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The Solar Neutrino Problem and Gravitationally Induced Long-wavelength Neutrino Oscillation

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We have reexamined the possibility of explaining the solar neutrino problem through long-wavelength neutrino oscillations induced by the breakdown of the weak equivalence principle of general relativity. We found that this can be a viable solution to the solar neutrino problem.

Nature seems to be most strongly in agreement with neutrino oscillations. The compelling evidences coming from solar neutrino experiments [1–4], that span over two decades, and from atmospheric neutrino experiments [5] are difficult, if not impossible, to be accommodated without admitting neutrino flavor conversion. Nevertheless the dynamics underlying such conversion is yet to be established and in particular does not have to be a priori related to the electroweak force.

The interesting idea that gravitational forces may induce neutrino mixing and flavor oscillations if the weak equivalence principle of general relativity is violated, was proposed by Gasperini [6] and independently by Halprin and Leung [7] about a decade ago, and thereafter, many works have been performed on this subject [8–14]. In Ref. [15] this was shown to be phenomenologically equivalent to velocity oscillations of neutrinos due to a possible violation of Lorentz invariance [16]. So even a tiny breakdown of the space-time structure of special and/or general relativity may lead to neutrino oscillations even if they are strictly massless.

Some theoretical insight on the type of gravitational potential that could violate the weak equivalence principle can be found in Ref. [17]. A discussion on the departure from exact Lorentz invariance in the standard model Lagrangian in a perturbative framework is developed in Ref. [18].

Several authors have investigated the possibility of solving the solar neutrino problem (SNP) by such gravitationally induced neutrino oscillations [9–11], generally finding it necessary, in this context, to invoke the MSW like resonance [7] since they conclude that it is impossible that this type of long-wavelength vacuum oscillation could explain the specific energy dependence of the data [9,10].

Recently these neutrino oscillation mechanisms have been investigated [12–14] in the light of the experimental results from Super-Kamiokande (SK) on the atmospheric neutrino anomaly, obtaining stringent limits for the $\nu_\mu \to \nu_\tau$ channel.

We consider in this letter the possibility of explaining the most precise and recent solar neutrino data coming from gallium, chlorine and water Cherenkov detectors by means of neutrino mixing due to a "just-so" violation of the weak equivalence principle (VEP). We demonstrate that all the data can be well accounted for by the VEP induced long-wavelength neutrino oscillation in contrast to previous conclusions [9,10,15].

We assume that neutrinos of different species will incur different time delay due to the weak, static gravitational field in the intervening space on their way from the Sun to the Earth. Their motion in this gravitational field can be appropriately described by the parametrized post-Newtonian formalism [19] with a different parameter for each neutrino type. In this manner neutrinos that are weak interaction eigenstates and neutrinos that are gravity eigenstates will be related by a unitary transformation that can be parameterized, assuming only two neutrino flavors, by a single parameter, the mixing angle θ_G which can lead to flavour oscillation [6].

Let us briefly revise the formalism that will be used in this work. We will assume oscillations only between two species of massless neutrinos, either between active and active ($\nu_e \leftrightarrow \nu_\mu, \nu_\tau$) or active and sterile ($\nu_e \leftrightarrow \nu_s$, ν_s being an electroweak singlet) neutrinos.

The evolution equation for neutrino flavors α and β propagating through the gravitational potential $\phi(r)$ in the absence of matter is [6] :

$$i\frac{d}{dt}\begin{bmatrix}\nu_\alpha \\ \nu_\beta\end{bmatrix} = E\phi(r)\Delta\gamma \begin{bmatrix}\cos 2\theta_G & \sin 2\theta_G \\ \sin 2\theta_G & -\cos 2\theta_G\end{bmatrix}\begin{bmatrix}\nu_\alpha \\ \nu_\beta\end{bmatrix}, \tag{1}$$

where E is the neutrino energy; $\Delta\gamma$ is the quantity which measures the magnitude of VEP, it is the difference of the gravitational couplings between the two neutrinos involved normalized by the sum.

