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Abstract

It is shown that canonical quantization of the $4d$ Siegel superparticle yields massless Wess–Zumino model as an effective field theory. Quantum states of the superparticle are realized in terms of real scalar superfields which prove to be the sum of on-shell chiral and antichiral superfields.

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In recent years the connection between theories of first and second quantized particles was under extensive investigation. On the one hand, it presents a fertile ground to develop constrained dynamics methods. In particular, classical mechanics and quantum mechanics for theories with Grassmann odd variables have been developed [1-3]. On the other hand, it allows to get a deeper insight into the structure of path integral calculations via the comparison of proper time and BRST approaches [4-9].

One of the most interesting puzzles issued from the incorporation of (world volume) supersymmetry into the scheme seems to be the infinitely reducible constraints problem (see Ref. [10] for a review). Following the conventional approach [11], care treatment of such constraints requires an infinite ghost tower, which proved to be rather difficult to handle with [12-14]. Although cohomology of BRST operator can generally be evaluated [13,14], the expression for the effective action looks formal.

Recently, a recipe how to supplement infinitely reducible first class constraints up to a constraint system of finite stage of reducibility has been proposed [15]. In this letter, which is a starting point for the forthcoming path integral test of the approach [15], we consider canonical quantization of the $4d$ Siegel superparticle, the latter being the simplest model to which the technique [15] applies. As shown below, an effective field theory which corresponds to the first quantized Siegel superparticle is the massless Wess–Zumino model in the component form [16]. We expect that the knowledge of propagators of the first quantized theory will suggest considerable simplification in forthcoming path integral calculations at the second quantized level.

The action functional which describes the dynamics of the Siegel superparticle in $R^{4/4}$ superspace reads [17] (we use the spinor notation from Ref. 18)

$$S=\int d\tau \frac{1}{2e}\Pi^m\Pi_m + i\dot{\theta}\rho - i\bar{\rho}\dot{\bar{\theta}}, \tag{1}$$

with

$$\Pi^m=\dot{x}^m - i\theta\sigma^m\dot{\bar{\theta}} + i\dot{\theta}\sigma^m\bar{\theta} + i\psi\sigma^m\bar{\rho} - i\rho\sigma^m\bar{\psi}.$$

It is invariant under global supersymmetry transformations, as well as under local reparametrizations and κ-symmetry [17,19]. The coordinates $(x^m,\theta^\alpha,\bar{\theta}_{\dot{\alpha}})$ parametrize the standard $R^{4/4}$ superspace, while the variables e and $(\psi^\alpha,\bar{\psi}_{\dot{\alpha}})$ prove to be gauge fields for the local symmetries. The role of the pair $(\rho^\alpha,\bar{\rho}_{\dot{\alpha}})$ is to provide terms corresponding to (mixed) covariant propagator for fermions in the action (1) [20].

The application of the Dirac-Bergmann algorithm to the theory (1) results in the constraints [1]

$$p^2=0, \qquad p_m\tilde{\sigma}^m p_\theta=0, \qquad p_{\bar{\theta}}\tilde{\sigma}^m p_m=0, \tag{2}$$

which form a closed algebra and, hence, are first class. In Eq. (2) $(p_m,p_\theta,p_{\bar{\theta}})$ denote momenta canonically conjugate to the variables $(x^n,\theta^\alpha,\bar{\theta}^{\dot{\alpha}})$ respectively. The canonical Hamiltonian vanishes on the constraint surface in the full agreement with the reparametrization invariance of the model.

Some remarks are relevant here. First, from Eq. (2) it follows

$$p_\theta^2=0, \qquad p_{\bar{\theta}}^2=0. \tag{3}$$

This means that the C-constraint of the $10d$ $ABCD$-superparticle [21] (which removes negative norm states from the quantum spectrum) auto-

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[1] Here, we partially reduced the original phase space by imposing a covariant gauge $e=1,\psi=0,\bar{\psi}=0$ to the first class constraints $p_e=0, p_\psi=0, p_{\bar{\psi}}=0$ and omitting the canonical pairs (e,p_e), (ψ,p_ψ), $(\bar{\psi},p_{\bar{\psi}})$, (ρ,p_ρ), $(\bar{\rho},p_{\bar{\rho}})$ after introducing the associated Dirac bracket. The Dirac brackets for the remaining variables prove to coincide with the Poisson ones.

