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COVARIANT AND LIGHT-FRONT APPROACHES TO THE ρ-MESON ELECTROMAGNETIC FORM-FACTORS

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Covariant and Light-Front approaches to the ρ-meson electromagnetic form-factors *

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Abstract

The ρ-meson electromagnetic form-factors are calculated, both in a covariant and light-front frameworks with constituent quarks. The effect of the breakdown of rotational symmetry for the one-body current operator in the null-plane is investigated by comparing calculations within light-front and covariant approaches. This allows to choose the appropriate front-form prescription, among the several ones, to evaluate the ρ-meson form-factors.
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I. INTRODUCTION

Since Dirac [1], it is known that the light-front hypersurface given by $x^+ = x^0 + x^3 = 0$ (null-plane) is suitable for defining the initial state of a relativistic system. Relativistic models with null-plane wave-functions have becoming widely used in particle phenomenology [2]. They permit to calculate the matrix elements of certain operators in a framework of a fixed number of constituents, while mantaining a limited covariance, under transformations that keeps the null-plane invariant [3]. As the generators of rotations around x and y-axis do not belong to the stability group [4], the covariance of a composite null-plane wave-function with a fixed number of constituents can be broken.

This fact has consequences, for example, in the calculation of the electromagnetic form-factors of a composite spin one particle [5]. It is well known that, the $J^+(= J^0 + J^3)$ component of the current, looses its rotational invariance and consequently violates the angular condition [5,6]. The matrix elements are computed with a null-plane wave-function [6,7] in the Breit-frame, where the vector component of the momentum transfer is along the x-direction. If rotational symmetry around the x-axis is valid, then $J^+_{zz} = J^+_{yy}$, where the subscripts are the polarizations of the spin one particle in the cartesian instant-form spin basis [8]. Such requirement is the angular condition [5,6]. It can also be derived in the front-form spin basis, using general arguments of parity and rotational invariance of J^+ [9].

The breakdown of rotational symmetry, implies that, it does not exist an unique way to extract electromagnetic form-factors from the matrix elements of J^+, for composite systems with spin equal or higher than one. Consequently, in the literature, there are different extraction schemes for spin one form-factors [6–8,10].

Recently, the issue of the breakdown of rotational covariance for the one-body component of the J^+ current, has been discussed in the calculation of ρ-meson form-factors with constituent quarks [11,12]. In these works, it was stressed the importance of relativistic effects related to the constituent mass scale and the ρ-meson size. Such relativistic effects are smaller in a test case of a S-wave deuteron system and the violation of the angular condition is quite small [13].

The judgement of the different prescriptions for obtaining electromagnetic form-factors with a specific ρ-meson null-plane wave-function, could be done in principle by a comparision with the results of a covariant calculation, within the same model.

It is our aim in this work to calculate the ρ-meson form-factors from the covariant Feynman one-loop triangle-diagram for the '+' component of the current, in two ways. The first one corresponds to integrate directly in the four-dimensional phase-space and the second one corresponds to first integrate in the '-' component of the loop momentum ($k^- = k^0 - k^3$). This last procedure is equivalent to the use of a wave-function in the null-plane to obtain form-factors [14]. To deal with a finite value for the triangle-diagram, we introduce a covariant regulator, in a manner proposed in Ref. [15]. The covariant regularization generates a null-plane ρ-meson quark wave-function [15].

We compare the covariant and front-form results for the ρ-meson form-factors, and then, we are able to point out the appropriate prescription to evaluate the form-factors of the ρ-meson with the null-plane wave-function.

The plan of the work is the following: in section II is discussed the different extraction schemes for obtaining the spin one electromagnetic form-factors, from J^+, and the notation is defined. In section III, the matrix elements of J^+ are obtained from the Feynman triangle-diagram, and the covariant and front-form calculations of the form-factors are discussed. The covariant regularization is shown to be related to the null-plane wave-function. In section IV, it is presented the numerical results for the ρ-meson form-factors calculated in both frameworks and a summary of the main findings are given.

