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GREEN FUNCTIONS OF QUANTUM SPINOR FIELD IN FRW UNIVERSE WITH CONSTANT ELECTROMAGNETIC BACKGROUND

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Green functions of quantum spinor field in FRW Universe with constant electromagnetic background

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Spinor field with conformal coupling in Friedmann-Robertson-Walker (FRW) Universe of special type with constant electromagnetic field is discussed. Treating an external gravitational-electromagnetic background exactly, at first time the Fock-Schwinger proper-time representations for out-in, in-in, and out-out spinor Green functions are explicitly constructed as proper-time integrals over the corresponding (complex) contours.

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I. INTRODUCTION

It is quite well known fact that quantum field theory in an external background is, generally speaking, theory with unstable vacuum. The vacuum instability leads to many interesting features, among which particles creation from vacuum is one of the most beautiful non-perturbative phenomena in quantum field theory. Furthermore, in interacting theories the vacuum instability may lead to quantum processes which are prohibited if the vacuum is stable. One ought to say that all the above mentioned peculiarities can not be reveal in frames of the perturbation theory with regards to the external background, one has to treat it exactly. The latter has been realized long ago by Schwinger [1] on the example of quantum electrodynamics in the constant electric field. The particles creation in this case has been calculated explicitly.

In quantum field theory with unstable vacuum it is necessary to construct different kinds of Green functions (GF), e.g. besides the causal GF (out-in GF) one has to use so called in-in GF, out-out GF, and so on [2–4] (for a review and technical details see [5], see as well [21]). General methods of such GF construction in electromagnetic (EM) background have been developed in [3,4]. The possible generalization of the formalism to an external gravitational background has been given in ref. [6]. Since the paper [1] it is known that causal (out-in) GF may be presented as a proper-time integral over a real infinite contour. At the same time, in the instable vacuum the in-in and out-out GF differ from the causal one. One can show [7] that these functions may be presented by the same proper-time integral (with the same integrand) but over another contours in the complex proper-time plane. The complete set of GF mentioned is necessary for the construction of Furry picture in interacting theories, and even in non-interacting cases one has to use them to define, for example, the back reaction of particles created and to construct different kinds of effective actions.

It may be likely that early Universe (EU) is filled with some type of electromagnetic fields. For example, recently (see [8,9] and references therein) the possibility of existence and role of primordial magnetic fields in EU have been discussed. From another point the possibility of existence of electromagnetic field in the EU has been discussed long ago in [10,11]. It has been shown there that the presence of the electrical field in the EU significally increases the gravitational particle creation from the vacuum. In principle, this process may be considered as a source for the dominant part of the Universe mass.

Having in mind the above cosmological motivations it is getting interesting to study the quantum field theory in curved background with electromagnetic field (of special form to be able to solve the problem analytically). In the present paper we are going to consider a massive spinor field in the expanding FRW Universe with the scale factor $\Omega(\eta)$ (in terms

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of the conformal time) $\Omega(\eta) = b\eta + c$. Such a scale factor corresponds to the expanding radiation dominated FRW Universe. In terms of physical time t the corresponding metric may be written as following:

$$ds^2 = dt^2 - \Omega^2(t)(dx^2 + dy^2 + dz^2)\,, \tag{1}$$

where $\Omega^2(t) = 2b|t|$ (see [12]). Moreover, such FRW Universe will be filled by the constant electromagnetic field (the precise form of this field is given in the next section). Thus, we start from the spinor theory in above background. Making conformal transformation of spinor theory , we remain with the theory in flat background but with time-dependent mass. The Maxwell theory is conformally invariant, and one can start from the FRW Universe (1) with constant EM field from the very beginning, before the conformal transformation. In Sect. II we solve the Dirac equation for the theory in the constant EM field and with time-dependent mass to get complete sets of solutions classified as particles and antiparticles at $t \to \pm\infty$. Using them in the Sect. III we construct all necessary GF for the spinor field as the Fock-Schwinger type proper-time integrals. All GF have the same integrand and differ by the contours of integration in the complex proper-time plane. As far as we know that it is first explicit example for proper-time representation for complete set of spinor GF in gravitational and gravitational-electromagnetic background (for pure electromagnetic background it was calculated in [7]).

II. EXACT SOLUTIONS

In this section we will present exact solutions of the spinor field in the external constant uniform electromagnetic background. In addition, the former field will be considered in the time-dependent mass-like potential, which effectively reproduces effects of mass-like potential (QED-Ω):

$$(\mathcal{P}_\mu\gamma^\mu - M\,\Omega(\eta))\psi(x) = 0, \tag{2}$$

where

$$\mathcal{P}_\mu = i\partial_\mu - qA_\mu(x)\,,\ \ \partial_0 = \partial_\eta,\ x^0 = \eta + c/b;$$
$$[\gamma^\mu, \gamma^\nu]_+ = 2\eta^{\mu\nu},\quad \eta^{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1)\,;$$

$q = -|e|$ (for electron); $\psi(x)$ is the spinor field. The time-dependent potential term is chosen as $\Omega(\eta) = bx^0$. The time-independent scalar product of the solutions of the Dirac equation may be chosen in the conventional form

$$(\psi, \psi') = \int \bar{\psi}(x)\gamma^0\psi'(x)d\mathbf{x}. \tag{3}$$

As usual, it is convenient to present $\psi(x)$ in the form

$$\psi(x) = \left(\mathcal{P}_\mu\gamma^\mu + bMx^0\right)\phi(x)\,. \tag{4}$$

Then the functions ϕ have to obey the squared Dirac equation,

$$\left(\mathcal{P}^2 - \left(bMx^0\right)^2 - \frac{q}{2}\sigma^{\mu\nu}F_{\mu\nu} + ibM\gamma^0\right)\phi(x) = 0\,, \tag{5}$$
$$F_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)\,,\ \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\,.$$

The external electromagnetic field will be chosen as the following: parallel constant uniform electric and magnetic fields

$$F_{0D} = E,\ \ F^\perp_{\mu\nu} = H\left(\delta^2_\mu\delta^1_\nu - \delta^2_\nu\delta^1_\mu\right). \tag{6}$$

For such a field we select the following potentials:

$$A_0 = 0,\ A_D = Ex^0,\ A_i = A^\perp_i = -H\,x_2\,\delta^1_i,\ \ i = 1, 2. \tag{7}$$

In this case solutions ϕ of the equation (5) are related with ones ϕ' with some other choice of the potentials A'_μ for the same electromagnetic field, by the relation

$$\phi' = \exp\left(iq\int_{x_c}^{x}\left(A_\mu - A'_\mu\right)dx^\mu\right)\phi\,, \tag{8}$$

where the integral is taken along the line: $\partial_\nu \int_{x_c}^{x}\left(A_\mu - A'_\mu\right)dx^\mu = A_\nu - A'_\nu$.

