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Abstract: We study the problem of negative contributions to final momentum distribution during the freeze out through 3-dimensional hypersurfaces with space-like normal. We suggest a solution for this problem based on the mechanism of continuous emission of particles. We show how the final particle spectrum is obtained in a simple one-dimensional model.

1 Introduction

Fluid dynamical models, especially their simpler versions are very popular in heavy ion physics, because they connect directly collective macroscopic matter properties, like the Equation of State (EoS) or transport properties, to measurables.

Particles which leave the system and reach the detectors, can be taken into account via source (drain) terms in the 4-dimensional space-time based on kinetic considerations, or in a more simplified way via freeze out (FO) or final break-up schemes, where the frozen out particles are formed on a 3-dimensional hypersurface in space-time. This information is then used as input to compute measurables such as two-particle correlation, transverse-, longitudinal-, radial-, and cylindrical- flow, transverse momentum and transverse mass spectra, etc.

In this paper we concentrate on freeze out. A basic standard assumption in this case is that freeze out happens across a hypersurface as already mentioned, so it can be pictured as a discontinuity where the kinetic properties of the matter, such as energy density and momentum distribution change suddenly. The hypersurface is an idealization of a layer of finite thickness (of the order of a mean free path or collision time) where the frozen-out particles are formed and the interactions in the matter become negligible. The dynamics of this layer is described in different kinetic models such as Monte Carlo models [1, 2] or four-volume emission models [3, 4, 5, 6, 7]. In fact, the zero thickness limit of such a layer is an over-idealization of kinetic freeze out in heavy ion reactions, while it is applicable on more macroscopic scales like in astrophysics [1].

Two types of hypersurfaces are distinguished: those with a space-like normal vector, $d\sigma^\mu d\sigma_\mu = -1$ (e.g. events happening on a propagating 2-dimensional surface) and those with a time-like normal vector $d\sigma^\mu d\sigma_\mu = 1$ (a common example of which is an overall sudden change in a finite volume).

Once the freeze out surface is determined, one can compute measurables. Landau when drafting his hydrodynamical model[8], just evaluated the flow velocity distribution at freeze out, and this distribution served as a basis for all observables. This approach was used in early fluid dynamical simulations of heavy ion collisions also [9, 10, 11]. This procedure was improved to add thermal velocities to the flow velocities at freeze out, by Milekhin[12, 13] and later by Cooper and Frye [14]. This method is widely used, however it rises at least three problems [15].

First, in some cases before the 90's, the possible existence of discontinuities across hypersurfaces with time-like normal vectors was not taken into account or considered unphysical[2] [19, 20, 21, 22]. This point was studied recently [23] so we do not discuss it further.

Second, since the kinetic properties of the matter are different on the two sides of the front, the explicit evaluation of conservation laws across the freeze out surface should be taken into account which is not always easy to implement. In some (simple) cases [24, 25, 26], these conservation laws are enforced and discussed. For example in [24], it was pointed out that the

[1] On the other hand, if kinetic freeze out coincides with a rapid phase transition, like in the case of rapid deconfinement transition of supercooled quark-gluon plasma, the short freeze out hypersurface idealization may still be applicable even for heavy ion reactions. It is, however, beyond the scope of this work to study the freeze out dynamics and kinetics in this latter case.

[2] Taub[16] discussed discontinuities across propagating hypersurfaces, which have a space-like normal vector. If one applies Taub's formalism from 1948, to freeze out surfaces with time-like normal vectors, one gets a usual Taub adiabat but the equation of the Rayleigh line yields imaginary values for the particle current across the front. Thus these hypersurfaces were thought unphysical. However more recently, Taub's approach has been generalized to these hypersurfaces [17] (see also [18]) while eliminating the imaginary particle currents arising from the equation of the Rayleigh line. Thus it is possible to take into account conservation laws exactly across any surface of discontinuity with relativistic flow.

freeze out momentum distribution for hypersurfaces with time-like normal may become locally anisotropic. We remind the procedure that should be followed in section 2.

