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Publicação IF - 1396/99

Virtual states of light non-Borromean halo nuclei

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(December 21, 1999)

It is shown that the three-body non-Borromean halo nuclei like ^{12}Be, ^{18}C, ^{20}C have $p-$wave virtual states with energy of about 1.6 times the corresponding neutron-core binding energy. We use a renormalizable model that guarantees the general validity of our results in the context of short range interactions.

PACS 21.10.Dr, 21.45.+v, 24.30.Gd, 27.20.+n

Halo nuclei offer the opportunity to study the few-body aspects of the nuclear interaction with their peculiar three-body phenomena. Recently, attention was drawn to the possibility of existing Efimov states [1] in such systems, because some halo nuclei can be viewed as a three body system with two loosely bound neutrons and a core [2–4]. It was suggested [5] ^{18}C and ^{20}C as promising candidates to have Efimov states. In Ref. [6], by considering the critical conditions to allow the existence of one Efimov state, using the experimental values for the neutron separation energies ($^{19}C+n$ and $^{18}C+2n$) given in [7], it was concluded that ^{20}C could have such state.

The weakly bound Efimov states [1] appear in the zero angular momentum state of a three boson system and the number of states grows to infinity, condensing at zero energy as the pair interactions are just about to bind two particles in $s-$wave. Such states are loosely bound and their wave functions extend far beyond those of normal states. If such states exist in nature they will dominate the low-energy scattering of one of the particles with the bound-state of the remaining two particles. Such states have been studied in several numerical model calculations [5,8,9]. There were theoretical searches for Efimov states in atomic and nuclear systems without a clear experimental signature of their occurrence. [10–12].

The physical picture underlying such phenomena is related to the unusually large size of these light three-body halo nuclei. The core can be assumed structureless [5,13], considering that the radius of the neutron halo is much greater than the radius of the core. The large size scale of the orbit of the outer neutrons in halo nuclei comes from the small neutron separation energies, characterizing a weakly bound few-body system. Thus, the detailed form of the nuclear interaction is not important giving to the system universal properties as long some physical scales are known [6]. This situation allows the use of concepts coming from short-range interactions.

In the limit of a zero-range interaction the three-body system is parameterized by the physical two-body and three-body scales. In a renormalization approach of the quantum mechanical many-body model with the $s-$wave zero-range force, all the low-energy properties of the three-body system are well defined if one three-body and another two-body physical informations are known [14]. The three-body input can be chosen as the experimental ground state binding energy. All the detailed informations about the short-range force, beyond the low-energy two-body observables, are retained in only one three-body physical information in the limit of zero range interaction. The sensibility of the three body binding energy to the interaction properties comes from the collapse of the system in the limit of zero-range force, which is known as the Thomas effect [15].

The three-body scale vanishes as a physical parameter if angular momentum or symmetry do not allow the simultaneous presence of the particles close to each other. In three-body $p-$wave states, the particles interacting through $s-$wave potentials have the centrifugal barrier forbbiden the third particle to be close to the interacting pair. Consequently, the third particle just notice the asymptotic wave of the interacting pair, which is defined by a two-body physical scale, and the three-body scale is not seen by the system in these states. The observables of the three-body system in states which have non zero angular momentum are determined just by two-body scales. We look for special possibilities in $p-$wave like the virtual state. The trineutron system in $p-$wave presents a peculiar pole in the second energy sheet [16,17], when the neutron-neutron ($n-n$) is artificially bound. The value of the pole scale with the binding energy of the fictious $n-n$ system, as this is the only scale of the three-body system [17]. It is not forbbiden, in principle, to exist one virtual state of the three-body halo nuclei system in $p-$wave, and if it exists, it depends exclusively on the two-body scales: the binding energy of the neutron to the core and the $n-n$ virtual state energy.