II. ELECTROMAGNETIC CURRENT AND FORM-FACTORS

The general expression of the electromagnetic current of a spin-one particle has the form [5] :

$$J^{\mu}_{\alpha\beta} = [F_1(q^2) g_{\alpha\beta} - F_2(q^2)\frac{q_\alpha q_\beta}{2m^2_\rho}] P^\mu - F_3(q^2)(q_\alpha g^\mu_\beta - q_\beta g^\mu_\alpha) \ , \qquad (1)$$

where, m_ρ is the ρ-meson mass, q^μ is the momentum transfer and P^μ is the sum of the initial and final momentum.

We write the instant-form matrix elements of the J^+ component of the current, given by Eq. 1 in the Breit-frame, where $q^\mu = (0, q_x, 0, 0)$. The instant-form cartesian polarization four-vectors are given by:

$$\epsilon^\mu_x = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0) \ , \ \epsilon^\mu_y = (0,0,1,0) \ , \ \epsilon^\mu_z = (0,0,0,1) \ , \qquad (2)$$

for the initial ρ-meson polarization states and,

$$\epsilon'^\mu_x = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0) \ , \ \epsilon'^\mu_y = \epsilon_y \ , \ \epsilon'^\mu_z = \epsilon_z \ , \qquad (3)$$

for the final polarization states; $\eta = -q^2/4m^2_\rho$. The ρ-meson four-momentum in the Breit-frame are, $p^\mu_i = (p^0, -q_x/2, 0, 0)$ for the initial state, and $p^\mu_f = (p^0, q_x/2, 0, 0)$for the final state; $p^0 = m_\rho\sqrt{1+\eta}$.

We use the spherical polarization vectors, and in the instant-form spin basis are given by:

$$\epsilon_{+-} = -(+)\frac{\epsilon_x + (-)\epsilon_y}{\sqrt{2}} \ and \ \epsilon_0 = \epsilon_z \ . \qquad (4)$$

The "+" component of the electromagnetic current in the instant-form spin basis is written as:

$$J^+ = \frac{1}{2}\begin{pmatrix} J^+_{xx} + J^+_{yy} & -\sqrt{2}J^+_{zx} & J^+_{yy} - J^+_{xx} \\ \sqrt{2}J^+_{zx} & 2J^+_{zz} & -\sqrt{2}J^+_{zx} \\ J^+_{yy} - J^+_{xx} & \sqrt{2}J^+_{zx} & J^+_{xx} + J^+_{yy} \end{pmatrix} , \qquad (5)$$

where the spin projections are in the following order $m = (+, 0, -)$. The first and second subscripts of the current means the polarizations of the final and initial states, respectively.

The matrix elements of "+" component of the current in the instant-form spin basis, Eq. 5, are related to the matrix elements in the front-form spin basis. For notational convenience, we use I^+, to express the front-form matrix elements. The unitary transformation between these spin-basis is the Melosh rotation [9] (Appendix). The general form of the "+" component of the current in the front-form spin basis is written as [8]

$$I^{+}=\begin{pmatrix} I^{+}_{11} & I^{+}_{10} & I^{+}_{1-1} \\ -I^{+}_{10} & I^{+}_{00} & I^{+}_{10} \\ I^{+}_{1-1} & -I^{+}_{10} & I^{+}_{11} \end{pmatrix} . \tag{6}$$

We express the front-form matrix elements in terms of the instant-form matrix elements of the current, by using the results of the Appendix,

$$\begin{aligned} I^{+}_{11} &= \frac{J^{+}_{xx}+(1+\eta)J^{+}_{yy}-\eta J^{+}_{zz}+2\sqrt{\eta}J^{+}_{zx}}{2(1+\eta)} \\ I^{+}_{10} &= \frac{\sqrt{2\eta}J^{+}_{xx}+\sqrt{2\eta}J^{+}_{zz}+\sqrt{2}(\eta-1)J^{+}_{zx}}{2(1+\eta)} \\ I^{+}_{1-1} &= \frac{-J^{+}_{xx}+(1+\eta)J^{+}_{yy}+\eta J^{+}_{zz}-2\sqrt{\eta}J^{+}_{zx}}{2(1+\eta)} \\ I^{+}_{00} &= \frac{-\eta J^{+}_{xx}+J^{+}_{zz}+2\sqrt{\eta}J^{+}_{zx}}{(1+\eta)} . \end{aligned} \tag{7}$$