There are many possible sorces for ϕ, but it is generally believed that the Super Cluster contribution ($\phi \sim 3 \times 10^{-5}$) would be the dominant one [20]. Therefore, it seems reasonable to ignore any variation of ϕ over the whole solar system and take it as a constant [21]. In this case Eq. (1) can be analytically solved to give the survival

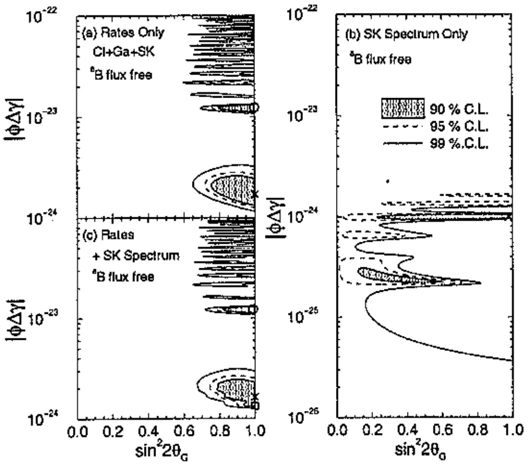

FIG. 1. Allowed region for $\sin^2 2\theta_G$ and $|\phi\Delta\gamma|$ for (a) the rates only, (b) SK spectrum only and (c) rates + SK spectrum combined. The best fit points are indicated by the crosses and the local best fit points in the other 90 % C.L. islands are indicated, in each plot, by the open circles. The "test point" which will be used in Fig. 2 and 4 is indicated by the open square (see also the text).

probability of ν_e produced in the Sun after traveling the distance L to the Earth:

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_G \sin^2 \frac{\pi L}{\lambda}, \qquad (2)$$

where the oscillation wavelength λ is given by,

$$\lambda = \left[\frac{\pi \text{ km}}{5.07}\right] \left[\frac{10^{-15}}{|\phi\Delta\gamma|}\right] \left[\frac{\text{MeV}}{E}\right], \qquad (3)$$

which in contrast to the wavelength for mass induced neutrino oscillations in vacuum, is inversely proportional to the neutrino energy.

In this case the survival probability is a function of two unknowns parameters that can be fitted, or constrained, by experimental data: $\Delta\gamma$ and $\sin 2\theta_G$. Since the value of the potential ϕ in our solar system is somewhat uncertain [21], we will adopt the procedure used by other authors and work with the product $\phi\Delta\gamma$.

We will perform a fit of the rates and SK recoil-electron spectrum but not take into account the day night effect (or zenith angle dependence) in SK. This is justified by the fact that day night variations can not be induced by this mechanism, and therefore, are irrelevant in determining the allowed parameter region. We will comment about the possible seasonal variations at the end.

We first examine the observed solar neutrino rates in the VEP framework. In order to do this we have calculated the theoretical predictions for gallium, chlorine and Super-Kamiokande water Cherenkov solar neutrino experiments, as a function of the two VEP parameters, using the solar neutrino fluxes predicted by the Standard Solar Model by Bahcall and Pinsonneault (BP98 SSM) [22] taking into account the eccentricity of the Earth orbit around the Sun.

We then have performed a χ^2 analysis to fit these parameters and an extra normalization factor f_B for the

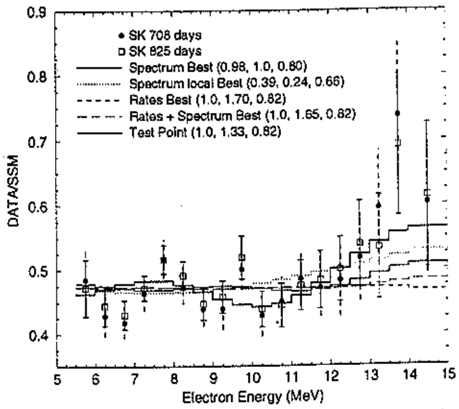

FIG. 2. Expected recoil-electron spectra at SK for the best fitted parameters of the VEP scenarios, which are indicated in the legend of the plot as ($\sin^2 2\theta_G$, $|\phi\Delta\gamma| \times 10^{24}$, f_B). The preliminary data from SK are also plotted.

^{8}B neutrino flux, to the most recent experimental results coming from Homestake [1] $R_{\rm Cl} = 2.56 \pm 0.21$ SNU, GALLEX [3] and SAGE [2] combined $R_{\rm Ga} = 72.5 \pm 5.5$ SNU and SK [4] $R_{\rm SK} = 0.475 \pm 0.015$ normalized to BP98 SSM. The definition of the χ^2 function to be minimized is the same as the one used in Ref. [23] which essentially follows the prescription given in Ref. [24] except that our theoretical estimatives were computed by convoluting the survival probability given in Eq. (2) with the absortion cross sections taken from Ref. [25] and the neutrino-electron elastic scattering cross section with radiative corrections [26] and the solar neutrino flux corresponding to each reaction, *pp*, *pep*, ^{7}Be, ^{8}B, ^{13}N and ^{15}O and other minor neutrino sources such as ^{17}F or *hep* neutrinos are neglected.