In this work, we search for the virtual state of the three-body halo nuclei in $p-$wave. We make use of the zero-range model which is well defined in $p-$wave, and the inputs are the energy of the bound state of the neutron to the core and the virtual $n-n$ state energy. We look for weakly bound $n-$core systems, in particular we look at ^{12}Be ($^{10}Be+2n$), ^{18}C ($^{16}C+2n$), and ^{20}C ($^{18}C+2n$). The zero-range model is analitically continued to the second sheet, in the complex energy plane, and there we seek for the solution of the homogeneous equation. In the case of Borromean halo nuclei such as ^{11}Li, our method does not work. However to get reed of the virtual state, the ^{10}Li core is made artificially bound, to allow the analytical continuation to the second energy sheet through the elastic cut.

The nuclei ^{12}Be, ^{18}C, and ^{20}C have an interesting non-Borromean nature with strong $n-n$ pairing in the ground-state. Specifically, ^{12}Be is $\{O^+, 23.6\text{ms}, E_n = 3169\text{KeV}\}$, ^{18}C is $\{O^+, 95\text{ms}, E_n = 4180\text{KeV}\}$, and ^{20}C is $\{O^+,\ ?\ , E_n = 3340\text{KeV}\}$, where the first number is the spin-parity of the ground state, the second is the mean lifetime and the third is the neutron separation energy. The lifetime of ^{20}C, shown by a question mark, is not available. The numbers should be compared to the one-neutron-less isotopes, ^{11}Be, ^{17}C, and ^{19}C, respectively given by $\{1/2^+, 13.81\text{ms}, E_n = 504\text{KeV}\}$, $\{\ ?\ , 193\text{ms}, E_n = 729\text{KeV}\}$, and $\{5/2^+(1/2^+),\ ?\ , E_n = 160(530)\text{KeV}\}$. The number in the round brackets, in ^{19}C refer to the recent measurement of Nakamura et al. [18]. Again, the question marks refer to not available results. The above nuclei are used to determine the neutron-core binding energies in our calculation to follow. Note that the $n-n$ pairing energies Δ_{nn} are in the range $2260 \leq \Delta_{nn} \leq 3400$ KeV. In our calculation of the $p-$wave virtual state, the pairing is taken inoperative and the only energy scales left are the neutron-core binding energy (E_{nc}) and the $n-n$ virtual state energy (E_{nn}) in the $p-$wave three-body virtual state (pygmy dipole state).

As the input energies are fixed in the renormalized model, a more realistic potential will not affect the generality of the present conclusions. The Pauli principle correction, between the halo and the core neutrons, affects essentially the ground state and it is weakened in the $p-$wave state due to the centrifugal barrier. We have to consider that this is a short-range phenomenon that occurs for distances less than the core size (about $\approx 3fm$ for light-halo nuclei). We believe that our results are valid even in the case where the spin of the core is non zero. The results show little dependence on the mass difference of the particles, in a sense explained together with the numerical results, enough to indicate that the dependence on the details of the interactions cannot be larger.

In other context, the three-nucleon system has been studied with zero-range force models [11]. They succeeded in explaining the qualitative properties of the three-nucleon system and described the known correlations between three-nucleon observables. The universality in the three-nucleon system means the independence of the correlations to the details of the short-range nucleon-nucleon potentials [11].

Here we use a notation appropriate for halo nuclei, n for neutron and c for core, but we would like to point out that our approach is applicable to any three-particle system that interact via $s-$wave short-range interactions, where two of the particles are identical. The $s-$wave interaction for the $n-c$ potential is justified in the present analysis, because the $p-$wave virtual state if exists it should have a small energy, just being sensitive to the properties of the zero angular momentum two-particle state in the relative coordinates. It also was observed in Ref. [19], when discussing ^{11}Li, that even the three-body wave-function with an $s-$wave $n-n$ correlation produces a ground state of the halo nuclei with two or more shell-model configurations.

The energies of the two particle subsystem, E_{nn} and E_{nc} can be virtual or bound. However, the extension to the second energy sheet, will be done through the cut of the elastic scattering of the neutron and the bound neutron-core subsystem. Thus, we are going to use the value of the virtual state energy E_{nn}=143 KeV and the binding energy of the neutron to the core E_{nc} in our calculations. We vary the core mass to study the light halo nuclei like ^{11}Li, ^{12}Be, ^{18}C and ^{20}C.