The *third* problem is a conceptual problem arising in the Cooper-Frye freeze out description when we apply it to a hypersurface with space-like normal: it is the problem of negative contributions (see section 2). This is the main subject of this paper. This problem appears in all freeze out calculations up to now we are aware of, and to our knowledge it was not satisfactorily discussed yet in the literature. It was recognized by some of those who applied the Cooper-Frye freeze out description before [25, 26, 27]. A possible solution was presented in part 2 of ref.[26] for noninteracting massless particles, in 1+1 dimension. In section 3 we generalize the results from [26] and in section 4 we suggest an improved solution to this problem.

2 Conservation laws across idealized freeze out discontinuities

In the zero width limit of the freeze out domain (freeze out surface), the energy - momentum tensor changes discontinuously across this surface. Consequently, the four-vector of the flow velocity may also change [17, 28, 29]. These changes should be discussed in terms of the conservation laws.

The invariant number of conserved particles (world lines) crossing a surface element, $d\sigma^\mu$, is

$$dN = N^\mu \, d\sigma_\mu \ , \tag{1}$$

and the total number of all the particles crossing the FO hyper-surface, S, is

$$N = \int_S N^\mu \, d\sigma_\mu \ . \tag{2}$$

If we insert the kinetic definition of N^μ

$$N^\mu = \int \frac{d^3p}{p^0} \, p^\mu \, f_{FO}(x,p) \ ,$$

into eq. (1) we obtain the Cooper-Frye formula[14]:

$$E\frac{dN}{d^3p} = \int f_{FO}(x,p) \, p^\mu d\sigma_\mu \ , \tag{3}$$

where $f_{FO}(x,p)$ is the post FO phase space distribution of frozen-out particles which is not known from the fluid dynamical model. The problem is to choose its form correctly. Usually one assumes that the pre-FO momentum distribution as well as the post FO distribution are both local thermal equilibrium distributions boosted by the local collective flow velocity on the actual side of the freeze out surface, although the post FO distribution need not be a thermal distribution. Parametrizing the post FO distribution as thermal, $f_{FO}(x,p;T,n,u^\mu)$, and knowing the pre-FO baryon current and energy-momentum tensor, N_0^μ and $T_0^{\mu\nu}$, we can calculate the post freeze out quantities N^μ and $T^{\mu\nu}$ from the relations [16, 17]

$$[N^\mu \, d\sigma_\mu] = 0 \quad \text{and} \quad [T^{\mu\nu} \, d\sigma_\mu] = 0, \tag{4}$$

across a surface element[3] of normal vector $d\sigma^\mu$. Here $[A] \equiv A - A_0$.This fixes the parameters, T, n, u^μ, of our post FO momentum distribution, $f_{FO}(x,p;T,n,u^\nu)$.

We can now remind briefly what the problem of negative contributions to the Cooper-Frye formula is and a possible way out. For a FO surface with time-like normal, both p^μ and $d\sigma^\mu$ are time-like vectors, thus

$$p^\mu d\sigma_\mu > 0,$$

and the integrand in the integral (3) is always positive. For a FO surface with space-like normal, p^μ is time-like and $d\sigma^\mu$ is space-like, thus $p^\mu d\sigma_\mu$ can be both positive and negative. (Note that p^μ may point now both in the post- and pre- FO directions.) Thus the integrand in the integral (3) may change sign in the integration domain, and this indicates that part of the distribution contributes to a current going back, into the front while another part is coming out of the front. On the pre-FO side p^μ is unrestricted and $p^\mu d\sigma_\mu$ may have both signs, because we are supposing that pre-FO phase is in the thermal equilibrium. However, in the zero width limit of the FO front, it is difficult to understand such a situation. What happens actually is that internal rescatterings occur inside the finite FO domain and feed particles back to the pre-FO side to maintain the thermal equilibrium there. On the post-FO side, however, we do not allow rescattering and back scattering any more. If a particle has passed the freeze out domain it cannot scatter back. In other words, the post-FO distribution should have the form[25, 26],

$$f^*_{FO}(x,p,d\sigma^\mu) = f_{FO}(x,p)\Theta(p^\mu \, d\sigma_\mu). \tag{5}$$

[3] In numerical calculations the local freeze out surface can be determined most accurately via self-consistent iteration [26, 30].

