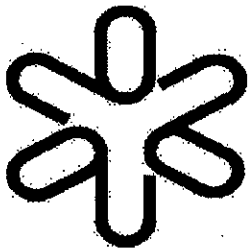


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forward scattering amplitudes at two loops**

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Retarded thermal Greens functions and forward scattering amplitudes at two loops

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Abstract

In this paper, we extend our earlier one loop analysis to two loops and give a simple diagrammatic description for the retarded Greens functions at finite temperature, in terms of forward scattering amplitudes of on-shell thermal particles. We present a simple discussion, which can be easily generalized to any field theory, of the temperature dependent parts of the retarded two and three point functions in scalar field theory and QED. As an application of our result at two loops, we show how the infrared singularities in the thermal part of the retarded photon self-energy, cancel in QED₂ in the limit of vanishing electron mass.

I. INTRODUCTION

In recent years, there has been an increased interest in the study of field theories at finite temperature for a variety of reasons [1–5]. This has led to a better understanding of the formalism of such theories at finite temperature. In particular, we understand now that, in addition to the conventional imaginary time formalism, physical phenomena at finite temperature can equally well be described by the real time formalism where the time variable is not traded in for the temperature of the system [6–8].

The brute force calculations in field theories at finite temperature are quite tedious, particularly, in the real time formalism where there is a necessity to double the degrees of freedom of a theory which leads to $(2^n - 1)$ independent causal amplitudes at n -th order. Therefore, it is quite important to find simple calculational methods in such theories for them to be useful. Furthermore, even though there exist various formal arguments showing the equivalence of the two distinct formalisms [9–12] (imaginary time and real time), it is still necessary to check explicitly that the two formalisms give the same physical quantities, at least in lower orders.

Amplitudes calculated in the imaginary time formalism naturally give rise to retarded (advanced) amplitudes with appropriate analytic continuation to the Minkowski space. In an earlier paper [13], we gave a very simple diagrammatic representation of such amplitudes (retarded) in the real time formalism, at one loop, which is calculationally quite trivial and which also explicitly verifies the agreement of the two formalisms to this order. In this paper, we would like to extend our earlier results to two loops. Namely, we would give a simple diagrammatic description of the retarded two loop amplitudes at finite temperature in the real time formalism which is calculationally much simpler (than a brute force calculation) and which also explicitly verifies the agreement of the imaginary time and the real time formalisms up to two loops. This description expresses the temperature dependent part of the retarded Green functions in terms of forward scattering amplitudes for on-shell particles of thermal medium. These amplitudes are multiplied by the appropriate statistical factors

and integrated over the three-momenta of the thermal particles (for details and alternative approaches related to this method see the references [14]).

The paper is organized as follows. In section II, we analyze the temperature dependent part of the retarded self-energy for the ϕ^4 theory up to two loops in the real time formalism and give a simple diagrammatic representation for it. We explicitly evaluate the retarded self-energy and show that it coincides with the results obtained earlier using the imaginary time formalism. In section III, we analyze the temperature dependent part of the retarded photon self-energy in QED at two loops and obtain a simple diagrammatic representation for it. We also analyze briefly and find a simple diagrammatic representation for the temperature dependent retarded three point function at two loops in QED. (The diagrams for the amplitudes, in this theory, are more like those in the ϕ^3 model.) Since our discussion is quite general, it is easy to extend these results to the temperature dependent part of any retarded n -point amplitude at two loops (for any bosonic or fermionic theory). In section IV, we discuss, as an example of our method, how the infrared mass singularities in the retarded photon self-energy, cancel at finite temperature for the Schwinger model [15] at two loops. Finally, we present a brief conclusion in section V.