Let us discuss first the physical motivations behind the Eq. (2). For $b = 0$ one has usual flat case with mass $m = cM$. In this case solutions of the Dirac equations and number of

Green functions were found in [16,7]. The case $b \neq 0$ is of special interest for us. In this case one can consider spinor field in the conformally-flat Universe (with scale factor $\Omega(\eta) = b\eta + c$) filled by the constant EM field. Making the standard conformal transformation of the gravitational metric and spinor field, we come to the theory in flat space-time with time-dependent mass. The corresponding field equation is given by (2) (EM field should not be transformed under the conformal transformation). Note that such conformal transformation may be used also for interacting theories [14,13]. Hence, Eq. (2) actually corresponds to the quantum spinor field in the expanding FRW-Universe with constant EM field.

To construct the above mentioned generalized Furry picture in QED, and in QED-Ω, as well, with an external field one has to find special sets of exact solutions of the equation (2), namely, two complete and orthonormal sets of solution: $\left\{{}_{\pm}\psi_{\{n\}}(x)\right\}$ which describes particles (+) and antiparticles (−) in the initial time instant ($x^0 \to -\infty$), and $\left\{{}^{\pm}\psi_{\{n\}}(x)\right\}$ which describes particles (+) and antiparticles (−) in the final time instant ($x^0 \to +\infty$). According to the general approach [3] , which can be light generalized to QED-Ω, such solutions obey the following asymptotic conditions

$$
\begin{aligned}
&H_{o.p.}(x^0)\ {}_{\zeta}\psi_{\{n\}}(x) = {}_{\zeta}\varepsilon\ {}_{\zeta}\psi_{\{n\}}(x), \quad ,\mathrm{sgn}\ {}_{\zeta}\varepsilon = \zeta,\ x^0 \to -\infty\,,\\
&H_{o.p.}(x^0)\ {}^{\zeta}\psi_{\{l\}}(x) = {}^{\zeta}\varepsilon\ {}^{\zeta}\psi_{\{l\}}(x),\ \mathrm{sgn}\ {}^{\zeta}\varepsilon = \zeta,\ \ x^0 \to +\infty\,,
\end{aligned}
\tag{9}
$$

where $\zeta, \{n\}$ and $\zeta, \{l\}$ are a complete sets of quantum numbers which characterize solutions ${}_{\zeta}\psi_{\{n\}}(x)$ and ${}^{\zeta}\psi_{\{l\}}(x)$ respectively, $H_{o.p.}(x^0) = \gamma^0(M\Omega(\eta) - \gamma^i\mathcal{P}_i)$ is one-particle Dirac Hamiltonian in convenient external field gauge $A_0(x) = 0$; ${}^{+}\varepsilon$, ${}_{+}\varepsilon$ are particle quasi-energies and $|{}^{-}\varepsilon|$ and $|{}_{-}\varepsilon|$ are antiparticles quasi-energies. All the information about the processes of particles scattering and creation by an external field (in zeroth order with respect to the radiative corrections) can be extracted from the decomposition coefficients (matrices) $G\left({}_{\zeta}|{}^{\zeta'}\right)$,

$$
{}^{\zeta}\psi(x) = {}_{+}\psi(x)G\left({}_{+}|{}^{\zeta}\right) + {}_{-}\psi(x)G\left({}_{-}|{}^{\zeta}\right)\ . \tag{10}
$$

The matrices $G\left({}_{\zeta}|{}^{\zeta'}\right)$ obey the following relations,

$$
\begin{aligned}
&G\left({}_{\zeta}|{}^{+}\right)G\left({}_{\zeta}|{}^{+}\right)^{\dagger} + G\left({}_{\zeta}|{}^{-}\right)G\left({}_{\zeta}|{}^{-}\right)^{\dagger} = \mathbf{I},\\
&G\left({}_{+}|{}^{+}\right)G\left({}_{-}|{}^{+}\right)^{\dagger} + G\left({}_{+}|{}^{-}\right)G\left({}_{-}|{}^{-}\right)^{\dagger} = 0\,,
\end{aligned}
\tag{11}
$$

where $\mathbf{I}$ is the identity matrix.

As was already said, when quantum fields are considered in time-dependent backgrounds (electromagnetic or gravitational ones) one has to construct different Green functions. To this end one has to find the sets of exact solutions of the equation (2) $\left\{{}_{\pm}\psi_{\{n\}}(x)\right\}$ and $\left\{{}^{\pm}\psi_{\{n\}}(x)\right\}$. Here we are going to describe such solutions. The functions $\phi(x)$ can be presented in form:

$$
\phi_{p_3p_1n\xi r}(x) = \phi_{p_3n\xi r}(x_{\|})\,\phi_{p_1nr}(x_{\perp})v_{\xi r}\quad , \tag{12}
$$

where nonzero $x^i_{\perp} = x^i$, $\ i = 1,2,\ \ x^{\mu}_{\|} = x^{\mu}$, $\mu = 0,3$; $\{p_3, p_1, n, \xi, r\}$ is a complete set of quantum numbers. Among them p_3 and p_1 are momenta of the continuous spectrum, n is integer quantum umber, $\xi = \pm 1$ and $r = \pm 1$ are spin quantum numbers.

