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CHIRAL SYMMETRY, η-χ MIXING AND THE C_K/C_π RATIO

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ABSTRACT

The C_K/C_π ratio has been evaluated as a function of the usual η-X mixing angle and is found to be in quite good agreement with experiment. This calculation is based upon chiral symmetry algebra, equal wave-function renormalization constants for the nine pseudoscalar mesons and the possibility that the η-X mixing be generated from the vacuum when this vacuum is SU(3) non invariant. No use of the symmetry breaking Hamiltonian density is required. The consistency between this calculation and another one based on PCAC and giving C_K/C_π as function of the π,K and η masses is discussed, and it is argued that in this case the Hamiltonian density must contain a term which explicitly takes mixing into account.

I. Introduction

Recently, by considering the vacuum non invariance under SU(3) in the Gell-Mann, Oakes, Renner[1] and Glashow, Weinberg [2] model for the divergences of the axial currents, the C_K/C_π ratio has been evaluated as a function of the pseudoscalar masses[3.4]. Although the result obtained was elegant and in quite good agreement with the available experimental results, the link between the usual η-X mixing theory and the SU(3) non invariance of the vacuum was not established. The same formula for the C_K/C_π ratio has also been obtained by considering operator product expansions at short distances[5]. In this last work, the η-X mixing has been exploited, but there is a disagreement of about ten percent between the calculated mixing angle and that arising from the usual η-X mixing theory with quadratic masses.

In Sec. II of this paper a different evaluation of the C_K/C_π ratio is presented-this time as a function of the mixing angle of the usual η-X mixing theory-and is in quite good agreement with experiment. An interesting feature of this calculation is that the knowledge of the symmetry breaking Hamiltonian density is not required. In Sec. III, the consistency of these calculations and those of refs.[3-5] is discussed and it is concluded that the effective Hamiltonian density which breakes SU(3) $\otimes$ SU(3) must have the following form

$$\mathcal{H}' = u_o + cu_8 + \varepsilon v_8 \, v_o$$

where ε is necessarily different from zero in order to avoid inconsistencies. In Sec. IV the results are discussed and conclusions are presented.

II. Usual η-X mixing and a formula for the C_K/C_π ratio.

The constants C_π, C_K and C_8* are defined by

$$\langle o| \, A^\mu_\pi(\underset{\sim}{y},t)\,|\pi(q)\rangle \equiv iC_\pi q^\mu e^{i(\underset{\sim}{q}\cdot\underset{\sim}{y}-q_o t)} \tag{1}$$

$$\langle o| \, A^\mu_K(\underset{\sim}{y},t)\,|K(q)\rangle \equiv iC_K q^\mu e^{i(\underset{\sim}{q}\cdot\underset{\sim}{y}-q_o t)} \tag{2}$$

$$\langle o| \, A^\mu_8(\underset{\sim}{y},t)\,|\eta(q)\rangle \equiv iC_8 \cos\theta q^\mu \, e^{i(\underset{\sim}{q}\cdot\underset{\sim}{y}-q_o t)} \tag{3}$$

$$\langle o| \, A^\mu_8(\underset{\sim}{y},t)\,|X(q)\rangle \equiv iC_8 \sin\theta \, q^\mu \, e^{i(\underset{\sim}{q}\cdot\underset{\sim}{y}-q_o t)} \tag{4}$$

The angle θ, which appears in the last two definitions, is the usual η-X mixing angle, is value being given by

$$\tan\theta = \pm \sqrt{\frac{M_8^2 - M_\eta^2}{M_X^2 - M_8^2}} \tag{5}$$

$$M_8^2 = \frac{4}{3} M_K^2 - \frac{1}{3} M_\pi^2. \tag{6}$$

Information about C_π, C_K, C_8 and tanθ is obtained from the vacuum expectation value of the axial charges and the fields of the pseudoscalar mesons, under the assumption of saturation by one particle intermediate states. Let Q_i^A be the axial charge and ϕ_j be the field of the pseudoscalar meson, then

