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GREEN-SCHWARZ TYPE FORMULATION OF THE ELEVEN DIMENSIONAL SUPERSTRING ACTION WITH "NEW SUPERSYMMETRY" S - ALGEBRA

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Green-Schwarz type formulation of the eleven dimensional superstring action with " new supersymmetry " S - algebra.

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Abstract

A manifestly Poincare invariant formulation for $D = 11$ superstring action is proposed. The action is invariant under a local fermionic κ-symmetry as well as under a number of global symmetries, which turn out to be on-shell realization of the known "new supersymmetry" S-algebra. Canonical quantization of the theory leads to a quantum state spectrum, which can be identified with that of the type IIA superstring. Besides, a mechanical model is constructed , which is a zero tension limit of the $D = 11$ superstring and which incorporates all essential features of the latter. A corresponding action invariant under off-shell closed realization of the S-algebra is obtained.

1 Introduction

Green-Schwarz (GS) approach [1] to the construction of manifestly super Poincare invariant actions for extended objects implies the invariance under the local κ-symmetry [2], which eliminates half of the initial fermionic

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variables. It provides free dynamics for physical sector variables as well as supersymmetric spectrum of quantum states. The requirement of consistency of the manifest super Poincare invariance with local κ-symmetry leads to rather rigid restrictions on possible dimensions of the target and worldvolume spaces in which the action can be formulated. These restrictions are enumerated in the known brane scan [3,4], which prohibits, in particular, the Green-Schwarz type formulation for $D = 11$ superstring action already at the classical level. According to the brane scan the only permitted dimensions are 3, 4, 6 and 10.

It would be an intriguing task to avoid this no-go theorem in relation to recent progress in understanding of the eleven-dimensional nature of the known superstring theories (see [4-9] and references therein). In the strong coupling limit of M-theory $R^{11} \to \infty$, where R^{11} is the radius of the 11th dimension, the vacuum is eleven-dimensional Minkowski and the effective field theory is $D11$ supergavity, which is viewed now as strong coupling limit of ten-dimensional type IIA superstring [5]. Since $D11$ Poincare symmetry survives in this special point in the moduli space of M-theory vacua ("uncompactified M-theory" according to Ref. [8]), one may ask of the existence of a consistent $D11$ quantum theory with $D11$ supergravity being its low energy limit. One possibility might be the supermembrane action [10-12], but in this case one faces the problem of a continuous spectrum for the first quantized supermembrane [13-15]. By analogy with the ten-dimensional case, where the known field theories can be obtained as low energy limit of the corresponding superstrings [16,8], a different natural candidate might be a $D11$ superstring.

Several ways are known to avoid the no-go theorem, either by considering space-time with non standard signature [17-20], or by introducing higher spin worldvolume fields into the action [22-28][1]. In the present pa-

[1]In recent works [29,30] $D = 11$ action with second-class constraints instead of the κ-symmetry was

per we chose another line. Since the brane scan is based on demanding of super Poincare invariance, there might exist $D=11$ GS type superstring actions for which the supergroup is different from the super Poincare [4]. To elucidate how it may work note that the crucial point of GS formulation for the case of superstring is the γ-matrix identity

$$\Gamma^{\mu}_{\alpha(\beta}(C\Gamma^{\mu})_{\gamma\delta)}=0, \qquad (1)$$

which holds in $D=3,4,6,10$. It provides the existence of both global supersymmetry and local κ-symmetry for the action [1]. An eleven-dimensional analog of Eq.(1) has the form [3,31,32]

$$10\Gamma^{\mu}_{\alpha(\beta}(C\Gamma^{\mu})_{\gamma\delta)}+\Gamma^{\mu\nu}_{\alpha(\beta}(C\Gamma^{\mu\nu})_{\gamma\delta)}=0, \qquad (2)$$

which contains antisymmetric product of γ-matrices[2]. It turns out to be applicable for the superstring case instead of Eq.(1), if one replaces the standard superspace 1-form

$$dx^{\mu}-i\bar{\theta}\Gamma^{\mu}d\theta \qquad (3)$$

by an other one, which contains the same product of Γ-matrices as in Eq.(2),

$$dx^{\mu}-i(\bar{\theta}\Gamma^{\mu\nu}d\theta)n_{\nu}. \qquad (4)$$

Appearance of the new variable $n^{\mu}(\tau,\sigma)$ seems to be an essential property of the construction [17-20,29,30,34-36]. An action, which may be constructed from these 1-forms, is not invariant under the standard supertranslations.

suggested. In was achieved by adding of an appropriately chosen terms to the GS action written in $D=11$. Since there is no κ-symmetry, identities of the type (1), (2) are not necessary for the construction, but the price is that only one half of supersymmetries survive in the resulting Poincare invariant action. Supersymmetry of quantum states spectrum for the model is under investigation now.

[2]Being appropriate for construction of the supermembrane action [10],this identity does not allow one to formulate $D=11$ super Poincare invariant action for superstring with desirable properties. As was shown by Curtright [31], the globally supersymmetric action based on this identity involves additional to $x^i,\theta_a,\bar{\theta}_a$ degrees of freedom in the physical sector. Moreover, it does not posses the κ-symmetry that could provide free dynamics [31,33].

As it will be shown the suitable generalization is the "new supersymmetry" [18-20]

$$\delta\theta=\epsilon, \qquad \delta x^{\mu}=i(\bar{\epsilon}\Gamma^{\mu\nu}\theta)n_{\nu}. \qquad (5)$$

The algebra of the corresponding generators is different from the super Poincare and may be written as [17-20]

$$\{Q_{\alpha},Q_{\beta}\}\sim\Gamma^{\mu\nu}P_{\mu}n_{\nu}. \qquad (6)$$

It is known as S-algebra previously discussed in the M-theory context [17] (see [21] for discussion of the general case). To understand why it may be interesting, note that in special Lorentz reference frame, where $n^{\mu}=(0,\cdots 0,1)$, eq.(5) reduces (see Appendix for our γ-matrix notations) to the following one:

$$\begin{aligned}&\delta\theta^{\alpha}=\epsilon^{\alpha}, \qquad \delta\bar{\theta}_{\alpha}=\bar{\epsilon}_{\alpha},\\&\delta x^{\bar{\mu}}=-i\bar{\epsilon}_{\alpha}\tilde{\Gamma}^{\bar{\mu}\alpha\beta}\bar{\theta}_{\beta}-i\epsilon^{\alpha}\Gamma^{\bar{\mu}}_{\alpha\beta}\theta^{\beta}, \qquad \delta x^{11}=0,\end{aligned} \qquad (7)$$

where $\theta=(\bar{\theta}_{\alpha},\theta^{\alpha}),\mu=(\bar{\mu},10),\bar{\mu}=0,1,\cdots,9$. Equation (7) coincides exactly with the standard $D=10,N=2$ supersymmetry transformations. Thus, one can treat the new supersymmetry (5) as a way to rewrite the $D=10,N=2$ supersymmetry in "eleven dimensional notations", and the corresponding action might be related to type IIA superstring. The possibility of lifting the known ten-dimensional models to the manifestly invariant eleven dimensional form is now under intensive investigation [18-20,29,30,35,36], and the main problem here is to find an appropriate Lagrangian formulation with the variable n^{μ} treated on equal footing with all other ones. From the previous discussion it is clear that the most preferable might be a formulation where the gauge $n^{\mu}=(0,\cdots,0,1)$ would be possible. Unfortunately, it is unknown how to introduce pure gauge variable with the desired properties [18-20,34-36]. Below, we propose $D=11$ superstring action, in which only zero modes of the auxiliary variables survive in the sector of physical degrees of freedom. Since the state spectrum of a string is formed by the action on a vacuum of oscillator modes only, one expects that the presence of zero modes for the case is not essential. This fact will be demonstrated within the canonical quantization framework in Sec.2.

As compared to Refs.[18-20,35,36], an advantage of the present formulation is that the explicit Lagrangian action for $D = 11$ superstring will be presented. Moreover, since the variable $n^\mu(\tau, \sigma)$ is treated on equal footing with other ones, global symmetry transformations form a superalgebra in the usual sense, without appearance of nonlinear in generators terms in the right hand side of Eq.(6) (see below). Thus, true form of the S-algebra will be obtained.

The work is organized as follows. In Sec.2 classical Hamiltonian analysis for the bosonic part of the $D = 11$ superstring action is carried out. We demonstrate that zero modes of the auxiliary variables surviving in the sector of physical degrees of freedom do not make contributions to the quantum state spectrum of the theory. In Sec.3 an action of the $D = 11$ superstring and its local and global symmetries are presented. In Sec.4 we show that physical degrees of freedom of the theory obey free equations of motion. The canonical quantization of the theory leads to the quantum state spectrum identical to that of the type IIA GS superstring. In Sec.5 zero-tension limit of the superstring action is studied. We present $D = 11$ action for mechanics system, which is invariant under local κ-symmetry as well as under off-shell closed realization of S-algebra of global symmetries. In the result, a model-independent form of the S-algebra will be presented. Appendix contains our γ-matrix conventions for $D = 11$.

