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B.V. Carlson
L.F. Canto
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COULOMB EXCITATION OF A DAMPED OSCILLATOR AND THE BRINK-AXEL MECHANISM

B.V. Carlson[1], L.F. Canto[2], S. Cruz-Barrios[3], M.S. Hussein[4], and A.F.R. de Toledo Piza[4]
[1]*Departamento de Física, Instituto Tecnológico da Aeronáutica - CTA, 12228-900 São José dos Campos, SP, Brazil*
[2]*Instituto de Física da Universidade Federal do Rio de Janeiro, CP 68528, 21945-970 Rio de Janeiro, RJ, Brazil*
[3]*Departamento de Física Aplicada, Universidad de Sevilla, 41080 Sevilla, Spain*
[4]*Instituto de Física, Universidade de São Paulo, CP 66318, 05315-970 São Paulo, SP, Brazil*
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Multiple Coulomb excitation of collective nuclear modes is examined in the context of an extremely schematic dynamic model consisting of an harmonic oscillator, taken to represent the relevant collective mode, damped by linear coupling to a "bath" consisting also of harmonic oscillators, and forced by an applied external pulse. The (well known) exact solution of the model allows for an estimate cross-sections which takes into account the joint effect of *all* excitation mechanisms leading to definite excitation energy domains, including the Brink-Axel mechanism, which occurs very naturally in this type of model. The semi-classical estimate of cross-sections leading to the two phonon domain shows enhancement with respect to the corresponding values obtained in the case of no damping. The magnitude of the enhancement decreases as the beam energy increases (or, equivalently, as the impact parameter averaged time width of the external pulse decreases), for model parameters chosen to conform to the appropriate nuclear orders of magnitude.

I. INTRODUCTION

In a frequently used description of Giant Resonances in atomic nuclei one associates them to special "doorway" states in the host nucleus which are spread through the effect of couplings to other nearby non-collective states. A similar picture can be extended also to double excitations such as those that have been observed for the giant isovector dipole mode [1,2]. The convenience of this type of picture stems from the fact that it can be readily implemented in terms of suitable decompositions of the phase space of the nuclear system through the use of appropriate projection operators, leading to the proper sorting of the various dynamical processes involved. In the sorting process one usually introduces energy averaged amplitudes corresponding to the doorway effects, which require the subsequent calculation of fluctuation contributions to be incoherently added to the results obtained from them.

When the various doorway states involved imply the presence of a vibrational band, however, as it is the case when one considers multi-phonon excitations of a giant isovector dipole mode, an alternate picture suggests itself in which one is led to single out not just a given state, or a group of states, but rather a collective *degree of freedom* [3]. Unlike in the case of a set of doorways, to which one associates a certain subspace of the full quantum phase space which still involves all the nuclear degrees of freedom, in this case one is led to formulate the dynamics in a *factorized* phase space, involving on the one side the relevant collective degree of freedom and, on the other side, the remaining ones, considered as being "non collective" in the adopted sense. The resulting picture is that of a damped oscillator, the non-collective degrees of freedom playing the role of a reservoir which exchanges energy with the collective mode. The dynamics of such a damped oscillator can be represented, albeit somewhat schematically, by a Hamiltonian of the form

$$H_n = \omega_d d^\dagger d \otimes \mathbf{1}_k + \mathbf{1}_d \otimes \sum_k \mid \epsilon_k\rangle \epsilon_k \langle \epsilon_k \mid + \sum_{kk'} \left(\mid \epsilon_{k'}\rangle g_{k'k} d \langle \epsilon_k \mid + \mid \epsilon_k \rangle g^*_{k'k} d^\dagger \langle \epsilon_{k'} \mid \right)$$

where the first two terms represent the collective linear mode of frequency ω_d and the reservoir with spectrum $\{\epsilon_k\}$ and eigenstates $\{\mid \epsilon_k\rangle\}$ respectively, while the last term couples these two subsystems. The ground state of the coupled system will have simple properties when the coupling can be seen as restricted so that $g_{k'k} \equiv 0$ unless $\epsilon_{k'} > \epsilon_k$. In this case it reduces in fact to the product state $\mid 0_d\rangle \otimes \mid \epsilon_0\rangle$, this latter ket being the reservoir ground state. In fact, when this condition does not apply one has a correlated ground state which contains components involving combined excitations of the two subsystems.