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*when used as indexes, π stands for 1,2,3 and K for 4,5,6,7.

$$C_{\pi} = -i <o|\left[Q^{A}_{\pi}, \phi_{\pi} \right]|o> \tag{7}$$

$$C_{K} = -i <o|\left[Q^{A}_{K}, \phi_{K} \right]|o> \tag{8}$$

$$C_{8}\cos\theta = -i <o|\left[Q^{A}_{8}, \phi_{\eta} \right]|o> \tag{9}$$

$$C_{8} \operatorname{sen}\theta = -i <o|\left[Q^{A}_{8}, \phi_{X} \right]|o> \tag{10}$$

To evaluate the left hand side it is necessary to know two things: 1. the representation of SU(3) $\otimes$ SU(3) to which the physical particles belong; 2. the relation between the fields of that representation and those of the physical particles.

With respect to 1, it is assumed that the fields of the theory belong to the $(3,\overline{3}) + (\overline{3},3)$ representation of SU(3) $\otimes$ SU(3). About 2, some considerations must be made. It is known that the η and X physical states are mixtures of an SU(3) singlet and octet. In the usual η-X mixing theory, where the vacuum is assumed to be a scalar of SU(3), this fact is taken into consideration when fields composed of singlet and octet parts are used as interpolating fields for the physical η and X particles. When the possibility of SU(3) non-invariance of the vacuum is considered, a field belonging to a pure SU(3) multiplet can create, nevertheless, from this vacuum, a mixed state. In other words, the SU(3) non-invariance of the vacuum can generate the η-X mixing. Because of this it is assumed that the following identifications can he made

$$v_{\pi} = \sqrt{Z_{\pi}}\ \phi_{\pi}; \qquad v_{K} = \sqrt{Z_{K}}\ \phi_{K}; \qquad v_{8} = \sqrt{Z_{8}}\ \phi_{\eta};$$

$$v_{o} = \sqrt{Z_{o}}\ \phi_{X}; \tag{11}$$

where the v's are the pseudoscalar fields of the $(3,\overline{3})$ + $(\overline{3},3)$ representation. Furthermore, the wave-function renormalization constants are all assumed to be equal:

$$Z_\pi = Z_K = Z_8 = Z_o \equiv Z \tag{12}$$

These assumptions are to be taken as working hypotheses; the quite good agreement of the results with experiment will be used as an "a posteriori" justification of them.

It is now straightforward to obtain from (8-10), and the above assumptions, the values of C_π, C_K, $C_8 \cos\theta$ and $C_8 \sin\theta$ as functions of

$$\lambda_o \equiv \langle o|\ u_o\ |o\rangle$$

and

$$\lambda \equiv \langle o|\ u_8\ |o\rangle / \lambda_o :$$

$$\epsilon_\pi = \left(\sqrt{\frac{2}{3}} + \frac{\lambda}{\sqrt{3}}\right) \frac{\lambda_o}{\sqrt{Z}} \tag{13}$$

$$C_K = \left(\sqrt{\frac{2}{3}} - \frac{\lambda}{2\sqrt{3}}\right) \frac{\lambda_o}{\sqrt{Z}} \tag{14}$$

$$C_8 \cos\theta = \left(\sqrt{\frac{2}{3}} - \frac{\lambda}{\sqrt{3}}\right) \frac{\lambda_o}{\sqrt{Z}} \tag{15}$$

$$C_8 \sin\theta = \sqrt{\frac{2}{3}}\,\lambda\ \frac{\lambda_o}{\sqrt{Z}} \tag{16}$$

From (15) and (16) λ can be obtained as a function of tan θ

$$\lambda = \frac{\sqrt{2}\ \tan\theta}{\sqrt{2} + \tan\theta} \tag{17}$$

Dividing (1) by (1) and then substituting the value of λ given by (1), we obtain

$$\frac{C_K}{C_\pi} = \frac{\sqrt{2} - \frac{\lambda}{2}}{\sqrt{2} + \lambda} = \frac{1 + \frac{\tan\theta}{2\sqrt{2}}}{1 + \frac{2\tan\theta}{\sqrt{2}}} \tag{18}$$