2 Bosonic part of the action and its spectrum.

As was mentioned in the Introduction we need to get in our disposal an auxiliary space-like vector variable. As a preliminary step to such a construction we discuss an action of the bosonic string modified by some additional terms with the above mentioned variable. The aim of this section is to show that the additional terms describe trivial degrees of freedom. Although zero modes of the auxiliary variables survive in the physical sector, they do not make contribution to the quantum state spectrum of the model. An action for the $D = 11$ superstring will be obtained in the next section as a supersymmetrization of the above mentioned bosonic action.

The action which will be examined is

$$S = \int d^2\sigma \left\{ \frac{-g^{ab}}{2\sqrt{-g}}\partial_a x^\mu \partial_b x^\mu - \epsilon^{ab}\xi_a(n^\mu \partial_b x^\mu) - n^\mu \epsilon^{ab}\partial_a A^\mu_b - \phi(n^2+1) \right\}. \quad (8)$$

Here $n^\mu(\sigma^a)$ is $D11$ vector and $d2$ scalar, $A^\mu_a(\sigma^b)$ is $D11$ and $d2$ vector, while $\phi(\sigma^a)$ is a scalar. In Eq.(8) we have set $\epsilon^{ab} = -\epsilon^{ba}, \epsilon^{01} = -1$ and it also supposed that all the variables are periodic on the interval $\sigma \subset [0, \pi]$ functions. From the equation of motion $\delta S/\delta\phi = 0$ it follows that n^μ is a space-like vector.

Let us discuss the dynamics of the model. For this aim the Hamiltonian formalism seems to be the most appropriate, since second-class constraints will arise and must be taken into account.The total Hamiltonian constructed by means of standard procedure [37,38] has the form

$$H = \int d\sigma \left\{ -\frac{N}{2}[\hat{p}^2 + (\partial_1 x)^2] - N_1(\hat{p}\partial_1 x) - \xi_0(n\partial_1 x) + (n\partial_1 A_0) + \right.$$
$$\left. +\phi(n^2+1) + \omega^{ab}(\pi_g)_{ab} + \lambda_\phi \pi_\phi + \lambda_{\xi a}\pi_\xi{}^a + \lambda^\mu_0 p^\mu_0 + \lambda^\mu_1(p^\mu_1 - n^\mu) + \lambda^\mu_n p^\mu_n \right\}, \quad (9)$$

where

$$\hat{p}^\mu \equiv p^\mu + \xi_1 n^\mu, \qquad N \equiv \frac{\sqrt{-g}}{g^{00}}, \qquad N_1 \equiv \frac{g^{01}}{g^{00}}, \quad (10)$$

and p^μ, p^μ_a, p^μ_n, $(\pi_g)_{ab}$, π^a_ξ, π_ϕ are momenta conjugated to the variables x^μ, A^μ_a, n^μ, g^{ab}, ξ_a, ϕ respectively; λ_* are Lagrange multipliers corresponding to the primary constraints. The complete set of constrains can be found and presented as follows

$$p^\mu_n = 0, \qquad n^\mu - p^\mu_1 = 0; \tag{11}$$

$$\pi^1_\xi = 0, \qquad \xi_1 - (p_1 p) = 0; \tag{12}$$

$$(\pi_g)_{ab} = 0, \qquad \pi_\phi = 0, \qquad \pi^0_\xi = 0, \qquad p^\mu_0 = 0; \tag{13}$$

$$(p_1)^2 = -1, \qquad \partial_1 p^\mu_1 = 0; \tag{14}$$

$$H_0 \equiv (p_1 \partial_1 x) = 0, \qquad H_\pm \equiv (\hat{p}^\mu \pm \partial_1 x^\mu)^2 = 0; \tag{15}$$

Constraints (11),(12) are of second-class, while the remaining ones are first-class. An appropriate gauge fixing for the constraints (13) is

$$g^{ab} = \eta^{ab}, \qquad \phi = \frac{1}{2}, \qquad \xi_0 = 0, \qquad A^\mu_0 = \int_0^\sigma d\sigma' \xi_1 \hat{p}^\mu. \tag{16}$$

After introducing of Dirac brackets, which correspond to second-class set (11)-(13),(16), the canonical pairs of variables (n^μ, p^μ_n), (ξ_a, π^a_ξ), $(g_{ab}, (\pi_g)_{ab})$, (ϕ, π_ϕ), (A^0_μ, p^μ_0) can be omitted. The Dirac brackets for the remaining variables coincide with the Poisson ones [38]. The choice in (16) simplifies the subsequent analysis of (A^μ_1, p^μ_1)-sector, since the Hamiltonian equations of motion for these variables look now as

$$\partial_0 A^\mu_1 = p^\mu_1, \qquad \partial_0 p^\mu_1 = 0. \tag{17}$$

In order to find an appropriate gauge fixing for the constraints (14) let us consider Fourier decomposition of periodical in the interval $\sigma \subset [0, \pi]$ functions

$$\begin{aligned} A^\mu_1(\tau,\sigma) &= Y^\mu(\tau) + \sum_{n\neq 0} y^\mu_n(\tau) e^{i2n\sigma}, \\ p^\mu_1(\tau,\sigma) &= P^\mu_y(\tau) + \sum_{n\neq 0} p^\mu_n(\tau) e^{i2n\sigma}. \end{aligned} \tag{18}$$

Then the constraint $\partial_1 p^\mu_1 = 0$ is equivalent to $p^\mu_n = 0$, $n \neq 0$, and an appropriate gauge condition is $y^\mu_n = 0$, or, equivalently, $\partial_1 A^\mu_1 = 0$. Thus physical degrees of freedom in the sector (A^μ_1, p^μ_1) are zero modes of these variables and the corresponding dynamics is

$$\begin{aligned} A^\mu_1(\tau,\sigma) &= Y^\mu + P^\mu_y \tau, \\ p^\mu_1(\tau,\sigma) &= P^\mu_y = const, \qquad (P_y)^2 = -1. \end{aligned} \tag{19}$$

Since there are no of oscillator variables, this sector of the theory may be considered as describing a point-like object, which propagates freely according to Eq.(19). Dynamics of the remaining variables is governed now by the equations

$$\partial_0 x^\mu = -p^\mu - (P_y p) P^\mu_y, \qquad \partial_0 p^\mu = -\partial_1 \partial_1 x^\mu. \tag{20}$$

In addition, the constraints

$$H_0 \equiv (P_y \partial_1 x) = 0, \qquad H_\pm \equiv (p^\mu + (P_y p) P^\mu_y \pm \partial_1 x^\mu)^2 = 0, \tag{21}$$

hold, which obey the following algebra

$$\begin{aligned} \{H_\pm, H_\pm\} &= \pm 4[H_\pm(\sigma) \pm (P_y p) H_0(\sigma) + (\sigma \to \sigma')]\partial_\sigma \delta(\sigma - \sigma'), \\ \{H_+, H_-\} &= 4[(P_y p) H_0(\sigma) + (\sigma \to \sigma')]\partial_\sigma \delta(\sigma - \sigma'), \\ \{H_0, H_\pm\} &= \pm 2 H_0(\sigma')\partial_\sigma \delta(\sigma - \sigma'). \end{aligned} \tag{22}$$

On the $D = 10$ hyperplane selected by the constraint $H_0(\sigma) = 0$ it reduces to the standard Virasoro algebra. Note also that the variable $x^\mu(\tau, \sigma)$ obeys the free equation $(\partial^2_\tau - \partial^2_\sigma)x^\mu = 0$ as a consequence of Eqs.(20),(21).

