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## **Particle-hole level densities in deformed nuclei**

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# Particle-hole level densities in deformed nuclei

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Microscopic Combinatorial approach is used to calculate the state and level densities with fixed exciton numbers, in some actinide nuclei. Deformed Saxon-Woods shell model was used as a basis from which all possible configurations were generated. The pairing interaction was taken into account by applying the BCS theory to each configurations. Both the spin and parity distributions were obtained, considering the deformation of the nucleus. Relevance of the result to parity nonconservation studies involving epithermal neutrons on  $^{238}\text{U}$  and  $^{232}\text{Th}$  is discussed.

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## I. INTRODUCTION

Adequate description of the nuclear level density of an excited nuclear state with a fixed number of quasiparticles is a basic ingredient of the statistical analysis of nuclear reactions. On the other hand, it is well known that the exciton dependent state density

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used in the various pre-equilibrium models is lacking in considering the adequate particle-hole combination and its relation with the particular structure of the nuclei involved in the reactions.

Analytical expressions for particle-hole excitation density can be obtained by means of methods of combinatorial or statistical mechanics in terms of the noninteracting particle model using the equidistant spacing approximation for the single-particle states. The main deficiency of the formulae commonly used to estimate exciton level densities comes from the assumption of equidistant spacings for the single particle levels [1] [2]. Usually only few-exciton configurations contribute significantly to pre-equilibrium emission and, in some cases, the population of low energy configurations is relevant as well. Therefore statistical approaches and equidistant spacings for the single particle levels do not seem to be adequate for pre-equilibrium calculations in some instances when shell structure effects are observed [3].

In a recent work [4], two of the authors studied fundamental symmetry breaking on nuclear reaction and suggested that the observation of sign correlations in the longitudinal asymmetry of polarized neutron scattering from  $^{232}\text{Th}$  at epithermal energies may constitute the first direct evidence for local 2p-1h doorways. The discovery of sign correlation in parity non-conserving (PNC) epithermal neutron-induced compound-nucleus reactions involving the heavy nucleus  $^{232}\text{Th}$  have prompted intensive theoretical discussion concerning their origin. The great interest in the TRIPLE data [5], stems from the fact that the statistical theory of these reactions, although it predicts the possibility of large PNC at individual resonances as the data also show, rules out any sign correlations in the longitudinal asymmetry, contrary to what the data show. More recently [6], data were obtained with smaller error bars for  $^{232}\text{Th}$  and  $^{238}\text{U}$ . Feshbach and collaborators [4], started with the hypothesis that a very natural mechanism that could account for this sign correlations is to assume that the compound nuclear PNC process occurs through a single isolated p-wave local doorway which coupler to nearly s-resonances. In such studies, an important quantity which could determine how isolated the p-wave resonances are, is the 2p-1h density of states. It has been

a common practice to use the equidistant model to calculate the np-nh level densities, which is reasonable in spherical nuclei. No thorough investigation of the case of deformed nuclei is available. Sire nuclei such as U and Th are strongly deformed, one would be tempted to calculate the exciton level densities more accurately using a deformed mean field.

Few years ago M. Herman and G. Reffo [7], [8] evaluated a realistic few quasiparticle state densities with fixed exciton numbers using combinatorial calculations in spherical and some deformed nuclei. Their calculations were carried out in the basis of single particle orbitals derived from the harmonic oscillators defined by parameter due to Seeger and Howard [10]. Calculation of spin-dependence particle-hole level density were carried out only for spherical nuclei. For these nuclei the Gaussian-Wigner type spin factor was used. A direct test of the validity of this procedure is afforded by calculations made by Herman and Reffo. They found that the Gaussian formula works very well for configurations containing at least three excitons. However, in the case of deformed nuclei there are no comprehensive calculations with take into account the role of the collective rotational degrees of freedom for the evaluation of the particle-hole level density, with the question of the spin factor still open.

It is generally assumed, that levels of different parities have an equiprobable condition. This is the Ericson's prediction [9] for a non-interacting many fermion system. The possibility of considerable irregularities occurring in the distribution of positive-and negative-parity levels is demonstrated in particular by the results of combinatorial calculations of level density carried out within the framework of the non-interacting particle model [25] as well as those based on the more rigorous quasi-particle-phonon model, which takes into account the collective excitations of nuclei [26] [27]. Previous combinatorial microscopical calculations [7] show that the results have evidently oscillatory character around the equal probability value. The fluctuation are observed throughout a wide energy range ( $\sim 30$  MeV), even though on the average the amplitude of the fluctuations decreases with increasing excitation energy. The fluctuations are reduced with the increase of the deformation, exciton number and mass of the nucleus. It is an indication that in most of the applications, the assumptions

of equal parity distribution seems, to be justified. However, certain channels, particularly sensitive to the parity distribution (e.g. radiative channels), have to be treated with caution.

Another interesting question is related to the deformation parameters used in the calculation of single-particle spectra. Herman et al in [8] studied the effect of nuclear deformation on few-quasiparticle state-densities. It is shown that nuclear deformation tends to suppress strong fluctuations observed in the similar calculations performed in the space of spherical shell model basis vectors. The conclusion of Hermann and Reffo have the qualitative character, since some fine details of particular structure of studied nuclei ( $^{27}\text{Al}$ ,  $^{100}\text{Mo}$ ,  $^{170}\text{Er}$ , ) are not taken in to account. In our view, the first step should be the right selection of deformation parameters with the calculations of single particle spectrum in the equilibrium deformation. The study of fine nuclear structure effects, of order of PNC need the most accurate calculation of particle hole level density, which is very sensitive to the single-particle spectrum.

The paper is devoted to the development of a more accurate method for the calculation of particle-hole level densities for deformed nuclei by thoroughly considering the rotational degree of freedom. Particular effort is made in applying the method to actinide nuclei used in parity no-conservation studies induced by epithermal neutrons. The final intent is to evaluate the various uncertainties, constraints or difficulties related to the level density question and their impact on nuclear reaction predictions.

In sect.2 we outline very briefly the formalism with the description of the general method which improves our previous mode. The preliminary treatments. The problems of the spin and parity distributions is presented in sect.3. In sect.4 we discuss some results for two nuclei belonging to the actinide region. Finally, we present the main conclusions in sect.5.

## II. PARTICLE-HOLE LEVEL DENSITY CALCULATION'S METHOD

In order to calculate the 2p-1h doorway states densities coupled to total angular momentum  $J$  we used the microscopic combinatorial method [7] [8] [11] at excitation energy  $E$ ,

considering the axial deformation of the compound nucleus. The calculation was performed in the space of realistic shell-model single particle states, including pairing interaction in terms of the BCS theory.

The first step to start the single-particle spectrum calculation is to fix the value of the equilibrium deformation of the nucleus using an optimal set of potential parameters [14]. In the shell model approach, based on mean-field potentials, the Strutinsky method is usually used [15]. We have calculated the extreme points of the potential energy along the path from the equilibrium deformation to the more unstable form, using the BARRIER code [12] which includes the Strutinsky prescription and the Pashkevich parametrization for the nuclear shape [21]. The calculation was performed taking into account quadrupole and hexadecapole deformation parameters, which is a reasonable approximation to describe the equilibrium ground state. It is important to note that for more complex nuclear shapes, higher order deformation parameters should be included in the calculations [20].

The other quantity which needs to be accurately determined is the pairing strength parameter  $G_{n,p}$ . There are few methods commonly used to fix the value of  $G$ . Some authors [22], [23], take the pairing strength as a free parameter chosen to fit the calculated level densities to the experimental number of resonances at the neutron binding energy. This procedure certainly gives the appropriate level densities, but the pairing strength parameter derived this way is not related directly to the pairing interaction and may hide several inadequacies of the applied theory. We have chosen the method of relating the  $G$  parameter to the difference of masses of neighboring nuclei.

In order to obtain intrinsic state densities all configurations with a specified number of particles and holes are generated within the assumed set of the single-particle levels by means of the permutation enumeration algorithm. For each of them a proper coupling of the spin projections is performed to obtain the nuclear states. The state density  $w(E,M)$  is found by counting states with the angular momentum projection  $M$  falling in the 0.5 MeV interval centered at the excitation energy. The pairing interaction was considered by applying the BCS model to each configuration so that a better understanding of the pairing correlations

in few quasiparticle states could be achieved.

We also used an improved formulae for the spin factor of particle-hole level density. Owing to the axial-symmetry of the ground states of these deformed nuclei we will use the spin factor formula which contains the rotational degree of freedom contribution, proposed by S. Bjornholm, A. Bohr and B.R. Mottelson in Ref [24]. The statistical microscopical method [11] is used for to obtaining the parallel and perpendicular spin-cutoff factors in a consistent way.