Since it is known by experiment that $C_K/C_\pi > 1$, the minus sign can he chosen in eq.(5), leading to

$$\frac{C_K}{C_\pi} = \frac{1 - \left[\dfrac{4M_K^2 - M_\pi^2 - 3M_\eta^2}{8(3M_X^2 - 4M_K^2 + M_\pi^2)}\right]^{1/2}}{1 - \left[\dfrac{2(4M_K^2 - M_\pi^2 - 3M_\eta^2)}{3M_X^2 - 4M_K^2 + M_\pi^2}\right]^{1/2}} \tag{19}$$

When the experimental values of the pseudoscalar masses[6] are introduced into the above expression, $C_K/C_\pi = 1.25$ is obtained, agreeing very well with the measured value 1.27 ± 0.03.[7]

It must be noted that the above formula for C_K/C_π exhibits the correct behavior in the limit of exact SU(3) symmetry, that is, $\tan\theta \to 0$ leads to $C_K = C_\pi$. As pointed out earlier, it is not necessary to know the symmetry breaking Hamiltonian density to do the calculation; the only hypotheses are the

$(3,\bar{3}) + (\bar{3},3)$ representation for the fields, η-X mixing generated by the SU(3) non-invariance of the vacuum and equal wave-function renormalization constants for the <u>nine</u> pseudoscalar mesons. The quite good agreement between the consequences of these hypotheses and experiment is used as an "a posteriori" justification for them.

Now it would be interesting to reproduce some of the results of refs. (3-5) using the hypotheses already made, and check the consistency between those results and ours.

III. Axial current divergences and another formula for the C_K/C_π ratio.

Let $$\mathcal{H}' = u_o + cu_8 + \varepsilon\, v_8\, v_o \qquad (20)$$

be the symmetry breaking Hamiltonian density; where u_o, u_8, v_o and v_8 are fields belonging to the $(3,\bar{3}) + (\bar{3},3)$ representation of SU(3) ⊗ SU(3). The term $\varepsilon\, v_8\, v_o$ is introduced to take η-X mixing into consideration in the effective Hamiltonian density.

In this model the axial current divergences are given by

$$\partial_\mu A_i^\mu = i\left[\mathcal{H}',\ Q_i^A\right] = -i\left[Q_i^A,\ u_o + cu_8 + \varepsilon\, v_8\, v_o\right] =$$

$$= \left(\sqrt{\tfrac{2}{3}} + cd_{8ii}\right) v_i + \sqrt{\tfrac{2}{3}}\,\delta_{i8}\, c\, v_o + \varepsilon \quad d_{8ii}\, v_o\, u_i + \sqrt{\tfrac{2}{3}}\,\delta_{i8}\, v_o\, u_o$$
$$+ \sqrt{\tfrac{2}{3}}\, v_8\, u_i \qquad (21)$$

To evaluate the expectation values of $\partial\mu\, A_i^\mu$ between the vacuum and states of one pseudoscalar particle it is necessary to know the matrix elements:

$$\langle o|\, v_o\, u_i\, |\, P_j\rangle \text{ and } \langle o|\, v_8\, u_i\, |\, P_j\rangle .$$

The use of the pole approximation, which proved to be so fruitfull in Sec.II, gives

$$<o|\ v_o\ u_i|\ P_j> = <o|v_o|\ X><X|\ u_i|\ P_j> = \sqrt{Z}\ <X|u_i|\ P_j> \qquad (22)$$

$$<o|\ v_8\ u_i|\ P_j> = <o|v_8|\ \eta><\eta|\ u_i|\ P_j> = \sqrt{Z}\ <\eta|u_i|\ P_j> \qquad (23)$$

For the matrix elements that appear on the r.h.s. the following ansatz is made:

$$<P_n\ |\ u_i\ |\ P_j\ > = \ a\ d_{ijn} \qquad (24)$$

when i, j and n are all different from zero.

$$<P_n\ |\ u_i\ |\ P_j\ > = \ b\ d_{ijn} \qquad (25)$$

when at least one index is zero.