To proceed further it is useful to impose the gauge condition

$$(P_y \partial_1 p) = 0, \tag{23}$$

to the constraint $H_0 = 0$. By virtue of Eqs.(20),(23) one finds, in particular, that $(P_y p) = (P_y P)$, where P^μ is the zero mode of $p^\mu(\tau, \sigma)$. Then the solution to Eq.(20) (for the case of closed world sheet) reads

$$\begin{aligned} x^\mu(\tau,\sigma) &= X^\mu - \tfrac{1}{\pi}(P^\mu + (P_yP)P_y^\mu)\tau + \\ &\tfrac{i}{2\sqrt{\pi}}\sum \tfrac{1}{n}[\bar\alpha_n^\mu e^{i2n(\tau+\sigma)} + \alpha_{-n}^\mu e^{-i2n(\tau-\sigma)}], \\ p^\mu(\tau,\sigma) &= \tfrac{1}{\pi}P^\mu + \tfrac{1}{\sqrt{\pi}}\sum[\bar\alpha_n^\mu e^{i2n(\tau+\sigma)} - \alpha_{-n}^\mu e^{-i2n(\tau-\sigma)}], \end{aligned} \tag{24}$$

which is accompanied by the constraints

$$P_y^\mu \bar\alpha_n^\mu = 0, \qquad P_y^\mu \alpha_{-n}^\mu = 0, \tag{25}$$

$$\begin{aligned} H_+ &= \tfrac{8}{\pi}\sum_{-\infty}^{\infty} L_n e^{i2n(\tau-\sigma)}, \qquad & L_n &\equiv \tfrac{1}{2}\sum_{-\infty}^{\infty} \alpha_{n-k}^\mu \alpha_k^\mu = 0, \\ H_- &= \tfrac{8}{\pi}\sum_{-\infty}^{\infty} \bar L_n e^{i2n(\tau+\sigma)}, \qquad & \bar L_n &\equiv \tfrac{1}{2}\sum_{-\infty}^{\infty} \bar\alpha_{n-k}^\mu \bar\alpha_k^\mu = 0, \end{aligned} \tag{26}$$

where $\alpha_0^\mu = -\bar\alpha_0^\mu \equiv \frac{1}{2\sqrt{\pi}}(P^\mu + (P_yP)P_y^\mu)$.

From Eq.(25) and the equality $(P^\mu + (P_yP)P_y^\mu)P_y^\mu = 0$ for the momenta of the center of mass, it follows that the sector (x^μ, p^μ) of the theory describes, in fact, a closed string, which lives on the (D-1)-dimensional hyperplane orthogonal to the P_y^μ - direction.

Using the zero modes X^μ, P^μ, Y^μ, P_y^μ, one can construct the following combinations

$$\mathcal{X}^\mu \equiv X^\mu - \frac{1}{2}\frac{P_yY}{P_yP}P_y^\mu, \qquad \mathcal{P}^\mu \equiv P^\mu + (P_yP)P_y^\mu, \tag{27}$$

which obey the Poisson brackets

$$\{\mathcal{X}^\mu, \mathcal{P}^\nu\} = \eta^{\mu\nu}, \qquad \{\mathcal{X}^\mu, \mathcal{X}^\nu\} = \{\mathcal{P}^\mu, \mathcal{P}^\nu\} = 0. \tag{28}$$

Thus, the quantities $\mathcal{P}^\mu$, $\mathcal{L}^{\mu\nu} = \frac{1}{2}(\mathcal{X}^\mu\mathcal{P}^\nu - \mathcal{X}^\nu\mathcal{P}^\mu)$ are generators of the Poincare group. This allows one to obtain the mass formulae for physical states. We adopt the Gupta-Bleuler prescription by requiring that physical states be annihilated by half of the operators $: L_n :$, $: \bar L_n :$

$$(L_n - a\delta_{n,0}) \mid phys >= (\bar L_n - a\delta_{n,0}) \mid phys >= 0, \qquad n > 0. \tag{29}$$

By virtue of Eq.(26) for n=0 one finds the mass of the states

$$m^2 = \mathcal{P}^2 = -4\pi \left\{ \sum_{n>0} (\alpha_{-n}^\mu \alpha_n^\mu + \bar\alpha_{-n}^\mu \bar\alpha_n^\mu) + 2a \right\}. \tag{30}$$

As it should be, the mass of a state is determined by oscillator excitations of $x^\mu(\tau,\sigma)$ –string only, zero modes of the sector (A_1^μ, p_1^μ) do not make any contribution into this expression.

In order to describe the spectrum of the superstring suggested below, it is useful to consider also noncovariant quantization in an appropriately chosen coordinate system. By making use of a Lorentz transformation one can consider coordinate system where $P_y^\mu = (0, ...0, 1)$. This breaks manifest $SO(1, D-1)$ covariance up to $SO(1, D-2)$ one. In this basis Eq.(20)-(23) are reduced to

$$\partial_0 x^{D-1} = 0, \qquad \partial_0 p^{D-1} = 0; \tag{31}$$

$$\partial_0 x^{\bar\mu} = -p^{\bar\mu}, \qquad \partial_0 p^{\bar\mu} = -\partial_1\partial_1 x^{\bar\mu}, \qquad (p^{\bar\mu} \pm \partial_1 x^{\bar\mu})^2 = 0; \tag{32}$$

where $\mu = (\bar\mu, D-1)$. Thus, zero modes of the theory (8) along the direction P_y^μ decouples from $(D-1)$-dimensional sector (32), while oscillator modes along the direction P_y^μ are absent as a consequence of the equations $(P_y\partial_1 x) = (P_y\partial_1 p) = 0$.

Let us discuss the obtained results. Classically, the bosonic D - dimensional theory (8) can be considered as describing a composite object. The sector of the auxiliary variables (A_1^μ, p_1^μ) corresponds to a point-like object. The only physical degrees of freedom of the sector are zero modes Y^μ, P_y^μ which describe propagation of a free particle, see Eq.(19). The sector of variables (x^μ, p^μ) describes the closed string (32),(30), which lives on (D-1) - dimensional hyperplane orthogonal to P_y^μ - direction (the constraints (15), which relate the particle and the closed string mean that the latter one has no component of center of mass momenta as well as of oscillator excitations in the P_y^μ - direction, see Eqs.(24),(25)).

Next let us look at the spectrum of the theory. The ground state of the full theory $\mid p_{y0}, p_0, 0 >=\mid p_{y0} >\mid p_0 >\mid 0 >$ is a direct product of vacua, corresponding to the sectors (Y^μ, P_y^μ), (X^μ, P^μ), $(\alpha_n^\mu, \bar{\alpha}_n^\mu)$, which obey $P_y^2 \mid p_{y0} >= - \mid p_{y0} >$, $P^\mu \mid p_0 >= p_0^\mu \mid p_0 >$, $\alpha_n^\mu \mid 0 >= \bar{\alpha}_n^\mu \mid 0 >= 0$ for $n > 0$. The excitation levels are then obtained by acting with $n < 0$ oscillators on the ground state. From the mass formulae (30), and Eq.(32) it follows that the quantum state spectrum of the theory (8) coincides with that of the (D-1) - dimensional closed bosonic string. One notes that zero modes Y^μ, P_y^μ manifest themselves in additional degeneracy of the continuous energy spectrum only.

3 Action of D=11 superstring and its symmetries

As the $D = 11$ superstring action we propose the following supersymmetric version of (8):

$$S = \int d^2\sigma \left\{ \frac{-g^{ab}}{2\sqrt{-g}} \Pi_a^\mu \Pi_{b\mu} - i\varepsilon^{ab}(\partial_a x^\mu - \frac{i}{2}\bar{\theta}\Gamma^{\mu\nu}n_\nu\partial_a\theta)(\bar{\theta}\Gamma_\mu\partial_b\theta) - \right.$$
$$\left. -\varepsilon^{ab}\xi_a(n_\mu\Pi_b^\mu) - n_\mu\varepsilon^{ab}\partial_a A_b^\mu - \phi(n^2+1)\right\}, \qquad (33)$$

where θ is a 32-component Majorana spinor of $SO(1,10)$, ξ_a is a $d = 2$ vector and $\Pi_a^\mu \equiv \partial_a x^\mu - i\bar{\theta}\Gamma^{\mu\nu}n_\nu\partial_a\theta$. The role of the last two terms was explained in the previous section. The third term is crucial for existence of local κ-symmetry and, at the same time, it provides a split of x^{10} coordinate from the physical sector.[3]

Let us describe global symmetry structure of the action (33). Bosonic symmetries are the $D = 11$ Poincaré transformations in the standard realization, and additional transformations with antisymmetric parameter

[3]On-shell states carry only ten-dimensional momenta, which turn out to be common property for known $D = 11, 12$ formulations[18-20,39].