II. RETARDED SELF-ENERGY IN ϕ^4 THEORY

Let us consider the theory of a self-interacting real scalar field described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 \quad (1)$$

As is well known, at finite temperature, in the real time formalism (we will follow the closed time path formalism although everything can be carried over to thermo field dynamics equally well), one needs to double the degrees of freedom, which leads to a 2×2 matrix structure for the propagator

$$G_{ab}(q) = G_{ab}^{(0)}(q) + G_{ab}^{(\beta)}(q) \quad a, b = \pm \quad (2)$$

Here $G_{ab}^{(0)}$ represents the zero temperature propagator while $G_{ab}^{(\beta)}$ is the on-shell, temperature dependent part of the propagator and, in the closed time path formalism, all the four components of the temperature dependent part of the propagator are identical. Explicitly,

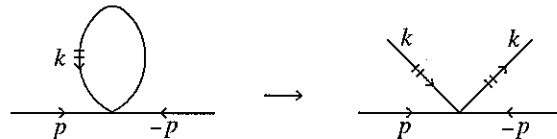
$$G^{(0)}(q) = \begin{pmatrix} \frac{1}{q^2 - m^2 + i\epsilon} & -2i\pi\theta(-q^0)\delta(q^2 - m^2) \\ -2i\pi\theta(q^0)\delta(q^2 - m^2) & -\frac{1}{q^2 - m^2 - i\epsilon} \end{pmatrix}$$

$$G^{(\beta)}(q) = -2i\pi n_B(|q^0|)\delta(q^2 - m^2) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad (3)$$

where $n_B(|q^0|)$ stands for the bosonic distribution function (and the propagator really is iG_{ab}). Following ref. [13], we introduce the following diagrammatic representation for the two parts of the propagator

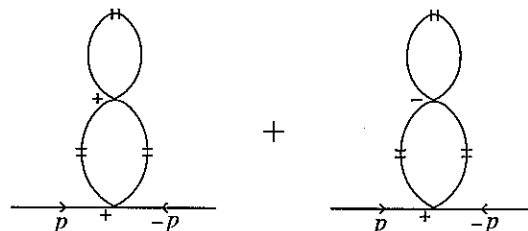
$$G_{ab}^{(0)}(q) = a \xrightarrow{q} b; \quad G_{ab}^{(\beta)}(q) = a \xrightarrow{+} b \quad (4)$$

and the temperature dependent part of the self-energy, at one loop, is easily obtained to be (this also coincides with the retarded self-energy to this order) the forward scattering amplitude for a single on-shell thermal particle



$$\quad \quad \quad (5)$$

In going beyond one loop, we note that there are two kinds of diagrams for the self-energy at two loops in this theory, namely, the penguin and the rising sun diagrams. Let us analyze each class of the diagrams (which involve three internal propagators) separately. (The real time method for obtaining retarded Greens functions is explained in section III of reference [13].) For the penguins, it is straightforward to check that all the diagrams for the retarded self-energy where all three internal propagators are cut cancel, namely,



$$\quad \quad \quad (6)$$

This follows trivially from the fact that the cut propagators are the same, independent of their thermal indices, as is clear from Eq. (3) and the fact that the vertices with the $-$ thermal index have a negative sign relative to those with the $+$ thermal index. Some of the diagrams with two internal cut propagators can also be easily seen to add up to zero

$$\begin{array}{c} \text{Diagram 1} \end{array} + \begin{array}{c} \text{Diagram 2} \end{array} = 0. \quad (7)$$

However, not all the diagrams with two internal cut propagators add up to zero and all the diagrams with a single cut propagator do contribute. Thus, all the temperature dependent contribution coming from the penguin diagrams to the two loop retarded self-energy can be seen to be

$$\begin{array}{c} \text{Diagram 1} \end{array} + \begin{array}{c} \text{Diagram 2} \end{array} + \begin{array}{c} \text{Diagram 3} \end{array} + \begin{array}{c} \text{Diagram 4} \end{array} + \begin{array}{c} \text{Diagram 5} \end{array} + \begin{array}{c} \text{Diagram 6} \end{array}, \quad (8)$$

where graphs containing permuted cuts on the internal propagators in the lower bubble are to be understood.

Next, let us look at the rising sun diagrams for the self-energy at two loops. Once again, in the retarded self-energy, all the diagrams with three internal cut propagators add up to zero

$$\begin{array}{c} \text{Diagram 1} \end{array} + \begin{array}{c} \text{Diagram 2} \end{array} = 0 \quad (9)$$

quadratic in the statistical factor corresponding to two cut lines can be thought of simply as the forward scattering amplitude of two on-shell thermal particles. The terms which are linear in the statistical factor or have a single cut propagator, on the other hand, correspond to the forward scattering amplitudes of a single on-shell thermal particle with all possible one loop self-energy and vertex corrections where the corrections are retarded and have to be evaluated at zero temperature.