The angular condition $J^{+}_{zz} = J^{+}_{yy}$ can be written in front-form spin basis (see Appendix), giving its usual form [6,9]

$$\Delta(q^2)=(1+2\eta)I^{+}_{11}+I^{+}_{1-1}-\sqrt{8\eta}I^{+}_{10}-I^{+}_{00}=0 . \tag{8}$$

In general, the impulse approximation to the electromagnetic current does not satisfy such condition [6,11–13], this fact led to different extraction schemes of the form-factors from the matrix elements of the current [6–8,10,12]. Let us review the prescriptions existing in the literature for calculating the form-factors for spin-one particle, from the matrix elements $I^{+}_{m'm}$.

The charge, G_0, magnetic, G_1, and quadrupole, G_2, form-factors are obtained from linear combinations of the covariant form-factors, F_1, F_2 and F_3 [8], see Appendix. Below, we give the different prescriptions for obtaining the form-factors, which are also written in terms of matrix elements in the cartesian instant-form spin-basis. Such spin basis is used because it facilitates the algebraic manipulations of the covariant amplitude for the photon absorption process, and it is completly equivalent to the front-form spin basis.

In reference [6], the elimination of the matrix element I^{+}_{00}, gives the following prescription to calculate the form-factors:

$$\begin{aligned} G^{GK}_0 &= \frac{1}{3}[(3-2\eta)I^{+}_{11}+2\sqrt{2\eta}I^{+}_{10}+I^{+}_{1-1}]=\frac{1}{3}[J^{+}_{xx}+2J^{+}_{yy}-\eta J^{+}_{yy}+\eta J^{+}_{zz}] \\ G^{GK}_1 &= 2[I^{+}_{11}-\frac{1}{\sqrt{2\eta}}I^{+}_{10}]=J^{+}_{yy}-J^{+}_{zz}+\frac{J^{+}_{zx}}{\sqrt{\eta}} \\ G^{GK}_2 &= \frac{2\sqrt{2}}{3}[-\eta I^{+}_{11}+\sqrt{2\eta}I^{+}_{10}-I^{+}_{1-1}]=\frac{\sqrt{2}}{3}[J^{+}_{xx}+J^{+}_{yy}(-1-\eta)+\eta J^{+}_{zz}] . \end{aligned} \tag{9}$$

In Ref. [7], they have obtained

$$\begin{aligned} G^{CCKP}_0 &= \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I^{+}_{11}+I^{+}_{00})+5\sqrt{2\eta}I^{+}_{10}+(2\eta-\frac{1}{2})I^{+}_{1-1}] \\ &= \frac{1}{6}[2J^{+}_{xx}+J^{+}_{yy}+3J^{+}_{zz}] \\ G^{CCKP}_1 &= \frac{1}{(1+\eta)}[I^{+}_{11}+I^{+}_{00}-I^{+}_{1-1}-\frac{2(1-\eta)}{\sqrt{2\eta}}I^{+}_{10}]=\frac{J^{+}_{zx}}{\sqrt{\eta}} \\ G^{CCKP}_2 &= \frac{\sqrt{2}}{3(1+\eta)}[-\eta I^{+}_{11}-\eta I^{+}_{00}+2\sqrt{2\eta}I^{+}_{10}-(\eta+2)I^{+}_{1-1}]=\frac{\sqrt{2}}{3}[J^{+}_{xx}-J^{+}_{yy}] \end{aligned} \tag{10}$$

The prescrition of Brodsky and Hiller [10], to obtain the form-factors is:

$$\begin{aligned} G^{BH}_0 &= \frac{1}{3(1+\eta)}[(3-2\eta)I^{+}_{00}+8\sqrt{2\eta}I^{+}_{10}+2(2\eta-1)I^{+}_{1-1}] \\ &= \frac{1}{3(1+2\eta)}[J^{+}_{xx}(1+2\eta)+J^{+}_{yy}(2\eta-1)+J^{+}_{zz}(3+2\eta)] \\ G^{BH}_1 &= \frac{2}{(1+2\eta)}[I^{+}_{00}-I^{+}_{1-1}+\frac{(2\eta-1)}{\sqrt{2\eta}}I^{+}_{10}] \\ &= \frac{1}{(1+2\eta)}[\frac{J^{+}_{zx}}{\sqrt{\eta}}(1+2\eta)-J^{+}_{yy}+J^{+}_{zz}] \\ G^{BH}_2 &= \frac{2\sqrt{2}}{3(1+2\eta)}[\sqrt{2\eta}I^{+}_{10}-\eta I^{+}_{00}-(\eta+1)I^{+}_{1-1}] \\ &= \frac{\sqrt{2}}{3(1+2\eta)}[J^{+}_{xx}(1+2\eta)-J^{+}_{yy}(1+\eta)-\eta J^{+}_{zz}] \end{aligned} \tag{11}$$

According to Ref. [8], the electromagnetic form-factors, are obtained from the matrix elements J^{+}_{xx}, J^{+}_{zx} and J^{+}_{yy},

$$\begin{aligned} G^{FFS}_0 &= \frac{1}{3(1+\eta)}[(2\eta+3)I^{+}_{11}+2\sqrt{2\eta}I^{+}_{10}-\eta I^{+}_{00}+(2\eta+1)I^{+}_{1-1}]=\frac{1}{3}[J^{+}_{xx}+2J^{+}_{yy}] \\ G^{FFS}_1 &= G^{CCKP}_1 \\ G^{FFS}_2 &= G^{CCKP}_2 . \end{aligned} \tag{12}$$

The low-energy ρ-meson observables, mean square radius, magnetic moment and quadrupole moment, are given by [7],

$$< r^2 >= \lim_{q^2\to 0} \frac{6(G_0(q^2)-1)}{q^2} \ , \ \mu = \lim_{q^2\to 0} G_1(q^2) \ , \ Q_2 = \lim_{q^2\to 0} 3\sqrt{2}\frac{G_2(q^2)}{q^2} \ , \tag{13}$$

respectively.

III. COVARIANT AND FRONT-FORM CURRENTS

The ρ-meson electromagnetic form-factors are obtained in the impulse approximation. It includes only one-body current operator, and the amplitude for the photon absorption is given by the Feynman triangle-diagram, with the photon leg attached to one of the quarks. We compute only the "good" component of the current (J^+), which is diagonal in the null-plane Fock-state. The pair creation diagram is supressed for J^+ [16].

The spinor structure of the $\rho - q\bar{q}$ vertex, is written in the following form,

$$\Gamma^\mu(k,k') = \gamma^\mu - \frac{m_\rho}{2}\frac{k^\mu + k'^\mu}{p.k + m_\rho m - \imath\epsilon} \ . \tag{14}$$

where, the ρ-meson is on-mass-shell, and its four momentum is $p^\mu = k^\mu - k'^\mu$, the quark momenta are given by k^μ and k'^μ, and their mass by m. Eq. 14 reduces to the vertex given in Ref. [17] for a on-mass-shell quark. This vertex corresponds to a relative S-state quark-antiquark wave-function [5,17]. Above, we wrote the spinor structure of the vertex. The complete null-plane wave-function comes from the regularization factor and the denominator of the propagator, as it will be clear in the following.

The impulse approximation to J^+, is given by the Feynman triangle-diagram, and we assume the constituent quark as a Dirac pointlike particle,

$$J^+_{ji} = \imath\int \frac{d^4k}{(2\pi)^4}\frac{Tr[\epsilon'^{\alpha}_j\Gamma_\alpha(k,k-p_f)(\not k - \not p_f + m)\gamma^+(\not k - \not p_i + m)\epsilon^\beta_i\Gamma_\beta(k,k-p_i)(\not k + m)]}{((k-p_i)^2 - m^2 + \imath\epsilon)(k^2 - m^2 + \imath\epsilon)((k-p_f)^2 - m^2 + \imath\epsilon)}$$
$$\times\Lambda(k,p_f)\Lambda(k,p_i) \ , \tag{15}$$

where J^+_{ji} is written in the cartesian instant-form spin basis, and ϵ'^{α}_j is the final polarization four-vector (Eq.3) and ϵ^β_i is the initial four-vector polarization (Eq.2), the subscripts i and j stand for x, y and z.