matically holds in four dimensions. Second, the constraints (2) just coincide with the first-class ones of the $4d$ Casalbuoni-Brink–Schwarz (CBS) superparticle [22]. In Ref. [23] they have been used to covariantly quantize the model within the framework of the Gupta-Bleuler method. We would like to stress, however, that the naive omitting of second class constraints intrinsic in the CBS theory (which generally leads to Siegel's model [17,13]) in the approach of Ref. [23] will not reproduce the result of the Dirac quantization presented below. It is worth mentioning also Ref. [24], where the technique of quantization with a complex Hamiltonian has been applied to establish a precise relation between on-shell massive chiral superfields and the corresponding particle mechanics. The massless limit of the procedure, however, leads to ghost excitations in the quantum spectrum [24] and, hence, is ill defined. Third, a covariant gauge to Eq. (2) is known to be problematic in the original phase space (the conventional noncovariant gauge choice is $x^+ = \tau p^+$, $\theta\sigma^+ = 0$, $\sigma^+\bar{\theta} = 0$). By this reason, we refrain from fixing a gauge and proceed to covariant quantization.

Since commutation relations for the variables (x^n, p_m), $(\theta_\alpha, p_{\theta_\alpha})$, $(\bar{\theta}_\alpha, p_{\bar{\theta}_\alpha})$ are canonical, we can realize them in the coordinate representation $\hat{x}^n = x^n$, $\hat{p}_m = -i\partial_m$, $\hat{\theta}^\alpha = \theta^\alpha$, $\hat{p}_{\theta_\alpha} = i\partial_\alpha$, $\hat{\bar{\theta}}^{\dot{\alpha}} = \bar{\theta}^{\dot{\alpha}}$, $\hat{p}_{\bar{\theta}_{\dot{\alpha}}} = i\bar{\partial}_{\dot{\alpha}}$ on a Hilbert space whose elements are chosen to be real scalar superfields

$$\begin{aligned} V(x,\theta,\bar{\theta}) &= A(x) + \theta\psi(x) + \bar{\theta}\bar{\psi}(x) + \theta^2 F(x) + \bar{\theta}^2 \bar{F}(x) + \\ &+ \theta\sigma^n\bar{\theta}C_n(x) + \bar{\theta}^2\theta\lambda(x) + \theta^2\bar{\theta}\bar{\lambda}(x) + \theta^2\bar{\theta}^2 D(x). \end{aligned} \tag{4}$$

In what follows we assume the standard boundary condition

$$V(x,\theta,\bar{\theta}) \underset{x\to\pm\infty}{\longrightarrow} 0\,. \tag{5}$$

The physical states in a complete Hilbert space are defined in the usual way [25,26]

$$\begin{aligned} &\hat{p}^2|\text{phys}\rangle = 0, \\ &\tilde{\sigma}^n\hat{p}_n\hat{p}_\theta|\text{phys}\rangle = 0, \\ &\tilde{\sigma}^n\hat{p}_n\hat{p}_{\bar{\theta}}|\text{phys}\rangle = 0. \end{aligned} \tag{6}$$

In the representation chosen this yields

$$\begin{aligned} &\tilde{\sigma}^n\partial_n\psi = 0, \qquad \tilde{\sigma}^n\partial_n\bar{\psi} = 0, \\ &\Box A = 0; \end{aligned} \tag{7.a}$$

$$\begin{aligned} &(\tilde{\sigma}^m\sigma^n)^{\dot{\alpha}}{}_{\dot{\beta}}\partial_m C_n = 0, \qquad (\sigma^n\tilde{\sigma}^m)_\alpha{}^\beta\partial_m C_n = 0, \\ &\Box C_n = 0, \end{aligned} \tag{7.b}$$

with all other component fields vanishing due to the boundary condition (5). In obtaining Eq. (7) we used the identity

$$\mathrm{Tr}\,(\sigma^n\tilde{\sigma}^m) = -2\eta^{nm}. \tag{8}$$

It is instructive then to simplify Eq. (7.b). Taking a trace in the first equation and making use of Eq. (8) one finds

$$\partial^n C_n = 0, \tag{9}$$

which (with the use of the relation $\sigma^n\tilde{\sigma}^m + \sigma^m\tilde{\sigma}^n = -2\eta^{nm}$) allows one to rewrite Eq. (7.b) in the form

$$\begin{aligned} &(\sigma^{mn})_\alpha{}^\beta\partial_m C_n = 0, \qquad (\tilde{\sigma}^{mn})^{\dot{\alpha}}{}_{\dot{\beta}}\partial_m C_n = 0, \\ &\Box C_n = 0 \end{aligned} \tag{10}$$