All these terms are rather simple to evaluate, by doing the energy integrals with the help of the delta functions in Eq. (3). One then obtains from the diagrams which depend on the external momentum, that the relevant temperature dependent part at two loops is given by

$$\begin{aligned} \Sigma_R^{(\beta)}(p_0, \vec{p}) &= \frac{\lambda^2}{16} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{1}{\omega_k} \frac{1}{\omega_q} \frac{1}{\omega_r} \{n_B(\omega_k) [D(\omega_k, \omega_q, \omega_r) + D(-\omega_k, \omega_q, \omega_r)] \\ &\quad + n_B(\omega_k)n_B(\omega_q) [D(\omega_k, \omega_q, \omega_r) + D(-\omega_k, \omega_q, \omega_r) \\ &\quad + D(\omega_k, -\omega_q, \omega_r) - D(\omega_k, \omega_q, -\omega_r)]\}, \end{aligned} \quad (13)$$

where $\omega_k \equiv \sqrt{\vec{k}^2 + m^2}$, $\vec{r} = \vec{k} + \vec{q} - \vec{p}$ and

$$D(\omega_k, \omega_q, \omega_r) = \frac{1}{p_0 + i\epsilon + \omega_k + \omega_q + \omega_r} + \frac{1}{-(p_0 + i\epsilon) + \omega_k + \omega_q + \omega_r}. \quad (14)$$

Such a form of the retarded thermal self-energy is rather convenient, particularly for the calculation of its imaginary part which is related to the plasma damping rate. The above result agrees with the one obtained by Parwani in ref. [16], after analytic continuation from the imaginary time formalism. In this way, one can establish an explicit equivalence between the two formalisms up to two loops much like in the one loop case.

III. RETARDED GREENS FUNCTIONS IN QED

Let us next consider the QED theory described by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\cancel{\partial} - m + eA)\psi, \quad (15)$$

where A_μ is the photon field, $F_{\mu\nu}$ the field strength tensor and ψ is the Dirac field. Apart from a matrix factor and the replacement of $(-n_B)$ by the fermionic distribution function

n_F , the structure of the electron propagator is similar to that in Eqs. (2) and (3). However, let us note that, for the purpose of our discussion, the actual form of the propagator is irrelevant, all that is crucial is that the temperature dependent part of any propagator (bosonic or fermionic) is the same for all the thermal indices in the closed time path formalism. (Consequently, our discussion will be quite general without restrictions to a particular theory.) Assuming that the largest time vertex is the one connected with the external line p , the retarded thermal self-energy of photons at one loop is given by the forward scattering amplitude of a single on-shell thermal particle

$$\left[\text{Diagram} \right]_{Ret}^{\beta} \rightarrow \text{Diagram } R + \text{Diagram } A \quad (16)$$

Note that the propagator with momentum flowing outwards from the largest time vertex is retarded while the one with momentum flowing towards this vertex is advanced (see ref. [13] for details).

In going beyond one loop, we note that there are again two classes of diagrams - ones containing overlapping divergences and the others with the structure of one loop diagrams with a self-energy correction for one of the internal propagators. Each of these classes of diagrams contains five internal propagators. As before, it is easy to check that the sum of all diagrams containing five cut propagators, four cut propagators as well as three cut propagators individually vanish which follows from the fact that the cut propagators are the same irrespective of their thermal indices and that vertices with $-$ thermal index have an extra negative sign. Some of the graphs with two internal cut propagators also cancel, but not all of them. And all the graphs with a single cut propagator contribute. Adding them all up and remembering that a cut propagator is on-shell and hence can be thought of as an external thermal particle, we again find that the temperature dependent part of the retarded self-energy, at two loops, can be written as

$$\left[\text{Diagram 1} + \text{Diagram 2} \right]_{Ret}^{(\beta)} \rightarrow$$

(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

(j) (k) (l)

(17)

