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**THE TRANSIENT DISTINGUISHABILITY OF
IDENTICAL PARTICLES**

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Abstract

In this paper irreversible processes are explained not in terms of the maximum probability principle of Statistical Mechanics but, instead, exclusively in terms of the relativistic uncertainty principle. According to this principle, indistinguishability of identical particles is shown to be the result of a non-reversible, non-potential, fundamental interaction. The constituent particles of a thermodynamic system removed from its condition of equilibrium are shown to be in states of incomplete indistinguishability and therefore, susceptible to interact irreversibly until the state of equilibrium is attained.

Résumé

Dans cet article on propose une explication des phénomènes irréversibles qui n'est pas considéré comme une conséquence du principe de maxime probabilité de la Mécanique Statistique mais du principe d'incertitude relativiste. D'accord ce principe, l'indiscernabilité des particules identiques est le résultat d'une interaction fondamentale irréversible et non potentiel. On démontre que les particules qui composent un système thermodynamique hors d'équilibre sont trouvés dans des états d'indiscernabilité incomplète, donc susceptibles aux effets de ces interactions irréversibles qui ont place jusqu'à sa extinction, dans l'état d'équilibre.

Key-words

Irreversibility - Second Law of Thermodynamics - Uncertainty - Indistinguishability - Entropy - Boltzmann's Principle - Equilibrium - Relativistic Quantum Mechanics - Time's arrow

1. The Relativistic Uncertainty Principle

It is known that in the relativistic domain of quantum phenomena the uncertainty principle of Heisenberg must be reconsidered because of the added restrictions of special relativity [1]. We intend to show in this paper that the uncertainty principle thus modified allows a consistent explanation for irreversible phenomena.

1.1. Indistinguishability

While, in the strict framework of formal quantum mechanics, indistinguishability of identical particles is sometimes acknowledged at the outset, from an heuristic standpoint it can be seen not as something primary but as a consequence of the uncertainty principle (see, for instance, [2]).

The usual picture from where this last conclusion is drawn, consists of an ideal experiment where two free identical particles are observed. If, as the outcome of a measurement process, the positions of these two particles are determined with a finite precision, then, as a consequence of this measurement, and according to the uncertainty principle, their velocities will suffer an unknown modification. This modification is such that if, in any posterior instant, one of these particles happens to be detected somewhere in the space, one cannot identify which one of the particles has arrived at this point. It is said that the particles are completely indistinguishable.

Figure 1 shows two particles, *A* and *B*, whose positions in space have been determined by a measurement process. Suppose that in a second measurement, two particles, *P* and *Q*, are detected. According to the uncertainty principle, all the information the second measurement provides is that either the outcome $\{A \rightarrow P, B \rightarrow Q\}$ or $\{A \rightarrow Q, B \rightarrow P\}$ took place, although nothing can be said about which one has really occurred. It is then impossible to determine to which one of the two detected particles *P* or *Q*, the particle *A* (or *B*) corresponds.

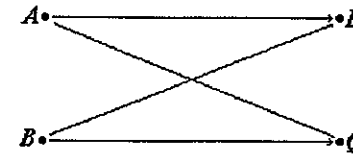


Figure 1

Let us denote respectively by ψ_A and ψ_B the wave functions of the particles *A* and *B*, each one considered independent of the other. The non relativistic quantum mechanical description of the two-particle system composed of *A* and *B* is usually made in terms of a common wave function Ψ obtained by the symmetric or anti-symmetric mathematical composition of ψ_A and ψ_B [2]:

$$\Psi = \frac{1}{\sqrt{2}} [\Psi_A(\xi_A)\Psi_B(\xi_B) \pm \Psi_A(\xi_B)\Psi_B(\xi_A)] \quad (1)$$

where ξ_A and ξ_B represent the coordinates required to completely describe the wave functions of the particles.

1.2. Relativistic indistinguishability

In this section we will try to show that the composition of the two wave functions ψ_A and ψ_B in (1) is not just a mathematical construction but the expression of a fundamental physical interaction existing between *A* and *B*.