The regularization function,

$$\Lambda(k,p) = \frac{N}{((k-p)^2 - m_R^2 + \imath\epsilon)^2} \ , \tag{16}$$

was chosen to turn Eq.15 finite. The special form of the regulator, allows to identify a null-plane wave-function similar to the one proposed for the pion in Ref. [15]. They have used a monopole form-factor, instead of a dipole. The normalization factor N is found by imposing $G_0(0)=1$.

The covariant calculation of the form-factors, is performed with Eq. 15, which is analitically integrated in the k^0 complex-plane. The integration over $\vec{k}$ is done numerically. The angular condition is satisfied exactly by the covariant calculation, as it should be. Also for $q^2 = 0$, $J^+_{xx}(0) = J^+_{yy}(0) = J^+_{zz}(0)$. The matrix elements of the current satisfy current conservation, $q^\mu J_\mu(q^2) = 0$, as we verified explicitly.

The front-form calculation, corresponds to integrate analitically in the complex-plane of k^- variable [14]. The pair diagrams are not present with $q^+ = 0$. The pole which contributes to the integration is

$$k^- = \frac{k_\perp^2 + m^2 - \imath\epsilon}{k^+} \ , \tag{17}$$

for $p^+ > k^+ > 0$, where $p^+ = p^0$ is the energy of the ρ-meson in the Breit-frame. This pole belongs to the lower complex semi-plane, where no other pole is present. Eq. 17, is the on-mass-shell condition for the spectator quark, in the process of photon absorption.

The null-plane wave-function of the ρ-meson appears after the substitution of the on-mass-shell condition, Eq.17, in the propagator of the quark that absorbs the photon and in the corresponding regulator,

$$\frac{1}{((k-p)^2 - m^2 + \imath\epsilon)((k-p)^2 - m_R^2 + \imath\epsilon)^2} = \frac{1}{(1-x)}\frac{1}{(1-x)^2(m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2} \ , \tag{18}$$

where, $x = k^+/p^+$. The free quark-antiquark mass squared is given by

$$M_0^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m^2}{1-x} - p_\perp^2 . \tag{19}$$

The function M_R^2 is given by

$$M_R^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m_R^2}{1-x} - p_\perp^2 \; . \tag{20}$$

The null-plane wave-function is obtained from the combination of denominators in Eq.18 with the spinor structure of the vertex, Eq.14. We leave out the phase-space factor $1/(1-x)$. The resulting expression is evaluated in the center of mass system,

$$\Phi_i(x, \vec{k}_\perp) = \frac{N^2}{(1-x)^2(m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2} \vec{\epsilon}_i . [\vec{\gamma} - \frac{\vec{k}}{\frac{M_0}{2} + m}] \; , \tag{21}$$

the polarization state is given by $\vec{\epsilon}_i$. The wave-function corresponds to a S-wave state [17].

IV. DISCUSSION

The constituent quark model for the ρ-meson null-plane wave-function has two parameters, the constituent quark mass, m, and the regulator mass, m_R. The ρ-meson mass is 0.77 GeV. In this case, the composite wave-function corresponds to a bound state, which imposes a lower bound for the regulator and constituent quark masses, such that

$$m \; > \; \frac{m_\rho}{2} \; , \; m_R + m \; > m_\rho \; .$$

The scale of the model is obtained by adjusting the parameters to get a mean square radius of about 0.35 fm^2, and $G_2(q^2 \sim 5GeV^2) \; \sim$ -0.25, as calculated in Ref. [12], with point-like constituent quarks. They used a wave-function in the null-plane which is dominated by one-gluon exchange at short distances and linear confinement at large distances.

In the non-relativistic limit, the quadrupole form-factor vanishes for a S-state wave-function. The non-zero values of G_2 are a consequence of the relativistic nature of the model, and for this reason we consider it in the parameter fit. We used the covariant calculation for the form-factors to get the parameters $m = 0.43$ GeV and $m_R = 1.8$ GeV.