$b^{\mu\nu} = -b^{\nu\mu}$,

$$\delta_b x^\mu = b^\mu{}_\nu n^\nu,$$
$$\delta_b A_a^\mu = -b^\mu{}_\nu \left(\varepsilon_{ab}\frac{g^{bc}}{\sqrt{-g}}\Pi_c{}^\nu - \xi_a n^\nu + i(\bar{\theta}\Gamma^\nu\partial_a\theta)\right). \qquad (34)$$

The following fermionic supersymmetry transformations also take place:

$$\delta\theta = \epsilon, \qquad \delta x^\mu = i\bar{\epsilon}\Gamma^{\mu\nu}n_\nu\theta, \qquad (35)$$
$$\delta A_a^\mu = i\varepsilon_{ab}\frac{g^{bc}}{\sqrt{-g}}\Pi_{c\nu}(\bar{\epsilon}\Gamma^{\mu\nu}\theta) - \frac{5}{6}(\bar{\epsilon}\Gamma^{\nu\mu}\theta)(\bar{\theta}\Gamma_\nu\partial_a\theta) +$$
$$+\frac{1}{6}(\bar{\epsilon}\Gamma_\nu\theta)(\bar{\theta}\Gamma^{\nu\mu}\partial_a\theta).$$

One can prove that the complete algebra of symmetry transformations is on-shell closed up to the equation of motion $\partial_a n^\mu = 0$ and up to the trivial transformations $\delta A_a^\mu = \partial_a\rho^\mu$ (see Eq.(38) below) with field-dependent parameter ρ^μ, as it usually happens in component formulations of supersymmetric models without auxiliary fields. In Sec.5 an off-shell closed version of these transformations will be obtained for the case of $D = 11$ superparticle. The only nontrivial commutator is[4]

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_b, \qquad b^{\mu\nu} = -2i(\bar{\epsilon}_1\Gamma^{\mu\nu}\epsilon_2). \qquad (36)$$

Let us note that one needs to use the $D = 11$ Fierz identities to prove Eq.(36) for A_a^μ variable

$$(\Gamma^\mu)_{\alpha(\beta}(C\Gamma^{\mu\nu})_{\gamma\delta)} + (\Gamma^{\mu\nu})_{\alpha(\beta}(C\Gamma^\mu)_{\gamma\delta)} = 0. \qquad (37)$$

A relation of Eq.(35) to the $D = 10, N = 2$ supersymmetry has been described in the Introduction.

Local bosonic symmetries for the action (33) are $d = 2$ reparametrizations (with the standard transformation lows for all the variables except

[4]To elucidate relation between Eqs.(36) and (6) let us point a simple analogy: algebra of the Lorentz generators $M^{\mu\nu} = \frac{1}{2}(x^\mu p^\nu - x^\nu p^\mu)$ can be written either as $[M^{\mu\nu}, M^{\rho\sigma}] = \eta^{\mu\rho}M^{\nu\sigma} + \ldots$ or $[M^{\mu\nu}, M^{\rho\sigma}] = -\eta^{\mu\rho}p^\sigma x^\nu + \ldots$. The second case may be considered as corresponding to Eq.(6).

the variable ϕ, which transforms as a density, $\phi'(\sigma') = \det(\partial\sigma'/\partial\sigma)\phi(\sigma)$), Weyl symmetry, and the following transformations with parameters $\rho^\mu(\sigma^a)$ and $\omega_a(\sigma^b)$,

$$\delta A^\mu_a = \partial_a \rho^\mu + \omega_a n^\mu, \qquad \delta\phi = -\tfrac{1}{2}\epsilon^{ab}\partial_a\omega_b. \tag{38}$$

These symmetries are reducible since their combination with parameters of a special form ($\omega_a = \partial_a\omega, \rho^\mu = -\omega n^\mu$) is a trivial symmetry, $\delta_\omega A^\mu_a = -\omega\partial_a n^\mu, \delta_\omega\phi = 0$ (note that $\partial_a n^\mu = 0$ is one of the equations of motion). Thus, Eq.(38) includes 12 essential parameters, which correspond to the primary first-class constraints $p^\mu_0 = 0, \pi_\phi = 0$ (see below).

The action is also invariant under a pair of local fermionic κ-symmetries. To describe them let us consider the following ansatz:

$$\begin{aligned} &\delta\theta = \pm\Pi_{d\mu}S^\pm\Gamma^\mu\kappa^{\mp d}, \qquad \delta x^\mu = -\delta\bar\theta\Gamma^{\mu\nu}n_\nu\theta, \\ &\delta g^{ab} = 8i\sqrt{-g}P^{\pm ca}(\partial_c\bar\theta S^\mp\kappa^{\mp b}), \end{aligned} \tag{39}$$

where

$$S^\pm = \frac{1}{2}(1 \pm n_\mu\Gamma^\mu), \quad \kappa^{\mp d} \equiv P^{\mp dc}\kappa_c, \quad P^{\mp dc} = \frac{1}{2}\left(\frac{g^{dc}}{\sqrt{-g}} \mp \varepsilon^{dc}\right). \tag{40}$$

Note that on-shell (where $n^2 = -1$) the operators ${S^\pm}_\alpha{}^\beta$ form a pair of projectors in θ-space. Let us remember also that the $d = 2$ projectors $P^\pm$ obey the following properties: $P^{+ab} = P^{-ba}$, $P^{\mp ab}P^{\mp cd} = P^{\mp cb}P^{\mp ad}$. After tedious calculations with the use of latter properties and the Fierz identities (37) a variation of the action (33) under the transformations (39) can be presented in the form

$$\delta S = -\varepsilon^{ab}\partial_a n_\nu G^\nu_b + (n^2+1)H + \varepsilon^{ab}(n_\mu\Pi^\mu_b)F_a, \tag{41}$$

where

$$\begin{aligned} {G_b}^\nu \equiv &-i\varepsilon_{bc}\frac{g^{cd}}{\sqrt{-g}}(\delta\bar\theta\Gamma^{\mu\nu}\theta)\Pi_{d\mu} + \frac{1}{2}(\delta\bar\theta\Gamma^{\mu\nu}\theta)(\bar\theta\Gamma_\mu\theta) - \\ &-\frac{1}{2}(\delta\bar\theta\Gamma_\mu\theta)(\bar\theta\Gamma^{\mu\nu}\partial_b\theta) + i\xi_b(\delta\bar\theta\Gamma^{\mu\nu}\theta)n_\nu, \\ H \equiv &+i\frac{g^{ab}}{\sqrt{-g}}(\partial_a\bar\theta\Gamma^\mu\tilde\kappa^\mp)\Pi_{b\mu}, \\ F_a \equiv &\, i[\varepsilon_{ac}\frac{g^{cd}}{\sqrt{-g}}(\partial_d\bar\theta\Gamma^\mu\tilde\kappa^\mp)n_\mu + (\partial_a\bar\theta\tilde\kappa^\mp)\mp \\ &\mp 2\varepsilon_{ab}P^{\pm cd}(\partial_c\bar\theta\Gamma^\mu\kappa^{\mp b})\Pi_{d\mu}], \qquad \tilde\kappa^\mp \equiv \Pi_{a\mu}\Gamma^\mu\kappa^{\mp a}. \end{aligned} \tag{42}$$

All terms in Eq.(41) can evidently be canceled by the corresponding variations of the auxiliary fields,

$$\delta A^\nu_b = G^\nu_b, \qquad \delta\phi = H, \qquad \delta\xi_a = F_a. \tag{43}$$

In the end one can see that the eleven dimensional superstring action (33) is invariant under the transformations from Eq.(39) supplemented by ones from Eq.(43). Let us stress that three last terms in the action (33) turn out to be essential for achieving this local κ-symmetry. Since in Eq.(39) there appeared the double projectors ($S^\pm$ and $\Pi_{a\mu}\Gamma^\mu$) acting on the θ-space, the total number of essential parameters is $8+8$.

As a cheek-up of our calculations note that after the substitution $n^\mu = (0,\cdots 0,1)$ the equations (39) are reduced to the ten-dimensional κ-symmetry transformations of the GS superstring action

$$\begin{aligned} &\delta\theta^\alpha = -P^{-cd}\Pi^{\bar\mu}{}_d\tilde\Gamma^{\bar\mu\,\alpha\beta}\bar\kappa_{c\beta}, \qquad \delta\bar\theta_\alpha = P^{+cd}\Pi^{\bar\mu}{}_d\Gamma^{\bar\mu}_{\alpha\beta}\kappa_c{}^\beta, \\ &\delta x^\mu = i\theta^\alpha\Gamma^{\bar\mu}_{\alpha\beta}\delta\theta^\beta + i\bar\theta_\alpha\tilde\Gamma^{\bar\mu\,\alpha\beta}\delta\bar\theta_\beta, \\ &\delta g^{ab} = 8i\sqrt{-g}\{P^{-ca}(\partial_c\bar\theta\kappa^{+b}) - P^{+ca}(\partial_c\theta\bar\kappa^{-b})\}. \end{aligned} \tag{44}$$

4 Dynamics of the D=11 superstring and D=10 type IIA GS superstring.

In this Section we are going to demonstrate that the dynamics of physical variables in the theory (33) is governed by free equations. In the coordinate system, where $n^\mu = (0, \cdots 0, 1)$, the variables and the corresponding equations can be identified with the ones of type IIA GS superstring (modulo center of mass type variables discussed in Sect.2). As a result, quantum state spectrum of the theory (33) coincides with that of the type IIA GS superstring. This conclusion is independent on the frame chosen since the initial action has $D = 11$ Poincare invariance.