When we try to extend our previous analysis to the relativistic domain (where the values of the particles' velocities are bounded by the velocity of light), we can readily conclude that, whatever the amplitude of the perturbation produced upon the movements of the particles by their original observation, indistinguishability should not be taken (as opposed to the non-relativistic case) as an instantaneous consequence of uncertainty.

In fact, according to special relativity, the movement of each particle is constrained to its light cone. Figure 2 illustrates the light cones of two particles, A and B, originally independent, and the instant t_0 when the light cones of the two particles begin to intersect.

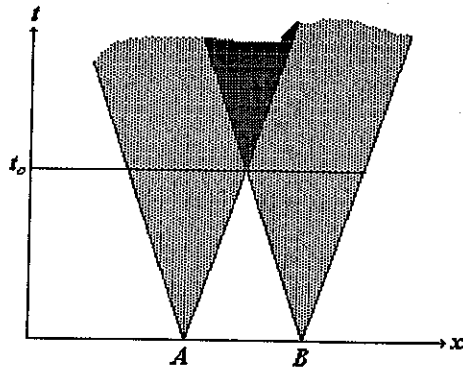


Figure 2

We can readily see, by observing figure 2, that before the instant t_0 the wave function ψ_A of particle A is completely independent of particle B; the same for the wave function ψ_B in relation to particle A. During the initial time interval $(0, t_0)$ there is no physical possibility for the existence of a symmetric or anti-symmetric composition (1) of the wave functions ψ_A and ψ_B of the two particles, i.e., during this period the particles cannot be indistinguishable. Such possibility only exists after the moment when the two light cones begin to intersect, that is, an interaction begins to take place.

Since the initial description of the system (requiring two independent wave functions ψ_A and ψ_B) does not coincide with its final description (made in terms of the common wave function (1)), then the existence of a transient process must necessarily be assumed and since there is no known (or conceivable) physical process whose outcome would restore two indistinguishable particles to their original identities, this process is irreversible.

1.3. Uncertainty Reactions

Such a transient process in which two indistinguishable particles come out as the result of a collision of two distinguishable ones can be visualized in the same way we see a chemical reaction in which a compound molecule comes out of independent atoms. We shall hereafter name these processes *uncertainty reactions* for short. These essentially relativistic reactions are the manifestation of what we should recognize as a *non-reversible, non-potential, fundamental interaction*.

2. Indistinguishability in combinatorial analysis

The same conclusion we arrived at in section 1.2, i.e., that indistinguishability must be taken not as a property of particles but as the result of an interactive process, can also be reached when we examine a simple mathematical combinatorial system from the standpoint of physics.

Let us consider a system formed by ideal coins. One might stress that from a purely logical standpoint, the quality of being indistinguishable cannot be assigned to a single coin. Indistinguishability is only meaningful when applied to two or more coins, i.e., *indistinguishability does not manifest when a single coin is considered in the absence of others*.

We should therefore understand indistinguishability not as some physical property possessed by a coin (something that always existed together with the coin) but instead as an emergent property that appears as the effect of putting two or more coins together. From the standpoint of Physics, emergent properties are usually identified with the result of a process of an interactive nature. Since it is a well established fact that there is no instantaneous interaction in nature, such a process must necessarily take a finite time to elapse.

3. The entropy of a monatomic gas

In this section, a derivation of the entropy of a monatomic gas - which requires no *ad-hoc* correction, is free of any paradoxes and provides the explanation on how equilibrium is reached - is obtained under the assumption of the transient distinguishability of its constituent particles.

3.1. Irreversible processes in microcanonical systems

Let us consider, as a preliminary discussion, an elementary, abstract microcanonical system composed of two independent identical systems, each one containing a single ideal coin that can take, in the absence of other, the values head (H) or tail (T) as its only configurations.

