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constant electromagnetic background**

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Quantum spinor field in the FRW universe with a constant electromagnetic background

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The article is a natural continuation of our paper *Quantum scalar field in FRW Universe with constant electromagnetic background*, Int. J. Mod. Phys. A12, 4837 (1997). We generalize the latter consideration to the case of massive spinor field, which is placed in FRW Universe of special type with a constant electromagnetic field. To this end special sets of exact solutions of Dirac equation in the background under consideration are constructed and classified. Using these solutions representations for out-in, in-in, and out-out spinor Green functions are explicitly constructed as proper-time integrals over the corresponding contours in complex proper-time plane. The vacuum-to-vacuum transition amplitude and number of created particles are found and vacuum instability is discussed. The mean values of the current and energy-momentum tensor are evaluated, and different approximations for them are

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presented. The back reaction related to particle creation and to the polarization of the unstable vacuum is estimated in different regimes.

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I. INTRODUCTION

The article is a natural continuation of our paper [1]. Here we generalize the latter consideration to the case of a massive spinor field, which is placed in Friedmann-Robertson-Walker (FRW) Universe of special type with constant electromagnetic field. First, we ought to repeat briefly a motivation for such an activity.

It is quite well known fact that quantum field theory in an external background is, generally speaking, theory with unstable vacuum. The vacuum instability leads to many interesting features, among which particle creation from the vacuum is one of the most beautiful non-perturbative phenomenon. One has to treat it exactly in regard to the external field. The latter has been realized long ago by Schwinger [2]. The particle creation effects together with back reaction issue are important in black hole physics and in dynamics of the early Universe (EU) (see, for example, [3,4] and references therein).

In quantum field theory with unstable vacuum it is necessary to construct different kinds of Green functions (GF), e.g. besides the causal GF (out-in GF) one has to use so called in-in GF, out-out GF, and so on [5-7] (for a review and technical details see [8]). General methods of such GF construction in electromagnetic background have been developed in [6,7]. A possible generalization of the formalism to an external gravitational background has been given in Ref. [9]. Since Ref. [2] it has been known that causal (out-in) GF may be presented as a proper-time integral over a real infinite contour. At the same time, in

the unstable vacuum case the in-in and out-out GF differ from the causal one. It was shown [10,11] there are examples of external fields (electromagnetic with constant uniform invariants) when these functions may be presented by the same proper-time integrals (with the same integrand) but over another contours in the complex proper-time plane. Then, it is not difficult to compare contributions from the in-in GF and from the causal one. The complete set of GF mentioned is necessary for the construction of the Furry picture in interacting theories, and even in noninteracting cases one has to use them to define, for example, the back reaction of particles created and to construct different kinds of effective actions (EA) (for a general introduction to EA in background field method, see [12], and for review of modern generalizations, see [13]). The such proper-time representation of GF may be the necessary step in the study of chiral symmetry breaking in QED and the four-fermion models under the action of gravitational and electromagnetic fields (see [14] and references therein). From another point, in-in GF which gives the origin to in-in EA maybe used in more realistic theories, like GUT theories with scalars, spinors and vectors in order to analyse the properties of above electro-gravitational background in the EU. For example, one of extremely interesting questions there is: can we realise the asymptotic conformal invariance phenomenon (which means that theory becomes approximately conformally invariant at large curvature) [15] even for in-in EA, or in other words for mean values in EU. Taking into account our recent study of in-in GF structure for scalars [1] it looks quite interesting next application of above calculation.

It may be likely that EU is filled with some type of electromagnetic fields. For example, recently (see [16,17] and references therein) the possibility of existence and role of primordial magnetic fields in EU have been discussed. From another point the possibility of existence of electromagnetic field in the EU has been discussed long ago in [18,19]. It has been shown

there that the presence of the electrical field in the EU increases significantly the gravitational particle creation from the vacuum. In principle, this process may be considered as a source for the dominant part of the Universe mass.

Bearing in mind the above cosmological motivations it is becoming interesting to study the quantum field theory in curved background with electromagnetic field (of a special form in order to solve the problem analytically). In the present paper we are going to consider a massive spinor field placed in the expanding FRW Universe with the scale factor $\Omega(\eta)$ (in terms of the conformal time) $\Omega^2(\eta) = b^2\eta^2 + a^2$. Such a scale factor corresponds to the expanding radiation-dominated FRW Universe. In terms of physical time t the corresponding metric may be written as follows:

$$ds^2 = dt^2 - \Omega^2(t)(dx^2 + dy^2 + dz^2), \quad (1)$$

where for small times $|t| \ll a^2/b$, $\Omega^2(t) \simeq a^2[1 + (bt/a^2)^2]$, and for large times $|t| \gg a^2/b$, $\Omega^2(t) \simeq 2b|t|$ (see [18]). Moreover, such FRW Universe will be filled by the constant electromagnetic field.