Performing the standard Hamiltonian analysis for the theory (33), one finds a pair of second-class constraints $p_n^\mu = 0$, $p_1^\mu - n^\mu = 0$ among primary constraints of the theory. Then the variables (n^μ, p_n^μ) can be omitted after introducing the associated Dirac bracket (see Sec. 2). The Dirac brackets for the remaining variables coincide with the Poisson ones, and the total Hamiltonian may be written as

$$H = \int d\sigma^1 \Big\{ -\frac{N}{2}(\hat{p}^2 + \Pi_{1\mu}\Pi_1^\mu) - N_1 \hat{p}_\mu \Pi_1^\mu + p_{1\mu}\partial_1 A_0^\mu - \xi_0 (p_{1\mu}\partial_1 x^\mu) + \\ + \phi(p_1^2 + 1) + \lambda_\phi \pi_\phi + \lambda_{0\mu} p_0^\mu + \lambda^{ab}(\pi_g)_{ab} + \lambda_{\xi a} p_\xi{}^a + L_\alpha \lambda_\theta{}^\alpha \Big\}, \qquad (45)$$

where $p^\mu, p_0^\mu, p_1^\mu, p_\xi^a, (\pi_g)_{ab}$ are momenta conjugated to the variables x^μ, A_0^μ, A_1^μ, ξ_a, g_{ab} respectively; λ_* are Lagrange multipliers corresponding to the primary constraints, and the following notations are used

$$N = \frac{\sqrt{-g}}{g^{00}}, \quad N_1 = \frac{g^{01}}{g^{00}}, \quad \hat{p}^\mu = p^\mu - i\bar{\theta}\Gamma^\mu\partial_1\theta + \xi_1 p_1^\mu,$$

$$L_\alpha \equiv \bar{p}_{\theta\alpha} - i(p_\mu - \frac{i}{2}\bar{\theta}\Gamma_\mu\partial_1\theta)(\bar{\theta}\Gamma^{\mu\nu})_\alpha p_{1\nu} - i(\partial_1 x^\mu - \frac{i}{2}\bar{\theta}\Gamma^{\mu\nu}p_{1\nu}\partial_1\theta)(\bar{\theta}\Gamma_\mu)_\alpha = 0. \qquad (46)$$

Poisson brackets for the fermionic constraints are:

$$\{L_\alpha, L_\beta\} = 2i\left[(\hat{p}^\mu + \Pi_1^\mu)(C\Gamma^\mu S^+)_{\alpha\beta} - (\hat{p}^\mu - \Pi_1^\mu)(C\Gamma^\mu S^-)_{\alpha\beta}\right], \qquad (47)$$

from which it follows that half of the latter are of first-class. The complete system of constraints can be presented in the form

$$p_{\xi 1} = 0, \qquad \xi_1 - (pp_1) + i(\bar{\theta}\Gamma^\mu\partial_1\theta)p_{1\mu} = 0; \qquad (48.a)$$

$$(\pi_g)_{ab} = 0, \qquad \pi_\phi = 0, \qquad p_{\xi 0} = 0, \qquad p_0^\mu = 0; \qquad (48.b)$$

$$\partial_1 p_1^\mu = 0, \qquad (p_1^\mu)^2 = -1; \qquad (48.c)$$

$$H_0 \equiv \partial_1 x^\mu p_{1\mu} = 0, \qquad H_\pm \equiv (\hat{p}^\mu \pm \Pi_1^\mu)^2 = 0, \qquad L_\alpha = 0. \qquad (48.d)$$

Besides, some equations for the Lagrange multipliers have been determined in the course of Dirac procedure,

$$\lambda_n^\mu = 0, \qquad \lambda_1^\mu = \partial_1 A_0^\mu + 2\phi p_1^\mu + Q^\mu; \qquad (49)$$

$$(\hat{p}_\mu - \Pi_{1\mu})\Gamma^\mu S^-(\lambda_\theta - \partial_1\theta) = 0, \\ (\hat{p}_\mu + \Pi_{1\mu})\Gamma^\mu S^+(\lambda_\theta + \partial_1\theta) = 0; \qquad (50)$$

where

$$Q^\mu \equiv -N\xi_1\hat{p}^\mu - N_1\xi_1\Pi_1^\mu - \xi_0\partial_1 x^\mu - \\ - \left[ip_\nu\bar{\theta}\Gamma^{\nu\mu} + \tfrac{1}{2}(\bar{\theta}\Gamma_\nu\partial_1\theta)\bar{\theta}\Gamma^{\nu\mu} + \tfrac{1}{2}(\bar{\theta}\Gamma^{\nu\mu}\partial_1\theta)\bar{\theta}\Gamma_\nu\right]\lambda_\theta; \qquad (51)$$

and the Eq.(50) was obtained from the condition $\{L_\alpha, H\} = 0$. The constraints (48.a-c) were considered in Sect.2 and we do not repeat the corresponding analysis here. Doing the gauge fixing (16) and solving of the (A_1^μ, p_1^μ)-sector similar to the Eq.(19), one can see that the dynamics of the remaining variables is governed by equations of motion of the form

$$\partial_0 x^\mu = -(p^\mu + (pP_y)P_y^\mu) - i(\bar{\theta}\Gamma^{\mu\nu}\lambda_\theta)P_{y\nu}, \\ \partial_0 p^\mu = -\partial_1\left[\partial_1 x^\mu - i(\bar{\theta}\Gamma^{\mu\nu}\partial_1\theta)P_{y\nu} + i\bar{\theta}\Gamma^\mu\lambda_\theta\right], \\ \partial_0\theta^\alpha = -\lambda_\theta^\alpha, \qquad (52)$$

together with the constraints (48.d). Equations for $\bar{p}_\theta$-variables are omitted since they are a consequence of the constraints $L_\alpha = 0$ and other equations.

Similarly to GS superstring, physical variables of the theory (33) obey free equations of motion. To demonstrate this let us consider the following decomposition for θ-variable, $\theta = \theta^+ + \theta^-$, where $\theta^\pm$ are spinors of opposite S-chirality[5]

$$\theta^\pm \equiv S^\pm \theta, \qquad S^\mp \theta^\pm = 0. \tag{53}$$

By virtue of Eq.(50), the last equation from (52) can be rewritten as

$$(\hat{p}_\mu + \Pi_{1\mu})\Gamma^\mu(\partial_0 - \partial_1)\theta^+ = 0, \qquad (\hat{p}_\mu - \Pi_{1\mu})\Gamma^\mu(\partial_0 + \partial_1)\theta^- = 0. \tag{54}$$

Further, the following conditions

$$\Gamma^+\theta^+ = 0, \qquad \Gamma^+\theta^- = 0, \tag{55}$$

turn out to be an appropriate gauge fixing for the first-class constraints, which can be extracted from the equations $L_\alpha = 0$. Then $\Gamma^+\lambda_\theta^\mp$-projections vanish, $\Gamma^+\lambda_\theta^\pm = 0$, while for $\Gamma^-\lambda_\theta^\pm$-projections one finds as a consequence of Eq.(50),[6]

$$\Gamma^-\lambda_\theta^+ = -\Gamma^-\partial_1\theta^+, \qquad \Gamma^-\lambda_\theta^- = \Gamma^-\partial_1\theta^-. \tag{56}$$

Besibes, the following identities

$$\begin{aligned} &\bar{\theta}\Gamma^+\lambda_\theta = \bar{\theta}\Gamma^i\lambda_\theta = 0, \\ &(\bar{\theta}\Gamma^{+\mu}\lambda_\theta)P_{y\mu} = (\bar{\theta}\Gamma^{i\mu}\lambda_\theta)P_{y\mu} = 0, \\ &(\bar{\theta}\Gamma^{+\mu}\partial_1\theta)P_{y\mu} = (\bar{\theta}\Gamma^{i\mu}\partial_1\theta)P_{y\mu} = 0, \end{aligned} \tag{57}$$

hold in the gauge (54), where $i = 1, 2, \cdots, 8, 10$.

Thus, we have, in fact, a situation similar to $D = 10$ GS superstring, and the corresponding analysis coincides with the well known case [1,16]. Physical variable sector contains the transverse components $x^i, i = 1, \cdots, 8$ of the coordinate x^μ (see Sect.2), and a pair of 32-component spinors $\theta^\pm$ constrained by the equations (53),(55). By virtue of Eqs.(52)-(57) one gets

[5] In the basis where $n^\mu = P_y^\mu = (0, \cdots, 0, 1)$ the S-chiral spinors $\theta^\pm$ can be identified with $D = 10$ Majorana-Weyl spinors of opposite chirality $\theta^+ = (\bar{\theta}_\alpha, 0), \theta^- = (0, \theta^\alpha)$. Also, in this basis $S^\pm$-projectors commutes with the light-cone $\Gamma^\pm$-matrices.

