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The graviton self-energy at finite temperature depends on fourteen structure functions. We show that, in the absence of tadpoles, the gauge invariance of the effective action imposes three non-linear relations among these functions. The consequences of such constraints, which must be satisfied by the thermal graviton self-energy to all orders, are explicitly verified in general linear gauges to one loop order.

The non-linear relation imposed by gauge invariance on the thermal self-energy of gluons, has been recently discussed by Weldon in an interesting paper [1]. He proved that in QCD, the Slavnov-Taylor identities [2,3] require a non-linear constraint among the structure functions which occur at finite temperature. In this brief report, we show that a similar behavior occurs in the gauge theory of gravity. In this case, local gauge invariance leads to three non-linear relations which restrict the form of the thermal self-energy of gravitons.

The Einstein theory of gravity is described by the Lagrangian density [4–6]

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{-g}R, \tag{1}$$

where $\kappa^2 = 32\pi G$, G is the Newton constant and R is the Ricci scalar. The graviton field $h_{\mu\nu}$ can be defined in terms of the metric tensor as

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \tag{2}$$

Using this parametrization, the Einstein action will be invariant under the gauge transformation [7]

$$\begin{aligned}\delta h_{\mu\nu} &= \left[\delta_\mu^\lambda\partial_\nu + \delta_\nu^\lambda\partial_\mu + \kappa\left(h_\mu^\lambda\partial_\nu + h_\nu^\lambda\partial_\mu + \partial^\lambda h_{\mu\nu}\right)\right]\xi_\lambda \\ &\equiv G^{(0)\,\lambda}_{\mu\nu}\xi_\lambda,\end{aligned} \tag{3}$$

where ξ_λ is an infinitesimal gauge parameter.

In this gauge theory, the corresponding identities which occur at finite temperature, differ from those at $T = 0$ because of the appearance of one-particle graviton functions (tadpoles). Their thermal contribution is purely leading, being proportional to $T^{2(n+1)}$ at the n-loop order. Hence, we may assume that the tadpoles are important, in the Ward identities, only for the leading thermal contributions to the graviton self-energy. (To one-loop order, for example, the tadpoles can be neglected for the purpose of studying the sub-leading T^2, $\log(T)$ and $T = 0$ contributions).

At finite temperature, the graviton self-energy may depend on the four-velocity u_α of the plasma, so that it can be a linear combination of the 14 independent tensors given in Table I. This contains three traceless tensors $T^A_{\alpha\beta,\,\mu\nu}$, $T^B_{\alpha\beta,\,\mu\nu}$ and $T^C_{\alpha\beta,\,\mu\nu}$, which are transverse with respect to the wave 4-vector k_μ . They are also, respectively, completely transverse, partially transverse and longitudinal with respect to the spatial component $\vec{k}$ [8]. These tensors depend individually on the plasma four-velocity, but their sum is a Lorentz covariant tensor which is independent of u_α

$$\begin{aligned}&\left(T^A + T^B + T^C\right)_{\alpha\beta,\,\mu\nu} = \\ &\frac{1}{2}\left(P_{\alpha\mu}P_{\beta\nu} + P_{\alpha\nu}P_{\beta\mu}\right) - \frac{1}{3}P_{\alpha\beta}P_{\mu\nu},\end{aligned} \tag{4}$$

where $P_{\alpha\beta} = \eta_{\alpha\beta} - k_\alpha k_\beta/k^2$.

In terms of this basis, which is convenient for our purpose, the graviton self-energy can be parametrized as

$$\begin{aligned}\Pi_{\alpha\beta,\,\mu\nu} = \Pi_A T^A_{\alpha\beta,\,\mu\nu} + \Pi_B T^B_{\alpha\beta,\,\mu\nu} + \Pi_C T^C_{\alpha\beta,\,\mu\nu} \\ + \sum_{i=4}^{14}\Pi_i T^i_{\alpha\beta,\,\mu\nu}\end{aligned} \tag{5}$$

$$\begin{aligned}
T^{1,2,3}_{\alpha\beta,\,\mu\nu} &= T^{A,B,C}_{\alpha\beta,\,\mu\nu} \\
T^{4}_{\alpha\beta,\,\mu\nu} &= \eta_{\alpha\beta}\eta_{\mu\nu} \\
T^{5}_{\alpha\beta,\,\mu\nu} &= u_\mu u_\nu \eta_{\alpha\beta} + u_\alpha u_\beta \eta_{\mu\nu} \\
T^{6}_{\alpha\beta,\,\mu\nu} &= [u_\beta\,(k_\nu\eta_{\alpha\mu} + k_\mu\eta_{\alpha\nu}) + k_\beta\,(u_\nu\eta_{\alpha\mu} + u_\mu\eta_{\alpha\nu}) + \\
&\quad u_\alpha\,(k_\nu\eta_{\beta\mu} + k_\mu\eta_{\beta\nu}) + k_\alpha\,(u_\nu\eta_{\beta\mu} + u_\mu\eta_{\beta\nu})]\,/k\cdot u \\
T^{7}_{\alpha\beta,\,\mu\nu} &= [k_\nu u_\alpha u_\beta u_\mu + k_\mu u_\alpha u_\beta u_\nu + \\
&\quad k_\beta u_\alpha u_\mu u_\nu + k_\alpha u_\beta u_\mu u_\nu]\,/k\cdot u \\
T^{8}_{\alpha\beta,\,\mu\nu} &= [k_\beta k_\nu\eta_{\alpha\mu} + k_\beta k_\mu\eta_{\alpha\nu} + k_\alpha k_\nu\eta_{\beta\mu} + k_\alpha k_\mu\eta_{\beta\nu}]\,/k^2 \\
T^{9}_{\alpha\beta,\,\mu\nu} &= [k_\mu k_\nu u_\alpha u_\beta + k_\alpha k_\beta u_\mu u_\nu]\,/k^2 \\
T^{10}_{\alpha\beta,\,\mu\nu} &= (k_\beta u_\alpha + k_\alpha u_\beta)\,(k_\nu u_\mu + k_\mu u_\nu)\,/(k\cdot u)^2 \\
T^{11}_{\alpha\beta,\,\mu\nu} &= [k_\beta k_\mu k_\nu u_\alpha + k_\alpha k_\mu k_\nu u_\beta + \\
&\quad k_\alpha k_\beta k_\nu u_\mu + k_\alpha k_\beta k_\mu u_\nu]\,/(k^2\,k\cdot u) \\
T^{12}_{\alpha\beta,\,\mu\nu} &= (k_\alpha k_\beta k_\mu k_\nu)\,/k^4 \\
T^{13}_{\alpha\beta,\,\mu\nu} &= (k_\mu k_\nu\eta_{\alpha\beta} + k_\alpha k_\beta\eta_{\mu\nu})\,/k^2 \\
T^{14}_{\alpha\beta,\,\mu\nu} &= [(k_\nu u_\mu + k_\mu u_\nu)\,\eta_{\alpha\beta} + (k_\beta u_\alpha + k_\alpha u_\beta)\,\eta_{\mu\nu}]\,/(k\cdot u)
\end{aligned}$$

TABLE I. A basis of 14 independent tensors.