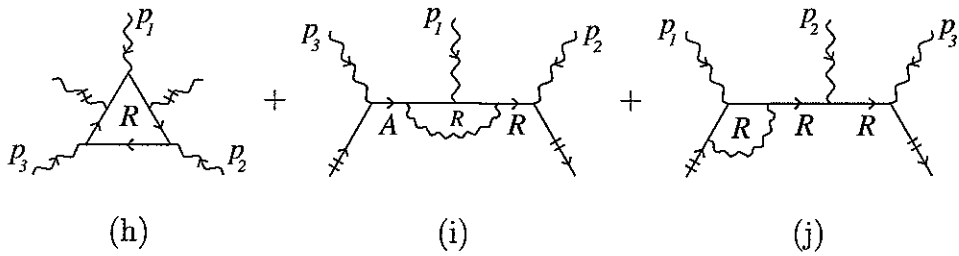
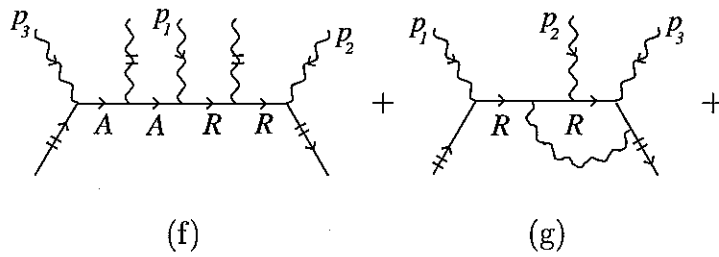
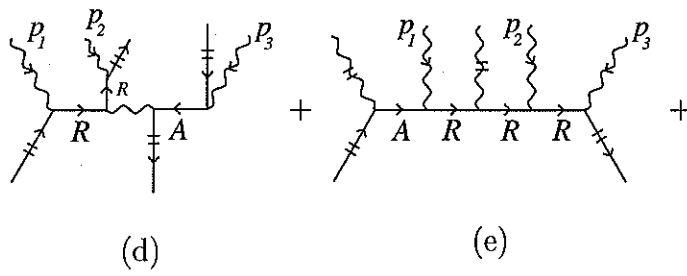
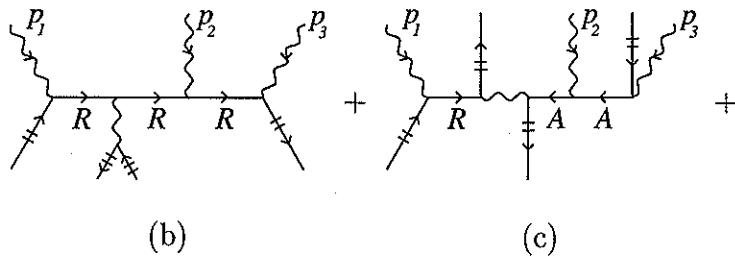
Here, all other diagrams obtained by the permutation of external lines are to be understood. Once again, it is clear that the temperature dependent part of the two loop retarded self-energy contains terms which are quadratic in the statistical factors as well as ones which are linear in the statistical factor. The terms which are quadratic in the statistical factor, come from diagrams with two cut propagators each of which can have a simple description of a forward scattering amplitude for two on-shell thermal particles. On the other hand, those

which are linear in the statistical factor, come from diagrams with a single cut propagator and can be described in terms of forward scattering amplitudes of a single on-shell thermal particle with all possible one loop self-energy and vertex corrections. As in the last section, we note that these corrections are retarded and are to be evaluated at zero temperature. This is, indeed, a very simple description of the two loop retarded self-energy and much simpler to evaluate than the brute force calculation would entail.

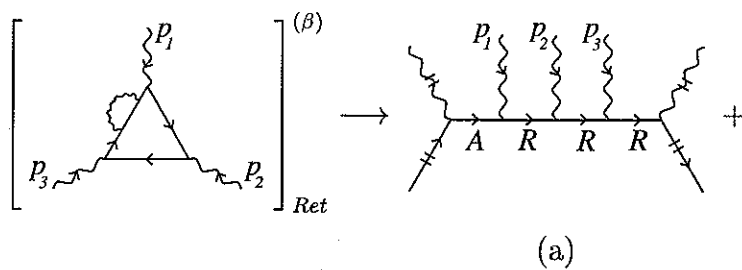
Let us evaluate next the temperature dependent part of the retarded three point function, at two loops order (although the three photon vertex vanishes in QED by charge conjugation, our analysis does also apply to a theory like QCD, where the three gluon vertex is non-zero). Without going into too much detail, let us note that there will be two classes of diagrams contributing to this - one containing diagrams which can be thought of as a vertex correction to the one loop three point function and the other with graphs which contain a one loop self-energy correction in one of the internal propagators of the one loop three point vertex function. The diagrams in each of these two classes of graphs consist of six internal propagators and it is straightforward to check that the sum of graphs with six cut propagators, five cut propagators, four cut propagators as well as three cut propagators individually vanish. Some of the graphs with two cut propagators also add up to zero, but not all. Furthermore, all the graphs with a single cut propagator contribute. The non-vanishing diagrams can again be given a simple graphical representation by remembering that a cut line is on-shell and can be thought of as an external thermal particle (of the medium). We thus get, for example, the graphical equations