Here $v_{\xi r}$ are some constant orthonormal spinors, $v^{\dagger}_{\xi r}v_{\xi r'} = \delta_{rr'}$. The eq.(5) allows one to subject these spinors to some supplementary conditions,

$$
\Xi v_{\xi r} = \xi v_{\xi r},\ \Xi = \gamma^0(qE\gamma^3 - bM)/\rho,\ \ \rho = \sqrt{(qE)^2 + (bM)^2}; \tag{13}
$$

$$
Rv_{\xi r} = rv_{\xi r},\ \ R = \mathrm{sgn}\,(qH)\,i\gamma^1\gamma^2. \tag{14}
$$

The function $\phi_{p_1nr}(x_{\perp})$ obeys the following equations

$$
(\gamma\mathcal{P}_{\perp})^2\phi_{p_1nr}(x_{\perp}) = -\omega\phi_{p_1nr}(x_{\perp}),\ \ P^i_{\perp} = P^i,\ \ i = 1,2\,, \tag{15}
$$

$$
\omega = \begin{cases} |qH|(2n + 1 - r),\ n = 0, 1, \ldots\ , & H \neq 0 \\ p_1^2 + p_2^2\ , & H = 0 \end{cases}\,,
$$

$$
\mathcal{P}_1\phi_{p_1nr}(x_{\perp}) = p_1\phi_{p_1nr}(x_{\perp}), \tag{16}
$$

$$
\int \phi^{*}_{p_1nr}(x_{\perp})\phi_{p_1'n'r}(x_{\perp})d\mathbf{x}_{\perp} = \delta_{nn'}\delta\,(p_1 - p_1')\ . \tag{17}
$$

If $H \neq 0$, a solution of these equations is

$$\phi_{p_1nr}(x_\perp) = \left(\frac{\sqrt{|qH|}}{2^{n+1}\pi^{\frac{3}{2}}n!}\right)^{1/2} \exp\left\{-ip_1x^1 - \frac{|qH|}{2}\left(x^2+\frac{p^1}{qH}\right)^2\right\}\mathcal{H}_n\left[\sqrt{|qH|}\left(x^2+\frac{p^1}{qH}\right)\right],$$

where $\mathcal{H}_n(x)$ are the Hermite polynomial with integer $n = 0,1,\ldots$. If $H = 0$, the discrete quantum numbers n have to be replaced by the momenta p_2, and the corresponding function has the form

$$\phi_{p_1nr}(x_\perp) = (2\pi)^{-1}\exp\left\{-i\left(p_1x^1 + p_2x^2\right)\right\}.$$

Let us write

$$\phi_{p_3n\xi r}(x_\parallel) = \frac{1}{\sqrt{2\pi}}e^{-ip_3x^3}\phi_{p_3n\xi r}(x^0) \quad , \tag{18}$$

where

$$\phi_{p_3n\xi r}(x^0) = \phi_{p_3n\xi r}(x^0,p_z)|_{p_z=0} \, ,$$

and $\phi_{p_3n\xi r}(x^0,p_z)$ is a solution of equation

$$\left[\left(i\frac{\partial}{\partial\tilde\eta}\right)^2 - (p_z-\rho\tilde\eta)^2 - \rho\lambda - i\rho\xi\right]\phi_{p_3n\xi r}\left(x^0,p_z\right) = 0\,, \tag{19}$$

with

$$\tilde\eta = x^0 - \frac{1}{\rho^2}qEp_D, \quad \rho\lambda = p_D^2\frac{(bM)^2}{\rho^2} + \omega\,.$$

One can form the two complete sets $\{{}_\pm\phi_{p_3n\xi r}(x^0,p_z)\}$ and $\{{}^\pm\phi_{p_3n\xi r}(x^0,p_z)\}$ of the solutions of equation (19) by using functions

$$\begin{aligned} {}_{+}^{-}\phi_{p_3n\xi r}\left(x^0,p_z\right) &= C_\xi D_{\nu-\xi/2}[\pm(1-i)\tau], \\ {}^{+}_{-}\phi_{p_3n\xi r}\left(x^0,p_z\right) &= C'_\xi D_{-\nu-1+\xi/2}\left[\pm(1+i)\tau\right]\,, \\ \tau &= \frac{1}{\sqrt\rho}\left(\rho\tilde\eta - p_z\right),\ \nu = \frac{i\lambda}{2}-\frac{1}{2}\,. \end{aligned} \tag{20}$$

The same solutions were first studied in [16]. Then, solutions of equation (5) $\phi(x)$ can be presented in the form:

$$\begin{aligned} {}_\pm\phi_{p_3p_1n\xi r}(x) &= {}_\pm\phi_{p_3p_1n\xi r}(x,p_z)|_{p_z=0}, \\ {}_\pm\phi_{p_3p_1n\xi r}(x,p_z) &= \frac{1}{\sqrt{2\pi}}e^{-ip_3x^3}\ {}_\pm\phi_{p_3n\xi r}(x^0,p_z)\,\phi_{p_1nr}(x_\perp)v_{\xi r}, \end{aligned} \tag{21}$$

and in the same form with $(\pm)$ indices above.

One can verify the solutions of the Dirac equation with different ξ, namely, $(\mathcal{P}_\mu\gamma^\mu + bMx^0)\ {}_\pm\phi_{p_3,p_1,n,+1,r}(x)$ and $(\mathcal{P}_\mu\gamma^\mu + bMx^0)\ {}_\pm\phi_{p_3,p_1,n,-1,r}(x)$, or $(\mathcal{P}_\mu\gamma^\mu + bMx^0)\ {}^\pm\phi_{p_3,p_1,n,+1,r}(x)$ and $(\mathcal{P}_\mu\gamma^\mu + bMx^0)\ {}^\pm\phi_{p_3,p_1,n,-1,r}(x)$ are linearly dependent for each sign "+" or "−". Thus, to construct the complete sets it is enough to use only the following solutions:

$$ {}_\pm\psi_{p_3p_1nr}(x) = \left(\mathcal{P}_\mu\gamma^\mu + bMx^0\right)\ {}_\pm\phi_{p_3,p_1,n,+1,r}(x)\,, \tag{22}$$

$$ {}^\pm\psi_{p_3p_1nr}(x) = \left(\mathcal{P}_\mu\gamma^\mu + bMx^0\right)\ {}^\pm\phi_{p_3,p_1,n,+1,r}(x) \tag{23}$$