[6] From equation $B_\mu\Gamma^\mu\Psi = 0$ subject to condition $\Gamma^+\Psi = 0$ it follows, in particular, that $B^+\Gamma^-\Psi = 0$.

that the physical variables obey the free equations

$$\partial_0 x^i = -(p^i + (pP_y)P_y^i), \qquad \partial_0 p^i = -\partial_1\partial_1 x^i; \tag{58}$$

$$(\partial_0 - \partial_1)\Gamma^-\theta^+ = 0, \qquad (\partial_0 + \partial_1)\Gamma^-\theta^- = 0. \tag{59}$$

To analyze the quantum state spectrum for the theory under consideration let us follow on the SO(8) covariant procedure described in Sect.2. In the basis where $P_y^\mu = (0, \cdots, 0, 1)$ the gauge conditions (55) are equivalent to $\Gamma^+\theta = 0$ with the solution being $\theta = (\theta_a, 0, 0, \bar{\theta}_{\dot{a}})$, where $\theta_a, \bar{\theta}_{\dot{a}}$ are SO(8) spinors of opposite chirality. Then the equations (59) reduce to $(\partial_0 - \partial_1)\theta_a = 0, (\partial_0 + \partial_1)\bar{\theta}_{\dot{a}} = 0$ ones, while the second equation from (48.d) coincides with the ten-dimensional Virasoro constraints. They lead to the standard mass formulae and one can see that the quantum state spectrum of the theory (33) can be identified with that of type IIA GS superstring.

5 D=11 mechanical system with off-shell closed new supersymmetry S-algebra.

Being zero-tension limit of the GS superstring, the Casalbioni-Brink-Schwarz superparticle incorporates all its essential features [40,41]. It allows one to study corresponding problems in a more simple framework of the mechanical model. In a similar fashion, in this Section a point-like analog for the $D = 11$ superstring is presented and discussed. The action is invariant under local κ-symmetry as well as under a number of global symmetries with on-shell closed algebra of commutators. Its off-shell closed version will be obtained by a slight modification of the initial action, which allows one to extract a true form of the S-algebra. Being model-independent, it may be used now as a basis for systematic construction of various $D = 11$ models.

Our starting point is the following $D = 11$ Lagrangian action

$$S = \int d\tau \left\{ \frac{1}{2e}\Pi^\mu \Pi_\mu + n^\mu \dot{z}^\mu - \phi(n^2+1) \right\}, \qquad (60)$$
$$\Pi^\mu \equiv \dot{x}^\mu - i(\bar{\theta}\Gamma^{\mu\nu}\dot{\theta})n_\nu - \xi n^\mu,$$

with all the variables being functions on the evolution parameter τ. Note that the last two terms are, in fact, an action for bosonic particle $z^\mu(\tau)$ written in the first-order form.

Global bosonic symmetries of the action (60) are $D = 11$ Poincare transformations (with the variable n^μ being inert under the Poincare shifts), and the following transformations

$$\delta_b x^\mu = b^\mu{}_\nu n^\nu, \qquad \delta_b z^\mu = -\frac{1}{e} b^\mu{}_\nu \Pi^\nu, \qquad (61)$$

with antisymmetric parameter $\omega^{\mu\nu} = -\omega^{\nu\mu}$. There is also a global symmetry with a fermionic parameter ϵ^α,

$$\delta_\epsilon \theta = \epsilon, \qquad \delta_\epsilon x^\mu = -i(\bar{\theta}\Gamma^{\mu\nu}\epsilon)n_\nu, \qquad \delta_\epsilon z^\mu = -\frac{i}{e}(\bar{\epsilon}\Gamma^{\mu\nu}\theta)\Pi_\nu. \qquad (62)$$

The algebra of the corresponding commutators turns out to be on-shell closed and looks as follows:

$$[\delta_{b1}, \delta_{b2}]x^\mu = 0, \qquad [\delta_{b1}, \delta_{b2}]z^\mu = \frac{1}{e} b_1{}^\mu{}_\nu(\delta_{b2}\Pi^\nu) - (1 \leftrightarrow 2);$$
$$[\delta_{\epsilon1}, \delta_{\epsilon2}]\theta = 0, \qquad [\delta_{\epsilon1}, \delta_{\epsilon2}]x^\mu = \delta_b x^\mu, \qquad (63)$$
$$[\delta_{\epsilon1}, \delta_{\epsilon2}]z^\mu = \delta_b z^\mu + \left[\frac{i}{e}(\bar{\epsilon}_1\Gamma^{\mu\nu}\theta)(\delta_{\epsilon2}\Pi_\nu) - (1 \leftrightarrow 2)\right], b^{\mu\nu} \equiv -2i(\bar{\epsilon}_1\Gamma^{\mu\nu}\epsilon_2);$$
$$[\delta_\epsilon, \delta_b]\theta = 0, \qquad [\delta_\epsilon, \delta_b]x^\mu = 0,$$
$$[\delta_\epsilon, \delta_b]z^\mu = -\frac{1}{e} b^\mu{}_\nu(\delta_\epsilon \Pi^\nu) + \frac{i}{e}(\bar{\epsilon}\Gamma^{\mu\nu}\theta)\delta_b \Pi_\nu.$$

Commutators with the Poincare transformations are omitted here since they have the standard form. All the extra terms in the right hand side of Eq.(63) contain $\delta\Pi^\mu \sim \dot{n}^\mu$ and vanish on-shell, where $\dot{n}^\mu = 0$. To find off-shell closed version of these transformations let us note that all extra terms

arise owing to the variation of the Π^μ-term. The latter appears, in its turn, due to variation of the variable z^μ. Following the standard ideology [42,43], these terms can be canceled by replacing $\Pi^\mu \to (\Pi^\mu - B^\mu)$ in Eqs.(61),(62), where the auxiliary variable B^μ has the same transformation properties as Π^μ, $\delta B^\mu = \delta\Pi^\mu$. The resulting off-shell closed version of the global symmetries is

$$\delta_\epsilon \theta = \epsilon, \qquad \delta_\epsilon x^\mu = -i(\bar{\theta}\Gamma^{\mu\nu}\epsilon)n_\nu,$$
$$\delta_\epsilon z^\mu = -i(\bar{\epsilon}\Gamma^{\mu\nu}\theta)\left[\frac{1}{e}\Pi_\nu - B_\nu\right], \qquad \delta_\epsilon B^\mu = \frac{i}{e}(\bar{\epsilon}\Gamma^{\mu\nu}\theta)\dot{n}_\nu; \qquad (64)$$

$$\delta_b x^\mu = b^\mu{}_\nu n^\nu, \qquad \delta_b z^\mu = -\omega^\mu{}_\nu(\frac{1}{e}\Pi^\nu - B^\nu), \qquad \delta_b B^\mu = \frac{1}{e}\omega^\mu{}_\nu \dot{n}^\nu, \qquad (65)$$

while the final form of the action, which is invariant under these transformations, looks as follows:

$$S = \int d\tau \left\{ \frac{1}{2e}\Pi^\mu \Pi_\mu + n^\mu \dot{z}^\mu - \phi(n^2+1) - \frac{1}{2}B^2 \right\}. \qquad (66)$$

Thus, S-algebra consist of Poincare subalgebra $(M^{\mu\nu}, P^\mu)$, and includes generators of the new supertranslations Q_α as well as second-rank Lorentz tensor $Z_{\mu\nu}$, corresponding to transformation (65). The only nontrivial commutator is

$$\{Q_\alpha, Q_\beta\} = 2i(C\Gamma^{\mu\nu})_{\alpha\beta} Z_{\mu\nu}. \qquad (67)$$

Note, that it is not a modification of the super Poincare algebra, but essentially different one, since the commutator of the supertranslations leads to Z-transformation instead of the Poincare shift.