### A. The single-particle spectra

In the last few years, several calculations of single particle level schemes have been carried out both as a function of nucleon numbers and as a function of nuclear shape in connection with the studies of nuclear deformation potential energy surfaces. Simple global parameters for the underlying model potential have also been estimated on the basis of fits to nuclear ground state masses and fission barrier heights. Based on these and other single particle level schemes and the partition function approach, several numerical calculations of nuclear level densities have been carried out.

In this work the nuclear shape parametrization is carried out using the Cassini parametrization [21] According to this the deforming shape (up to and beyond its separation into two fragments) can be conveniently described by the Cassini ovaloids figures

Considering only axially symmetric nuclear shapes, the Cassini ovaloids are taken as the first approximation to the nuclear shape. The deviation from the ovaloid shape is given by an expansion into a series of Legendre polynomials. Geometrically, the family of Cassini ovaloids is defined by [?]:

$$r^2(z, \epsilon) = \sqrt{(a^4 + 4(cz)^2)} - (c^2 + z^2 - \epsilon^2). \quad (1)$$

In this equation,  $r$  and  $z$  are cylindrical coordinates;  $\epsilon$  is a dimensionless quantity such that  $c = \epsilon R_0^2$ ;  $c$  stands for the square distance from the focus of the Cassini ovaloids to the origin of

coordinates; and  $a$  is a dimensionless parameter which completely defines the shape taking into account volume conservation.

In the plane containing the symmetry axis one can define a system of coordinates  $(R, x)$  such that the coordinate line  $R$  is constant. This is a Cassini ovaloid, where  $0 \leq R < \infty$  and  $-1 \leq x \leq 1$ . The  $(R, x)$  coordinates are related to the cylindrical ones  $(r, z)$  by the following equations

$$R(z, r) = \sqrt[4]{[(z^2 + r^2)^2 - 2\epsilon R_0^2 \cdot (z^2 - r^2) + \epsilon^2 R_0^2]} \quad (2)$$

$$x(z, r) = \frac{\text{sign}(z)}{\sqrt{2}} \left[ 1 + \frac{z^2 - r^2 - \epsilon R_0^2}{R^2(z, r)} \right]^{\frac{1}{2}} \quad (3)$$

In this system of coordinates, the basic shape of the nucleus is described by these equations, where  $R$  is constant, determining the Cassini ovaloids. Thus, the nuclear shape can be defined as a curve  $R(x)$  that does not intersect any straight line  $x = \text{constant}$  at more than one point. Accordingly, we can expand the function  $R(x)$  into multipoles, giving

$$R(x) = R_0[1 + \beta_m Y_{m0}(x)]. \quad (4)$$

The set of parameters  $(\epsilon, \beta)$  completely determine the nuclear shape. The details of this parametrization are given in [12]. As an example, we show in Figure 1  $\{\epsilon, \alpha_4\}$  as functions of  $\{\beta_2, \beta_4\}$ . As is clearly seen in this figure, it is difficult to establish an analytical connection between the two set of parameters. This relation was obtained by a least-square fit of the parameters  $\beta_2, \beta_4$ , for the harmonic spherical expansion, to our shapes described by the Cassini ovaloids. Using this figure it is possible to establish a connection between the two set of parameters to describe the same nuclear shape, but for more complex shapes more coefficients are needed in the harmonic spherical expansion.

In order to obtain the single-particle energies and wave functions, the Hamiltonian matrix elements are calculated with the wave functions of a deformed axially symmetric oscillator potential. The basis cut-off energy is determined in such a way that the negative energy eigenvalues of the Woods-Saxon potential do not change when adding more harmonic oscillator shells.



As usual, the real potential  $V(r)$  is expected to follow roughly the density distribution. One of the most used radial dependences, is that of the Woods-Saxon potential, which takes into account the nuclear potential and density distribution. This potential involves the parameters  $V_0$ ,  $r_0$  and  $a$ , describing the depth of the central potential, the radius and the diffuseness parameters, respectively.

A definition of the radial dependence of the potential for a deformed nucleus, with an arbitrary shape of the surface, was proposed by Pashkevich [21]. According to Pashkevich, the nuclear potential is given by

$$V(r, z, \epsilon, \hat{\beta}) = \frac{V_0}{1 + \exp \frac{\text{dist}(r, z, \epsilon, \hat{\beta})}{a}} \quad (5)$$

where  $\text{dist}(r, z, \epsilon, \hat{\beta})$  is the distance between a point and the nuclear surface,  $a$  is the diffuseness parameter and  $V_0$  the depth of the potential well.

The Woods-Saxon-type potentials, with the spin-orbit interaction proportional to the potential gradient, are the most appropriate from the physical point of view. The spin-orbit interaction is given by:

$$V_{so}(r, z, \epsilon, \hat{\beta}) = \lambda \left( \frac{\hbar}{2Mc} \right)^2 \nabla V(r, z, \epsilon, \hat{\beta}) \cdot (\vec{\sigma} \times \vec{p}) \quad (6)$$

where  $\lambda$  denotes the strength of the spin-orbit potential and  $M$  is the nucleon mass. The vector-operator  $\vec{\sigma}$  stands for Pauli matrices and  $\vec{p}$  is the linear momentum operator.

The Coulomb potential is assumed to be that corresponding to the nuclear charge  $(Z - 1)e$ , and uniformly distributed inside the nucleus. In short, the depth of the central potential is parametrized as

$$V = V_0[1 + \kappa' T_z], \quad \kappa' = \frac{2k}{A} \quad (7)$$

with  $T_z$  is the z-component of the isospin and  $k$  is an adjustable parameter.

The single-particle spectra were obtained by means of the new version of the BARRIER code [12], based in a WSBETA code [13], using axial deformed Saxon-Woods nuclear potential well defined by parameters, due to Chepurnov [14].

## B. Microscopic Combinatorial approach for level densities with fixed exciton numbers

The combinatorial method to obtain the level densities provides the possibility of direct counting of the levels with a fixed number of particle and holes. We start from the finite set of single particle states derived from the shell model with appropriately nuclear model potential, whose detailed description has been made above. Usually 100 neutron and 100 proton orbitals were used in our calculation. It allows us to calculate the particle-hole state densities theory for deformed nuclei as Th<sup>233</sup> and U<sup>238</sup>, in energy range below 10 MeV.

The pairing interaction was taken into account by applying the BCS theory to each configurations. All residual interactions but pairing are neglected.

The total energy of each configuration is determined through the superconductivity theory. Configuration dependence is introduced into BCS theory by the blocking method as proposed by Wahlborn [16], to allow for blocking of more than one orbital.

Accordingly for each generated configuration, a set of two BCS equations is solved

$$\begin{aligned} N &= 2 \sum_i' U_{is}^2 \\ \frac{2}{G} &= \sum_i' [(\varepsilon_i - \lambda_s)^2 + \Delta_s^2]^{-1/2} \end{aligned} \quad (8)$$

where

$$U_{is}^2 = \frac{1}{2} \left( 1 - \frac{(\varepsilon_i - \lambda_s)}{[(\varepsilon_i - \lambda_s)^2 + \Delta_s^2]} \right) \quad (9)$$

Here  $\varepsilon_i$  is the single particle energies,  $N$  stands for the number of paired nucleons, and  $\lambda_s$  and  $\Delta_s$  are the chemical potential and correlation function for a given configuration that are supposed to be determined.

The total configuration energy, according to the BCS model

$$E_s = \sum_J \varepsilon_i + 2 \sum_i' U_{is}^2 \varepsilon_i - \frac{\Delta_s^2}{G} \quad (10)$$

The excitation energy is calculated in turn as the difference between the total energy of a configuration and the total energy of the ground state where the first summation included only blocked orbitals.

The particle-hole level density were obtained by means of the ICAR and CONV codes [17]

### C. Collective degrees of freedom. Spin dependence

Collective phenomena in nuclei are receiving considerable attention in the analysis of spectroscopic data on the characteristics of low-lying levels. Various microscopic methods of describing the structure of collective levels [28] [29] [30] are also widely used at present to consider the interrelationship between the collective excitations and the single-particle motion of nucleons in a self consistent nuclear potential.

Strictly speaking, any separation of collective variables should be accompanied by a corresponding decrease in the number of internal degrees of freedom. But since collective motions are formed owing to deep-lying nucleons, while internal excitations are determined basically by the single-particle levels adjacent to the Fermi surface, exclusion of the extra degrees of freedom in the low-temperature region should not strongly affect the density of the internal excited states. Under these conditions, adiabatic consideration can be fully justified, at least as a first step in the analysis of the rotational increases, in the level density of nuclei.