$$\left[\begin{array}{c} \text{Diagram: Triangle with external lines } P_1, P_2, P_3 \text{ and internal lines} \\ \text{Ret} \end{array} \right]^{(\beta)} \rightarrow \text{Diagram: Chain of four retarded propagators } R \text{ with external lines } P_1, P_2, P_3 + \dots$$

(a)



(18)



(b) P_1 P_2 P_3 R R R R

(c) P_3 P_2 P_1 A A R R

(d) P_2 P_3 P_1 A A A A

(e) P_3 P_1 P_2 A A A R

(f) P_1 P_2 P_3 R

(g) P_1 P_2 P_3 R

(h) P_1 P_2 P_3 R R R R

(i) P_1 P_3 P_2 R R R R

(j) P_2 P_1 P_3 A R R R

(19)

In the above forward scattering amplitudes, all other relevant permutations of the external lines are to be understood. Here, we have assumed that the vertex with external momentum p_1 corresponds to the largest time vertex. Then, all propagators with momentum flowing towards this vertex are advanced, whereas the propagators with momentum flowing outward from the largest time vertex are retarded. Once again, we note the familiar pattern. The temperature dependent part of the retarded three point function, at two loops, consists of

two types of terms. The ones which are quadratic in the statistical factor correspond to simple forward scattering amplitudes of two on-shell thermal particles (of the medium) whereas the ones which are linear in the statistical factor correspond to the forward scattering amplitude of a single thermal on-shell particle with all possible one loop self-energy and vertex corrections. Such retarded one loop corrections, which are calculated at zero temperature, may contain ultraviolet divergences and must be renormalized as usual.

From the analysis of these two loop amplitudes so far, it is quite straightforward to find a simple, general diagrammatic description of the two loop retarded n -point amplitudes. *The temperature dependent part of a general two loop retarded n -point amplitude can be written in terms of two classes of diagrams. The first class would involve simple forward scattering amplitudes for two on-shell thermal particles (two cut lines) and would correspond to terms which are quadratic in the statistical factor. The second class of diagrams would involve forward scattering amplitudes of a single on-shell thermal particle (one cut line) with all possible one loop self-energy and vertex corrections, where these corrections are retarded and are to be evaluated at zero temperature.*

IV. APPLICATION

In this section, we will study the QED₂ model at finite temperature and, as an application of the general method, show how the infrared mass singularities of the temperature dependent retarded photon self-energy cancel at two loops in the limit of vanishing fermion mass. This gauge theory, which is described by the Lagrangian density (15) in 1 + 1 dimensions, is well behaved in the ultraviolet region. For simplicity, we will work in the Feynman gauge although the choice of gauge is immaterial, since the finite temperature results are gauge invariant in the Schwinger model.

If we look at the photon self-energy in this theory, then, from our discussions, we recognize that the temperature dependent part of the retarded self-energy, at one loop, can be written in terms of the forward scattering amplitude of a thermal on-shell fermion as shown in

Eq. (16). The two diagrams give identical contribution, since inverting the direction of the internal momentum flow interchanges the advanced and retarded propagators. Unlike in higher dimensions, in 1 + 1 dimensions, there is only one transverse structure for the polarization tensor at finite temperature. Furthermore, there are special identities for the Dirac trace in 1 + 1 dimensions which make the one loop calculation quite simple and give

$$\begin{aligned}\Pi_{R(1)}^{(\beta)}(p^0, p^1) &= -\frac{2e^2m^2}{\pi} \int d^2k \frac{n_F(|k^0|)}{(k^1 + p^1)^2 - m^2 + i\epsilon(k^0 + p^0)} \delta(k^2 - m^2) \\ &= -\frac{e^2m^2}{2\pi} \int dk^1 \frac{n_F(\omega_k)}{\omega_k \omega_{k+p}} \left[\frac{1}{\omega_k + p^0 - \omega_{k+p} + i\epsilon} - \frac{1}{\omega_k + p^0 + \omega_{k+p} + i\epsilon} \right. \\ &\quad \left. + \frac{1}{\omega_k - p^0 - \omega_{k+p} - i\epsilon} - \frac{1}{\omega_k - p^0 + \omega_{k+p} - i\epsilon} \right]\end{aligned}\quad (20)$$