II. DRIVEN COUPLED OSCILLATORS

The standard semi-classical picture of the multiple Coulomb excitation of the damped Giant Isovector mode can be schematically described by a Hamiltonian of this type, to which one adds furthermore an external driving term for

$$S_1 \equiv \sum_\nu \omega_\nu \mid x_{0\nu} \mid^2 = \omega_d \qquad \text{and} \qquad \sum_\nu \omega_\nu^2 \mid x_{0\nu} \mid^2 - S_1^2 = \sum_k \mid g_k \mid^2 . \tag{2.4}$$

The first of these simply equates the collective frequency ω_d to the centroid of the fine structure frequencies ω_ν, while the second relates the mean-square deviation of the fine-structure frequencies to the sum of the absolute squares of the coupling constants g_k. In the case of reservoir oscillator frequencies forming an endless picket-fence and k-independent couplings g_k, this quantity diverges as the weights $\mid x_{0\nu} \mid^2$ decrease as ω_ν^{-2} when $\omega_\nu \gg \omega_d$.

Eq. (2.3) describes a set of *independent* driven oscillators, which can thus be dealt with separately, one by one. The relevant initial state in the present context is that in which all oscillators are in their respective ground states. In order to obtain the solution one e.g. considers the Heisenberg equation of motion for the operator $c_\nu(t) = U_\nu(t) c_\nu U_\nu^\dagger(t)$, $U_\nu(t)$ being the full evolution operator for the corresponding normal mode, which reads

$$i\, \dot{c}_\nu(t) = \omega_\nu c_\nu(t) + x_{0\nu}^* f(t)$$

and is readily solved as

$$c_\nu(t) = e^{-i\omega_\nu t} c_\nu - i\, x_{0\nu}^* e^{-i\omega_\nu t} \int_0^t dt' e^{i\omega_\nu t'} f(t')$$

revealing that, besides acquiring the usual harmonic phase, $c_\nu(t)$ undergoes a coherent displacement due to the action of the driving force, the corresponding amplitude being $\exp(-i\omega_\nu t)\alpha_\nu(t)$, with

$$\alpha_\nu(t) \equiv -i\, x_{0\nu}^* \int_0^t dt' e^{i\omega_\nu t'} f(t'). \tag{2.5}$$

It is also an easy matter to use the initial condition and write down the state at time t of this oscillator in the Schrödinger picture. One gets

$$U_\nu(t) \mid 0_\nu\rangle = \mid e^{-i\omega_\nu t} \alpha_\nu(t)\rangle \equiv e^{-\frac{|\alpha_\nu(t)|^2}{2}} \sum_{n=0}^{\infty} \frac{\left[e^{-i\omega_\nu t}\alpha_\nu(t)\right]^n}{\sqrt{n!}} \mid n_\nu\rangle \tag{2.6}$$

where the states $\mid n_\nu\rangle$ are the usual phonon number eigenstates for the ν-th normal mode. In the case of the short pulses one considers in connection with the semi-classical treatment of Coulomb excitation, letting the pulse start sufficiently late (after $t = 0$), one gets for $\alpha_\nu(T)$ at asymptotically large times T essentially the Fourier transform of the pulse, a time independent quantity.

III. INCLUSIVE EXCITATION PROBABILITY AND THE BRINK-AXEL MECHANISM

The next point to be discussed concerns the quantities, relevant for the Coulomb excitation problem, that one wishes to obtain from this general solution of the driven, damped oscillator model. First and foremost are various excitation probabilities, which eventually become translated to corresponding cross-sections. The "elementary", excitation probabilities that correspond to distinguishable nuclear excitation processes are excitations to a definite phonon number of each of the normal mode oscillators (i.e., definite "fine structure" nuclear excitations). Actual measurements are however more inclusive, and correspond to the probability of exciting certain energy bands which can be associated with total phonon number in the collection of normal modes. The calculation of these excitation probabilities is completely straightforward in terms of Eq. (2.6), given the fact that the complete state of the model system is simply a product state of coherent states of this sort. The probability for exciting a single phonon in normal mode ν (while all other normal modes remain in their respective ground states) is calculated simply as

$$P_1^{(\nu)}(t) = \mid \langle 0 \ldots 1_\nu 0 \ldots \mid \left[\prod_{\nu'} \mid e^{-i\omega_\nu t}\alpha_\nu(t)\rangle\right] \mid^2 = e^{-\sum_{\nu'} |\alpha_{\nu'}(t)|^2} \mid \alpha_\nu(t) \mid^2$$