The impact of different models for spherical nuclei on the consideration of collective enhancement has been studied in Refs. [32] [31]. Herman and Reffo [7], have considered only internal degree of freedom at low energies without account of the rotation and the vibration enhancement can not be considered for these heavy nuclei. The validity of the statistical law describing the spin distribution of nuclear levels must be reconsidered when applied to levels with fixed exciton numbers.

For spherical nuclei the formula for the spin distribution function reads

$$R(J) = \frac{2J + 1}{2(2\pi)^{\frac{1}{2}}\sigma^3} e^{-[(J+\frac{1}{2})^2/2\sigma^2]} \quad (11)$$

which is derived under the assumption of a Gaussian distribution of spin projections  $M$ . While it is very likely to be true when the number of levels is high enough, this assumption

Fig 1a.

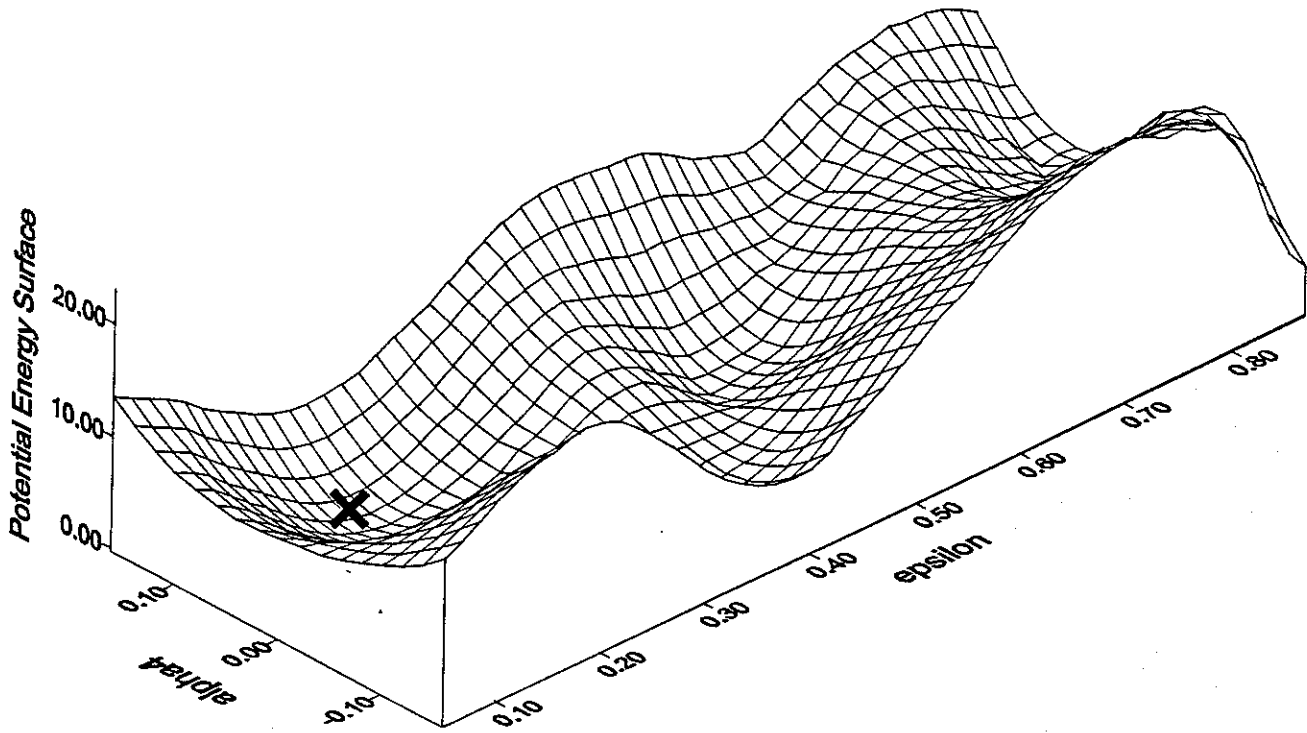
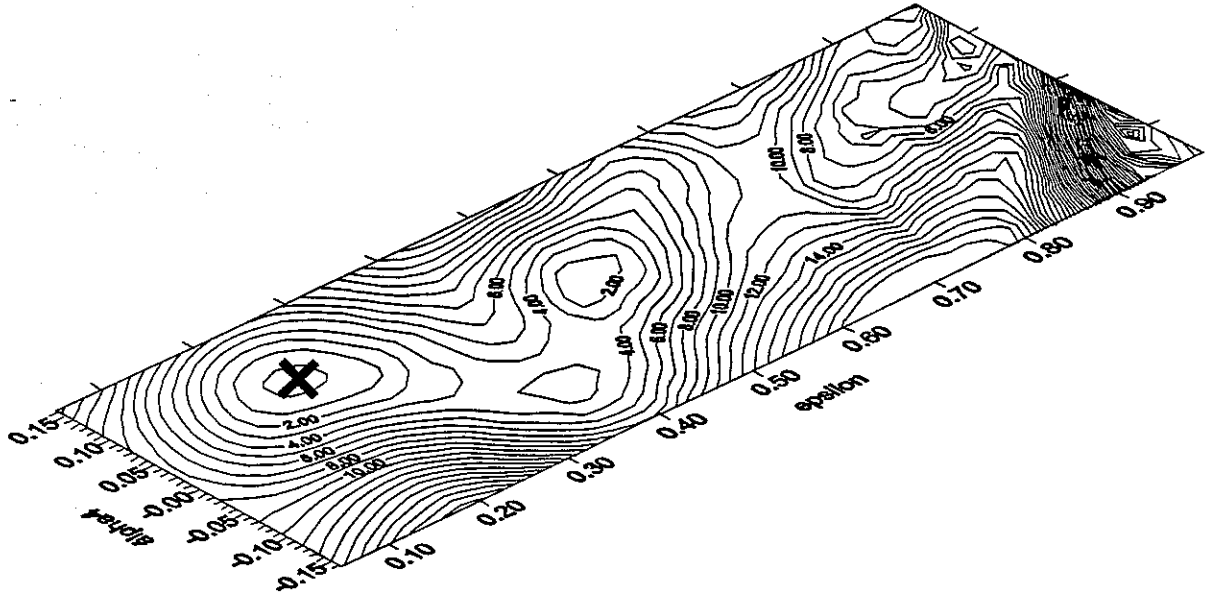


Fig. 1b.