Multiplying the first equality in Eq. (10) by $(\sigma^{kl})_\beta{}^\alpha$ and taking into account the identity

$$\mathrm{Tr}\,\sigma^{mn}\sigma^{kl} = -\frac{1}{2}(\eta^{mk}\eta^{nl} - \eta^{ml}\eta^{nk}) - \frac{i}{2}\epsilon^{mnkl}, \tag{11}$$

one gets

$$\partial_k C_l - \partial_l C_k = -i\epsilon_{klmn}\partial^m C^n, \tag{12}$$

which, together with its complex conjugate, implies

$$\partial_m C_n - \partial_n C_m = 0, \qquad \epsilon_{klmn}\partial^m C^n = 0. \tag{13}$$

The only solution to Eqs. (9), (13) is

$$C_n = \partial_n B, \tag{14}$$

with B the on-shell massless real scalar field

$$\Box B = 0. \tag{15}$$

Thus, physical states of the first quantized Siegel superparticle look like

$$V_{\text{phys}}(x,\theta,\bar{\theta}) = A(x) + \theta\psi(x) + \bar{\theta}\bar{\psi}(x) + \theta\sigma^n\bar{\theta}\partial_n B(x), \tag{16}$$

with A, B the on-shell massless real scalar fields (irreps of the Poincaré group of helicity 0) and $\psi, \bar{\psi}$ the on-shell massless spinor fields (helicities 1/2 and −1/2, respectively). Note that together they fit to form two irreducible representations of the super Poincaré group of superhelicities 0 and −1/2 [27].

It is worth mentioning that Eq. (6) can be rewritten in the manifestly superinvariant form

$$\tilde{\sigma}^{n\dot{\alpha}\alpha}\partial_n D_\alpha V = 0, \qquad \tilde{\sigma}^{n\dot{\alpha}\alpha}\partial_n \bar{D}_{\dot{\alpha}} V = 0, \tag{17}$$

where D_α, $\bar{D}_{\dot{\alpha}}$ are the covariant derivatives, or as a single massless Dirac equation

$$\gamma^n \partial_n \Psi = 0, \tag{18}$$

with $\Psi \equiv \begin{pmatrix} D_\alpha V \\ \bar{D}^{\dot{\alpha}} V \end{pmatrix}$ a (superfield) Majorana spinor.

An effective field theory which reproduces equations (7.a), (15) is easy to write

$$S = \int d^4x \{\frac{1}{2}\partial^m A \partial_m A + \frac{1}{2}\partial^m B \partial_m B + i\psi\sigma^m\partial_m\bar{\psi}\}, \tag{19}$$

which is invariant under global (on-shell) supersymmetry transformations

$$\begin{aligned} &\delta A = \epsilon\psi + \bar{\epsilon}\bar{\psi}, \qquad \delta B = i\epsilon\psi - i\bar{\epsilon}\bar{\psi}, \\ &\delta\psi = i(\sigma^n\bar{\epsilon})\partial_n A + (\sigma^n\bar{\epsilon})\partial_n B, \\ &\delta\bar{\psi} = -i(\epsilon\sigma^n)\partial_n A + (\epsilon\sigma^n)\partial_n B. \end{aligned} \tag{20}$$

In Eq. (19) we recognize the massless Wess–Zumino model in the component form [16].

As is known, the superfield formulation of the massless Wess–Zumino model involves chiral and antichiral superfields [16,18],

$$S = \int d^8z\, \Phi\bar{\Phi}, \tag{21.a}$$

$$\bar{D}_{\dot{\alpha}}\Phi = 0, \tag{21.b}$$

$$D_\alpha\bar{\Phi} = 0. \tag{21.c}$$

The equations of motion read

$$D^2\Phi = 0, \tag{22.a}$$

$$\bar{D}^2\bar{\Phi} = 0. \tag{22.b}$$

Let us show that the real scalar superfield (4) satisfying the constraints (17) is the sum of on-shell massless chiral ((21.b), (22.a)) and antichiral ((21.c), (22.b)) superfields.