The zero-range three-body integral equations for the bound state of two identical particles and a core, is written as generalization of the three-boson equation [20]. It is composed by two coupled integral equations in close analogy to the case of $s-$wave separable potential model presented in Ref. [13]. The antisymmetrization of the two outer neutrons is satisfied since the spin couples to zero [5]. In our approach the potential form factors and corresponding strengths are replaced, in the renormalization procedure, by the two-body binding energies, E_{nn} and E_{nc}. In the case of bound systems, these quantities are the separation energies. We distinguish these two cases by the following definition:

$$K_{nn} \equiv -\sqrt{E_{nn}}, \quad K_{nc} \equiv \sqrt{E_{nc}}, \tag{1}$$

where + refers to bound and − to virtual state-energies. Our units will be such that $\hbar = 1$ and the nucleon mass, $m_n = 1$.

After partial wave projection, the $\ell-$wave coupled integral equations for the three-body system consisting of two neutrons and a core ($n-n-c$) are:

$$\chi^{\ell}_{nn}(q) = 2\tau_{nn}(q; E; K_{nn}) \int_0^{\infty} dk G^{\ell}_1(q, k; E)\chi^{\ell}_{nc}(k) \tag{2}$$

$$\chi_{nc}^{\ell}(q) = \tau_{nc}(q;E;K_{nc}) \int_0^{\infty} dk \left[G_1^{\ell}(k,q;E)\chi_{nn}^{\ell}(k) + A_c G_2^{\ell}(q,k;E)\chi_{nc}^{\ell}(k) \right], \tag{3}$$

where

$$\tau_{nn}(q;E;K_{nn}) = \frac{1}{\pi}\left[\sqrt{E + \frac{A_c+2}{4A_c}q^2} - K_{nn}\right]^{1}, \tag{4}$$

$$\tau_{nc}(q;E;K_{nc}) = \frac{1}{\pi}\left(\frac{A_c+1}{2A_c}\right)^{3/2}\left[\sqrt{E + \frac{A_c+2}{2(A_c+1)}q^2} - K_{nc}\right]^{1}, \tag{5}$$

$$G_1^{\ell}(q,k;E) = 2A_c k^2 \int_1^1 dx \frac{P_\ell(x)}{2A_c(E+k^2) + q^2(A_c+1) + 2A_c qkx}, \tag{6}$$

$$G_2^{\ell}(q,k;E) = 2k^2 \int_1^1 dx \frac{P_\ell(x)}{2A_c E + (q^2+k^2)(A_c+1) + 2qkx}. \tag{7}$$

In the above equations, A_c is the core mass number and E is modulus of the energy of the three-body halo state. As we are interested in the ℓ-th angular momentum three-body state, the Thomas collapse is forbidden and the integration over momentum extends to infinity. For $\ell > 0$ the short-range three-body scale are not seen by the system, while for $\ell = 0$ the renormalization of the Faddeev equations is necessary. In the renormalization procedure, for $\ell = 0$ a subtraction should be performed in the Faddeev equations and the momentum scale, which represents the subtraction point in the integral equation [14], qualitatively represents the inverse of the interaction radius [11]. The subtraction point goes to infinity as the radius of the interaction decreases. The three-body model is renormalizable for $\ell = 0$, requiring only one three-body observable to be fixed [14], which is the physical meaning of the subtraction performed in the Faddeev equations, together with the two-body low-energy physical informations. The scheme is invariant under renormalization group transformations. However, for $\ell > 0$, the original equations as given by (2) and (3) are well defined and the three-body observables are completly determined by the two-body physical scales corresponding to K_{nn} and K_{nc}.