Here, we have identified the space components of p^μ and k^μ with p^1 and k^1 respectively, n_F is the fermionic distribution function and $\omega_k = \sqrt{(k^1)^2 + m^2}$. Clearly, this vanishes as $m \rightarrow 0$.

In going beyond one loop, let us note that the infrared mass singularities in the retarded photon self-energy come from the graphs with a one loop self-energy correction in the internal fermion propagator, particularly when the internal photon propagator is cut (irrespective of whether there are other cuts in the diagram). Consider, for example, the following set of thermal diagrams

$$\left[\begin{array}{ccc} \text{(a)} & \text{(b)} & \text{(c)} \end{array} \right]_{Ret} \quad (21)$$

The cut photon line would give an infrared divergent contribution of the form (coming from $k \rightarrow 0$)

$$I(T/\lambda) = \int \frac{d^2k}{(2\pi)^2} \frac{1}{e^{|k^0|/T} - 1} \delta(k^2 - \lambda^2) = \frac{1}{(2\pi)^2} \left[\frac{\pi T}{2\lambda} + \frac{1}{2} \ln \frac{\lambda}{T} + \dots \right] \quad (22)$$

Here, λ is an infrared regulating photon mass which has to be set to zero at the end. Once we have isolated this factor, the photon momentum can be safely set to zero in the rest of the diagrams.

As we have seen earlier, at two loops, only graphs with two cut propagators or a single cut propagator can give a non-vanishing contribution. A little bit of analysis shows that in graphs with a single cut on the photon propagator, which as shown in Fig. (17-g) contain a retarded fermion loop calculated at zero temperature, the infrared divergent contributions cancel. The graphs with two cut propagators, shown in the set (21), can be expressed in terms of the forward scattering amplitudes (a), (b) and (c), indicated in the graphical equation (17). To evaluate these, we note that in consequence of special kinematic relations which hold in $1 + 1$ dimensions, there will again appear in the numerator an overall factor m^2 (compare with Eq. (20)). One may naively think that also in this case, this factor would be sufficient to ensure the vanishing of the whole contribution, in the limit $m \rightarrow 0$. However, such an argument would be fallacious at two loop order, where the infrared mass singularities of the theory begin to manifest. In this case, there are individual contributions whose denominators behave like $1/m^2$ as $m \rightarrow 0$. Such terms would then give rise to contributions proportional to $I(T/\lambda)$, which are independent of m . Therefore, a more complete analysis is necessary in order to ascertain the behavior of the theory in the limit of vanishing fermion mass.

To this end, we note that the infrared behavior of the diagrams in the set (21) is determined by the regions $k \rightarrow 0$ and $q, m \ll p$ (we assume that $p^2 \neq 0$). Then, in the corresponding forward scattering amplitudes (17-a, b, c)), we expand the retarded and advanced propagators into a sum of principal values and delta functions. Adding all such contributions, it turns out that the infrared divergent part of the above diagrams is given by

$$\tilde{\Pi}_{R(2)}^{(\beta)}(\lambda, p) = \frac{16e^4}{p^2} m^2 I(T/\lambda) \int d^2q n_F(|q^0|) (q^2 + m^2) \left\{ \pi^2 [\delta(q^2 - m^2)]^3 - 3 \left[\mathcal{P} \left(\frac{1}{q^2 - m^2} \right) \right]^2 \delta(q^2 - m^2) \right\}. \quad (23)$$