Eq.s (23) and (24) now be comes ,

$$\begin{aligned} &<o|\ v_o\ u_i|\ P_j> \ = \sqrt{Z}\ \ b\ d_{oij} \\ &<o|\ v_8\ u_i|\ P_j> \ = \sqrt{Z}\ \ a\ d_{8ij},\ \text{for } j\neq 0; \\ &<o|\ v_8\ u_i|\ \chi\ > \ = \sqrt{Z}\ \ b\ d_{8io}. \end{aligned} \qquad (26)$$

Using PCAC, definitions (1-4), eqs. (21) and (26) and the same hypotheses made earlier, it is possible to write, for particles on the mass shell,

$$<o|\ \partial_\mu\ A^\mu_\pi\ |\ \pi> \equiv C_\pi\ M^2_\pi = \left\{\sqrt{\frac{2}{3}} + \frac{1}{\sqrt{3}}\left[c\ +\varepsilon\sqrt{\frac{2}{3}}\,(a+b)\right]\right\}\sqrt{Z}, \ (27)$$

$$<o|\ \partial_\mu\ A^\mu_K\ |\ K> \equiv C_K\ M^2_K = \left\{\sqrt{\frac{2}{3}} - \frac{1}{2\sqrt{3}}\left[c+\varepsilon\sqrt{\frac{2}{3}}\,(a+b)\right]\right\}\sqrt{Z}, \ (28)$$

$$<o|\ \partial_\mu\ A_8^\mu\ |\ \eta> \equiv C_8\ \cos\theta\ M_\eta^2 = \left\{ \sqrt{\tfrac{2}{3}} - \frac{1}{\sqrt{3}} \left[c+\varepsilon \sqrt{\tfrac{2}{3}}(a+b) \right] \right\} \sqrt{Z}, \quad (29)$$

$$<o|\ \partial_\mu\ A_8^\mu\ |\ \chi> \equiv C_8\ \sin\theta\ M_\chi^2 = \sqrt{\tfrac{2}{3}} \left[c+\varepsilon \sqrt{\tfrac{2}{3}}\ (2\ b) \right] \sqrt{Z}\ . \quad (30)$$

From these equations a "corrected" Gell-Mann Okubro mass formula can be obtained

$$3C_8\ \cos\theta\ M_\eta^2 - 4\ C_K\ M_K^2 + C_\pi\ M_\pi^2 = 0 \quad (31)$$

Equations (14-16) lead to

$$3C_8\ \cos\theta - 4\ C_K + C_\pi = 0 \quad (32)$$

The elimination of $3C_8\cos\theta$ from these equations gives another formula for the C_K/C_π ratio:

$$\frac{C_K}{C_\pi} = \frac{M_\eta^2 - M_\pi^2}{4(M_\eta^2 - M_K^2)} \quad (33)$$

whose numerical value is 1.25, also in very good agreement with experiment. This formula is contained in refs.(3-5).

The consistency between this calculation and that of Sec.II can be fully exhibited. To do this, eqs.(27-29)are devided by eqs. (13-15) and then the parameters $\left[c+\varepsilon\ \frac{2}{3}\ (a+b)\right]$, λ and Z/λ_o are obtained as function of the pseudoscalar masses:

$$\left[c+\varepsilon\ \frac{2}{3}\ (a+b) \right] = -\sqrt{2}\ \frac{M_\eta^2\ M_K^2 + 3M_K^2\ M_\pi^2 - 4M_\eta^2\ M_\pi^2}{M_\eta^2\ M_K^2 - 3M_K^2\ M_\pi^2 + 2M_\eta^2\ M_\pi^2} \quad (34)$$

$$\lambda = \sqrt{2}\ \frac{3M_\eta^2 - 4M_K^2 + M_\pi^2}{3M_\eta^2 - 2M_K^2 - M_\pi^2} \tag{35}$$

$$\frac{Z}{\lambda_o} = \frac{M_\eta^2\ M_K^2 - 3M_K^2\ M_\pi^2 + 2M_\eta^2\ M_\pi^2}{3M_\eta^2 - 2M_K^2 - M_\pi^2} \tag{36}$$