Chosing coefficients in (20) as follows:

$$C_{+1} = (2\rho)^{-1/2}\exp\left(-\frac{\pi\lambda}{8}\right),\ C'_{+1} = (\rho\lambda)^{-1/2}\exp\left(-\frac{\pi\lambda}{8}\right), \tag{24}$$

one gets the two complete sets $\{{}_\pm\psi_{p_3p_1nr}(x\}$ and $\{{}^\pm\psi_{p_3p_1nr}(x)\}$ of orthonormalized solutions of the equation (2). These solutions are classified as particles (+) and antiparticles (−) at $x^0\to\pm\infty$ according to the asymptotic forms of the quasienergies of these solutions: ${}_\zeta\varepsilon = \zeta\rho|x^0|$ and ${}^\zeta\varepsilon = \zeta\rho|x^0|$ (see [15] for additional arguments advocating such a classification). It is agree with classification [16] of the similary solutions in QED.

One can find decomposition coefficients $G\left({}_\zeta|{}^{\zeta'}\right)$ of the out-solutions in the in-solutions defined by (10), and using (5) and (13) see that $G\left({}_\zeta|{}^{\zeta'}\right)$ are diagonal,

$$G\left({}_\zeta|{}^{\zeta'}\right)_{ll'} = \delta_{l,l'}\,g\left({}_\zeta|{}^{\zeta'}\right)\,,\ l = (p_3,p_1,n,r)\,,\ l' = (p_3{}',p_1{}',n',r')\,, \tag{25}$$

where

$$g({}_\zeta|{}^{\zeta'}) = {}_\zeta\phi^*_{p_3,n,+1,r}\left(x^0,p_z\right)\, i\,\overleftrightarrow{\partial_0}\left(i\partial_0 - \rho\tilde\eta\right)\ {}^{\zeta'}\phi_{p_3,n,+1,r}\left(x^0,p_z\right)\,. \tag{26}$$

III. GREEN FUNCTIONS

The perturbation theory with respect to the radiative interaction for the matrix elements of the processes has the usual Feynman structure also in an external field creating pairs [3–5]. The Feynman diagrams have to be calculated by means of the causal propagator

$$S^c(x,x') = c_v^{-1} i < 0, out|T\psi(x)\bar{\psi}(x')|0, in >, \quad c_v = < 0, out|0, in > , \tag{27}$$

where $\psi(x)$ is quantum spinor field in the generalized Furry representation, satisfying the Dirac equation (2), $|0, in >$ and $|0, out >$ are the initial and the final vacuum in this representation, and c_v is the vacuum to vacuum transition amplitude. The propagator $S^c(x,x')$ obeys the equation

$$(\mathcal{P}_\mu\gamma^\mu - M\Omega(\eta))\; S^c(x,x') = -\delta^{(4)}(x-x') , \tag{28}$$

and is a Green function of the equation. Other important singular function is the commutation function

$$S(x,x') = i\left[\psi(x), \bar{\psi}(x')\right]_+ . \tag{29}$$

It obeys the homogeneous Dirac equation (2) and the initial condition

$$S(x,x')|_{x_0=x_0'} = i\gamma^0\delta(\mathbf{x}-\mathbf{x}'). \tag{30}$$

The commutation function $S(x,x')$ is at the same time the propagation function of the Dirac equation, i.e. it connects solutions of the equation in two different time instants.

QED, and QED-Ω with unstable vacuum have a number of peculiarities. Thus, for instance, in the calculation of the expectation values and the total probabilities Green functions of different types from (27) appear [3–5]:

$$\begin{aligned}
S^c_{in}(x,x') &= i < 0, in|T\psi(x)\bar{\psi}(x')|0, in >,\\
S^-_{in}(x,x') &= i < 0, in|\psi(x)\bar{\psi}(x')|0, in >,\\
S^+_{in}(x,x') &= i < 0, in|\bar{\psi}(x')\psi(x)|0, in >,\\
S^{\bar{c}}_{in}(x,x') &= i < 0, in|\psi(x)\bar{\psi}(x')T|0, in >,\\
S^c_{out}(x,x') &= i < 0, out|T\psi(x)\bar{\psi}(x')|0, out >,
\end{aligned} \tag{31}$$

where the symbol of the T-product acts on both sides: it orders the field operators to the right of its and antiorders them to the left. The function $S^c_{in}(x,x')$, $S^c_{out}(x,x')$ obey the equation (28), $S^{\mp}(x,x')$ satisfy the equation (2) and $S^{\bar{c}}_{in}(x,x')$ obeys the equation

$$(\mathcal{P}_\mu\gamma^\mu - M\Omega(\eta))\; S^{\bar{c}}_{in}(x,x') = \delta^{(4)}(x-x') . \tag{32}$$

Besides, all these different kinds of the Green functions are used to represent various matrix elements of operators of current and energy-momentum tensor, and effective action beginning with zeroth order with respect to radiative interaction.