In order to simplify the expression in the curly bracket, we make use of the identity

$$\frac{1}{(q^2 - m^2 + i\epsilon)^3} = \mathcal{P} \left[\left(\frac{1}{q^2 - m^2} \right)^3 \right] - \frac{i\pi}{2} \delta''(q^2 - m^2), \quad (24)$$

where the derivatives in the delta function are with respect to its argument. From the imaginary part of this equation, one can see that the relation (23) can be written as

$$\tilde{\Pi}_{R(2)}^{(\beta)}(\lambda, p) = -\frac{8e^4}{p^2} m^2 I(T/\lambda) \int d^2q n_F(|q^0|) (q^2 + m^2) \delta''(q^2 - m^2). \quad (25)$$

Performing the q_0 integration and neglecting terms of order (m/T) , one finds after some calculation that Eq. (25) becomes

$$\begin{aligned} \tilde{\Pi}_{R(2)}^{(\beta)}(\lambda, p) &= \frac{8e^4}{p^2} m^2 I(T/\lambda) \int_0^\infty \frac{d\omega_q}{\omega_q} n_F(\omega_q) \left[\frac{2}{\omega_q^2} - \frac{3m^2}{\omega_q^4} \right] + \dots \\ &\simeq \frac{8e^4}{p^2} m^2 I(T/\lambda) \left[\frac{1}{m^2} \left(1 - 3 \times \frac{1}{3} \right) \right] + \dots = 0 + \frac{e^4}{p^2} I(T/\lambda) \mathcal{O}(m/T). \end{aligned} \quad (26)$$

It is interesting to note how the individual terms, which would give infrared divergent contributions independent of m , cancel in the complete amplitude. A similar behavior also occurs when the photon propagator is uncut.

The above results show that, in the limit $m \rightarrow 0$, the infrared mass singularities in the retarded photon self-energy cancel at finite temperature, at two loops. Due to the presence of infrared divergences associated with a massless photon, one cannot directly conclude that contributions like those on the right-hand side of (26) necessarily vanish as $m \rightarrow 0$. On the other hand, if we resum the theory, we know that the photon becomes massive so that we can replace the infrared regulator λ by the photon mass in the theory, namely $m_\gamma = e/\sqrt{\pi}$. We then see that all such contributions would indeed cancel in the limit $m \rightarrow 0$, which is consistent with our earlier analysis [13]. In view of this behavior of the infrared domain and due to the presence of an overall factor of m^2 in all other regions, the complete thermal amplitudes in QED₂ will therefore reduce, as $m \rightarrow 0$, to those of the massless Schwinger model. Presumably, this occurs because in the transition from a massive to a massless fermion, the number of fermionic degrees of freedom is conserved. Therefore, as far as the physical (advanced/retarded) amplitudes are concerned, it appears that in the limit of vanishing fermion mass, the thermal QED₂ theory may describe a free gas of massive bosons at finite temperature.

V. CONCLUSION

In this paper, we have extended our earlier one loop results on the calculation of temperature dependent parts of retarded Greens functions to two loops. We have analyzed the temperature dependent part of the retarded two and three point functions in scalar field theory and in QED. We have given a simple representation of these functions in terms of forward scattering amplitudes of on-shell thermal particles. This is computationally much easier than brute force calculations and leads to the same result as calculated from the imaginary time formalism, thereby establishing an explicit agreement between the two formalisms up to this order. From these analyses, we find that the temperature dependent part of the retarded Greens functions, at two loops, can have a simple description in terms of two classes of diagrams. The first one consists of simple forward scattering amplitudes of two on-shell thermal particles, corresponding to terms quadratic in the statistical factor. The second class, characterized by terms which are linear in the statistical factor, involves forward scattering amplitudes of a single on-shell thermal particle. These contain all possible one loop self-energy and vertex corrections, which are retarded and have to be calculated at zero temperature. As an application of these results, we have shown how the infrared singularities in the temperature dependent part of the retarded photon self-energy cancel at two loops in QED₂, in the limit of vanishing fermion mass.

We expect that the above representation of retarded Greens functions at finite temperature can be generalized for any theory to N -loop order. Namely, these functions may be described in terms of forward scattering amplitudes of N on-shell thermal particles, amplitudes of $N - 1$ on-shell thermal particles with all possible retarded one loop self-energy and vertex corrections evaluated at zero temperature, and so on. Such a representation leads to a simple and systematic way of calculating retarded (advanced) thermal amplitudes, which is rather useful especially in the case of non-Abelian gauge theories [14,17].

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