In Sec. II the value

$$\lambda = \frac{\sqrt{2}\ \tan\theta}{\sqrt{2} + \tan\theta} \tag{17}$$

was obtained. Using eqs.(5-6), this value becomes

$$\lambda = -\sqrt{2}\ \frac{\sqrt{4M_K^2 - 3M_\eta^2 - M_\pi^2}}{\sqrt{6M_X^2 - 8M_K^2 + 2M_\pi^2} - \sqrt{4M_K^2 - 3M_\eta^2 - M_\pi^2}} \tag{37}$$

Substitution for the experimental masses in eqs. (37) and (35) gives respective by $\lambda = -1.44\sqrt{2}$ and $\lambda = -1.43\sqrt{2}$. So the evaluation of λ by two different ways gives the same result, showing the consistency between calculations of Secs. II and III.

Using eqs. (34-36) in eq.(30) dbvided by eq.(16) it is possible to write

$$M_\chi^2 = \frac{c+\varepsilon\sqrt{\frac{2}{3}}\ (2b\ )}{\lambda}\ \frac{Z}{\lambda_o} = \frac{\left[c+\varepsilon\sqrt{\frac{2}{3}}\ (a+b)\right] + \varepsilon\sqrt{\frac{2}{3}}\ (b-a)}{\lambda}\ \frac{Z}{\lambda_o}$$

$$= \frac{\left[M_\eta^2\ M_K^2 - 4M_\eta^2\ M_\pi^2 + 3M_K^2\ M_\pi^2\right] + \frac{\varepsilon\ (a-b)}{\sqrt{3}}\left[M_\eta^2 M_K^2 - 3M_K^2 M_\pi^2 + 2M_\eta^2 M_\pi^2\right]}{4M_K^2 - 3M_\eta^2 - M_\pi^2} \tag{38}$$

Equating this expression to the measured M_χ^2 value, it is possible to obtain the numerical value

$$\frac{\varepsilon\ (b-a)}{\sqrt{3}} = -0.18,$$

which shows that it is impossible to have $\varepsilon = 0$ in the SU(3) 0 SU(3) - breaking Hamiltonian density.

On the other hand, the consistency conditions do not limit the range of values of c, so it is possible, for instance, to have $c=0$, when SU(3) would be broken by the mixing term, or $c=-\sqrt{2}$, with now the mixing term being responsible for the SU(2) $\otimes$ SU(2) breaking.

IV. CONCLUSION

In this work the C_K/C_π ratio has been calculated twice: firstly, as a function of the usual η-χ mixing angle; and secondly as a function of the masses of the π,K and η mesons. The numerical results of both calculations are equal and in good agreement with experiment. This fact is the main support to the hypotheses that have been made and are summed up below.

Although many representations of SU(3) $\otimes$ SU(3) can lead to the formula for C_K/C_π depending on π, K and η masses,[4] it has been shown to be essential to the first calculation of C_K/C_π that the fields of the theory transform according to the $(3,\bar{3}) + (\bar{3},3)$ representation.

The possibility of η-χ mixing be generated from the vacuum was used and, besides being the simplest explanation for η-χ mixing, it also proved to be a "sine qua non" condition for both calculations.

The wave function renormalization constants were assumed equal for the <u>nine</u> pseudoscalar mesons and the good results of Sec.II depend strongly on this assumption, but for the second C_K/C_π calculation it is not important. To achieve consistency it has been necessary to introduce into the Hamiltonian density a term which explicitly describes mixing. At this point one might be tempted to say that this term could be dropped and different wave function renormalization constants for the singlet and octet pseudoscalar fields might lead to a consistent result.

It must be noted, however, that this would introduce an error of ten percent on the η-χ mixing angle.

Finally it must be remarked that the properties of the scalar mesons have not been used, simply because they are not needed to obtain the results of this paper.

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