The analytic continuation of the scattering equations for separable potentials to the second energy sheet, has been extensively discussed by Glöckle [16] and, in the case of the zero-range three-body model [20], by Frederico et al. [21]. The integral equations on the second energy sheet are obtained by the analytical continuation through the two-body elastic scattering cut due to neutron scattering on the bound neutron-core subsystem. The elastic scattering cut comes through the pole of the neutron-core elastic scattering amplitude in Eq.(5). In the next, we perform the analytic continuation of Eqs. (2 - 7) to the second energy sheet. The spectator function $\chi_{nc}^{\ell}(k)$ is substituted by $\chi_{nc}^{\ell}(k)/\left[E_v - E_{nc} + \frac{A_c+2}{2(A_c+1)}k^2\right]$, where E_v is the modulus of the virtual state energy. The resulting coupled equations in the second energy sheet are given by:

$$\begin{aligned}\chi_{nn}^{\ell}(q) = {} & \tau_{nn}(q;E_v;K_{nn})\frac{4i(A_c+1)}{\pi q(A_c+2)}G_1^{\ell}(q,-ik_v;E_v)\chi_{nc}^{\ell}(-ik_v) \\ & + 2\tau_{nn}(q;E_v;K_{nn})\int_0^{\infty} dk \frac{G_1^{\ell}(q,k;E_v)\chi_{nc}^{\ell}(k)}{E_v - E_{nc} + \frac{A_c+2}{2(A_c+1)}k^2},\end{aligned} \tag{8}$$

$$\begin{aligned}\chi_{nc}^{\ell}(q) = {} & \overline{\tau}_{nc}(q;E_v;K_{nc})\frac{2iA_c(A_c+1)}{\pi q(A_c+2)}G_2^{\ell}(q,-ik_v;E_v)\chi_{nc}^{\ell}(-ik_v) \\ & + \overline{\tau}_{nc}(q;E_v;K_{nc})\int_0^{\infty} dk \left(G_1^{\ell}(k,q;E_v)\chi_{nn}^{\ell}(k) + \frac{A_c G_2^{\ell}(q,k;E_v)\chi_{nc}^{\ell}(k)}{E_v - E_{nc} + \frac{A_c+2}{2(A_c+1)}k^2} \right),\end{aligned} \tag{9}$$

where, the on-energy-shell momentum at the virtual state is $k_v = \sqrt{\frac{2(A_c+1)}{A_c+2}(E_v - E_{nc})}$, and

$$\overline{\tau}_{nc}(q;E;K_{nc}) = \frac{1}{\pi}\left(\frac{A_c+1}{2A_c}\right)^{3/2}\left[\sqrt{E + \frac{A_c+2}{2(A_c+1)}q^2} + K_{nc}\right], \tag{10}$$

The cut of the elastic amplitude, given by the exchange of the core between the different possibilities of the bound core-neutron subsystems, is near the physical region of virtual state pole, due to the small value of E_{nc}. This cut is given by the values of imaginary k between the extreme poles of the free three-body Green's function, $G_2^\ell(q,k;E_v)$, given by Eq.(7), which appears in the first term of the right-hand side of Eq.(9),

$$2A_cE + (q^2+k^2)(A_c+1) + 2qkx = 0, \tag{11}$$

with $-1 < x < 1$, $q = k = -ik_{cut}$ and $E = \frac{A_c+2}{2(A_c+1)}k_{cut}^2 + E_{nc}$. Introducing the value of E substituting the imaginary k in Eq.(11), the cut is found at values of E satisfying

$$2\frac{A_c+1}{A_c}E_{nc} > E > 2\frac{A_c+1}{A_c+2}E_{nc}\,. \tag{12}$$

The virtual state energy E_v in the second energy sheet is found between the scattering threshold and the cut, $E_v < 2\frac{A_c+1}{A_c+2}E_{nc}$, which gives for $B_v = E_v - E_{nc} < \frac{A_c}{A_c+2}E_{nc}$.