Consequently, this distribution cannot be an ideal gas distribution. (On the pre-FO side, the distribution may or not be ideal). The conservation laws across a small element of the freeze out front with space-like normal take the form:

$$\int_S \left(\int \frac{d^3p}{p^0} \; f^*_{FO}(x,p,d\sigma^\gamma)\; p^\mu \right) \, d\sigma_\mu = \int_S N_0^\mu(x) \; d\sigma_\mu \; , \tag{6}$$

$$\int_S \left(\int \frac{d^3p}{p^0} \; f^*_{FO}(x,p,d\sigma^\gamma)\; p^\mu p^\nu \right) \, d\sigma_\mu = \int_S T_0^{\mu\nu}(x) \; d\sigma_\mu \; . \tag{7}$$

3 Conserved currents for cut Jüttner distribution

We now study the particular case where f_{FO} is a Jüttner (or relativistic Boltzmann [31]) distribution and so f^*_{FO} is a cut Jüttner distribution. This case for massless particles was considered in part 2 of ref.[26] (following [25]). The cut selects particles with momenta $p^\mu d\sigma_\mu > 0$, in the "Reference Frame of the Gas" (RFG). Thus, we have to observe that the RFG frame is *not* the Local Rest frame, and the velocity of the RFG frame, u^μ_{RFG}, is *not* the flow velocity of the post FO matter.

In the RFG frame, the baryon current reads

$$\begin{aligned} N^0 &= \tfrac{\tilde{n}}{4}\left[vA + a^2\mathcal{K}_2(a,b) - \tfrac{(b^2-a^2)^{3/2}}{3} e^{-b}\right] \stackrel{m=0}{\longrightarrow} \tilde{n}(\mu,T)\tfrac{v+1}{2}\, , \\ N^x &= \tfrac{\tilde{n}}{8}\left[\left(-v^2+1\right)A - a^2 e^{-b}\right] \stackrel{m=0}{\longrightarrow} \tilde{n}(\mu,T)\tfrac{-v^2+1}{4}\, , \end{aligned} \tag{8}$$

where $\tilde{n} = 8\pi T^3 e^{\mu/T}(2\pi\hbar)^{-3}$, $a = \frac{m}{T}$, so that $\hat{n}(\mu,T) = \tilde{n}a^2K_2(a)/2$ is the invariant scalar density of the symmetric massless Jüttner gas, $b = a/\sqrt{1-v^2}$, $v = d\sigma_0/d\sigma_x$, $A = (2+2b+b^2)e^{-b}$, and

$$\mathcal{K}_n(z,w) \equiv \frac{2^n(n)!}{(2n)!}\; z^{-n}\int_w^\infty dx\; (x^2-z^2)^{n-1/2}\;\; e^{-x}\; ,$$

i.e. $\mathcal{K}_n(z,z) = K_n(z)$. When evaluating the limits we used the relation $\mathcal{K}_n(a,b) \stackrel{a=b}{\longrightarrow} K_n(a) \stackrel{a=0}{\longrightarrow} 2^{n-1}(n-1)!a^{-n}$. This baryon current may then be Lorentz transformed into the Eckart Local Rest (ELR) frame of the post FO matter, which moves with $u^\mu_E = N^\mu/(N^\nu N_\nu)^{1/2} = \gamma_E(1, v_E, 0, 0)|_{RFG}$ in the RFG, or alternatively into the Rest Frame of the Freeze out front (RFF), where $d\sigma_\mu = (0,1,0,0)|_{RFF}$ and the velocity of the RFG is $u^\mu_{E,RFG} = \gamma_\sigma(1, v, 0, 0)|_{RFF}$. Then the Eckart flow velocity of the matter represented by the cut Jüttner distribution viewed from the RFF is $u^\mu_E = \gamma_c(1, v_c, 0, 0)|_{RFF}$, where $v_c = (v + v_E)/(1 + vv_E)$.