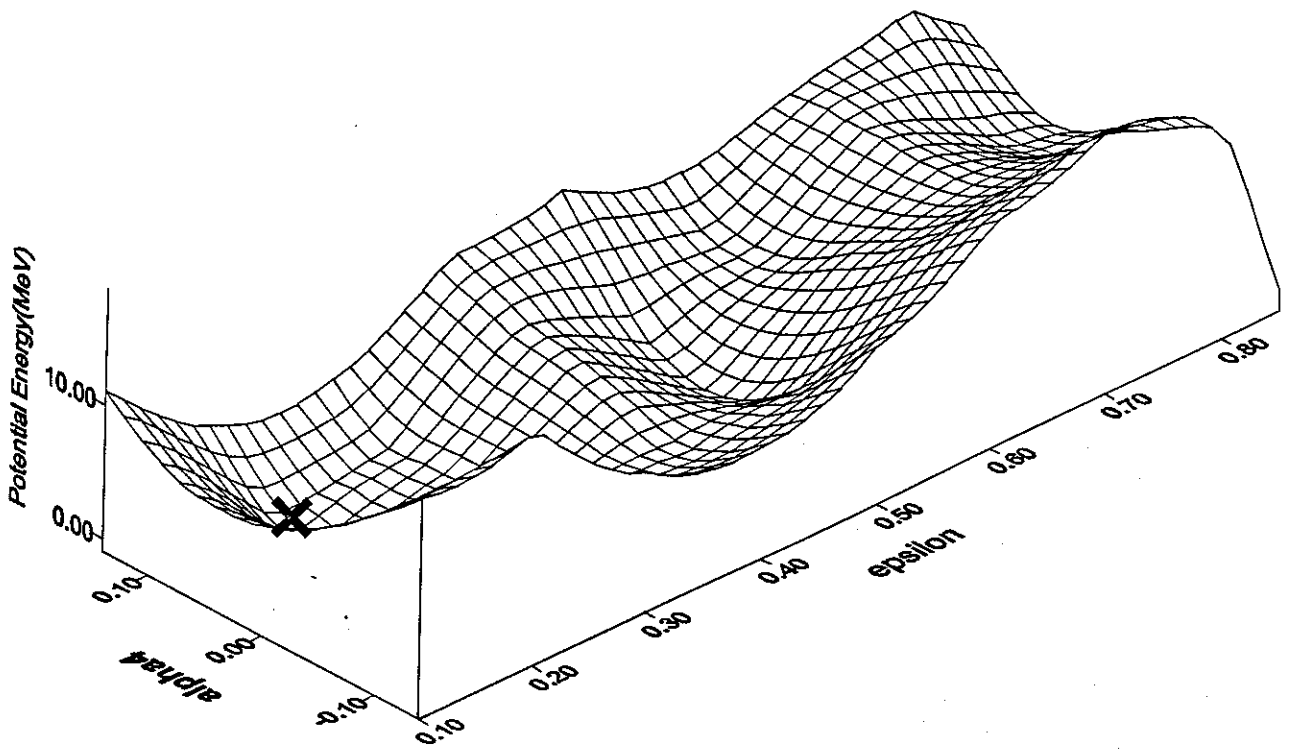
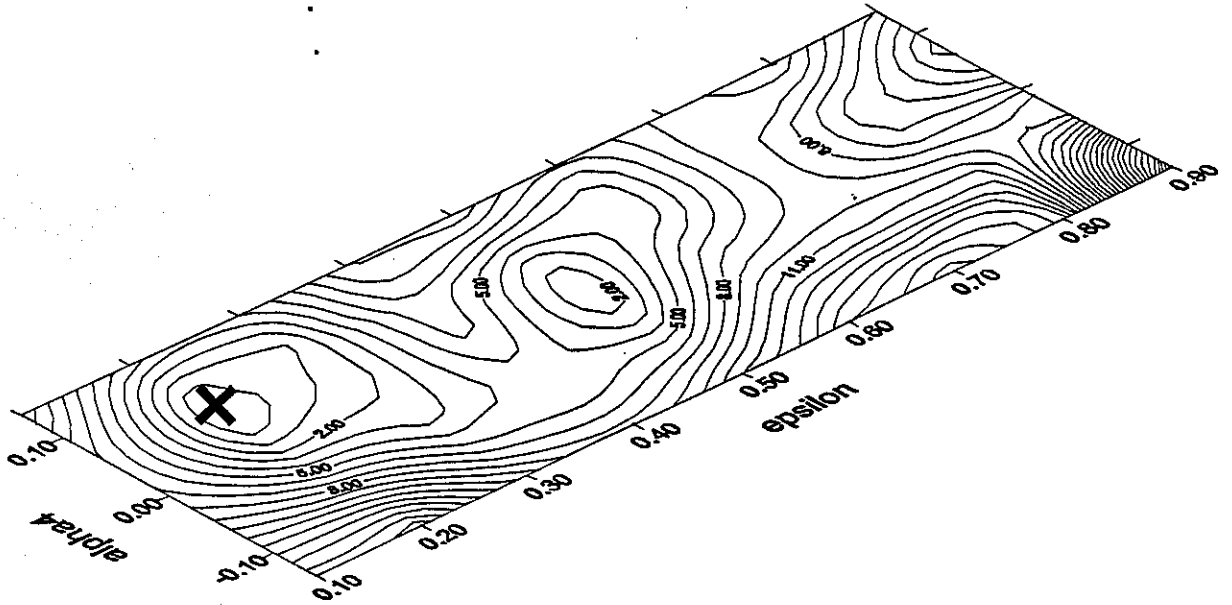