One can express the Green functions via the solutions (22) and (23) [3–5]:

$$S^c(x,x') = \theta(x_0-x_0')\,S^-(x,x') - \theta(x_0'-x_0)\,S^+(x,x'), \tag{33}$$

$$S(x,x') = S^-(x,x') + S^+(x,x'), \tag{34}$$

$$\begin{aligned}
S^-(x,x') &= i\int_{-\infty}^{+\infty} dp_3dp_1 \sum_{nr}\; {}^+\psi_{p_3p_1nr}(x) g\left({}_+|{}^+\right)^{-1}\; {}_+\bar{\psi}_{p_3p_1nr}(x'),\\
S^+(x,x') &= i\int_{-\infty}^{+\infty} dp_3dp_1 \sum_{nr}\; {}_-\psi_{p_3p_1nr}(x)\left[g\left({}_-|{}^-\right)^{-1}\right]^*\; {}^-\bar{\psi}_{p_3p_1nr}(x'),
\end{aligned} \tag{35}$$

$$S^c_{in}(x,x') = \theta(x_0-x_0')\,S^-_{in}(x,x') - \theta(x_0'-x_0)\,S^+_{in}(x,x'), \tag{36}$$

$$S^{\bar{c}}_{in}(x,x') = \theta(x_0'-x_0)\,S^-_{in}(x,x') - \theta(x_0-x_0')\,S^+_{in}(x,x'), \tag{37}$$

$$S^{\mp}_{in}(x,x') = i\int_{-\infty}^{+\infty} dp_3dp_1 \sum_{nr}\; {}_\pm\psi_{p_3p_1nr}(x)\,{}_\pm\bar{\psi}_{p_3p_1nr}(x'), \tag{38}$$

$$S^c_{out}(x,x') = \theta(x_0-x_0')\,S^-_{out}(x,x') - \theta(x_0'-x_0)\,S^+_{out}(x,x'), \tag{39}$$

$$S^{\mp}_{out}(x,x') = i\int_{-\infty}^{+\infty} dp_3dp_1 \sum_{nr}\; {}^\pm\psi_{p_3p_1nr}(x)\; {}^\pm\bar{\psi}_{p_3p_1nr}(x'). \tag{40}$$

where the symbol $\sum_{nr}$ means the summation over all discrete quantum numbers n, r (and the integration over the continuous p_2 if $H=0$). Using the relations between the Green functions and between the matrices $G\left({}_\zeta|{}^{\zeta'}\right)$ one can present the functions $S^{\mp}$, $S^{\mp}_{in}$ and $S^{\mp}_{out}$ as follows

$$\pm S^{\mp}(x,x') = S^c(x,x') \pm \theta(\mp(x_0-x_0'))S(x,x') \quad , \tag{41}$$

$$\pm S^{\mp}_{in}(x,x') = S^c_{in}(x,x') \pm \theta(\mp(x_0-x_0'))S(x,x') \quad , \tag{42}$$

$$\pm S^{\mp}_{out}(x,x') = S^c_{out}(x,x') \pm \theta(\mp(x_0-x_0'))S(x,x') \quad , \tag{43}$$

$$S^c_{in}(x,x') = S^c(x,x') - S^a(x,x') \quad , \tag{44}$$

$$S^c_{out}(x,x') = S^c(x,x') - S^p(x,x') \quad , \tag{45}$$

$$S^a(x,x') = -i\int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} {}_-\psi_{p_3p_1nr}(x)\left[g({}_+|^-)g({}_-|^-)^{-1}\right]^{\dagger} {}_+\bar{\psi}_{p_3p_1nr}(x') \quad , \tag{46}$$

$$S^p(x,x') = i\int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} {}^+\psi_{p_3p_1nr}(x)\left[g({}_+|^+)^{-1}g({}_+|^-)\right] {}^-\bar{\psi}_{p_3p_1nr}(x') \quad . \tag{47}$$

To calculate all kinds of the Green functions it is enough to take sums in $S^{\pm}(x,x')$ and $S^{a,p}(x,x')$ only. That will be done below.

The coefficients (26) do not depend on p_z, thus, one can present the functions $S^{\mp}$ and $S^{a,p}$ in the convenient form

$$S^{\mp,a,p}(x,x') = \int_{-\infty}^{+\infty} dz \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} e^{-ip_3y^3} S_Q^{\mp,a,p} , \quad y_\mu = x_\mu - x'_\mu ,$$
$$S_Q^{\mp,a,p} = S_Q^{\mp,a,p}\left(\tilde{\eta}, x_\perp, \tilde{\eta}', x'_\perp, z-z', p_D\right) , \tag{48}$$

$$S_Q^- = i\int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} {}^+\psi_{p_3p_1nr}(\tilde{\eta}, x_\perp, z, p_z) g\left({}_+|^+\right)^{-1} {}_+\bar{\psi}_{p_3p_1nr}\left(\tilde{\eta}', x'_\perp, z', p_z\right),$$
$$S_Q^+ = i\int_{-\infty}^{+\infty} dp_3 dp1 \sum_{nr} {}_-\psi_{p_3p_1nr}\left(\tilde{\eta}, x_\perp, z, p_z\right)\left[g\left({}_-|^-\right)^{-1}\right]^* {}^-\bar{\psi}_{p_3p_1nr}\left(\tilde{\eta}', x'_\perp, z', p_z\right),$$
$$S_Q^a = -i\int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} {}_-\psi_{p_3p_1nr}(\tilde{\eta}, x_\perp, z, p_z)\left[g({}_+|^-)g({}_-|^-)^{-1}\right]^{\dagger} {}_+\bar{\psi}_{p_3p_1nr}(\tilde{\eta}', x'_\perp, z', p_z),$$
$$S_Q^p = i\int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} {}^+\psi_{p_3p_1nr}(\tilde{\eta}, x_\perp, z, p_z)\left[g({}_+|^+)^{-1}g({}_+|^-)\right] {}^-\bar{\psi}_{p_3p_1nr}(\tilde{\eta}', x'_\perp, z', p_z), \tag{49}$$

where

$${}_\pm\psi_{p_3p_1nr}\left(\tilde{\eta}, x_\perp, z, p_z\right) = \left(\gamma^0 i\partial_0 + \gamma^3\left(p_3 - qEx^o\right) + \gamma_\perp\left(i\partial - qA\right) + bMx^0\right)$$
$${}_\pm\phi_{p_3p_1nr}\left(\tilde{\eta}, x_\perp, z, p_z\right),$$

$${}_\pm\phi_{p_3p_1nr}\left(\tilde{\eta}, x_\perp, z, p_z\right) = {}_\pm\phi_{p_3nr}(\tilde{\eta}, z, p_z)\,\phi_{p_1nr}(x_\perp)v_{+1,r},$$

$${}_\pm\phi_{p_3nr}(\tilde{\eta}, z, p_z) = \frac{1}{\sqrt{2\pi}} e^{-ip_zz} {}_\pm\phi_{p_3,n,+1,r}(x^0, p_z), \tag{50}$$

and in the same form with $(\pm)$ indices above.