The low-energy electromagnetic parameters, are calculated using the different front-form prescriptions and are compared with the covariant results. In Table I, we show the values of $< r^2 >$, μ and Q_2. The mean square radius, calculated in the front-form scheme has values

at most 10% higher than the covariant result of 0.37 fm^2. The magnetic moment obtained in the covariant calculation is 2.19, which can be compared with the non-relativistic value of 2. In Ref. [12] they obtained 2.26. The front-form calculations for the magnetic moment, give values with a spread of 15% above the covariant result. The quadrupole moment in the covariant calculation is 0.052 fm^2, somewhat higher than the value quoted in Ref. [12]. The front-form calculations are within 10% to 15% of the covariant result for the low-energy parameters.

In Fig. 1, we observe that the charge form-factor, G_0, is sensitive to the different front-form prescriptions. The calculations show a zero placed around 3 GeV^2 consistent with Ref. [12]. We found an increasing discrepancy among the several prescriptions and the covariant results, for momentum transfers above the zero crossing. The (GK) prescription gives results in agreement with the covariant calculation, while the (BH) results are about 30% below at higher q^2.

The differences between the various front-form calculations for the magnetic form-factor and the covariant results are not so pronounced, as shown in Fig. 2. At small momentum transfers the (FFS) prescription has a value about 15% higher than the covariant result, in agreement with the results of Table I. In the momentum range considered, the (GK) prescription is consistent with the covariant calculation.

The relativistic effects in the model are the origin of G_2, and thus it is more sensitive to the difference between the front-form prescriptions. In Fig. 3, the values of G_2 calculated in the front-form with prescriptions given by (CCKP) and (BH) are 20% lower than the covariant result. The calculations with (GK) combination of the currents, present the best consistency with the covariant results, among the four prescriptions tested.

We conclude that, in the scale of the ρ-meson bound state, tunned by a parametrization which reproduces the size and quadrupole form-factor, of an effective constituent quark model, which embodies gluon exchange and confinement; the prescription for a front-form calculation of the form-factors as given by the work of Grach and Kondratyuk [6] shows consistence with the covariant results. We have used a vertex for the ρ-meson, that was amenable to covariant integration and reproduced to some extend the size properties of a physically inspired null-plane wave-function.

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APPENDIX A

The Melosh rotation for spin 1 particle is given by:

$$R_M = \begin{pmatrix} \frac{(1+\cos\theta)}{2} & -\frac{\sin\theta}{\sqrt{2}} & \frac{(1-\cos\theta)}{2} \\ \frac{\sin\theta}{\sqrt{2}} & \cos\theta & -\frac{\sin\theta}{\sqrt{2}} \\ \frac{(1-\cos\theta)}{2} & \frac{\sin\theta}{\sqrt{2}} & \frac{(1+\cos\theta)}{2} \end{pmatrix}, \tag{A1}$$

where $\cos\theta = (\sqrt{1+\eta})^{-1}$ and $\sin\theta = -\sqrt{\frac{\eta}{(1+\eta)}}$.

The matrix elements of the current in the instant-form spin basis (J^+), Eq.5, and in front-form spin basis (I^+), Eq.6 are related by the Melosh rotation,

$$R^{\dagger}_{M} I^+ R^{\dagger}_{M} = J^+ . \tag{A2}$$

The instant-form matrix elements are expressed in terms of the front-form matrix elements as [8], using the above equation,

$$\begin{aligned} J^+_{xx} &= \frac{1}{1+\eta}[I^+_{11} + 2\sqrt{2\eta} I^+_{10} - \eta I^+_{00} - I^+_{1-1}] \\ J^+_{zx} &= \frac{\sqrt{2}}{1+\eta}[\frac{\sqrt{2\eta}}{2} I^+_{11} + (\eta-1) I^+_{10} + \sqrt{\frac{\eta}{2}} I^+_{00} - \frac{\sqrt{2\eta}}{2} I^+_{1-1}] \\ J^+_{yy} &= I^+_{11} + I^+_{1-1} \\ J^+_{zz} &= \frac{1}{1+\eta}[-\eta I^+_{11} + 2\sqrt{2\eta} I^+_{10} + I^+_{00} + \eta I^+_{1-1}] . \end{aligned} \tag{A3}$$