Fig.2

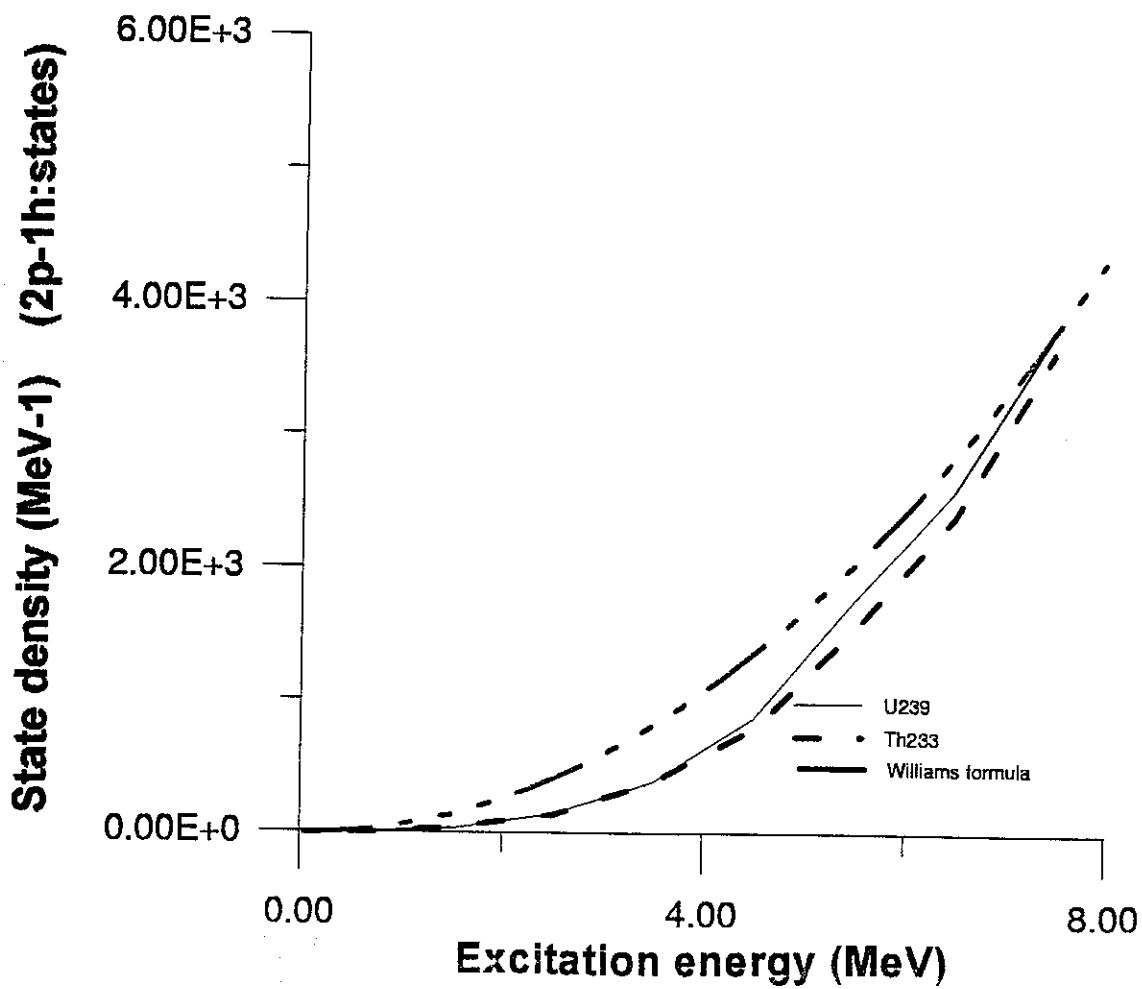


Fig.3

Spin - Cutoff Factors for axial deformed nucleus

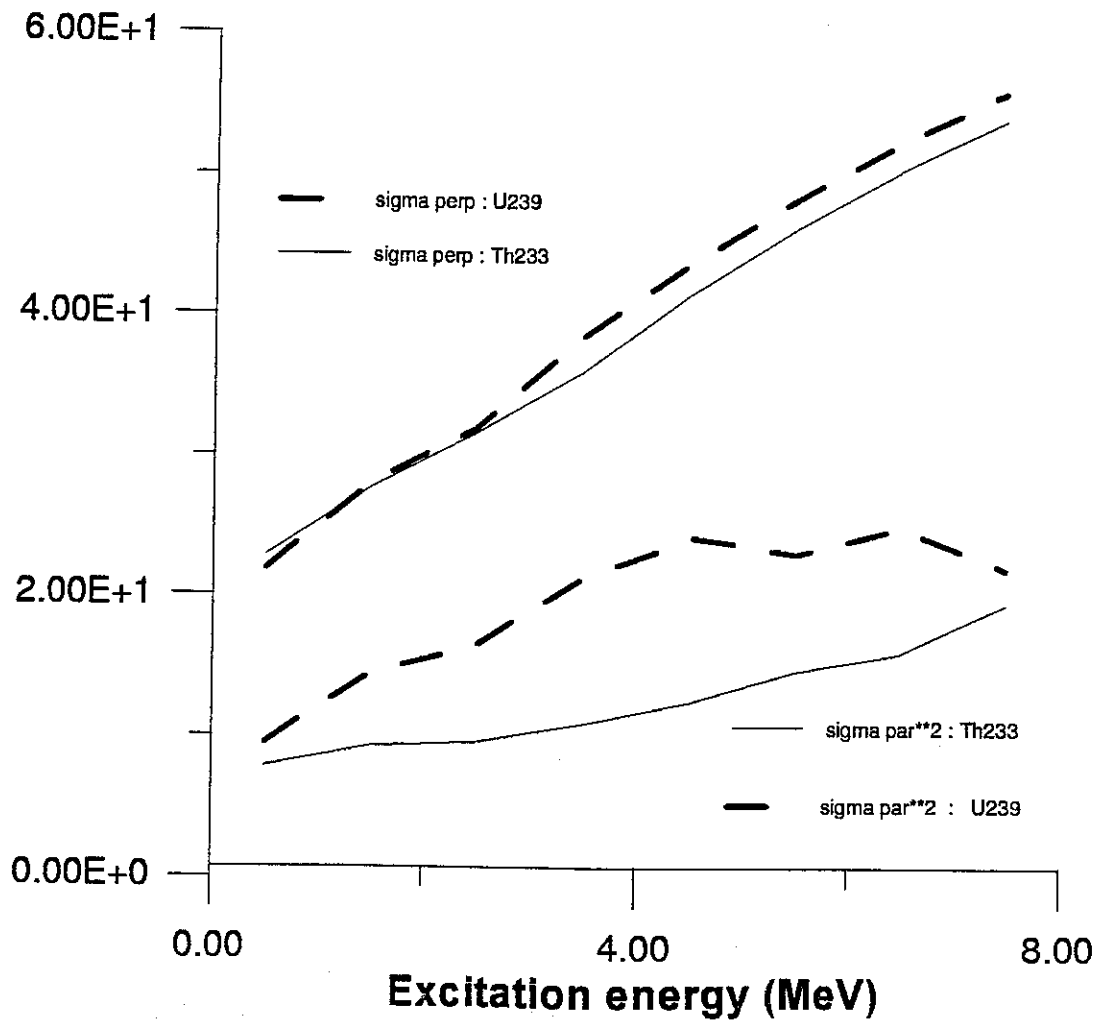
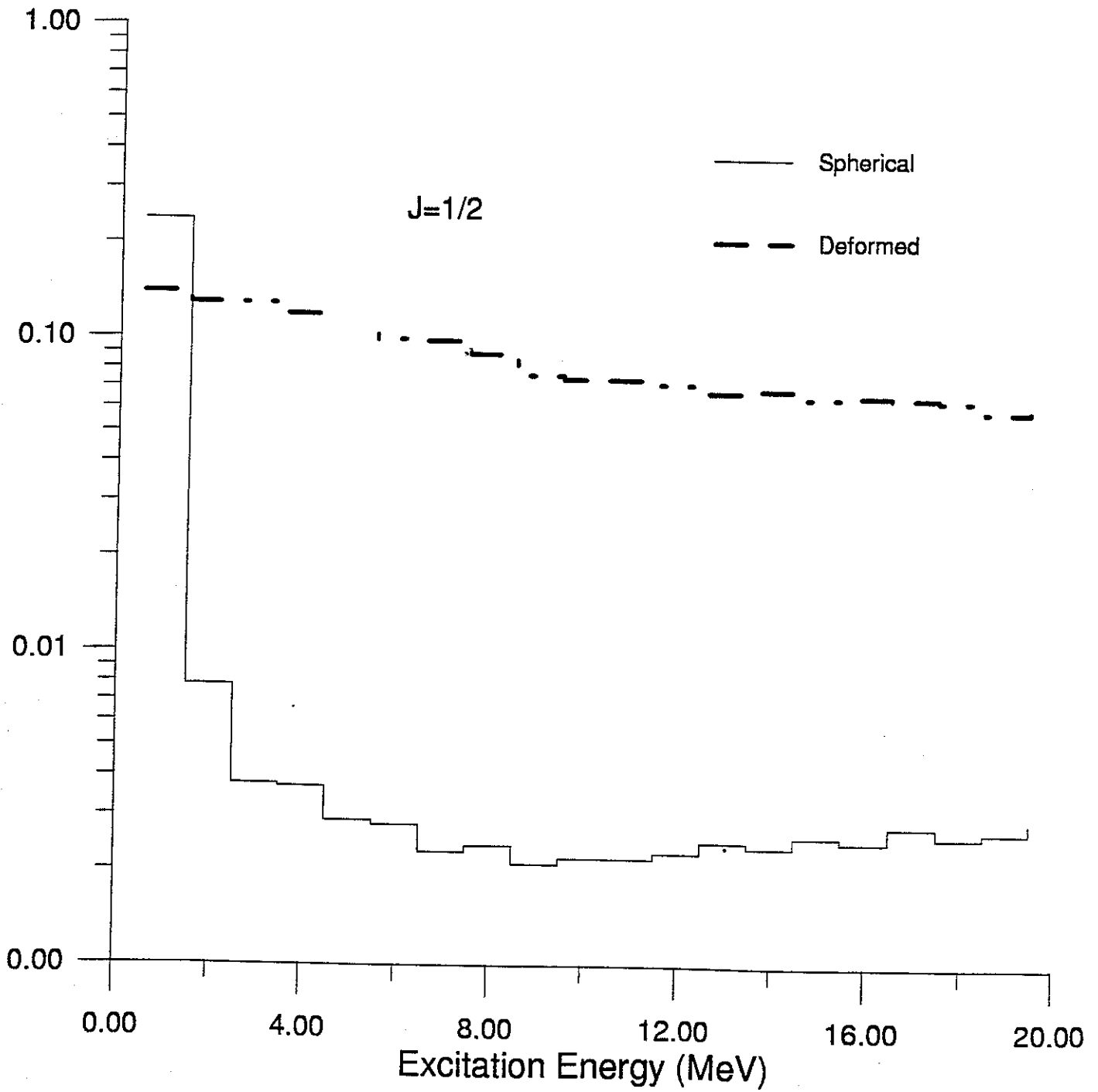
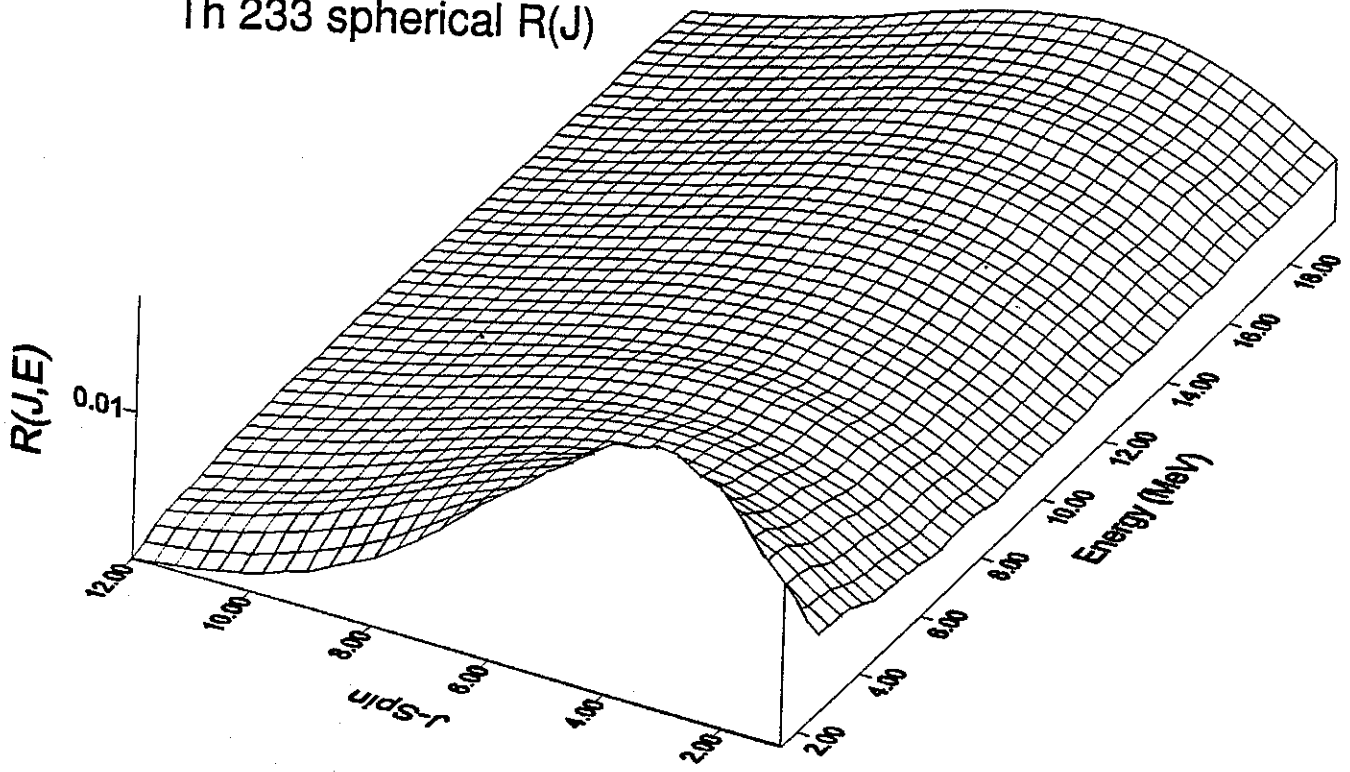


Fig. 4.

