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BRASIL

PUBLICAÇÕES

IFUSP/P-1247

DETERMINATION OF THE ELECTRIC FIELD GRADIENT TENSOR BY 2D NQR

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Novembro/1996

Single Graded-Response Neuron Model with Recurrent Excitation: Distributed Delay

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Abstract: The asymptotic behavior of a single graded response neuron model with a delayed self-excitatory connection is studied. The connection delay is distributed over a bounded interval. The dynamical system associated with the system generates a strongly monotone semi-flow, and therefore trajectories have a strong tendency to converge to equilibria. In fact, it is shown that the system is either globally asymptotically stable or bistable. In the latter case, the union of the basins of attraction of the two stable equilibria is an open and dense set. The two basins are separated by a codimension one, unordered Lipschitz manifold containing oscillating solutions.

1 Introduction

The time it takes for a signal to be transmitted from one neuron to another, referred to as the transmission delay or simply delay, can influence the behavior of neural networks. A number of studies have dealt with the local and global stability of networks with delay (Marcus and Westerwelt, 1989; Marcus *et al.*, 1991; Civalleri *et al.*, 1993; Roska *et al.*, 1992; Bélair, 1993; Burton, 1993; Roska *et al.*, 1993; Gopalsamy and He, 1994; Ye *et al.*, 1994). In some cases sufficient conditions have been derived for (almost) all trajectories to converge to stable equilibrium points in networks with delay. This can be of prime importance for neural network applications such as content addressable memories. However, even in such (quasi) convergent networks, the delay may induce changes in the transient regime and the boundary of the basins of attraction of the stable equilibrium points, as we have shown in the case of discrete fixed delays (Pakdaman *et al.*, 1996b and 1995a-b).

In this paper we study the dynamics of a single neuron with recurrent excitation with continuous distributed delay. In this system, likewise in the case of fixed discrete delays, most trajectories converge to stable equilibrium points, whatever the delay, thus constituting a more general example to illustrate how the delay may alter both the transient regime and the basin boundaries.

In section 2 we present the neuron model. The stationary dynamics of the trajectories are described in sections 3. In section 4 we show that the transient regime duration of a system with a single discrete delay provides information about the system with a continuous delay distribution. The duration of the transient regime is computed for different delays and different constant initial conditions in section 5.

2 The neuron model

The graded response model (GRM) is characterized by the neuron's activation a and a sigmoidal output function $\sigma(a)$. In the absence of inputs, the activation decays with a constant rate γ. For more details and references on the GRM see (Hopfield & Tank, 1986; Pasemann, 1993). We consider a neuron that has a delayed excitatory self-connection with a positive connection weight $w(u)$ and distributed delay u bounded by A. The neuron receives a constant input K.

The neuron activation evolves according to the following delay differential equation (DDE):

$$\frac{da}{dt}(t) = -\gamma a(t) + K + \int_{-A}^{0} w(u)\sigma(a(t+u))du \qquad (1)$$

Where σ is the sigmoidal function defined by:

$$\sigma(a) = \frac{1}{1+e^{-a}} \qquad (2)$$

and w is a positive continuous function[1]. We assume that $A = -\inf\{u, \text{ such that } w(u) > 0\}$ is a strictly positive real number. We note $W = \int_{-A}^{0} w(u)du > 0$.

Let $\mathcal{C}[-A,0]$ be the space of continuous real functions of the interval $[-A,0]$. For ϕ in $\mathcal{C}[-A,0]$, there exists a unique real function $a(t,\phi)$ on the interval $[-A,+\infty)$, such that

[1]This assumption can be relaxed to w being a positive measure such that: $A = -\inf\{u, \text{ such that } \int_{-A}^{-A+\epsilon} dw(u) > 0 \text{ for all } \epsilon > 0\}$ and $W = \int_{-A}^{0} dw(u)$ be strictly positive real numbers.

$a(t,\phi) = \phi(t)$ for $-A \leq t \leq 0$, and $a(t,\phi)$ satisfies equation (1) for $t \geq 0$ (Hale and Verduyn Lunel, 1993). For such a solution of the DDE, we note $a_t(\phi)$ the element of $\mathcal{C}[-A,0]$, defined by $a_t(\phi)(\theta) = a(t+\theta,\phi)$, for $-A \leq \theta \leq 0$.

