I}$, $\hat{p}_n = -i\partial_n\mathbf{I}$, $\hat{\psi}^n = \frac{i}{2}\Gamma^n_{(s)}$, where $\mathbf{I}$ is $2^d \times 2^d$ unit matrix, and $\Gamma^n_{(s)}$, are γ-matrices in $D = 2d+1$ dimensions. Besides of that, we have the following equations for the physical state vectors

$$\hat{\Phi}^{(2)}_1\mathbf{f}(x) = 0\,, \quad \hat{\Phi}^{(2)}_2\mathbf{f}(x) = 0\,, \tag{21}$$

where $\hat{\Phi}^{(2)}$ are operators, which correspond to the constraints (19). Taken into account the duality relation (5), one can write the second equation (21) in the form

$$\left(\hat{\mathcal{P}}_n\Gamma^n_{(s)} + s\kappa m\right)\mathbf{f}(x) = 0\,, \tag{22}$$

where $\hat{\mathcal{P}}_n = i\partial_n - gA_n(x)$. Thus, we get the Dirac equation in course of the operator quantization if we put $\kappa = -s$. By this choice of κ the first equation (21) is simply a consequence of the Dirac equation. It is the squared Dirac equation.

One of the new features of the action (15) consists in a new interpretation of the even variable χ as a composite bifermionic type variable. One ought to say that only from the point of view of the operator quantization, one may treat χ as an unique bosonic variable in the action (15). Indeed, one can believe that χ is simply an ordinary Lagrange multiplier as it occurred always before. Doing the Dirac procedure, bearing in mind this interpretation, one gets finally the same first class constraint (19) and the same Dirac brackets. Thus, the result of the Dirac quantization will be the same. However, as was demonstrated above, the new interpretation is necessary for the path integral construction, or for path integral quantization. Besides of that it provides the desirable supersymmetry of the pseudoclassical action. One ought also to say that the presence of the nonlocal supersymmetry (17) in the model (15) is a characteristic feature of the pseudoclassical theory since it was proved in [12] that for singular theories with bosonic variables any gauge transformations are local in time. Another question is how to interpret the constant κ in the action (15). This question is directly related with the well-known problem of classical inconsistency of some kind of constraints in pseudoclassical mechanics. Indeed, if one treats κ as an ordinary complex parameter, then from the classical point of view the constraint equation $\Phi_2^{(2)} = 0$ is inconsistent. Such a difficulty appears not for the first time in the pseudoclassical mechanics, (see for example [13]). Here the following point of view is possible. One can believe that in classical theory κ is an even, bifermionic type element of the Berezin algebra, $\kappa^2 = 0$. Then the above-mentioned constraint equation appears to be consistent in the classical theory. At the same time, as was pointed out in [14], one has to admit a possibility to change the nature of the parameters in course of transition from the pseudoclassical mechanics to the quantum theory (why we admit such a possibility for the dynamical variables?). Namely, in quantum theory the parameter κ appears to be a real number, whose possible values are defined by the quantum dynamics. For example, the path integral quantization of the action (15) demands $\kappa \to s$, whereas the operator quantizations demands $\kappa \to -s$, where $s = \pm 1$ defined an irreducible representation of the Clifford algebra, see (4). To get the same quantization for κ both in path integral and operator quantization (at given and fixed choice of the irreducible representations for γ matrices) one has to consider another action, which can be extracted from the alternative path integral representation (14).

REFERENCES

[1] M. Henneaux, C. Teitelboim, Ann. Phys. **143** (1982) 127; N.V. Borisov and P.P. Kulish, Teor. Math. Fiz. **51** (1982) 335; A.M. Polyakov, *Gauge Fields and Strings*, (Harwood, Chur, Switzerland, 1987); V.Ya. Fainberg and A.V. Marshakov, JETP Lett. **47** (1988) 565; Phys. Lett. **211B** (1988) 81; Nucl. Phys. **B306** (1988) 659; Proc. PhIAN **201** (1990) 139 (Nauka, Moscow, 1991); T.M. Aliev, V.Ya. Fainberg and N.K. Pak, Nucl. Phys. **B429** (1994) 321; E.S. Fradkin and D.M. Gitman, Phys. Rev. **D44** (1991) 3230; J.W. van Holten, NIKHEF-H/95-050

[2] E.S. Fradkin and D.M. Gitman, Phys. Rev. **D44** (1991) 3230

[3] F.A. Berezin, M.S. Marinov, Pisma Zh. Eksp. Theor. Fiz. **21** (1975) 678 [JETP Lett. **21** (1975) 320]; Ann. Phys. (N.Y.) **104** (1977) 336; L. Brink, S. Deser, B. Zumino, P. di Vecchia and P. Howe, Phys. Lett. **B64** (1976) 435; L. Brink, P. di Vecchia and P. Howe, Nucl. Phys. **B118** (1977) 76; R. Casalbuoni, Nuovo Cimento **A33** (1976) 115; 389; A. Barducci, R. Casalbuoni and L. Lusanna, Nuovo Cimento **A35** (1976) 377; A.P. Balachandran, P. Salomonson, B. Skagerstam and J. Winnberg, Phys. Rev. **D15** (1977) 2308

[4] M.S. Plyushchay, Mod. Phys. Lett. **A8** (1993) 937

[5] J.L. Cortés, M.S. Plyushchay and L. Velázquez, Phys. Lett. B **306** (1993) 34

[6] D.M. Gitman, A.E. Gonçalves and I.V. Tyutin, Preprint IFUSP/P-1144, Marco/1995, to be published in Yad. Phys.; D.M. Gitman and A.E. Gonçalves, Preprint IFUSP/P-1204, Marco/1996, hep-th/9603052, to be published in Int. J. Theor. Phys.

[7] D.M. Gitman and I.V. Tyutin, Preprint FIAN/TD/96-26, hep-th/9602048, to be published in Int. J. Mod. Phys.

[8] J. Schwinger, Phys. Rev. **82** (1951) 664

[9] D.M. Gitman, A.E. Gonçalves and I.V. Tyutin, Phys. Rev. **D50** (1994) 5439

[10] D.M. Gitman and A.E. Gonçalves, Preprint IFUSP/P-1158, Junho/1995, hep-th/9506010

[11] P.A.M. Dirac, *Lectures on Quantum Mechanics* (New York: Yeshiva University, 1964)

[12] D.M. Gitman and I.V. Tyutin, *Quantization of Fields with Constraints* (Springer-Verlag, 1990)

[13] P. Howe, S. Penati, M. Pernici and P. Townsend, Class. Quantum Grav. **6** (1989) 1125

[14] D.M. Gitman, A.E. Gonçalves and I.V. Tyutin, Preprint FIAN/TD/96-03, hep-th/9602151