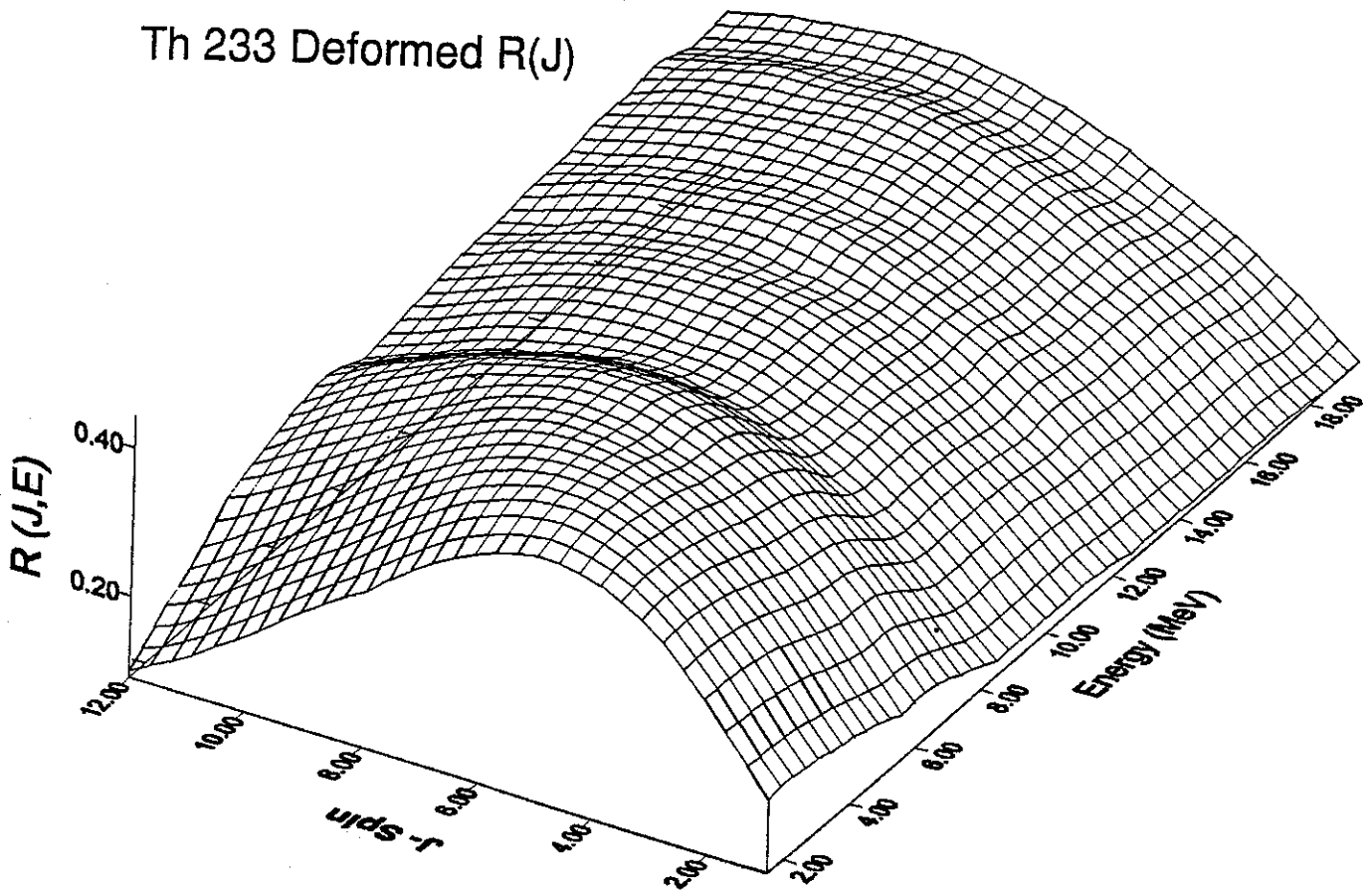




Th 233 spherical R(J)



Th 233 Deformed R(J)



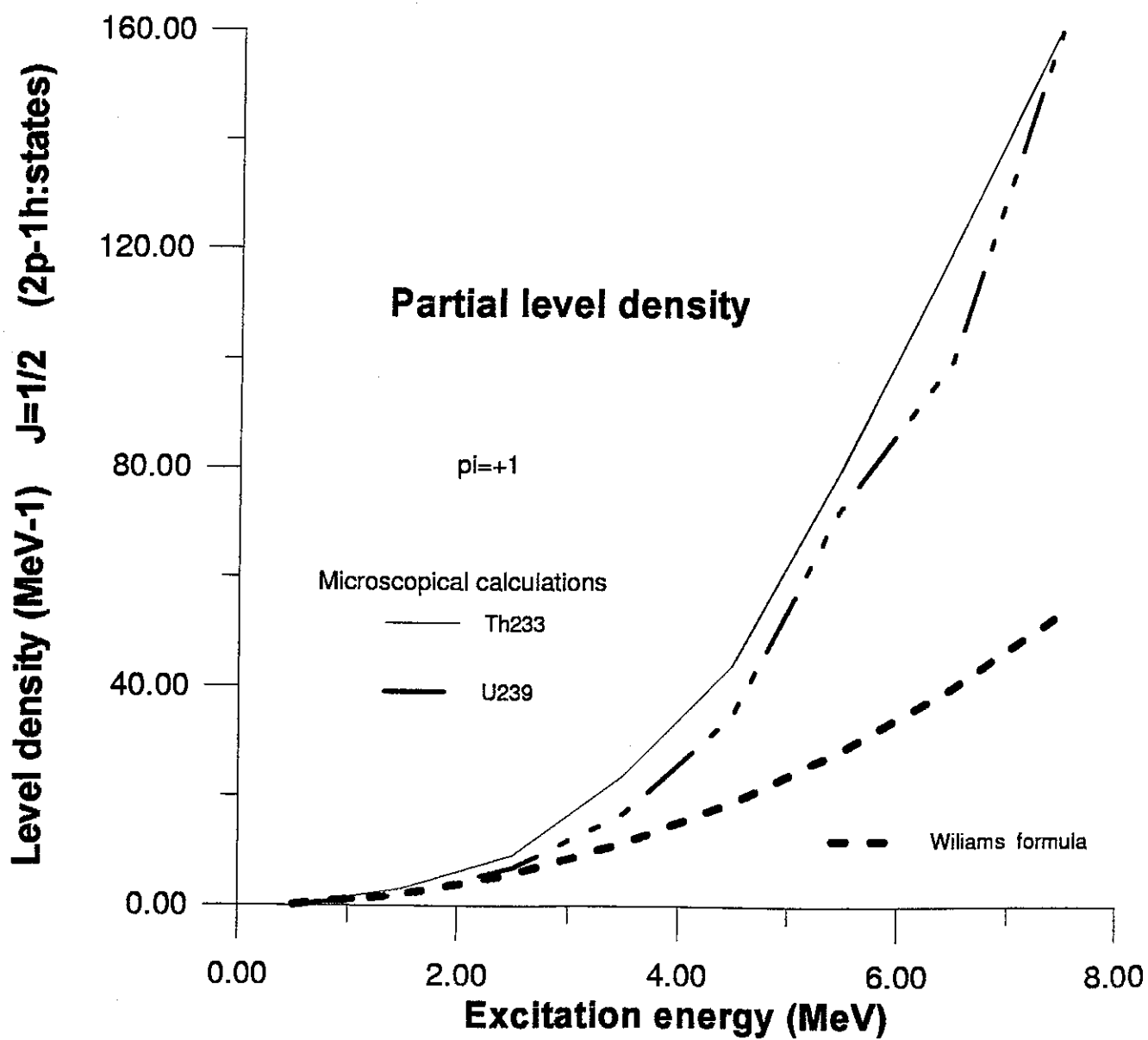


Fig. 6.