As long as the coins are maintained isolated or independent, the combined outcome of flipping the coins is one of the elements of the set

$$\Omega_1 = \{HH, HT, TH, TT\}$$

Although, if these coins are identical in the quantum meaning of the term, i.e., if they are of such a nature that when put together one cannot be distinguished from the other, then the outcome of throwing two such coins will be one of the elements of the set $\Omega_2 = \{hh, ht=th, tt\}$. Since neither the results 'HT' or 'TH' can be taken as equivalent to the single indistinguishable outcome 'ht=th', we can not consider Ω_2 as a subset of Ω_1 . In fact, $\Omega_1 \cap \Omega_2 \neq \Omega_2$.

Hence, when the conditions that keep two originally identical coins independent are removed, a non-reversible process takes place so that the number of possible configurations the system can take will decrease from the original four to the final value of three. This conclusion requires, as we shall see below, a re-formulation of the equilibrium criterion of Statistical Mechanics.

3.2. Gibbs' corrected derivation of entropy

The procedure usually adopted in the microcanonical approach to Thermodynamics for the calculation of the entropy S of an ideal monatomic gas is to take, as a model of the same, the superposition of N identical, mutually independent, non-relativistic quantum particles with mass m confined to a cubic box of volume V and with total energy U . The number Ω of different configurations the system can take, can be obtained from the solution of the Schrödinger's equation for the problem of one particle in a cube. The number Ω is equal to the number of solutions of the diophantine equation:

$$\sum_{i=1}^{3N} n_i^2 = \left(\frac{8m}{h^2}\right) UV^{2/3} \quad (2)$$

and the entropy S is obtained from the Boltzmann's Principle, $S = k \ln \Omega$. The asymptotic value for $k \ln \Omega$ is:

$$k \ln \Omega = Nk \ln \left[\frac{V}{h^3} \left(\frac{4m\pi U}{3N} \right)^{3/2} \right] + \frac{3}{2} Nk \quad (3)$$

In the traditional derivations of the entropy of the monatomic gas, since (3) does not satisfy the additivity condition for entropy, it is generally assumed that a correction factor (Gibbs' correction) is required. This correction, done under the assumption of the indistinguishability of particles is obtained by dividing Ω by $N!$ thus leading to the corrected value for the entropy [3].

3.3. Transient distinguishability and equilibrium

In the following, instead of introducing an *ad-hoc* correction factor for the derivation of entropy, we shall assume the transient distinguishability of identical particles as the motive power that drives the gas towards its equilibrium state.

Since according to phenomenological thermodynamics *additivity of entropy is not a condition of necessity but of equilibrium* it is legitimate to assume the value given by (3) as corresponding to a state of the gas out of equilibrium. In fact, this state has not been produced by a natural process but as the result of a mental construction where its N constituent particles were artificially *superposed* in the cube. As a consequence of this artificial superposition, a gas is formed whose particles are, at the moment of their superposition, *by construction*, completely independent and distinguishable.

By generalizing the process for the two-coins microcanonical system described in section 3.1 and taking into account that identity (3) corresponds to an extreme state of the system in which $k \ln \Omega$ assumes its maximum value, we can state that

as the system moves towards equilibrium, the value of $k \ln \Omega$ decreases monotonically until it attains its minimum value.

In other words, in its way to equilibrium, the system loses configurability. Since this fact is in complete disagreement with what is well established for entropy, $k \ln \Omega$ can not be set equal to the entropy S as it has been usually done in the formulation of the Boltzmann's principle.

Nevertheless $k \ln \Omega$ represents a legitimate thermodynamic function. In order to stress the difference between this function and entropy as well as its fundamental role in microcanonical theory, it is proper to give it a distinctive name, say the *complexity* of the system, and denote it by a different letter, say, L . Since neither L nor S are observable quantities, the existing relationship between them cannot be directly established by experience. The shortest path we found to solve this problem was to postulate the equation $S = L$ not as a *universal identity* but as the *condition for equilibrium*.