We will first discuss our results for active to active conversion. We present in Fig. 1 (a) the allowed region determined only by the rates with free f_B and in Table I the best fitted parameters as well as the $\chi^2_{\rm min}$ values for fixed and free f_B. We found for $f_B = 1$ that $\chi^2_{\rm min} = 1.49$ for 3-2=1 degree of freedom and for $f_B = 0.81$ that $\chi^2_{\rm min} = 0.32$ for 3-3=0 degree of freedom. We also have checked that the allowed region for fixed ^{8}B flux ($f_B = 1$) is rather similar to the one presented here and so we only give the values of the corresponding best fitted parameters for this case in Table I.

Next we perform a spectral shape analysis fitting the ^{8}B spectrum measured by SK [4] using the following χ^2 definition:

TABLE I. The best fitted parameters and χ^2_{min} for the VEP induced long-wave length neutrino oscillation solution to the SNP. The local best fit points in the 2nd 90 % C.L. islands are indicated in the parentheses.

Case	$\sin^2 2\theta_G$	$\lvert\phi\Delta\gamma\rvert \times 10^{24}$	f_B	χ^2_{min}
Rates ($f_B = 1$)	1.0 (1.0)	1.71 (12.3)	—	1.49 (1.88)
Rates	1.0 (1.0)	1.70 (12.4)	0.81 (0.81)	0.32 (0.71)
Spectrum	0.98 (0.39)	1.00 (0.24)	0.80 (0.66)	15.8 (19.8)
Combined	1.0 (0.99)	1.65 (12.2)	0.82 (0.82)	22.0 (23.0)

$$\chi^2 = \sum_i \left[\frac{S^{\rm obs}(E_i) - f_B S^{\rm theo}(E_i)}{\sigma_i} \right]^2, \qquad (4)$$

where the sum is performed over all the 18 experimental points $S^{\rm obs}(E_i)$ normalized by BP98 SSM prediction for the recoil-electron energy E_i, σ_i is the total experimental error and $S^{\rm theo}$ is our theoretical prediction that was calculated using the BP98 SSM ^{8}B differential flux, the $\nu - e$ scattering cross section [26], the survival probability as given by Eq. (2) taking into account the eccentricity as we did for the rates, the experimental energy resolution as in Ref. [27] and the detection efficiency as a step function with threshold $E_{\rm th} = 5.5$ MeV.

After the χ^2 minimization with $f_B = 0.80$ we have obtained $\chi^2_{\rm min} = 15.8$ for 18-3 =15 degrees of freedom. The best fitted parameters that can be found in Table I permit us to compute the allowed region displayed in Fig. 1 (b).

Finally we have performed a combined fit of the rates and the spectrum obtaining the allowed region presented in Fig. 1 (c). Again we can read from Table I the best fitted parameters. We observe that the combined allowed region is essentially the same as the one obtained by the rates alone. In all cases presented in Figs. 1 (a)-(c) we have two isolated islands of 90% C. L. allowed regions. See Table I for the fitted values corresponding to the local minimum in these islands. We note that only the upper corner of the Fig. 1 (c), for $|\phi\Delta\gamma| > 2 \times 10^{-23}$ and maximal mixing in the $\nu_e \to \nu_\mu$ channel can be excluded by CCFR [14], and moreover, there are no restrictions in the range of parameters we considered in the case of $\nu_e \to \nu_\tau$ or $\nu_e \to \nu_s$ oscillations.

In Fig. 2 we show the expected recoil-electron spectrum in SK for various fitted parameters of the VEP solution to the SNP. We see that the data from the spectrum alone can be quite well described by the VEP oscillation mechanism (thick solid line), whereas the prediction for the best fitted parameters from the rates alone and from the combined fit give flatter curves (dashed and long-dashed lines). Nevertheless parameters for a "test point" taken inside the 90 % C. L. region of Fig. 1 (c) can give rise to some spectral distortion (thin solid line).

We have performed the same analyses with rates as well as spectrum also for the $\nu_e \to \nu_s$ channel. Since the allowed regions as well as the fitted recoil-electron spectra obtained in this case are rather similar to the ones for active to active conversion, we do not present them here but only show the best fitted parameters and $\chi^2_{\rm min}$ values in Table II. Although the spectrum alone gives a comparable fit to the active to active case, we see that the rates can not be so well explained by this type of scenario and consequently the combination gives a worse fit. In spite of that this is still much better than the mass induced active to sterile vacuum oscillation solution to SNP.