so that the corresponding inclusive one phonon excitation probability is

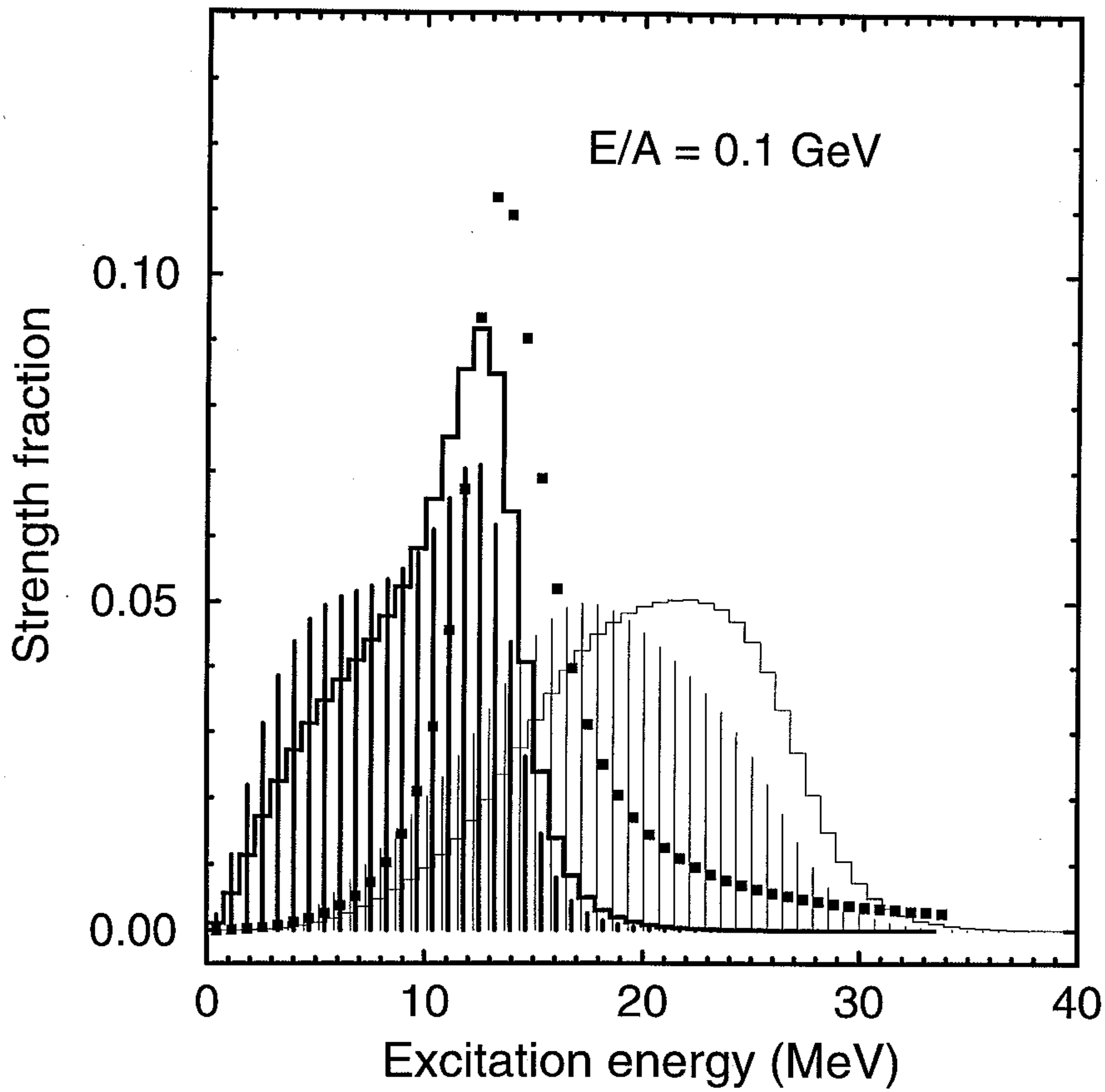
$$P_1(t) = \sum_\nu P_1^{(\nu)} = e^{-\sum_{\nu'} |\alpha_{\nu'}(t)|^2} \sum_\nu \mid \alpha_\nu(t) \mid^2 .$$