3.4. The macroscopic homogeneity principle

Since L is here recognized as a monotonic decreasing function of time we can not adopt, as a criterion for equilibrium, the maximum probability principle. It will be replaced here by a phenomenological criterion that we shall call the Macroscopic Homogeneity Principle. This criterion [7] states that:

in the equilibrium state, any thermodynamic extensive quantity can be expressed as a first-order homogeneous function of the remaining extensive quantities that completely determine the macroscopic state of the substance.

Since L can be identified as one of these extensive quantities, it must satisfy, in the equilibrium state, the equation:

$$L^* = \hat{\epsilon} L^*$$

where L^* denotes the equilibrium complexity, $\hat{\epsilon}$ is the Euler operator for homogeneous functions

$$\hat{\epsilon} = \sum_{i=1}^n \lambda_i \frac{\partial}{\partial \lambda_i}$$

and the λ_i 's stand for the extensive thermodynamic parameters such as U , V , N etc.

Defining the auxiliary function $y = L - S$, and taking into account that in the equilibrium S is also expressed as a first-order homogeneous function of the same extensive parameters, we obtain the differential equation:

$$\hat{\epsilon} y - y = \hat{\epsilon} L - L$$

whose solution allows the determination of the entropy S when L is given. It can be easily verified that, for the perfect gas whose complexity is given by (3), y coincides with the Gibbs' correction factor.

If we now define the *total complexity* of a system as the value of L corresponding to the state in which the system's constituent particles are completely distinguishable, then we can express the thermodynamic equilibrium in terms of the following principle:

The total complexity of a microcanonical system represents an upper bound for the different values the system's complexity can take. Progressive non reversible interactions among the system's constituent elements produce, in the course of time, a monotonic decrease in the value of the system's complexity until all pending interactions are consummated. The system then attains its state of equilibrium.

4. The equilibrium in a gas of bosons

Evidences of the existence of states of incomplete indistinguishability can also be found in the ideal gas of bosons. It is well known that Bose-Einstein statistics can be independently obtained either as a consequence of the equilibrium condition for a creation-annihilation process or as a mathematical consequence of the indistinguishability of identical particles.

4.1. Equilibrium in a creation-annihilation process

The movement of a gas of bosons towards the equilibrium state can be obtained from the restatement, in terms of a Markovian birth and death process, of A. Einstein's arguments for the equilibrium between matter and radiation [4]. A general Markovian birth and death process is described by the following system of differential equations [5,6]:

$$\begin{aligned} p_0'(t) &= -\lambda_0 p_0(t) + \mu_1 p_1(t) \\ p_n'(t) &= \lambda_{n-1} p_{n-1}(t) - (\lambda_n + \mu_n) p_n(t) + \mu_{n+1} p_{n+1}(t) \end{aligned} \quad (4)$$

where the $p_n(t)$ represent the probability of finding n elements in a population at time t , the λ_n 's are the elements' birth rates and the μ_n 's, their death rates. The equilibrium (stationary) probabilities $p_n(\infty)$ are obtained by equating $p_n'(t) = 0$ for all n .

Let us suppose that the rates of birth (*creation*) and death (*annihilation*) of the particles are independent of n , respectively $\lambda_n = \lambda$ and $\mu_n = \mu$ for all n . The equilibrium number of par-

ticles in the system can then be obtained as the expectation \bar{n} of the random variable n for the stationary solution $p_n = p_n(\infty)$ of the system of equations (4):

$$\bar{n} = \sum_{n=0}^{\infty} n p_n = \frac{1}{\frac{\mu}{\lambda} - 1} \quad (5)$$

We assume that both the birth and death rates λ and μ obey the Arrhenius' law:

$$\lambda = \xi e^{-\frac{E_2}{kT}} \quad \text{and} \quad \mu = \xi e^{-\frac{E_1}{kT}} \quad (6)$$

where E_1 and E_2 ($E_2 > E_1$) represent the energy levels of the substance with which the particles interact, k is the Boltzmann's constant, T is the absolute temperature and the Arrhenius' frequency factor ξ , is constant.