3 Asymptotic Stability

We first examine the local stability of the equilibria of DDE (1) and then the global stability.

3.1 Local stability

In this section, the local stability of the solutions taking a constant value, i.e. equilibrium points of the DDE (1), is studied. A function taking the value x, that is, $a(t) = x$ for all $t \geq -A$, is a solution of equation (1) if and only if x is a zero of Z, the right hand side of equation (1):

$$Z(x) = -\gamma x + K + W\sigma(x) \qquad (3)$$

The number and value of the zeros of Z depend on the values of the parameters (γ, W, K) (see also (Pasemann,1993)). The parameter set can be separated into two regions, one in which the equation has a unique zero, noted x_0, and another such that it has three zeros $x_1 < x_2 < x_3$.

More precisely we have:

- For $0 \leq W < 4\gamma$, Z has a unique zero noted x_0.
- For $W > 4\gamma$, let:

$$\begin{aligned} K_-(\gamma,W) &= -\gamma \mathrm{Log}\left(\frac{W-2\gamma+\sqrt{W(W-4\gamma)}}{2\gamma}\right) - \frac{W-\sqrt{W(W-4\gamma)}}{2}, \\ K_+(\gamma,W) &= \gamma \mathrm{Log}\left(\frac{W-2\gamma+\sqrt{W(W-4\gamma)}}{2\gamma}\right) - \frac{W+\sqrt{W(W-4\gamma)}}{2}, \end{aligned} \qquad (4)$$

1. For either $K < K_-$ or $K > K_+$, Z has a unique zero also noted x_0.

2. For $K_- < K < K_+$, Z has three zeros noted $x_1 < x_2 < x_3$.

For the study of the local exponential asymptotic stability of each equilibrium point, the real parts of the solutions λ of the characteristic equation (5) at the equilibrium point are examined.

$$\lambda + \gamma - \sigma'(x_i)\int_{-A}^{0} w(u)e^{\lambda u}du = 0 \ . \qquad (5)$$

3.1.1 The locally stable points

For the equilibria x_0, x_1 and x_3 the following inequality holds:

$$\gamma > W\sigma'(x_i) > 0 \quad \text{for } i \text{ in } \{0,1,3\} \ . \qquad (6)$$

From inequality (6) it can be deduced that the characteristic equation (5) admits a real strictly negative solution, noted $\mu_w(x_i)$, and that all its other solutions are complex with real parts smaller than $\mu_w(x_i)$. This ensures that these constant solutions be locally exponentially asymptotically stable (Hale and Verduyn Lunel, 1993).

Moreover we have:

$$-\gamma \le \mu_{W\delta_{-B}}(x_i) \le \mu_w(x_i) \le \mu_{W\delta_{-A}}(x_i) < 0 \ , \qquad (7)$$

where δ_X is the Dirac distribution at X, and B is defined by[2]: $B = -\sup\{u, \text{ such that } w(u) > 0\}$.

We define a continuous function v on the interval $[-\Delta - A, -\Delta]$ $(\Delta \ge 0)$ by $v(u) = w(u+\Delta)$. This represents the effect of increasing the delay. Then we have:

$$\mu_w(x_i) \le \mu_v(x_i) < 0 \ , \qquad (8)$$

and, in fact, $\mu_v(x_i)$ increases and tends to zero as Δ increases and tends to infinity.

In summary, the local asymptotic stability of the stable equilibrium points of the system does not depend on the delay.

3.1.2 The unstable point

At x_2 the situation is different, we have:

$$\gamma < W\sigma'(x_2) \ . \qquad (9)$$

From this inequality, it can be deduced that the characteristic equation admits a real strictly positive solution, noted $\nu(x_2)$, and all its other solutions are complex with real parts smaller than $\nu(x_2)$. Therefore the equilibrium point x_2 is a locally unstable point (Hale and Verduyn Lunel, 1993). Using the same notations and definitions as for μ we have:

$$0 < \nu_{W\delta_{-A}}(x_2) \le \nu_w(x_2) \le \nu_{W\delta_{-B}}(x_2) \ , \qquad (10)$$

and also:

$$0 < \nu_v(x_2) \le \nu_w(x_2) \ , \qquad (11)$$

and, in fact, $\nu_v(x_2)$ decreases and tends to zero as Δ increases and tends to infinity.