One can write the functions $S_Q^{\mp,a,p}$ as follows:

$$S_Q^{\mp,a,p} = \left(\gamma^0 i\partial_0 + \gamma^3\left(p_3 - qEx^o\right) + \gamma_\perp\left(i\partial - qA\right) + bMx^0\right)\Delta_Q^{\mp,a,p}, \tag{51}$$

where the functions $\Delta_Q^{\mp,a,p}$ obey equation

$$\left(\mathcal{P}_Q^2 - p_3^2\left(bM^2\right)/\rho^2 - i\rho\Xi - q/2F^{\perp}_{\mu\nu}\sigma^{\mu\nu}\right)\Delta_Q^{\mp,a,p} = 0,$$
$$\mathcal{P}_Q^2 = \left(i\frac{\partial}{\partial\tilde{\eta}}\right)^2 - \left(i\frac{\partial}{\partial z} - \rho\tilde{\eta}\right)^2 + \mathcal{P}_\perp^2,$$

The functions $\Delta_Q^{\mp,a,p}$ have the form of the corresponding GF of the squared Dirac equation in EM background [7], where $\tilde{\eta}$ is the time, z is the coordinate along the electric field, the mass $m_Q^2 = p_3^2\frac{(bM)^2}{\rho^2}$, the potential of the electromagnetic field is $A_z = \tilde{\eta}\frac{\rho}{q}$, and spin term $qE\gamma^0\gamma^3$ is changed to $\rho\Xi$. Then, after the similary calculation [7] one gets the following representation:

$$\pm\Delta_Q^{\mp} = \Delta_Q^c \pm \theta(\mp y_0)\Delta_Q \quad , \tag{52}$$

$$\Delta_Q^c = \int_{\Gamma_c} f_Q ds \quad , \ f_Q = f_Q\left(\tilde{\eta}, x_\perp, \tilde{\eta}', x'_\perp, z-z', p_D, s\right) , \tag{53}$$

$$\Delta_Q = \epsilon(y_0)\int_{\Gamma_c-\Gamma_2-\Gamma_1} f_Q ds \quad , \quad \epsilon(y_0) = \mathrm{sgn}(y_0) \quad , \tag{54}$$

$$\Delta_Q^a = \int_{\Gamma_a} f_Q ds + \theta(z'-z)\int_{\Gamma_3+\Gamma_2-\Gamma_a} f_Q ds , \tag{55}$$

$$\Delta_Q^p = \int_{\Gamma_a} f_Q ds + \theta(z-z')\int_{\Gamma_3+\Gamma_2-\Gamma_a} f_Q ds , \tag{56}$$

where $\theta(0) = 1/2$.

$$f_Q\left(\tilde{\eta}, \gamma_\perp, \tilde{\eta}_1, x_\perp, z-z', p_3, s\right) = e^{s\rho\Xi}\exp\left(-\frac{i}{2}qF^{\perp}_{\mu\nu}\sigma^{\mu\nu}s\right)$$
$$f_Q^{(0)}\left(\tilde{\eta}, \gamma_\perp, \tilde{\eta}_1, x_\perp, z-z', p_3, s\right), \tag{57}$$

$$f_Q^{(0)} = \exp\{-iq\int_{x'}^{x} A^{\perp}_\mu dx^\mu\} f_Q^{\parallel}\left(\tilde{\eta}, \tilde{\eta}', z-z', p_D, s\right) f_\perp\left(y_\perp, s\right) \quad , \tag{58}$$
$$f_\perp(y_\perp, s) = (4\pi)^{-2}\frac{qH}{\sin(qHs)}\exp\left\{-\frac{i}{4}y_\perp qF\coth(qFs)y_\perp\right\} ,$$

$$f_Q^{\|}\left(\tilde{\eta}, \tilde{\eta}', z, p_D, s\right) = \frac{\rho}{\sinh(\rho s)} \exp\left\{-i\frac{\rho}{2}(\tilde{\eta}+\tilde{\eta}')z - im_Q^2 s + i\frac{\rho}{4}\left[z^2 - (\tilde{\eta}-\tilde{\eta}')^2\right]\coth(\rho s)\right\} \quad , \tag{59}$$

whereas the following relations take place

$$i\frac{d}{ds}f_Q = \left(m_Q^2 - \mathcal{P}_Q^2 + i\rho\Xi + q/2F_{\mu\nu}^{\perp}\sigma^{\mu\nu}\right) f_Q \quad , \tag{60}$$

$$\lim_{s\to+0} f_Q = i\delta(\tilde{\eta}-\tilde{\eta}')\,\delta(z-z')\delta(y_\perp) \quad . \tag{61}$$

In accordance with (48) one can calculate the Gaussian integrals over p_3 and z for all the points s on the contours Fig.1. As a result one gets

$$S^{(\dots)}(x,x') = \left(\gamma^\mu \mathcal{P}_\mu + bMx^0\right)\Delta^{(\dots)}(x,x'), \tag{62}$$

$$\Delta^c(x,x') = \int_{\Gamma_c} f(x,x',s)ds \quad , \tag{63}$$

$$\Delta(x,x') = \epsilon(y_0)\int_\Gamma f(x,x',s)ds \quad , \tag{64}$$

$$\Delta^a(x,x') = -\Delta^{(1)}(x,x') - \Delta^{(2)}(x,x') \quad , \tag{65}$$

$$\Delta^p(x,x') = -\Delta^{(1)}(x,x') + \Delta^{(2)}(x,x') \quad , \tag{66}$$

$$\Delta^{(1)}(x,x') = -\frac{1}{2}\int_{\Gamma_3+\Gamma_2+\Gamma_a} f(x,x',s)ds \quad , \tag{67}$$

$$\Delta^{(2)}(x,x') = \int_{\Gamma_3+\Gamma_2-\Gamma_a} f_r(x,x',s)ds \quad , \tag{68}$$