The action (66) is also invariant under the local κ-symmetry transformations

$$\delta\theta = \Pi_\mu \Gamma^\mu \kappa, \qquad \delta x^\mu = i(\bar{\theta}\Gamma^{\mu\nu}\delta\theta)n_\nu,$$
$$\delta z^\mu = -\frac{i}{e}(\bar{\theta}\Gamma^{\mu\nu}\delta\theta)\Pi_\nu, \qquad \delta\xi = -2i(\dot{\bar{\theta}}\delta\theta),$$
$$\delta e = 4ie(\dot{\bar{\theta}}\Gamma^\mu \kappa)n_\mu. \qquad (68)$$

This fact is essential to confirm that physical sector variables obey free equations of motion. The Hamiltonian analysis for the model is very similar to that of the superstring action discussed above, and is as follows. One finds the total Hamiltonian

$$H = \frac{e}{2}p^2 + \xi(pp_z) + \phi(p_z^2+1) + \lambda_e\pi_e + \lambda_\xi p_\xi + \lambda_\phi\pi_\phi + \lambda_B^\mu p_{B\mu} + \lambda_{n\mu}p_n^\mu + \lambda_{z\mu}(p_z^\mu - n^\nu) + L_\alpha\lambda_\theta^\alpha, \tag{69}$$

and the constraints

$$p_n^\mu = 0, \qquad p_z^\mu - n^\mu = 0; \tag{70.a}$$

$$\pi_e = 0, \qquad \pi_\phi = 0, \qquad p_\xi = 0, \qquad p_B^\mu = 0; \tag{70.b}$$

$$p_z^2 = -1, \qquad (pp_z) = 0, \qquad p^2 = 0; \tag{70.c}$$

$$L_\alpha \equiv \bar{p}_{\theta\alpha} - i(\bar{\theta}'\Gamma^\mu)_\alpha p_\mu = 0, \tag{70.d}$$

where $\theta' \equiv p_{z\mu}\Gamma^\mu\theta$. The matrix of the Poisson brackets of fermionic constraints

$$\{L_\alpha, L_\beta\} = 2i(C\Gamma^{\mu\nu})_{\alpha\beta}p_\mu p_{z\nu}, \tag{71}$$

is degenerated on the constraints surface as a consequence of the identity $(\Gamma^{\mu\nu}p_\mu p_{z\nu})^2 = 4[(pp_z) - p^2p_z^2]\mathbf{1} = 0$. It means that half of the constraints are first-class. Also, from the condition $\{L_\alpha, H\} = 0$ one finds equation, which determine λ_θ-multipliers,

$$p_\mu\Gamma^\mu\lambda'_\theta = 0, \qquad \lambda'_\theta \equiv p_{z\mu}\Gamma^\mu\lambda_\theta. \tag{72}$$

After a gauge fixation for the first-class constraints (70.b) (and take into account the second-class constraints (70.a)), the canonical pairs (e,π_e), $(\phi,\pi_\phi), (\xi,p_\xi), (B^\mu,p_B^\mu), (n^\mu,p_n^\mu)$ can be omitted from the consideration. The dynamics of the remaining variables is governed by the equations

$$\dot{z}^\mu = p_z^\mu + i(\bar{\theta}\Gamma^{\mu\nu}\lambda_\theta)p_\nu, \qquad \dot{p}_z^\mu = 0; \tag{73.a}$$

$$\dot{x}^\mu = p^\mu - i(\bar{\theta}\Gamma^{\mu\nu}\lambda_\theta)p_{z\nu}, \qquad \dot{p}^\mu = 0; \tag{73.b}$$

$$\dot{\theta}^\alpha = -\lambda_\theta^\alpha, \qquad \dot{\bar{p}}_{\theta\alpha} = 0. \tag{73.c}$$

The next step is to impose a gauge for the first-class constraints which are contained among the equations (70.d),

$$\Gamma^+\theta' = 0. \tag{74}$$

By virtue of (72),(73.c) all the λ_θ-multipliers can be determined, $\lambda_\theta = 0$, and Eqs.(73.a-c) are reduced to free equations.

The resulting picture corresponds to zero-tension limit of the $D = 11$ superstring action (33). The above consideration of the physical sector allows one to treat the system as a composite one. It consist of the bosonic z^μ-particle (73.a) and the superparticle (73.b), (73.c), subjected to the constraints (70.c). Their free propagation is restricted by the kinematic constraint $(pp_z) = 0$, which means that the superparticle lives on $D = 10$ hyperplane orthogonal to the direction of motion of z^μ-particle.

6 Conclusion.

One can consider $D = 10$ GS superstring action as a lift of SO(8)-covariant formulation for superstring up to the manifestly SO(1,9)-invariant form. In this paper we have considered, in fact, the next step of such a lift, from SO(1,9) up to SO(1,10). The key point was that the action constructed is based on the superalgebra of global symmetries (34)-(36), (67), which is nonstandard super extension of the super Poincare one. It allows one to avoid restrictions of the brane scan followed from demanding of the super Poincare invariance. In the result, we have constructed $N = 1$ S-invariant action for $D = 11$ superstring with the quantum state spectrum identical to $D = 10$, type IIA GS superstring. The only difference is an additional infinite degeneracy in the continuous part of the energy spectrum, related with the zero modes Y^μ, P^μ_y. On the classical level these degrees of freedom may be identified with coordinate and momenta of a free propagating point-like object.

In accordance with the results of Refs.[18] and [20] one expects that critical dimension of the theory is $D = 11$. We hope that similar construction will work for lifting of the $D = 10$ type IIB string to corresponding (10,2) version (see also Ref.[20]). It will be also interesting to apply the scheme developed in this work for construction of the Lagrangian formulation for $(D-2,2)$ SYM equations of motion considered in [35,36].

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Appendix

In this Appendix we describe the minimal spinor representation of the Lorentz group $SO(1,10)$, which is known to have dimension $2^{[D/2]}$. To this aim, it is enough to find eleven 32×32 matrices Γ^μ satisfying the anticommutation relations $\Gamma^\mu\Gamma^\nu + \Gamma^\nu\Gamma^\mu = -2\eta^{\mu\nu}$, $\mu, \nu = 0, 1, \ldots, 10$, $\eta^{\mu\nu} = (+, -, \ldots, -)$. A convenient way is to use the well known 16×16 γ-matrices of $SO(1,9)$ group, which we denote as $\Gamma^m_{\alpha\beta}$, $\tilde{\Gamma}^{m\alpha\beta}$, $m = 0, 1, \ldots, 9$. Their

explicit form is:

$$\Gamma^0 = \begin{pmatrix} \mathbf{1}_8 & 0 \\ 0 & \mathbf{1}_8 \end{pmatrix}, \quad \tilde{\Gamma}^0 = \begin{pmatrix} -\mathbf{1}_8 & 0 \\ 0 & -\mathbf{1}_8 \end{pmatrix},$$
$$\Gamma^i = \begin{pmatrix} 0 & \gamma^i{}_{a\dot{a}} \\ \bar{\gamma}^i{}_{\dot{a}a} & 0 \end{pmatrix}, \quad \tilde{\Gamma}^i = \begin{pmatrix} 0 & \gamma^i{}_{a\dot{a}} \\ \tilde{\gamma}^i{}_{\dot{a}a} & 0 \end{pmatrix},$$
$$\Gamma^9 = \begin{pmatrix} \mathbf{1}_8 & 0 \\ 0 & -\mathbf{1}_8 \end{pmatrix}, \quad \tilde{\Gamma}^9 = \begin{pmatrix} \mathbf{1}_8 & 0 \\ 0 & -\mathbf{1}_8 \end{pmatrix}, \qquad \text{(A.1)}$$

where $\gamma^i{}_{a\dot{a}}$, $\bar{\gamma}^i{}_{\dot{a}a} \equiv (\gamma^i{}_{a\dot{a}})^{\mathrm{T}}$ are real $SO(8)$ γ-matrices [29],

$$\gamma^i\bar{\gamma}^j + \gamma^j\bar{\gamma}^i = 2\delta^{ij}\mathbf{1}_8, \qquad \text{(A.2)}$$

and $i, a, \dot{a} = 1, \ldots, 8$. As a consequence, the matrices Γ^m, $\tilde{\Gamma}^m$ are real, symmetric, and obey the anticommutation relation

$$\{\Gamma^m, \tilde{\Gamma}^n\} = -2\eta^{mn}\mathbf{1}, \qquad \text{(A.3)}$$

where $\eta^{mn} = (+, -, \ldots, -)$. Then a possible realization for the $D = 11$ γ-matrices is

$$\Gamma^\mu = \left\{ \begin{pmatrix} 0 & \Gamma^m \\ \tilde{\Gamma}^m & 0 \end{pmatrix}, \begin{pmatrix} \mathbf{1}_{16} & 0 \\ 0 & -\mathbf{1}_{16} \end{pmatrix} \right\}, \qquad \mu = 0, 1, \ldots, 10. \qquad \text{(A.4)}$$

The properties of Γ^m, $\tilde{\Gamma}^m$ induce the following relations for Γ^μ:

$$(\Gamma^0)^{\mathrm{T}} = -\Gamma^0, \qquad (\Gamma^i)^{\mathrm{T}} = -\Gamma^i,$$
$$(\Gamma^\mu)^* = \Gamma^\mu, \qquad \{\Gamma^\mu, \Gamma^\nu\} = -2\eta^{\mu\nu}\mathbf{1}_{32}, \qquad \text{(A.5)}$$