Depending on the delay distribution, the characteristic equation at x_2 may have other solutions with positive real parts. In fact, for a system with a single fixed delay A, (i.e. $w(u) = W\delta_{-A}$) there is an increasing sequence of delays A_k, defined by:

$$\begin{array}{l} \tan(A_k\sqrt{W^2\sigma'(x_2)^2-\gamma^2}) = \sqrt{W^2\sigma'(x_2)^2-\gamma^2}/\gamma \ , \\ \text{with} \quad 2k\pi\sqrt{W^2\sigma'(x_2)^2-\gamma^2} < A_k < (2k+1/2)\pi\sqrt{W^2\sigma'(x_2)^2-\gamma^2} \ , \end{array} \qquad (12)$$

[2] $B = -\sup\{u, \text{ such that } \int_{-B-\epsilon}^{-B} dw(u) > 0 \text{ for all } \epsilon > 0\}$

such that at A_k there is a pair of complex conjugate solutions of the characteristic equation crossing the imaginary axis from left to right. For a system with distributed delay, there is at least one sequence of increasing delays Δ_k at which a pair of complex conjugate solutions of the characteristic equation cross the imaginary axis, and:

$$\Delta_k = \Delta_0 + \frac{2k\pi}{\beta} , \qquad (13)$$

where Δ_0 is the delay at which the first crossing occurs and $+i\beta$ and $-i\beta$ ($\beta > 0$) are the solutions of the characteristic equation at all Δ_k.

The number of solutions with positive real parts determines the dimension of the unstable space of the unstable equilibrium point of the linearized equation, and it also gives some indication about the extent of instability of the nonlinear equation near this point (Hale and Verduyn Lunel, 1993). Therefore, increasing the delay renders the unstable point more unstable.

3.2 Return and escape times

The solutions of the characteristic equation at the equilibria change with the delay, even though for the stable equilibrium points their real parts remain negative for all delay values. This is important for evaluating the response of the system to perturbations. A system, stabilized at a locally stable equilibrium point, returns to it when perturbed with a characteristic return time $T_r(x_i, w)$ (Brauer, 1979a-b) and we have:

$$-B/\mu_{\delta_{-B}} = T_r(x_i, W\delta_{-B}) \leq T_r(x_i, w) \leq T_r(x_i, W\delta_{-A}) = -A/\mu_{W\delta_{-A}}(x_i) . \qquad (14)$$

$T_r(x_i, W\delta_{-B})$ is an increasing function of B tending to infinity.

In the same way we can define a characteristic escape time $T_e(x_2, w)$ for the unstable point

x_2:

$$B/\nu_{\delta_{-B}} = T_e(x_2, W\delta_{-B}) \leq T_e(x_2, w) \leq T_e(x_2, W\delta_{-A}) = A/\nu_{W\delta_{-A}}(x_2) , \qquad (15)$$

$T_e(x_2, W\delta_{-B})$ is an increasing function of B tending to infinity.

Therefore, the characteristic return and escape times close to the equilibria are lengthened and tend to infinity as the delay is increased.

3.3 Global stability

The DDE (1) generates an eventually strongly monotone semiflow:

Let ϕ_0 and ϕ_1 be two elements in $\mathcal{C}[-A, 0]$, then we say that ϕ_0 is larger (resp. strictly larger) than ϕ_1 noted $\phi_0 \geq \phi_1$ (resp. $\phi_0 >> \phi_1$) if for all θ in $[-A, 0]$ we have $\phi_0(\theta) \geq \phi_1(\theta)$ (resp. $\phi_0(\theta) > \phi_1(\theta)$). Then for ϕ_0 and ϕ_1 in $\mathcal{C}[-A, 0]$:

$$\text{if } \phi_0 \geq \phi_1 \text{ and } \phi_0 \neq \phi_1, \text{ then for } t > 2A \quad a_t(\phi_0) >> a_t(\phi_1) . \qquad (16)$$

This is illustrated in Fig. 1.

FIGURE 1 HERE

This property (16) strongly restricts the possible asymptotic behaviors of the solutions (Smith, 1987; Roska *et al.*, 1992).

- For $W < 4\gamma$, all solutions converge uniformly asymptotically to x_0.
- For $W > 4\gamma$
  1. For either $K < K_-$ or $K > K_+$, all solutions converge uniformly asymptotically to x_0.