The relations between the form-factors G_0, G_1 and G_2 and the covariant form-factors F_1, F_2 and F_3, are given by:

$$\begin{aligned} G_0 &= -\frac{2}{3} m_\rho \sqrt{1+\eta}[3F_1 + 2\eta(F_1 + F_3 + (1+\eta)F_2)] \\ G_1 &= 2 m_\rho \sqrt{1+\eta} F_3 \\ G_2 &= -4\frac{\sqrt{2}}{3} m_\rho \eta \sqrt{1+\eta}[F_1 + (1+\eta)F_2 + F_3] . \end{aligned} \tag{A4}$$

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FIGURES

FIG. 1. Charge form-factor $G_0(q^2)$ for the ρ-meson as a function of q^2, calculated with covariant and front-form schemes. The solid line is the covariant calculation. Results for the different front-form extraction schemes, Ref.[6] (GK) (dotted line) (it is not possible to distinguish from the covariant calculation), Ref.[7] (CCKP) (short-dashed), Ref. [8] (FFS) (dashed) and Ref.[10] (BH) (long-dashed).

FIG. 2. Magnetic form-factor $G_1(q^2)$ for the ρ-meson as a function of q^2, calculated with covariant and front-form schemes. The curves are labeled according to Fig.1 .

FIG. 3. Quadrupole form-factor $G_2(q^2)$ for the ρ-meson as a function of q^2, calculated with covariant and front-form schemes. The curves are labeled according to Fig.1 .

TABLES

TABLE I. Results for the low-energy electromagnetic ρ-meson observables, for the covariant (COV) and front-form calculations. The front-form extraction schemes to obtain the form-factors are given by Refs. (GK) [6], (CCKP) [7], (FFS) [8] and (BH) [10]. In the last column, the results of Ref. [12] are given.

MODEL	COV	GK	CCKP	BH	FFS	Ref.[12]
$< r^2 > (fm^2)$	0.37	0.37	0.38	0.40	0.39	0.35
μ	2.14	2.19	2.17	2.15	2.48	2.26
$Q_2(fm^2)$	0.052	0.050	0.051	0.051	0.058	0.024

Figure 1

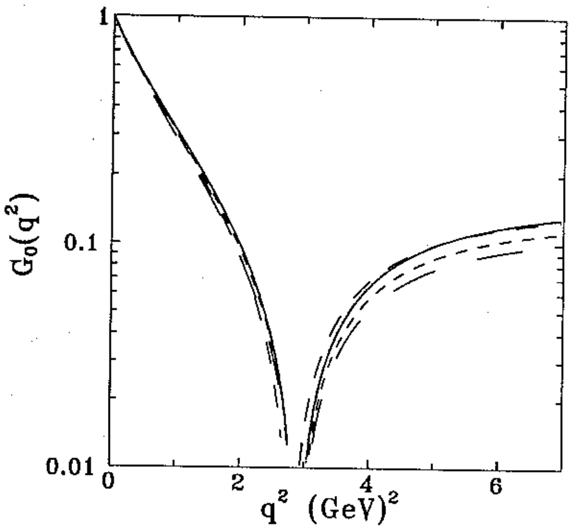

Figure 2

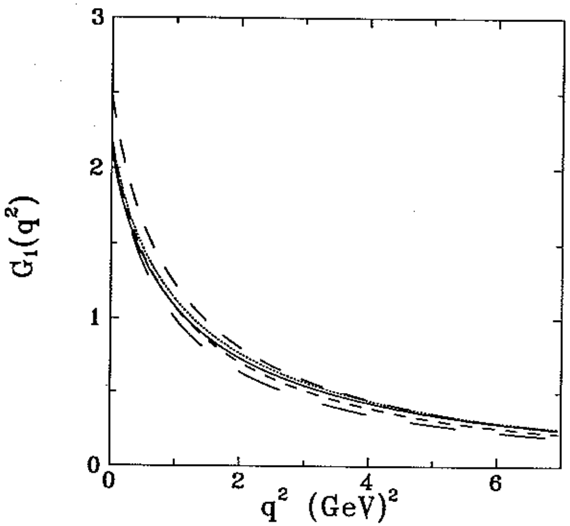

Figure 3