where

$$f_r(x,x',s) = \frac{1}{2\sqrt{\pi}}\gamma\left(\frac{1}{2},\alpha\right)f(x,x',s) \quad ,$$

$$\alpha = e^{-i\pi/2}\frac{1}{4s(bM)^2\omega}\left[(x_0+x'_0)\,s(bM)^2 + qEy^D\right]^2 \quad , \tag{69}$$

and $\gamma\left(\frac{1}{2},\alpha\right)$ is the incomplete gamma-function. Here

$$f(x,x',s) = \exp\left(-bM\gamma^0 s - \frac{i}{2}qF_{\mu\nu}\sigma^{\mu\nu}s\right)f^{(0)}(x,x',s) \tag{70}$$

$$f^{(0)}(x,x',s) = e^{iq\Lambda}f_{\|}\left(x_0,x'_0,y^D,s\right)\,f_\perp(y_\perp,s) \quad , \tag{71}$$

$$f_{\|}\left(x_0,x'_0,y^3,s\right) = \frac{\rho}{\sinh(\rho s)\omega^{1/2}}\exp\left\{i\frac{qE}{2}(x_0+x'_0)\,y^D - i\frac{\rho}{4}(x_0-x'_0)^2\coth(\rho s) - i(aM)^2 s + i\frac{\rho}{4\omega}y_3^2\coth(\rho s) - \frac{i}{4\omega}\left[(bM)^2 s\,(x_0+x'_0)^2 + 2qEy^3(x_0+x'_0)\right]\right\} \quad ,$$

$$\omega = \frac{(bM)^2}{\rho}s\coth(\rho s) + \frac{(qE)^2}{\rho^2} \quad . \tag{72}$$

Here only Λ depends on the choice of the gauge for the constant field, via the integral

$$\Lambda = -\int_{x'}^{x} A_\mu dx^\mu \quad , \tag{73}$$

which is taken along the line. To get the function f in an arbitrary gauge A', one has only to replace A by A' in the Λ.

One can see that

$$i\frac{d}{ds}f(x,x',s) = \left(\left(bMx^0\right)^2 - \mathcal{P}^2 + \frac{q}{2}\sigma^{\mu\nu}F_{\mu\nu} - ibM\gamma^0\right)f(x,x',s) \quad , \tag{74}$$

$$\lim_{s\to+0} f(x,x',s) = i\delta^{(4)}(x-x') \quad . \tag{75}$$

Thus, $f(x,x',s)$ is Fock-Schwinger function [1,18]. The contour $\Gamma_c - \Gamma_2 - \Gamma_1$ in (64) was transformed into Γ after the integration over p_D and z. Then the results are consistent with the general expression for the commutation function obtained in [19].

If $b \neq 0$, then the function $f(x,x',s)$ has three singular points on the complex region between contours $\Gamma_c - \Gamma_1$ and $\Gamma_a - \Gamma_3$ which are distributed at the imaginary axis: $\rho s_0 = 0$, $\rho s_1 = -i\pi$ and $\rho s_2 = -ic_2$. The latter point is connected with zero value of the function ω. We get an equation for c_2 from the condition $\omega = 0$,

$$c_2\tan(c_2 - \pi/2) - \left(\frac{qE}{bM}\right)^2 = 0\,, \tag{76}$$

where $\pi/2 < c_2 < \pi$. The position of this point depends on the ratio $qE/(bM)$, e.g. at $bM/(qE) \to 0$ one has $c_2 \to \pi$ and at $qE/(bM) \to 0$ one has $c_2 \to \pi/2$. Notice, in the case $E = 0$ it is convenient to put $c_2 = \pi/2 + 0$ because of the contour Γ_2 must be passed above the singular point s_2 in the case as well.

If $b = 0$, then $\omega = 1$ and the function $f(x, x', s)$ has only two singular points s_0 and s_1 on the above mentioned complex region. In this case the gauge invariant function $f_{\|}$ does not depend on on $x_0 + x'_0$. In this degenerate case it follows from (48), (55) and (56),

$$\Delta^a(x, x') = \int_{\Gamma_a} f(x, x', s)ds + \theta(-y^D) \int_{\Gamma_3+\Gamma_2-\Gamma_a} f(x, x', s)ds \,, \tag{77}$$

$$\Delta^p(x, x') = \int_{\Gamma_a} f(x, x', s)ds + \theta(y^D) \int_{\Gamma_3+\Gamma_2-\Gamma_a} f(x, x', s)ds \,. \tag{78}$$

Let us return to more interesting case $b \neq 0$. Our aim is to demonstrate that function $\Delta^{(2)}(x, x')$ from (68) can also be presented via a proper-time integral with the kernel $f(x, x', s)$ as it was done for all other Δ-functions. To this end let us transform the contour $\Gamma_3 + \Gamma_2 - \Gamma_a$ in (68) into two ones: Γ_1^a (see FIG.2) and $\Gamma_l + \Gamma_r$ (see FIG.3).

The radius of the contour Γ_1^a tends to zero. The contour $\Gamma_l + \Gamma_r$ is a infinitesimal radius clockwise circle around the singular point s_2. However, it is convenient to present it as a combination of two semicircles Γ_l and Γ_r placed on the left and the right sides of the imaginary axis respectively. The argument $\arg s'$ of the Γ_l radius is in the interval $\pi/2 \leq \arg s' \leq 3\pi/2$ and of the Γ_r radius is in the interval $-\pi/2 \leq \arg s' < \pi/2$. Then (68) can be rewritten in the form

$$\Delta^{(2)}(x, x') = \int_{\Gamma_l+\Gamma_r} f_r(x, x', s)ds + r(x, x') \quad , \tag{79}$$

$$r(x, x') = \int_{\Gamma_1^a} f_r(x, x', s)ds \,. \tag{80}$$

Taking into account [20]

$$\gamma(1/2, \alpha) = e^{-\alpha}\alpha^{1/2}\left[2 + (4/3)\alpha + 0\left(\alpha^2\right)\right] \quad ,$$

one gets

$$f_r\left(x, x', s' - i\frac{\pi}{\rho}\right) \underset{s'\to 0}{\longrightarrow} \text{const} \cdot \exp\{-\frac{i}{4s'}(x_0 - x'_0)^2\} \quad .$$