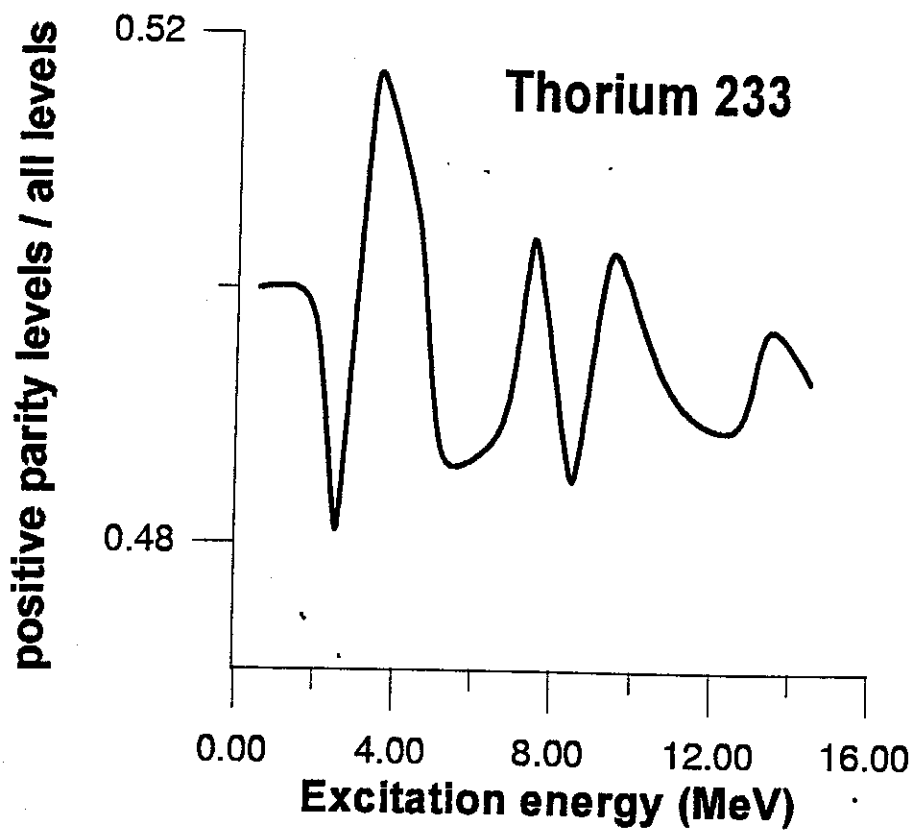
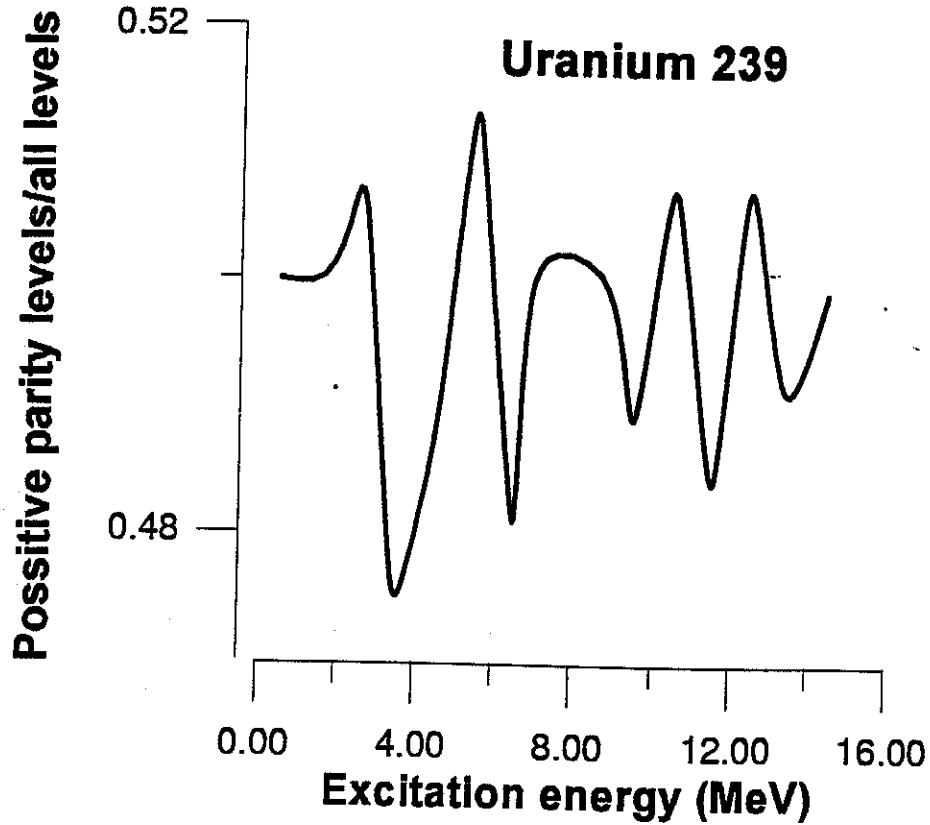


Fig.7

may not hold for levels with low exciton numbers, for which the density of states is too low for statistical treatment, to be justified.

In the case of spherical nuclei Herman and Reffo [7] found that Eq. (5) is valid for levels containing at least four and, to some extent, also three excitons.

The collective contribution to the level density of a deformed nucleus is defined by the symmetry order of nuclear deformation. For deformed nuclei the spectrum of its energy states will be determined not only by the internal excitations but also by the rotation of the nucleus as a whole. This rotation may lead to a considerable increase in the density of nuclear levels. Since the deformation of the nuclear potential removes the degeneracy of the basis vectors belonging to the same spin multiplet, we are no longer able to obtain the spin distribution of nuclear levels, according to Eq. (5).

Reffo et al [8] have studied the influence of nuclear deformation on the distributions of quasiparticle states. It is shown that deformation tends to suppress the strong fluctuations observed in the similar calculations performed in the space of spherical shell-model basis vectors. In a deformed nucleus, each intrinsic state gives rise to a rotational band on the total level spectrum, for a given angular momentum. It is therefore obtained by summing over a set of decomposition of the level spectrum, as for a spherical system. The experimental level data of the nuclei under study show that this prediction is well established. In that case, the following expression has been used for level-density calculation of axially-symmetric nuclei

$$R(J) = \frac{2I+1}{\sqrt{8\pi}\sigma_{\parallel}} e^{-\left(\frac{-I(I+1)}{2\sigma_{\perp}^2}\right)} \quad (12)$$

$$\rho_{2p1h}(U, J) = \omega_{2p1h}(U) \cdot R(J) .$$

In this formula the contribution of rotational states is taken into account. Here  $\omega_{2p1h}(U)$  is the microscopic level density of 2p-1h states.

The spin cut off parameters  $\sigma_{\perp}^2$  and  $\sigma_{\parallel}^2$  are calculated in the following way [19].

$$\sigma_{\perp}^2 = \frac{\mathfrak{S}_{\perp} t}{\hbar^2} \quad \sigma_{\parallel}^2 = \Omega^{-2} g t . \quad (13)$$

where  $t$  is the nuclear temperature and  $g$  is the single-particle level density near the Fermi energy, and  $\Omega^{-2}$  is the value of the average single-particle square projection on the symmetry axis of deformed nucleus.

The energy dependence of the moment of inertia  $\mathfrak{S}_{\perp}$  is approximated in the following way:

$$\mathfrak{S}_{\perp} = \begin{cases} (\mathfrak{S}_0 - \mathfrak{S}_{rig}) \left[1 - \frac{U}{U_{crit}}\right] & U < U_{crit} \\ \mathfrak{S}_{rig} & U > U_{crit} \end{cases} \quad (14)$$

$\mathfrak{S}_0$  is the moment of inertia in the ground state,  $U_{crit}$  is the maximum value of the transition energy from the superfluid to the normal state for neutron and proton system, and  $\mathfrak{S}_{rig}$  is the rigid body moment of inertia of the nucleus ( $\mathfrak{S}_{rig} = \frac{2}{5}Mr_o^2A^{\frac{2}{3}}$ ).

#### D. The parity distribution

According to Ericson [9], even and odd parity levels contribute equally to the level density. Being aware of the fact that the equal parity distribution is very questionable for levels with a fixed number of exciton, we have performed combinatorial calculations to investigate this problem. This distribution at a fixed value of energy can be described in terms of the asymmetry ratio:

$$A(U) = \frac{N^+(U)}{N^+(U) + N^-(U)} \quad (15)$$

where  $N^{\pi}$  are the number of levels with parity  $\pi = \pm 1$ .

### III. CALCULATIONS AND RESULTS

Based on the above described procedure, we consider in the following  $^{238}\text{U}$  and  $^{232}\text{Th}$ . Both nuclei are well deformed in their ground states and therefore, when they interact with epithermal neutrons the compound nucleus acquire reasonably high deformations. For this

reason, these two nuclei constitute a natural test of the procedure proposed in the present work.