The proper density (i.e. the density in the ELR frame) is obtained as

$$n(\mu, T, v) = \sqrt{N^\nu N_\nu} =$$

$$\frac{\tilde{n}}{4}\sqrt{\left[vA + a^2\mathcal{K}_2(a,b) - \frac{(b^2-a^2)^{3/2}}{3}e^{-b}\right]^2 - 4\left[A\left(-v^2+1\right) - a^2e^{-b}\right]^2}\,.$$

Note that the proper density of the cut Jüttner distribution, n, is reduced compared to the proper density of the complete spherical Jüttner distribution, $\tilde{n}$.

The energy momentum tensor in the RFG is

$$\begin{aligned}
T^{00} &= \tfrac{3\tilde{n}T}{2}\left\{\tfrac{a^2}{2}\mathcal{K}_2(a,b) + \tfrac{a^3}{6}\mathcal{K}_1(a,b) + Bv\right\},\\
T^{0x} &= \tfrac{3\tilde{n}T}{4}\left[(-v^2+1)B - \tfrac{a^2}{6}(b+1)e^{-b}\right\},\\
T^{xx} &= \tfrac{\tilde{n}T}{2}\left\{\tfrac{a^2}{2}\mathcal{K}_2(a,b) + \tfrac{v^3B}{3}\right\},\\
T^{yy} &= \tfrac{3\tilde{n}T}{4}\left\{v(1-v^2/3)B + \tfrac{a^2}{3}\mathcal{K}_2(a,b) - \tfrac{va^2}{6}(b+1)e^{-b} - \tfrac{va^3}{6}\mathcal{K}_1(a,b)\right\},
\end{aligned} \tag{9}$$

where $B = (1 + b + b^2/2 + b^3/6)e^{-b}$ and $T^{zz} = T^{yy}$. This energy-momentum tensor may then be Lorentz transformed into the Landau Local Rest (LLR) frame of the post FO matter, which moves with u^μ_L in the RFG, or into the Rest Frame of the Freeze out Front (RFF) where $d\sigma^\mu = (0, 1, 0, 0)$. Alternatively both can be transformed to the frame where we want to evaluate the conservation laws, eq. (4), and the parameters of the post FO, cut Jüttner distribution can be determined so, that it satisfies the conservation laws. In the massless limit the energy momentum tensor in the RFG is:

$$T^{00} = 3\tilde{n}T\,(v+1)\,/2\,,\quad T^{0x} = 3\tilde{n}T\,(-v^2+1)\,/4\,,$$

$$T^{xx} = \tilde{n}T\,(v^3+1)\,/2\,,\quad T^{yy} = \tfrac{\tilde{n}T}{4}\,(2+3v-v^3)\,,$$

and $T^{zz} = T^{yy}$.

Thus, (i) if we know the 3 parameters of the pre FO flow and (ii) the local freeze out surface from kinetic considerations, then assuming that the post FO distribution, $f^*_{FO}(p,x)$, is a cut Jüttner distribution, we can completely determine the parameters of the post FO matter from the conservation laws (6,7).

4 Freeze out distribution from kinetic theory

We can calculate the kinetic freeze out distribution based on the four volume emission models[4, 5, 6]. In order to illustrate the physical mechanism of this freeze out process, let us study a simple one-dimensional flow. We suppose an infinite tube where a stationary flow of a fluid is supplied from the left ($x < 0$) so that the freeze out occurs for the positive direction of x. Such an idealized model has the advantage of being not only simple but also useful to illustrate the basic roles of the conservation laws in a freeze out process. Because of the high symmetry of the problem, the conservation laws become very stringent.

In the four volume emission model, we introduce the escape probability

$$\mathcal{P}(\vec{r}, t, \vec{p}) \equiv e^{-\int_t^\infty \sigma v_{rel} n(\vec{r}+\vec{p}/Et, t)dt}, \tag{10}$$

where n is the total density, $v = p/E$, the velocity of the particle, σ, the total cross section and v_{rel}, the relative velocity. For a stationary one dimensional case, we can express it as [4]

$$\mathcal{P}(\vec{r}, t, \vec{p}) \to \mathcal{P}(x, \cos\theta) = e^{-\int_x^\infty \sigma n(x)\frac{dx}{\cos\theta}}, \tag{11}$$

where

$$\cos\theta = \frac{p^x}{p}, \tag{12}$$

for $\cos\theta \geq 0$. For $\cos\theta \leq 0$, the upper limit of the integral is $-\infty$, and for $x \to -\infty$, $n(x) \to$ *constant* and $\mathcal{P} \to 0$.