To understand why it is possible to fit the solar neutrino data we show in Fig. 3 (a) the survival probabilities for the best fitted parameters of the VEP induced oscillation. Due to the specific energy dependence of the probability assumed here we can actually strongly suppress the Be line and still keep the *pp* neutrino flux high enough to be in agreement with Ga data, and at the same time obtain $\sim$ 50 % reduction of the ^{8}B neutrino flux, which is in fact the required suppression pattern of the solar neutrino fluxes in order to get a good fit [28].

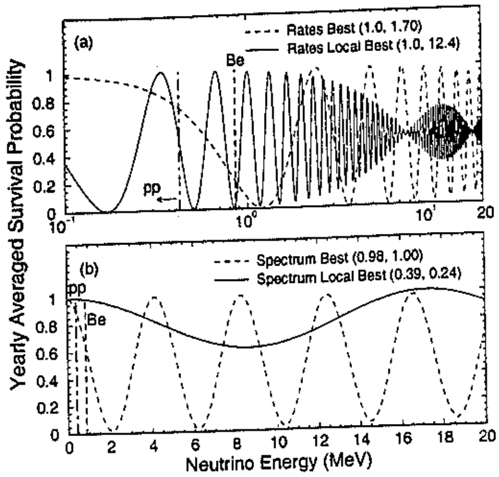

FIG. 3. Yearly averaged survival probability for the best fitted parameters, indicated in the parentheses as ($\sin^2 2\theta_G$, $|\phi\Delta\gamma| \times 10^{24}$) in the plots, which can explain well (a) the rates or (b) the SK spectrum by the VEP induced neutrino oscillation. The energies of the *pp* as well as ^{7}Be neutrinos are also indicated by the thick dot-dashed and dashed line, respectively.

Because of the contributions from the strong smearing in energy of the scattered electron and of the finite experimental energy resolution, the probability alone can not give us a precise insight on the spectral shape. We can only qualitatively expect some distortion for the probability in Fig. 3 (b).

TABLE II. Same as Table I but for the case of $\nu_e \to \nu_s$ conversion.

Case	$\sin^2 2\theta_G$	$\lvert\phi\Delta\gamma\rvert \times 10^{24}$	f_B	χ^2_{min}
Rates ($f_B = 1$)	1.0 (1.0)	1.80 (12.1)	—	3.06 (3.87)
Rates	1.0 (1.0)	1.80 (12.1)	0.94 (0.94)	2.96 (3.85)
Spectrum	0.88 (0.33)	1.01 (0.24)	0.84 (0.66)	15.6 (19.7)
Combined	1.0 (1.0)	1.66 (12.5)	0.94 (0.94)	24.7 (26.2)

Finally, let us discuss about the seasonal variation of the solar neutrino signal. In contrast to the usual vacuum oscillation solution to the SNP, in this scenario, no strong seasonal effect is expected in any of the present or future experiments, even the ones that will be sensitive to Be neutrinos such as Borexino [29] and Hellaz [30]. Contrary to the usual vacuum oscillation case, the oscillation length for the low energy *pp* and Be neutrinos are very large, comparable to or only a few times smaller than the Sun-Earth distance, so that the effect of the eccentricity in the oscillation probability is small. On the other hand, for higher energy neutrinos relevant for SK, the effect of the eccentricity in the probability could be large, but averaged out after the integration over a certain neutrino energy range. These observations are confirmed in Fig.

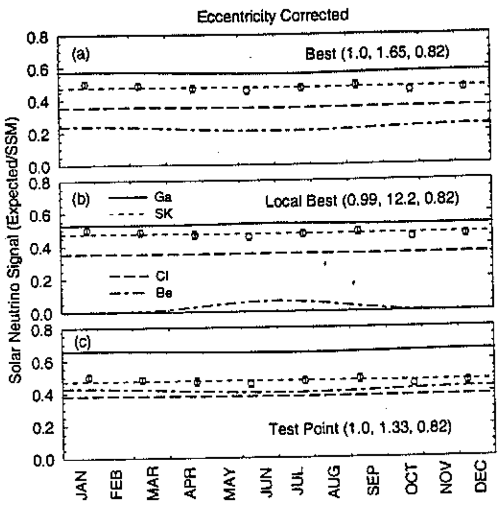

FIG. 4. Expected seasonal variations for the fitted parameters of VEP scenarios, indicated in the parentheses as ($\sin^2 2\theta_G$, $|\phi\Delta\gamma| \times 10^{24}$, f_B) in each plot. The preliminary data from SK are also plotted. Variations due to the eccentricity of the Earth orbit ($\sim 1/L^2$) were subtracted.

4 where we present the expected seasonal variations for the best fitted parameters of the VEP induced oscillation scenario.

In conclusion we found a new solution to the SNP which is comparable in quality of the fit to the other suggested ones.

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