In the limit of zero-ranged interaction the only physical scales of the three-body system for $p-$waves is E_{nn} and E_{nc}, implying that $B_v = E_{nc}\mathcal{F}(E_{nc}/E_{nn}, A_c)$, where $\mathcal{F}$ is a scaling function to be determined by the solution of Eqs. (8) and (9). However, because of the proximity of the cut to the scattering threshold, it is reasonable to believe that it should have a major importance for the formation of the virtual state, and $\mathcal{F}(E_{nc}/E_{nn}, A_c)$ be roughly independent of the ratio E_{nc}/E_{nn}. Another consequence of the dominance of the cut in the virtual state energy, should be a soft dependence on A_c of the ratio $B_v(A_c+2)/(E_{nc}A_c)$.

In figure 1, the results of the virtual state energy are shown in the form of the ratio $B_v(A_c+2)/(E_{nc}A_c)$ as a function of the core mass A_c, for $E_{nc} = E_{nn}$. The numerical values of the virtual $n-n$ and bound $n-c$ states energies can be choosen as being equal. The calculation are shown for a extreme variation of A_c between .001 and 1000, while the ratio changed by a factor of three. The other characteristic of the virtual state is the approximate independence of $B_v(A_c+2)/(E_{nc}A_c)$ on the ratio E_{nc}/E_{nn}, which is confirmed in figure 2, where calculations were performed for E_{nc}/E_{nn} between .01 and 1000.

The three-body halo nuclei ^{11}Li, ^{12}Be, ^{18}C and ^{20}C have the $p-$wave virtual state. In the case of ^{11}Li we artificially changed the virtual state of ^{10}Li to a bound state, just to give to the reader one value of the three-body virtual state in case the binding energy of the neutron to the core is some tenth's of KeV. In table I, our results are show. The $p-$wave virtual state energy scales with the binding energy of the neutron to the core, and it is roughly of about twice E_{nc}.

In summary, we have discussed the universal aspects of $p-$wave virtual states of three-body halo nuclei in the limit of a zero-range interaction. We show the existence of scaling properties of the three-body $p-$wave virtual state energy in respect to the energies of the $n-n$ virtual and $n-c$ bound states which determine the value of the $p-$wave virtual state energy. We conclude that the scaling function $\mathcal{F}(E_{nc}/E_{nn}, A_c)$ which gives the virtual state energy as $E_v = E_{nc}\left[1+\mathcal{F}(E_{nc}/E_{nn}, A_c)\right]$, is rougly independent on the ratio E_{nc}/E_{nn}, and approximately entirely determined by A_c. From knowledge of E_{nc}, we calculated the $p-$wave virtual state energies for ^{12}Be, ^{18}C and ^{20}C which came out to be about 1.6 times the neutron-core binding energy. These threshold dominated excited states, commonly called "pygmy resonances", are therefore not resonances at all and correspond to a manifestation of predominantly dipole final state interaction, just as in the two-body case of the most well known halo nucleus, the deuteron [22].

TABLE I. $p-$wave virtual state energy of light-halo nuclei. E_{nc} is the binding energy of the neutron to the core, considering the central value given in Ref. [7]. For ^{20}C we also use another value for the binding energy of a neutron to ^{19}C ($E_{nc} = 530\pm130$ KeV), as given in Ref [18].

Nucleus	A_c	E_{nc} (KeV)	E_v (KeV)	$B_v(A_c+2)/(E_{nc}A_c)$
^{11}Li	9	50	79.54	0.7221
^{12}Be	10	504	813.65	0.7373
^{18}C	16	729	1227.54	0.7694
^{20}C	18	162	274.36	0.7706
^{20}C	18	530	900.30	0.7763

Our thanks for support from Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and from Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) of Brazil.