It is this escape probability that determines the free particle distribution as a fraction of the total particle distribution

$$f_{free}(x,p) = \mathcal{P}f(x,p), \tag{13}$$

where

$$f(x,p) = f_{free}(x,p) + f_{int}(x,p) \tag{14}$$

and f_{int} is the interacting particle distribution. The total density $n(x)$ is given as

$$n(x) = \int d^3p f(x,p). \tag{15}$$

[4]We are considering fast particles so that $v_{rel} \sim v$. A formula similar to (11) is also obtained for massless particles.

It is obvious from Eq.(11) that if n is constant, then $\mathcal{P}$ becomes identically zero and the post-FO component can never emerge. If the system is truly one-dimensional for all x values, then the total density n should vanish for large x, otherwise $\mathcal{P}$ vanishes. However, this contradicts the conservation of flux in the stationary case: $N^1(x=\infty) = N^1(x=0) = \int d^3p(p^x/E)f(x=\infty,p) \neq 0$ is incompatible with $n(x=\infty) = \int d^3pf(x=\infty,p)) = 0$, since the velocity $p^x/E \leq 1$. Therefore, to get a stationary one-dimensional flow the system should have a finite size in the freeze out direction.

Suppose that there exists a boundary at $x = L > 0$, so that for $x > L$, the density falls off very rapidly and the escape probability is almost zero there. Such a situation happens for a semi-infinite tube open to the vacuum at $x = L$. We can write

$$\mathcal{P}(x,\cos\theta) = e^{-\int_x^L \sigma n(x) \frac{dx}{\cos\theta}} \tag{16}$$

for $\cos\theta \geq 0$. For $\cos\theta \leq 0$, the upper limit of the integral is $-\infty$ as before, so that for $x \to -\infty$, $n(x) \to$ *constant* and [5] $\mathcal{P} \to 0$. The free and interacting distributions are determined by

$$f_{free}(x,p) = \mathcal{P}f(x,p), \tag{17}$$

$$f_{int}(x,p) = (1-\mathcal{P})f(x,p). \tag{18}$$

In the following, we show that the above equations together with the conservation laws determine all the distribution functions when we assume a thermal spectrum for the interacting component.

First, note that all the distributions are specified if,

1. the interacting flow velocity, $v_{int}(x)$,
2. the interacting temperature, $T(x)$,
3. the interacting density, $n_{int}(x)$, and
4. the escape probability in the x direction,

$$P_0(x) \equiv e^{-\int_x^L \sigma n(x) dx}, \tag{19}$$

[5] We are using the frame of reference where all the distributions are stationary, that is, the frame in which the FO domain is at rest.

are known. To see this, first we write

$$\mathcal{P}(x, \cos\theta) = \{P_0(x)\}^{1/\cos\theta}, \tag{20}$$

and express the total and free distributions in terms of f_{int}.

$$\begin{aligned} f(x,p) &= \frac{1}{1-\mathcal{P}} f_{int}(x,p) = \frac{1}{1-\mathcal{P}} n_{int}(x) \frac{1}{Z} e^{-p^\mu u_\mu^{int}/T}, & (21) \\ f_{free}(x,p) &= \frac{\mathcal{P}}{1-\mathcal{P}} n_{int}(x) \frac{1}{Z} e^{-p^\mu u_\mu^{int}/T}, & (22) \end{aligned}$$