2. For $K_- < K < K_+$, the union of the basins of attraction of x_1 and x_3 is a dense open subset of $\mathcal{C}[-A,0]$. For ϕ in $\mathcal{C}[-A,0]$, there is a unique real number $b(\phi)$ such that $a_t(\phi + c)$ tends asymptotically to x_1 (resp. x_3) for all $c < b(\phi)$ (resp. $c > b(\phi)$); where for a real number c, we note $\phi + c$ the element of $\mathcal{C}[-A,0]$ defined by $(\phi + c)(t) = \phi(t) + c$, for $-A \leq t \leq 0$. $a(t, (\phi + b(\phi))$ oscillates around x_2, in the sense that the function $a(t, (\phi + b(\phi)) - x_2$ has at least one zero on each interval $kA < t < (k+1)A$, for $k \geq -1$. Properties of these oscillatory behaviors, such as periodicity, depend on the instability of x_2, which increases as the delay is increased beyond critical values (Arino and Séguier, 1979; Arino and Benkhalti, 1988).

   The function b from $\mathcal{C}[-A,0]$ to the real line is continuous. The boundary of the basins of attraction of the two locally stable equilibria is the closed set $\{\phi,\ \text{such that}\ \ b(\phi) = 0\}$.

4 Properties of solutions

In this section we show that for a system with distributed delay with a globally asymptotically stable equilibrium point, the duration of the transient regime of an orbit with an arbitrary initial condition is bounded by that of the solution of an equation with a single fixed delay, with properly chosen values of the delay, weight and constant initial condition. For a system with two stable and one unstable equilibrium points, the same result holds for initial conditions that are either larger or smaller than the unstable point x_2.

We identify constant functions with the value they take. The orbits of constant initial conditions are monotonous. When the DDE (1) has a globally asymptotically stable equilibrium

point x_0, the orbit of a constant initial condition $c > x_0$ (resp. $c < x_0$) decreases down (resp. increases up) to x_0:

$$\begin{array}{ll} a(t,c) \geq a(t',c) \geq x_0 & \text{for } -A \leq t \leq t' \text{ and } c \geq x_0\,, \\ a(t,c) \leq a(t',c) \leq x_0 & \text{for } -A \leq t \leq t' \text{ and } c \leq x_0\,. \end{array} \tag{17}$$

When the DDE (1) has three equilibrium points $x_1 < x_2 < x_3$, we have:

$$\begin{array}{ll} a(t,c) \geq a(t',c) \geq x_3 & \text{for } -A \leq t \leq t' \text{ and } c \geq x_3\,, \\ x_2 < a(t,c) \leq a(t',c) \leq x_3 & \text{for } -A \leq t \leq t' \text{ and } x_2 < c \leq x_3\,, \\ x_2 > a(t,c) \geq a(t',c) \geq x_1 & \text{for } -A \leq t \leq t' \text{ and } x_1 \leq c < x_2\,, \\ a(t,c) \leq a(t',c) \leq x_1 & \text{for } -A \leq t \leq t' \text{ and } c \leq x_1\,. \end{array} \tag{18}$$

And the orbit going through the constant initial condition $c = x_2$, is constant.

For a given constant initial condition c, we note $x(t,c)$ (resp. $y(t,c)$) the solution when the weight function is $W\delta_{-A}$, the weighted Dirac at $-A$ (resp. $W\delta_{-B}$, the weighted Dirac at $-B$), and as previously, $a(t,c)$ is the solution for a distributed delay with weight function $w(u)$. Then, a is bounded by x and y, that is:

$$\begin{array}{ll} y(t,c) \leq a(t,c) \leq x(t,c) & \text{for all } t \geq -A, \text{ when } a(t,c) \text{ is decreasing}\,, \\ x(t,c) \leq a(t,c) \leq y(t,c) & \text{for all } t \geq -A, \text{ when } a(t,c) \text{ is increasing}\,. \end{array} \tag{19}$$

Any given function ϕ in $\mathcal{C}[-A,0]$ is bounded, so that there are two constant functions in $\mathcal{C}[-A,0]$, taking the values m and M, such that $m \leq \phi \leq M$. As the DDE (1) generates an eventually strongly monotone semiflow, we have:

$$a_t(m) \leq a_t(\phi) \leq a_t(M) \quad \text{for all } t \geq -A\,. \tag{20}$$

From the previous two sets of inequalities (19)-(20) we deduce that the orbit of any initial condition is bounded between the orbits of two constant initial conditions of two delay equations with the same global weight, one with the shortest delay and the other with the longest delay.