Taking into account the Bohr's frequency condition

$$E_2 - E_1 = h\nu$$

where h is the Planck's constant and ν the characteristic frequency, then (5) can be identified with Planck's radiation law:

$$\bar{n} = \frac{1}{e^{\frac{h\nu}{kT}} - 1} \quad (7)$$

The time evolution of a bounding limit for the probability $p_n(t)$ of finding n particles in the system at time t is known [6] to be given by the expression

$$|p_n(t) - p_n(\infty)| \leq \alpha e^{-\beta t}$$

where:

$$\alpha = \frac{2 \left(\sqrt{\frac{\mu}{\lambda}} + 1 \right)}{\pi \left(\sqrt{\frac{\mu}{\lambda}} - 1 \right)^2} \left(\sqrt{\frac{\lambda}{\mu}} \right)^n \quad \text{and} \quad \beta = \left(\sqrt{\mu} - \sqrt{\lambda} \right)^2$$

Therefore, whatever the initial probability distribution of particles, in the final equilibrium condition the particles will become indistinguishable.

The Markovian birth and death process analysis of stationary states shows that equilibrium is characterized not in terms of the most probable subset of all states available to the system, as usually assumed in the traditional formulation of the Boltzmann's Principle, but as the stationary probability distribution function $p_n = p_n(\infty)$. In the case of bosons (whose birth and death rates are constant) p_n is given by the geometric probability distribution function:

$$p_n = p_n(\infty) = \left(1 - \frac{\lambda}{\mu} \right) \left(\frac{\lambda}{\mu} \right)^n = \left(1 - e^{-\frac{h\nu}{kT}} \right) e^{-\frac{nh\nu}{kT}} \quad (8)$$

4.2. Probability distribution of indistinguishable particles

Let us take the combinatorial system formed by the random placement of n indistinguishable balls in r cells. The asymptotic probability distribution function for $n \rightarrow \infty$, $r \rightarrow \infty$ so that the average number $\kappa = n/r$ of balls per cell remains constant is known to be [5]:

$$q_m = \left(\frac{1}{1+\kappa} \right) \left(\frac{\kappa}{1+\kappa} \right)^m \quad (9)$$

4.3. Equivalence between indistinguishability and equilibrium

Both probability distribution functions (8) and (9) are identical with $\kappa = \bar{n}$, as expected. Therefore the equivalence between the equilibrium state of the gas and the complete indistinguishability of its constituent particles can be formally established so that

when a gas of bosons is removed from its condition of equilibrium then its constituent particles are necessarily removed from the state of complete indistinguishability,

a result that can only be explained in terms of our previous findings.

5. Conclusions

In this paper we have shown that transient distinguishability is a consequence of the relativistic uncertainty principle where uncertainty can be understood as the manifestation of a *non-reversible, non-potential fundamental interaction* existing among identical particles so that indistinguishability results as the final outcome of their collisions. The picture we usually make of chemical reactions seems to be a suitable paradigm for a heuristic description of this interactive process.

It was also shown that transient distinguishability is a necessary hypothesis both for a paradox-free derivation of the entropy of a perfect gas and for the explanation of the non-equilibrium internal states of the constituent particles of a gas of bosons. The equilibrium state in these systems is characterized by the complete indistinguishability of their constituent identical particles as opposed to non-equilibrium states where incomplete indistinguishability is to be found. Besides, it was shown that when the system's constituent identical particles are in the extreme state of complete distinguishability, its complexity $L = k \ln \Omega$ attains its maximum value.

The time asymmetry observed in macroscopic natural phenomena (time's arrow) should then be explained not as a consequence of the movement of microscopic particles towards states of increasing probabilities but, instead, as the inexorable tendency of natural substances to lose configurability, i.e., to consume, in the course of time, all pending uncertainty reactions existing among identical particles giving rise to observable change. This change ends, microscopically, in the complete indistinguishability of identical particles and, macroscopically, in the homogeneous state of thermodynamic equilibrium.

6. References

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