where

$$u_\mu^{int} = \begin{pmatrix} \gamma \\ -\gamma v_{int} \end{pmatrix},$$

with $\gamma = 1/\sqrt{1 - v_{int}(x)^2}$ and Z is the normalization factor,

$$Z = Z(T) = \int d^3p e^{-p^\mu u_\mu^{int}/T}.$$

In our stationary regime, the conservation laws are expressed as

$$\begin{aligned} N^1(x) &= Const = N^1(0), & (23) \\ T^{01}(x) &= Const = T^{01}(0), & (24) \end{aligned}$$

and

$$T^{11}(x) = Const = T^{11}(0), \tag{25}$$

where

$$N^1(x) \equiv \int d^3p \frac{p^x}{p_0} f(x,p),$$

$$T^{01}(x) \equiv \int d^3p\, p^x f(x,p),$$

$$T^{11}(x) \equiv \int d^3p\, \frac{(p^x)^2}{p_0} f(x,p).$$

Once the initial values $N^1(0), T^{01}(0)$ and $T^{11}(0)$ are specified, these equations, together with Eq. (21), determine algebraically $T(x)$, $v_{int}(x)$ and $n_{int}(x)$, at each x as functions of $P_0(x)$.

On the other hand, from Eq.(19)

$$\frac{1}{P_0}\frac{dP_0}{dx} = \sigma n(x), \tag{26}$$

for $\cos\theta > 0$ and $P_0 = 0$ for $\cos\theta \leq 0$, but,

$$n(x) = \int d^3p f(x,p) = n_{int}(x) \int d^3p \frac{1}{1-\mathcal{P}} \frac{1}{Z} e^{-p^\mu u_\mu^{int}/T}, \qquad (27)$$

so that we get a differential equation for P_0,

$$\frac{1}{P_0}\frac{dP_0}{dx} = \sigma n_{int}(x) \int d^3p \frac{1}{1-\mathcal{P}} \frac{1}{Z} e^{-p^\mu u_\mu^{int}/T}, \qquad (28)$$

which can be solved, for example, from $x = 0$ to the right ($x > 0$) specifying some small value of P_0 till P_0 tends to unity. Note that P_0 is an increasing function of x, as expected. We can also solve the differential equation for $x < 0$, too. We have $P_0(x) \to 0$ for $x \to -\infty$.

To compute n, N^1, T^{01} and T^{11}, we need to know the integrals:

$$I_1[P_{int}, T, v_{int}] \equiv \int d^3p \frac{1}{1 - {P_0}^{1/\cos\theta}} e^{-p^\mu u_\mu^{int}/T},$$

$$I_2[P_{int}, T, v_{int}] \equiv \int d^3p \frac{p\cos\theta}{p_0} \frac{1}{1 - {P_0}^{1/\cos\theta}} e^{-p^\mu u_\mu^{int}/T},$$

$$I_3[P_{int}, T, v_0] \equiv \int d^3p \frac{p\cos\theta}{1 - {P_0}^{1/\cos\theta}} e^{-p^\mu u_\mu^{int}/T},$$

$$I_4[P_{int}, T, v_{int}] \equiv \int d^3p \frac{p^2\cos^2\theta}{p_0} \frac{1}{1 - {P_0}^{1/\cos\theta}} e^{-p^\mu u_\mu^{int}/T}.$$

for $\cos\theta \geq 0$. For $\cos\theta < 0$, all integrals should be set to zero. Note that these functions are not scalar, and the above expressions are valid in the frame where the density distributions are at rest [6].

5 Conclusions

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[6] In particular, if we want to evaluate the integrals in the local rest frame, $p \to p^*$, $p^\mu u_\mu \to p^{*0} = E^*$, $I_1[P_{int}, T, v_{int}] \equiv \int d^3p^* \frac{1}{1-\{P_0(x)\}^{1/\cos\theta}} e^{-E^*/T}$, etc, with $\cos\theta = \frac{p_x}{p} = \frac{\gamma p^* \cos\theta^* + \gamma v_{int} E^*}{\sqrt{E^{*2} - p^{*2}\cos^2\theta^* + (\gamma p^* \cos\theta^* + \gamma v_{int} E^*)^2}}$ and the limit of integral is restricted by $\gamma p^* \cos\theta^* + \gamma v_{int} E^* \geq 0$.

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