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FIG. 1

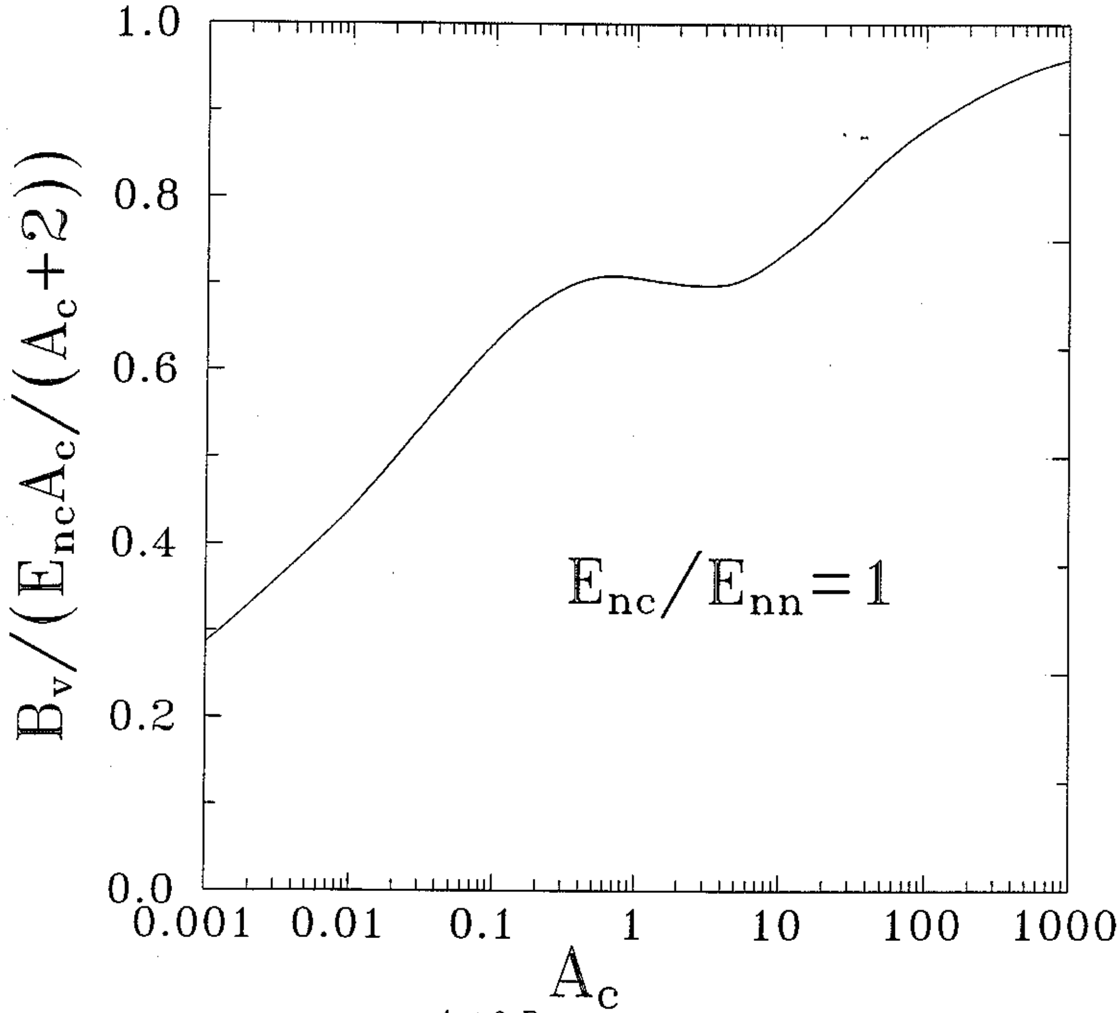

FIG. 1. Scaling plot of $\frac{A_c+2}{A_c}\frac{B_v}{E_{nc}}$ as a function of A_c for $E_{nc} = E_{nn}$.