Figures 1a and 1b show the potential energy surface calculated in the frame of the Strutinsky's Method [15] and the corresponding ground state deformation points for each case which are in agreement with the experimental data. As already mentioned, a comprehensive description of the level densities should include the characteristic of the nuclear configuration for different deformation parameters. In this sense, we show also other extreme points which define a possible path in the multidimensional space of the deformation degrees of freedom. This path could describe a decay-process of the compound system from its ground state and therefore has to be considered in calculations of cross sections in particular in reactions induced by epithermal neutrons.

Usually in calculations dealing with such highly deformed and excited compound nuclei where a drastic change of the nuclear shape ensues, one relies on extrapolations from the ground state structure properties. Of course, this represents a very crude procedure because in these regions the shell closure conditions are changing and one has to take into account the role of higher multipole coefficients to describe the nuclear shape.

The Chepurinov parameters, and the obtained deformation parameters allow us to reproduce the experimental low-lying quasi particle states of the nuclei and the ground state of neighboring nuclei.

The extremal points in Potential Energy Surface of  $\text{Th}^{233}$  are given in Table 1.

The single particle level density is a basic ingredient in the calculations, because the single particle spectra depend on the symmetry properties of the potential well. In nuclei deformed in their ground states the influence of the second minimum on the level density will be sharply decreased because of a smaller difference between deformation in two minima. This situation is showed in Table 2.

The calculations of the particle-hole level densities were carried out using the modified version of ICAR code [17]. The results of the calculation for 2p-1h states are shown in Fig.2 for positive parity in comparison with the Williams formula. The solid line represents

the Williams formula, while the dashed lines corresponds to calculation obtained with our method using the Cherpunov parameters of Wood-Saxon nuclear potential. The agreement is reasonable, though the Williams formula represents an average of the combinatorial level density.

The pairing strength parameter  $G$  is calculated from the difference of masses of neighboring nuclei as proposed by M. Herman [7]. The values obtained for  $^{232}\text{Th}$  and  $^{238}\text{U}$  are given in Table 3.

In Fig.3 we show the comparison of the behavior of the spin cutoff parameters  $\sigma_{\perp}^2$  and  $\sigma_{\parallel}^2$  as function of energy, for  $\text{Th}^{233}$  and  $\text{U}^{239}$ . The solid lines represent the results for the Th isotope using the approach described in eq.(13), while the dashed lines correspond to the calculations for the U isotope. In the case of  $\sigma_{\perp}$  the figure shows a similar behavior for both isotopes, which reflects the similarity of the moments of inertia calculated in our approach; this results from almost equal shapes found in the ground state. In the case of  $\sigma_{\parallel}$ , the calculations exhibit marked differences for the two isotopes. This fact reveals the need of taking into account the single-particle spectrum for each particular isotope, since for  $\sigma_{\parallel}^2$  this plays a crucial role.

In Fig.4 we show the behaviour of the spin factor distribution as a function of energy, for  $\text{Th}^{233}$  in the spherical and the deformed approaches. Note that the statistical assumption of a Gaussian distribution with spherical symmetry is not always justified, because for the cases of low  $J$  (in Fig.4 we show for  $J=\frac{1}{2}$ , see also Figs.5 and 6) it underestimates the calculations, which has an important influence on analyses such as those performed, in [4]. In Fig.5 we compare the influence of the spin cutoff parameters on the surface of the distribution  $R(E,J)$ , in comparison with the spherical formulae using eq.(11) for low energy and  $J$ -values. The absolute values differ by nearly two orders of magnitude for the spherical and deformed cases.

For small p-h excitations the exact combinatorial solutions described in Sect.II should be used. To take into account the rotational degrees of freedom, the rotational bands are built on each quasiparticle state, using the average values of the rotational constants  $A_r$  and

$B_r$  :

$$E(J, K^\pi) = E(K^\pi) + [J(J+1) - K^2] A_r + B_r \left(J + \frac{1}{2}\right) (-1)^{J+\frac{1}{2}} \delta_{K, \frac{1}{2}}$$

The low energy region is interesting from the point of view of its difference from the region where statistics is applicable. The number of levels is small and their energy and quantum number distributions may have strong fluctuations.

For large p-h excitations, the p-h level densities with spin distribution should be used with the approach described in Eq.(12). The width of the spin distribution can be determined using Eqs.(13) and (14). This procedure provides a convenient way for the analysis of deformed nuclei.

The results of the microscopic calculations of two-particle-one hole level densities for  $J=\frac{1}{2}$ , in  $\text{Th}^{233}$  and  $\text{U}^{238}$ , are shown in Fig. 6. The impact of collective rotational enhancement in the level density can result values 3 times large than those from the Williams formula using the standard parameters for these nuclei. This procedure can be useful in order to reevaluate the parameters of the Williams formula.

Finally, in Fig.7 the trend of the fraction of positive parity levels for the analyzed nuclei as a function of excitation energy (up to 15 MeV) is plotted. Here, this fraction is given by the parity asymmetry defined by Eq.(15) as a function of  $U$ , where  $N^\pm(U)$  is calculated by a direct counting of the levels. The regions of lower energies show the greatest fluctuations in  $A$  (eq. 15); although these fluctuations persist at higher energies, a slow smooth out is observed. This observed strong oscillatory character, not predicted by the Ericson hypothesis, must be taken into account in studies of PNC, for which the fine structure effects are important.

#### IV. CONCLUSION AND FINAL REMARKS

We have presented a realistic few quasiparticle level density approach for deformed nuclei, which should be preferred to the more phenomenological expression commonly used. The more realistic inclusion of the rotational degree of freedom improves the calculations in



the region of the actinide nuclei, where deformation is not negligible even in the ground state. The analysis of nuclear reactions in the region of the actinide nuclei, with epithermal neutrons, requires the consideration of nuclear deformations where the amplification of the collective movement must be taken into account. The deformation introduces significant fluctuations in the level densities. This prevents the use of a single analytical expression able to reproduce these densities in different mass regions. In this regard, a judicious analysis of the extreme points becomes essential. When one needs to analyze the shape of the nucleus corresponding to the external saddle point, it is necessary to use a procedure like the one here proposed, because the nuclear deformation for a fixed configuration of particles and holes neither suppresses the fluctuations that takes place in the level densities, nor the parity distribution. The importance of our calculation for an eventual quantitative understanding of the mechanism of PNC in epithermal neutron induced compound reactions using actinide nuclei is discussed.

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## VII. TABLE CAPTIONS

Table 1. Deformation parameters of extremal points in potential energy surface of  $^{233}\text{Th}$ .

Table 2. Single-particle level densities of extremal points in  $^{233}\text{Th}$ .

Table 3. Pairing constants of  $^{233}\text{Th}$  and  $^{238}\text{U}$ .

## VIII. FIGURE CAPTIONS

Figure 1a.  $^{233}\text{Th}$  Surface Potential Energy. See text for details.

Figure 1b.  $^{239}\text{U}$  Surface Potential Energy. See text for details.

Figure 2. Microscopic combinatorial 2p-1h level density corresponding to positive parity in comparison with the William formula. See text for details.

Figure 3. Spin cutoff parameters vs. excitation energy. See text for details.

Figure 4. The level density spin distribution factor  $R(J)$  for  $^{233}\text{Th}$  vs. excitation energy

Figure 5. Two-dimensional Plot of  $R(J)$  for  $^{233}\text{Th}$ . See text for details.

Figure 6. The 2p-1h level density for  $^{233}\text{Th}$  and  $^{239}\text{U}$ . The Microscopic Combinatorial Method results are compared with the equidistant Williams formula.

Figure 7. Percentage of positive parity levels in  $^{233}\text{Th}$  and  $^{239}\text{U}$ . See text for details.

## IX. TABLES

Table 1.

Def. param	1st Min.	1 Saddle	2d min.	3th min.	2 Saddle	3 Saddle
$\epsilon$	0.22	0.383	0.49	0.7213	0.7044	0.755
$\alpha_3$	0.0	0.0	0.0	$\pm 0.1179$	0.0	$\pm 0.087$
$\alpha_4$	0.071	-0.0827	0.0275	0.0386	0.0386	0.0386

Table 2.

Ext. point	$g_n$	$g_z$	$g^{lev}$
G.st	3.651	2.693	6.345
2 Min	3.705	2.669	6.374
2d Sadd Asymm	3.77	2.58	6.35
2d Sadd Symm	4.034	3.069	7.103
3rd Min	3.661	2.609	6.27

Table 3.

	Chepurnov parameters	Optimal parameters
Th <sup>233</sup>	20.25	18.5
U <sup>239</sup>	19.9	18.0

X.