Therefore, the orbits of constant initial conditions of equations with a single delay provides information about the solutions in the general case, and they can be used as a first approximation for estimating the duration of the transient regime.

5 The transient regime

For solutions of the DDE (1) converging to equilibrium points, the transient regime refers to the dynamics before the system stabilizes in its steady state. Practically, the transient regime ends when the state of the system cannot be distinguished from the equilibrium point with some given precision (Hubermann and Wolff, 1985).

FIGURE 2 HERE

Figure 2 shows the transient regime duration (TRD) for a globally asymptotically stable system and Figure 3 shows the TRD for the bistable system. In both cases, dashed and solid lines show the TRD as a function of the initial condition for short and long delays respectively. The figure is based on the numerical resolution of the DDE (1).

FIGURE 3 HERE

In the system considered in figure 2, we have $W < 4\gamma$, so that the system has one globally asymptotically stable equilibrium point $x_0 = 0$. A system with $W > 4\gamma$ and $K < K_-$ or $K > K_+$ behaves in the same way. In the system considered in figure 3, we have $W > 4\gamma$ and $K_- < K < K_+$, so that the system has two locally stable ($x_1 = -2.6$ and $x_3 = 2.6$) and one unstable ($x_2 = 0$) equilibrium points.

The TRD is an increasing function of the distance between the initial condition and the stable equilibrium point to which its orbit converges. This stems from the fact that the system preserves the order of initial conditions (section 3.3). Moreover, for a given initial

condition, the TRD increases with the delay (see also section 3.2).

It should be noted that for a bistable system (Fig. 3) the orbit of a constant initial condition c strictly smaller (resp. larger) than x_2 tends to x_1 (resp. x_3) and the orbit of the initial condition $c = x_2$ is constant (section 3.3) and that constant initial conditions close to the unstable equilibrium point x_2 tend to have long transient regimes (section 3.2).

6 Discussion and conclusion

In this paper, emphasis is put on some effects delay can have on transient responses without altering much the asymptotic behavior of a neural network. The results extend our previous work on the dynamics of a single neuron with recurrent excitation with a single discrete delay (Pakdaman *et al.*, 1996b). We have shown that the phase portrait of the system with distributed delay is similar to that of the system with a single discrete delay. Both systems are globally asymptotically stable when the nonlinear part is smaller than the linear part ($0 \leq W < 4\gamma$), and both are bistable when this is not the case. The attraction basins of the stable equilibria are not intertwined as they are separated by a smooth boundary which is an unordered codimension one manifold. Trajectories on the boundary are oscillating for both systems. However, for the case of a single discrete delay, the number of zeros on intervals of length equal to the delay are decreasing, thus behaving as a discrete Lyapunov function (Arino & Séguier, 1979; Arino, 1993). This leads to a Poincaré Bendixson Theorem for such systems (Mallet-Paret & Sell, 1996), which together with the non-existence of homoclinic orbits (Arino, 1993) shows that in the case of a single discrete delay, oscillatory solutions on the basin boundary are either damped or asymptotically periodic. Such detailed description of the trajectories on the boundary is not available for the case of distributed delays since the existence of the discrete Lyapunov function cannot be extended to this case.

In the case of discrete delays, studying the case of two-neuron, ring and irreducible networks (Pakdaman *et al.*, 1995a; 1996a; 1996c), we have shown that the results for the single neuron are representative of larger networks. In the same way, in the case of distributed delays, the results for a single neuron can be extended to larger networks.

Stability against perturbations is of prime importance for both living organisms and artificial systems evolving in changing environments, and so may be the speed of convergence to an attractor. Both these may be altered by the presence of delay. In our example, the delay slows down the system's response which can be detrimental for the performance of a neural network.

Acknowledgment: This work was partially supported by COFECUB under project U/C 9/94. One of us (CPM) is also partially supported by CNPq (the Brazilian Research Council).

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FIGURE LEGENDS

and

FIGURES

Figure 1: Examples of solutions

Examples of solutions for the bistable system are shown. It can be seen that solutions of constant initial conditions are monotonous increasing or decreasing depending on the relative position of the initial condition to the three equilibria, and that solutions of arbitrary initial conditions can be bounded by those of two appropriate constant initial conditions. Abscissae: time. Ordinates: activation.