The charge conjugation matrix C,

$$C \equiv \Gamma^0, \qquad C^{-1} = -C, \qquad C^2 = -\mathbf{1} \qquad \text{(A.6)}$$

can be used to construct the symmetric matrices $C\Gamma^\mu$, $(C\Gamma^\mu)^{\mathrm{T}} = C\Gamma^\mu$. One can introduce antisymmetrized products

$$\Gamma^{\mu\nu} = \frac{1}{2}(\Gamma^\mu\Gamma^\nu - \Gamma^\nu\Gamma^\mu), \qquad \text{(A.7)}$$

which have the following explicit form in terms of the corresponding $SO(1,9)$ and $SO(8)$ matrices:

$$
\begin{aligned}
\Gamma^{0i} &= \begin{pmatrix} \Gamma^{0i} & 0 \\ 0 & \tilde{\Gamma}^{0i} \end{pmatrix} = \left(\begin{array}{cc|cc} 0 & \gamma^i & & \\ \bar{\gamma}^i & 0 & \multicolumn{2}{c}{0} \\ \hline & & 0 & -\gamma^i \\ \multicolumn{2}{c|}{0} & -\bar{\gamma}^i & 0 \end{array}\right), \\
\Gamma^{09} &= \begin{pmatrix} \Gamma^{09} & 0 \\ 0 & \tilde{\Gamma}^{09} \end{pmatrix} = \left(\begin{array}{cc|cc} 1 & 0 & & \\ 0 & -1 & \multicolumn{2}{c}{0} \\ \hline & & -1 & 0 \\ \multicolumn{2}{c|}{0} & 0 & 1 \end{array}\right), \\
\Gamma^{ij} &= \begin{pmatrix} \Gamma^{ij} & 0 \\ 0 & \tilde{\Gamma}^{ij} \end{pmatrix} = \left(\begin{array}{cc|cc} \gamma^{ij} & 0 & & \\ 0 & \bar{\gamma}^{ij} & \multicolumn{2}{c}{0} \\ \hline & & \gamma^{ij} & 0 \\ \multicolumn{2}{c|}{0} & 0 & \bar{\gamma}^{ij} \end{array}\right), \\
\Gamma^{i9} &= \begin{pmatrix} \Gamma^{i9} & 0 \\ 0 & \tilde{\Gamma}^{i9} \end{pmatrix} = \left(\begin{array}{cc|cc} 0 & -\gamma^i & & \\ \bar{\gamma}^i & 0 & \multicolumn{2}{c}{0} \\ \hline & & 0 & -\gamma^i \\ \multicolumn{2}{c|}{0} & \bar{\gamma}^i & 0 \end{array}\right), \\
\Gamma^{0,10} &= \begin{pmatrix} 0 & -\Gamma^{0} \\ \tilde{\Gamma}^{0} & 0 \end{pmatrix} = \left(\begin{array}{cc|cc} & & \mathbf{1} & 0 \\ \multicolumn{2}{c|}{0} & 0 & \mathbf{1} \\ \hline \mathbf{1} & 0 & & \\ 0 & \mathbf{1} & \multicolumn{2}{c}{0} \end{array}\right), \\
\Gamma^{i,10} &= \begin{pmatrix} 0 & -\Gamma^{i} \\ \tilde{\Gamma}^{i} & 0 \end{pmatrix} = \left(\begin{array}{cc|cc} & & 0 & -\gamma^i \\ \multicolumn{2}{c|}{0} & -\bar{\gamma}^i & 0 \\ \hline 0 & \gamma^i & & \\ \bar{\gamma}^i & 0 & \multicolumn{2}{c}{0} \end{array}\right),
\end{aligned}
\tag{A.8}
$$

$$
\Gamma^{9,10} = \begin{pmatrix} 0 & -\Gamma^{9} \\ \tilde{\Gamma}^{9} & 0 \end{pmatrix} = \left(\begin{array}{cc|cc} & & -\mathbf{1} & 0 \\ \multicolumn{2}{c|}{0} & 0 & \mathbf{1} \\ \hline \mathbf{1} & 0 & & \\ 0 & -\mathbf{1} & \multicolumn{2}{c}{0} \end{array}\right), \tag{A.9}
$$

where $i = 1, 2, \ldots, 8$ and Γ^{0i}, Γ^{09}, $\Gamma^{0,10}$ are symmetric, whereas Γ^{ij}, Γ^{i9}, $\Gamma^{i,10}$, $\Gamma^{9,10}$ are antisymmetric. Besides, these matrices are real and, as a consequence of Eq. (A5), obey the commutation relations of the Lorentz algebra.

Under the action of the Lorentz group a $D = 11$ Dirac spinor is transformed as

$$\delta\theta = -\frac{1}{4}\omega_{\mu\nu}\Gamma^{\mu\nu}\theta. \tag{A.10}$$

Since $\Gamma^{\mu\nu}$ matrices are real, the reality condition $\theta^* = \theta$ is compatible with (A.10) which defines a Majorana spinor. To construct Lorentz-covariant bilinear combinations, one can note that

$$\delta\bar{\theta} = +\frac{1}{4}\omega_{\mu\nu}\bar{\theta}\Gamma^{\mu\nu}, \qquad \bar{\theta} \equiv \theta^{\mathrm{T}}C. \tag{A.11}$$

Then the combination $\bar{\psi}\Gamma^{\mu}\theta$ is a vector under the action of the $D = 11$ Lorentz group,

$$\delta(\bar{\psi}\Gamma^{\mu}\theta) = \omega^{\mu}{}_{\nu}(\bar{\psi}\Gamma^{\mu}\theta). \tag{A.12}$$

The following properties are also useful

$$
\begin{aligned}
\bar{\psi}\Gamma^{\mu_1}\cdots\Gamma^{\mu_k}\phi &= (-1)^k\bar{\phi}\Gamma^{\mu_k}\cdots\Gamma^{\mu_1}\psi \\
\bar{\psi}\Gamma^{\mu_1\cdots\mu_k}\phi &= (-1)^{\frac{k(k+1)}{2}}\bar{\phi}\Gamma^{\mu_1\cdots\mu_k}\psi.
\end{aligned}
\tag{A.13}
$$

It is possible to decompose a $D = 11$ Majorana spinor in terms of its $SO(1,9)$ and $SO(8)$ components. Namely, it follows from Eq. (A.8) that the decomposition

$$\theta = (\bar{\theta}_\alpha, \theta^\alpha), \tag{A.14}$$

where $\alpha = 1, \ldots, 16$, holds. Here θ and $\bar{\theta}$ are Majorana–Weyl spinors of opposite chirality with respect to the $SO(1,9)$ subgroup of the $SO(1,10)$ group. It follows from the third equation (A8) that in the decomposition

$$\theta = (\theta_a, \bar{\theta}'_{\dot{a}}, \theta'_a, \bar{\theta}_{\dot{a}}), a, \dot{a} = 1, \ldots, 8, \tag{A.15}$$

the pairs θ_a, θ'_a and $\bar{\theta}'_{\dot{a}}$, $\bar{\theta}_{\dot{a}}$ are $SO(8)$ spinors of opposite chirality.

It is convenient to define the $D = 11$ light-cone Γ-matrices

$$\Gamma^+ = \frac{1}{\sqrt{2}}(\Gamma^0 + \Gamma^9) = \sqrt{2}\left(\begin{array}{cc|cc} \multicolumn{2}{c|}{0} & \mathbf{1}_8 & 0 \\ & & 0 & 0 \\ \hline 0 & 0 & \multicolumn{2}{c}{0} \\ 0 & -\mathbf{1}_8 & & \end{array}\right),$$

$$\Gamma^- = \frac{1}{\sqrt{2}}(\Gamma^0 - \Gamma^9) = \sqrt{2}\left(\begin{array}{cc|cc} \multicolumn{2}{c|}{0} & 0 & 0 \\ & & 0 & \mathbf{1}_8 \\ \hline -\mathbf{1}_8 & 0 & \multicolumn{2}{c}{0} \\ 0 & 0 & & \end{array}\right),$$

$$\Gamma^i = \begin{pmatrix} 0 & \Gamma^i \\ \tilde{\Gamma}^i & 0 \end{pmatrix},$$

$$\Gamma^{10} = \begin{pmatrix} \mathbf{1}_{16} & 0 \\ 0 & -\mathbf{1}_{16} \end{pmatrix}, \tag{A.16}$$

Then the equation $\Gamma^+\theta = 0$ has a solution

$$\theta = (\theta_a, 0, 0, \bar{\theta}_{\bar{a}}). \tag{A.17}$$

Besides, under the condition $\Gamma^+\theta = 0$ the following identities:

$$\bar{\theta}\Gamma^+\partial_1\theta = \bar{\theta}\Gamma^i\partial_1\theta = \bar{\theta}\Gamma^{10}\partial_1\theta = 0, \qquad (\bar{\theta}\Gamma^\mu\partial_1\theta)\Gamma^\mu\theta = 0, \tag{A.18}$$

hold.

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