Hence, one can see that $r(x, x') = 0$, $\partial_0 r(x, x') = 0$ at any $x_0 - x'_0$. Moreover, using (74), it is easy to see that the distribution $r(x, x')$ obeys the equation (5). Thus, $r(x, x')$ is equal to zero identically. The function $f_r(x, x's)$ is 2π periodic function of the argument $\arg s'$ of the Γ_l and Γ_r radiuses. One needs to take into account the asymptotic decomposition [20]

$$\gamma(1/2, \alpha) = \sqrt{\pi} - e^{-\alpha}\alpha^{-1/2}\left[1 + O(\alpha^{-1})\right] \,, \quad x_0 > 0 \,. \tag{81}$$

which is valid in the region $-3\pi/2 < \arg\alpha < 3\pi/2$. Then, using (81) one gets from (79)

$$\Delta^{(2)}(x, x') = \frac{1}{2}\begin{cases} -\int_{\Gamma_l+\Gamma_r} f(x, x's)ds \,, & -5\pi/4 < \beta < -3\pi/4, \\ -\int_{\Gamma_l-\Gamma_r} f(x, x's)ds \,, & -3\pi/4 \leq \beta < -\pi/4, \\ \int_{\Gamma_l+\Gamma_r} f(x, x's)ds \,, & -\pi/4 \leq \beta \leq \pi/4, \\ \int_{\Gamma_l-\Gamma_r} f(x, x's)ds \,, & \pi/4 < \beta \leq 3\pi/4, \end{cases} \tag{82}$$

where $\beta = \arg\left[(x_0 + x'_0)c_2(bM)^2(-i) + \rho q E y^D\right]$.

One can verify that expression (82) is continuous in the boundaries of the β intervals. Then, using (74), one can demonstrate that the representation (82) obeys the equation (5). One can also verify that the representation $\Delta^{(1)}(x, x')$ (67) obeys the equation (5). Thus, all the Δ-Green functions considered here, excluding those marked by the index "c", are solutions of the equation (5). The important difference between basic Green functions $\Delta^c(x, x')$, $\Delta^{(1)}(x, x')$ and $\Delta(x, x')$, $\Delta^{(2)}(x, x')$ is that the first ones are symmetric under simultaneous change of sign in x_0, x'_0, x_D, x'_D and the seconds ones change sign in this case.

Note finally that using proper-time kernel $f(x, x', s)$ (70) one can easily construct Schwinger out-in effective action

$$\Gamma_{out-in} = \frac{1}{2}\left\{\int dx \int_0^\infty s^{-1} f(x, x, s)\, ds\right\}.$$

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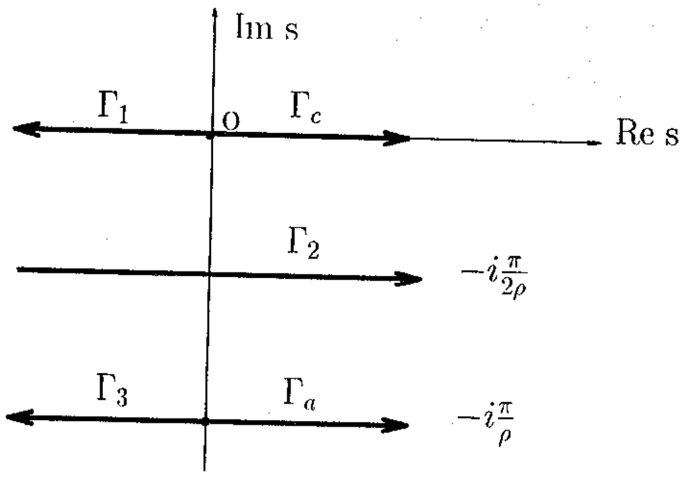

FIG. 1. Contours of integration $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_c, \Gamma_a$

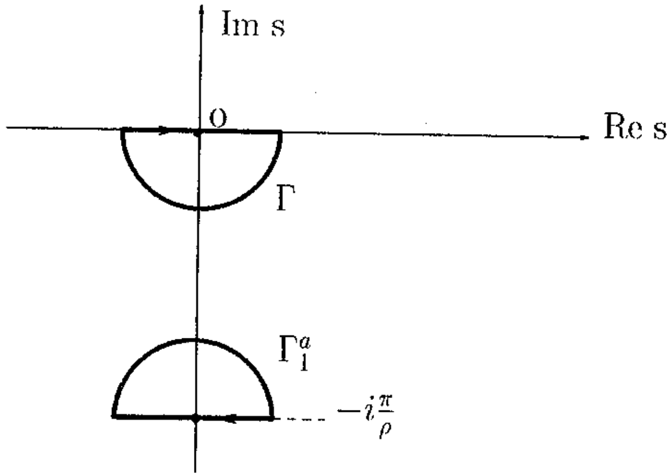

FIG. 2. Contours of integration Γ, Γ_1^a

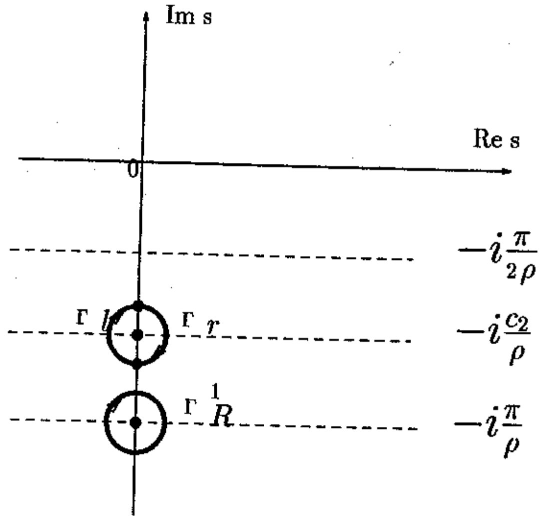

FIG. 3. Contours of integration Γ_R^1, Γ_l and Γ_r

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