FIG.2

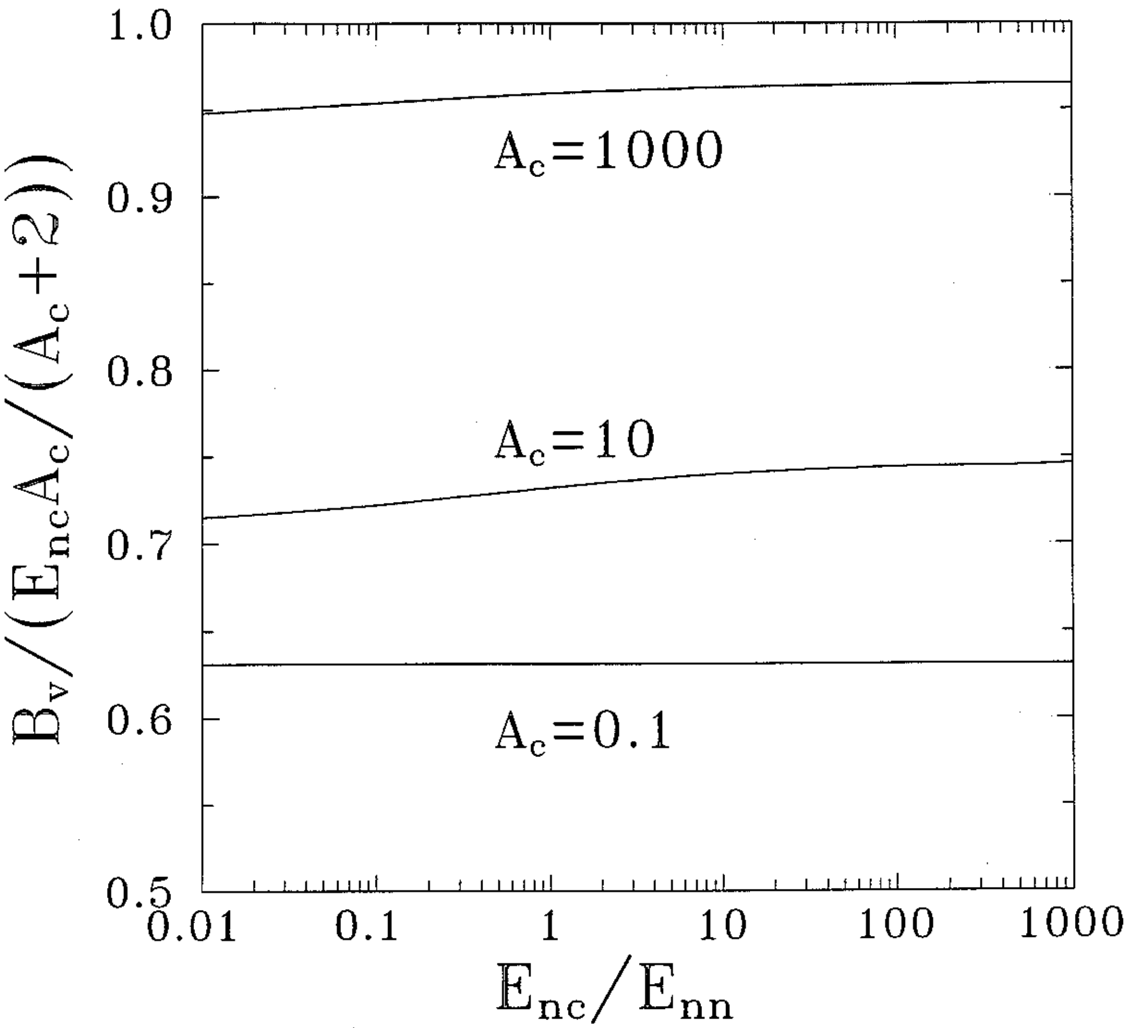

FIG. 2. Scaling plot of $\frac{A_c+2}{A_c}\frac{B_v}{E_{nc}}$ as a function of E_{nc}/E_{nn} for $A_c = 0.1$, 10 and 100.

Figure Captions

Fig. 1 Scaling plot of $\frac{A_c+2}{A_c}\frac{B_v}{E_{nc}}$ as a function of A_c for $E_{nc}=E_{nn}$.

Fig. 2 Scaling plot of $\frac{A_c+2}{A_c}\frac{B_v}{E_{nc}}$ as a function of E_{nc}/E_{nn} for $A_c = 0.1$, 10 and 100.