Let us consider Eqs. (21.b), (22.a). The first of them implies the decomposition [18,27]

$$\Phi(x,\theta,\bar{\theta}) = \alpha(x) + \theta\psi(x) + \theta^2 f(x) + i\theta\sigma^n\bar{\theta}\partial_n\alpha(x) + \frac{1}{2}\theta^2\bar{\theta}\tilde{\sigma}^n\partial_n\psi + \frac{1}{4}\theta^2\bar{\theta}^2\Box\alpha, \tag{23}$$

while the latter, being rewritten in the equivalent form

$$\tilde{\sigma}^{n\,\dot{\alpha}\alpha}\partial_n D_\alpha\Phi = 0, \tag{24}$$

yields

$$\tilde{\sigma}^n\partial_n\psi = 0, \qquad \Box\alpha = 0, \qquad f = 0. \tag{25}$$

In order to get Eqs. (24), (25) we used the identity

$$[D^2, \bar{D}_{\dot{\alpha}}] = -4i\sigma^n{}_{\alpha\dot{\alpha}}\partial_n D^\alpha, \tag{26}$$

and assumed the standard boundary condition. Note also that Eqs. (21.b), (24) together with the identity $\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma^n{}_{\alpha\dot{\alpha}}\partial_n$ imply

$$\Box\Phi = 0, \tag{27}$$

Thus, an on-shell chiral superfield can be written as

$$\begin{aligned} &\Phi(x,\theta,\bar{\theta}) = \alpha(x) + \theta\psi(x) + i\theta\sigma^n\bar{\theta}\partial_n\alpha(x), \\ &\Box\alpha(x) = 0, \qquad \tilde{\sigma}^n\partial_n\psi(x) = 0. \end{aligned} \tag{28}$$

Similarly, an on-shell antichiral superfield ((21.c), (22.b)) reads

$$\begin{aligned} &\bar{\Phi}(x,\theta,\bar{\theta}) = \bar{\alpha}(x) + \bar{\theta}\bar{\psi}(x) - i\theta\sigma^n\bar{\theta}\partial_n\bar{\alpha}(x), \\ &\Box\bar{\alpha}(x) = 0, \qquad \tilde{\sigma}^n\partial_n\bar{\psi}(x) = 0. \end{aligned} \tag{29}$$

Considering now the sum

$$\Phi + \bar{\Phi} = (\alpha + \bar{\alpha}) + \theta\psi + \bar{\theta}\bar{\psi} + \theta\sigma^n\bar{\theta}\partial_n i(\alpha - \bar{\alpha}), \tag{30}$$

and denoting

$$\alpha + \bar{\alpha} \equiv A, \qquad i(\alpha - \bar{\alpha}) \equiv B, \tag{31}$$

one arrives just at Eq. (16).

Thus, the real scalar superfield subject to the constraints (17) was proven to be the sum of on-shell chiral and antichiral superfields

$$V_{\text{phys}}(x,\theta,\bar{\theta}) = \Phi(x,\theta,\bar{\theta}) + \bar{\Phi}(x,\theta,\bar{\theta}). \tag{32}$$

As is known, on-shell massless scalar chiral superfields form massless irreducible representation of the super Poincaré group of superhelicity 0 [27].

Analogously, on-shell massless scalar antichiral superfields realize irrep of superhelicity $-1/2$. We conclude that quantum states of the first quantized Siegel superparticle form a reducible representation of the super Poincaré group which contains superhelicities 0 and $-1/2$.

To summarize, in this letter we have considered canonical quantization of the Siegel superparticle in $R^{4/4}$ superspace. Quantum states of the model were proven to be the sum of on-shell chiral and antichiral superfields. The corresponding effective field theory was shown to be the massless Wess–Zumino model. Because propagators of the theory are well known, it is tempting to reproduce Siegel's action within the framework of the proper-time approach, as well as, to compare the result with that of the straightforward BFV quantization combined with the scheme [15]. This work is currently in progress.

As was mentioned above the C-constraint of the $10d$ $ABCD$-superparticle [21] is not necessary in four dimensions. Note in this connection that an alternative possibility $(p_\theta + ip_n\sigma^n\bar{\theta})_\alpha(p_{\bar{\theta}} + i\theta\sigma^n p_n)_{\dot{\alpha}} = 0$, or $(D_\alpha \bar{D}_{\dot{\alpha}} - \bar{D}_{\dot{\alpha}} D_\alpha)V = 0$ at the quantum level, leads to the trivial solution $V = 0$ only (see, however, Ref. [28]). By this reason, it is not obvious to us how to extend the model (1) up to a theory equivalent to the $4d$ CBS superparticle along the lines of Ref. 21.

Due to the relation to superstring theory, the $10d$ case is of prime interest. The operatorial quantization presented in this work is rather specific in four dimensions. We hope, however, that BFV path integral quantization will proceed along the same lines both in $4d$ and $10d$. The results on this subject will be present in a separate publication.

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