Figure 2: TRD for the stable system, for constant initial conditions.

System with a globally asymptotically stable equilibrium point ($x_0 = 0$). *Model parameters:* $\gamma = 1$, $W = 2$ *and* $K = -1$. *Dashed line: short delay* $d = 0.5$. *Solid line: long delay* $d = 25$. *Abscissae: constant initial condition value; ordinate: duration of the transient regime.*

Figure 3: TRD for the bistable system, for constant initial conditions.

Bistable system with two locally asymptotically stable points ($x_1 \simeq -2.6$, $x_3 \simeq 2.6$) *and one unstable* ($x_0 = 0$) *equilibrium points. Model parameters:* $\gamma = 1$, $W = 6$ *and* $K = -3$. *Dashed line: short delay* $d = 0.5$. *Solid line: long delay* $d = 25$. *Abscissae: constant initial condition value; ordinate: duration of the transient regime.*

Figure 1

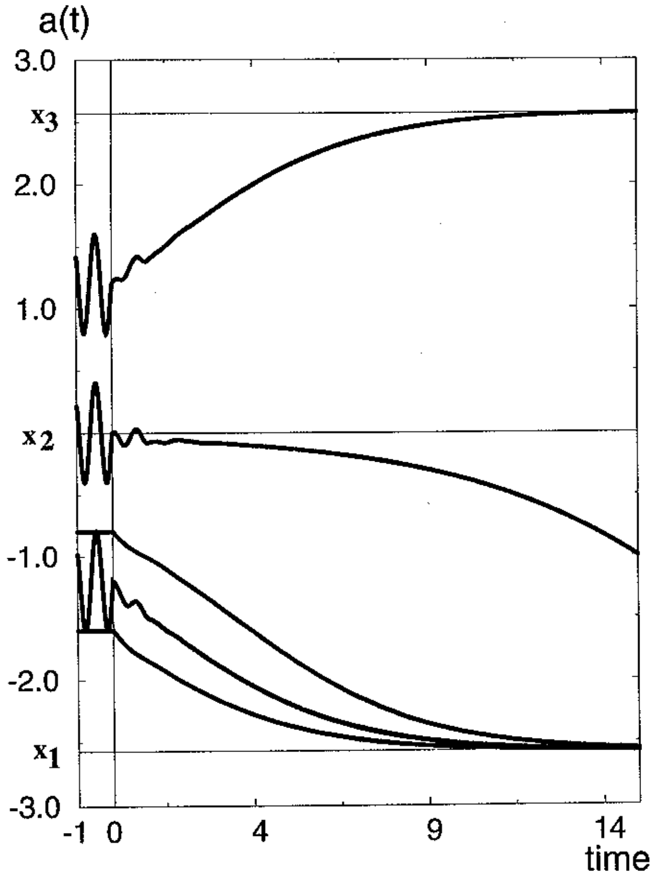

Figure 2

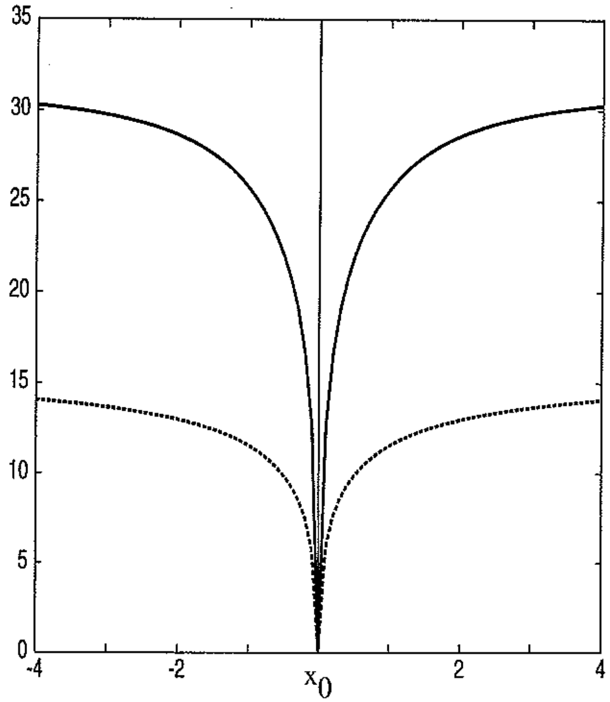

Figure 3