Table I

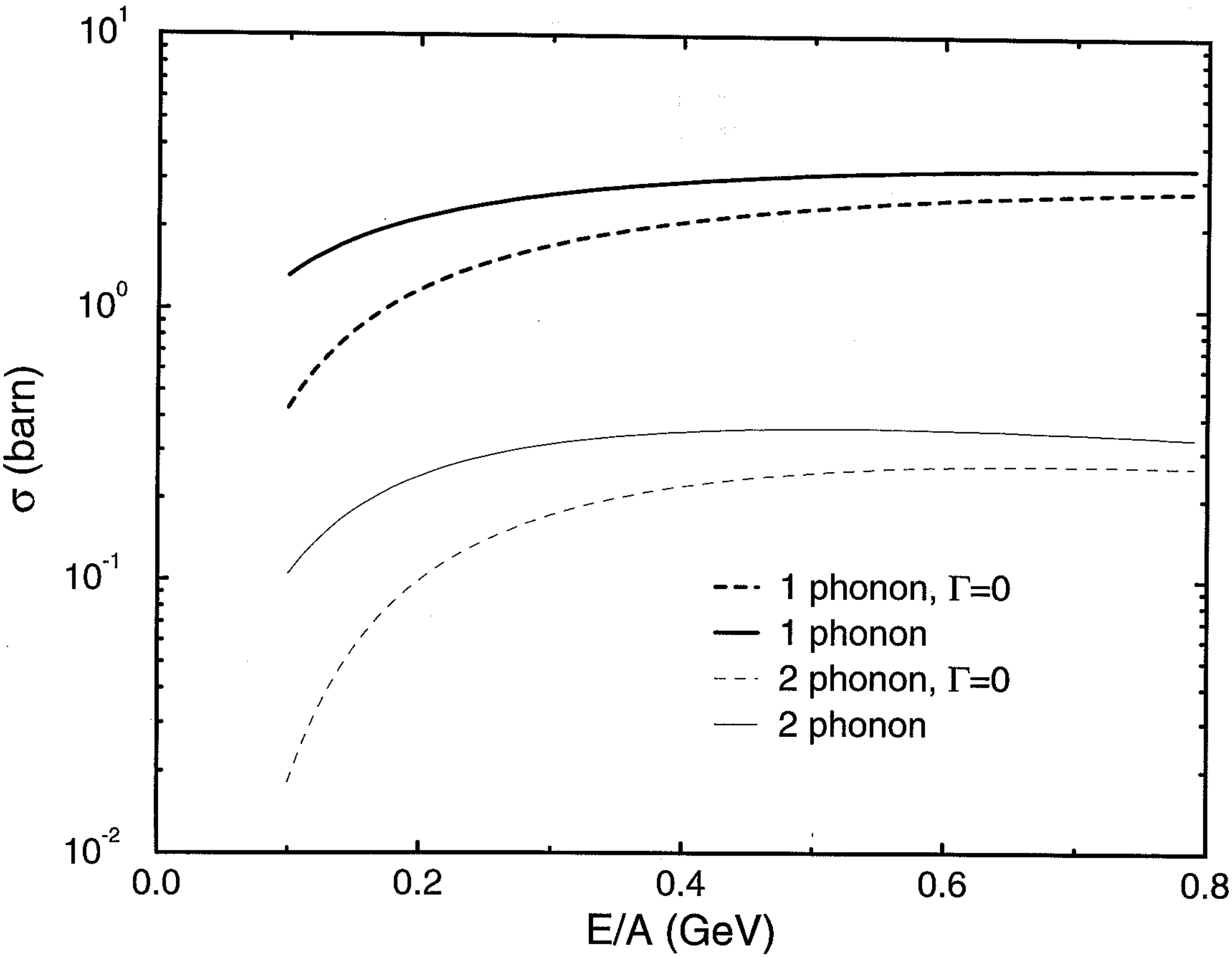
$$
\begin{array}{lccccccc}
\hline
\bullet\bullet\bullet & \cdots & & \cdots & & \cdots & & \cdots \\
 & f\updownarrow & & f\updownarrow & & f\updownarrow & & f\updownarrow \\
\textbf{3 phonons} & |3_d,\{0_k\}\rangle & \overset{g}{\longleftrightarrow} & |2_d,1_k\rangle & \overset{g}{\longleftrightarrow} & |1_d,2_k\rangle, |1_d,1_k,1_{k'}\rangle & \overset{g}{\longleftrightarrow} & \ldots \\
 & f\updownarrow & & f\updownarrow & & f\updownarrow & & \\
\textbf{2 phonons} & |2_d,\{0_k\}\rangle & \overset{g}{\longleftrightarrow} & |1_d,1_k\rangle & \overset{g}{\longleftrightarrow} & |0_d,2_k\rangle, |0_d,1_k,1_{k'}\rangle & & \\
 & f\updownarrow & & f\Updownarrow & & & & \\
\textbf{1 phonon} & |1_d,\{0_k\}\rangle & \overset{g}{\Longleftrightarrow} & |0_d,1_k\rangle & & & & \\
 & f\Updownarrow & & & & & & \\
\textbf{G}-\textbf{S} & |0_d,\{0_k\}\rangle & & & & & & \\
\hline
\end{array}
$$

TABLE I. Multi-phonon excitation paths for the collective damped oscillator. The notations f and g refer to the external pulse and to the coupling of the collective mode to the bath oscillators respectively (see Eq. (2.1)). The Brink-Axel route to the two-phonon domain is indicated by the thick arrows.

Carlson et al. Fig 1a



Carlson et al. Fig 2



Carlson et al. Fig 3b