Thus, we start from the theory of massive spinor in above background. Making a conformal transformation we remain with QED in flat background but with time-dependent mass (QED- Ω theory). In the Sect.II special sets of exact solutions of Dirac equation in QED- Ω theory are constructed and classified as corresponding to particles and antiparticles at $t \rightarrow \pm\infty$. In the Sect.III, using these solutions, representations for out-in, in-in, and out-out spinor GF are explicitly constructed as proper-time integrals over the corresponding contours in complex proper-time plane. As far as we know, it is a first explicit example for the proper-time representations for complete set of spinor GF in gravitational-electromagnetic background. In the Sect.IV we are interested in to reveal global features of the theory. The

vacuum-to-vacuum transition amplitudes and number of created particles are found and vacuum instability is discussed. It is seen the creation process is a coherent effect of both fields. The all mean values of the current and energy-momentum tensor are presented in the same manner as the proper-time integrals, and evaluated. The different approximations for them are investigated. The back reaction produced by both of particles created from a vacuum and polarization of an unstable vacuum estimated in different regimes. It is shown a behaviour of such components in time are quite different.

II. CLASSIFIED SETS OF EXACT SOLUTIONS

In this Section we study exact solutions of the Dirac equation in an external constant uniform electromagnetic background and in a time-dependent mass-like potential, which effectively reproduces effects of a gravitational background (solutions of the Dirac equation of QED- Ω theory),

$$(\mathcal{P}_\mu \gamma^\mu - M \Omega) \psi(x) = 0, \quad \Omega = \Omega(x_0) = \sqrt{a^2 + b^2 x_0^2}, \quad (2)$$

$$\mathcal{P}_\mu = i\partial_\mu - qA_\mu(x), \quad [\gamma^\mu, \gamma^\nu]_+ = 2\eta^{\mu\nu}, \quad \eta^{\mu\nu} = \text{diag}(1, -1, -1, -1),$$

where $x^0 = \eta$ is conformal time, q is charge of a particle, for example, $q = -|e|$ for electron. The time-independent inner product of the solutions of the equation (2) may be chosen as

$$(\psi, \psi') = \int \bar{\psi}(x) \gamma^0 \psi'(x) d\mathbf{x}. \quad (3)$$

As usual, it is convenient to present $\psi(x)$ in the following form

$$\psi(x) = (\mathcal{P}_\mu \gamma^\mu + M \Omega) \phi(x). \quad (4)$$

Then the functions ϕ have to obey the squared Dirac equation,

$$\left(\mathcal{P}^2 - (M\Omega)^2 - \frac{q}{2}\sigma^{\mu\nu}F_{\mu\nu} + iM\partial_0\Omega\gamma^0 \right) \phi(x) = 0, \quad (5)$$

$$F_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x), \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu].$$

The external electromagnetic field in our case consists of a constant uniform electric (E) and parallel to it magnetic (H) fields,

$$F_{03} = E, \quad F_{\mu\nu}^\perp = H \left(\delta_\mu^2 \delta_\nu^1 - \delta_\nu^2 \delta_\mu^1 \right). \quad (6)$$

For such a field we select the following potentials:

$$A_0 = 0, \quad A_3 = Ex^0, \quad A_i = A_i^\perp = -H x_2 \delta_i^1, \quad i = 1, 2. \quad (7)$$

For $b = 0$ we have a usual flat-space case with the mass $m = aM$. In this case particle-antiparticle classified solutions of the equation and all the GF were found in [22,10,11]. The case $b \neq 0$ is of special interest for us. In the case one can consider spinor field in the conformally-flat Universe (with scale factor Ω) filled by a constant uniform electromagnetic field. Making a standard conformal transformation of the gravitational metric and spinor field, we arrive to a theory in flat space-time with a time-dependent mass (QED- Ω). The corresponding field equation is given by (2) (an electromagnetic field should not be transformed under the conformal transformation). Note that the such a conformal transformation may be used also for interacting theories [12,15]. Thus, Eq. (2) is actually relevant to the quantum spinor field in the expanding FRW Universe with the external constant electromagnetic field.

Remember [18] that the expansion law with $a \neq 0$ is necessarily connected with a nonzero energy density of the cosmological substratum in the early, radiation-dominated phase of the Universe. The minimum value of Ω ($\Omega_{min} = a$) is caused by the strong interaction. It is difficult to find solutions of the equation (5) with the last spin term. Nevertheless, in many

interesting cases with strong background part of this term (for region of small $(x_0)^2 \lesssim (a/b)^2$) may be treated perturbatively starting from the reduced squared Dirac equation:

$$\left[\mathcal{P}^2 - (M\Omega)^2 - \frac{q}{2}\sigma^{\mu\nu}F_{\mu\nu} + ibM\gamma^0 \right] \phi(x) = 0, \quad (8)$$

where the asymptotic form of the last spin term is used ($\partial_0\Omega \rightarrow b$ for large enough x_0). If the intensity of an electric field is more than the characterizing parameter of a mass-like potential, $(qE)^2 \gg (bM)^2$, or its potential is not intense, $a^2M/b \gg 1$, then both the last spin term of equation (5) and of the equation (8) can be disregarded. If $(qE)^2 \lesssim (bM)^2$, one can not neglect the contribution from the term $iM\partial_0\Omega\gamma^0$. However, our main interest is to study an intense gravitational background when $a^2M/b \ll 1$. Then, as it may be seen from explicit solutions of the equation (8), one can calculate any spin contributions to matrix elements of operators using solutions of equation (8) with a relative accuracy of the order of a^2M/b . And one may use these sets as a basis to construct solutions of the equation (5) perturbatively, considering the term $iM(\partial_0\Omega - b)\gamma^0$ as a perturbation. Moreover, the large time ($(x_0)^2 \gg (a/b)^2$) asymptotic solutions of both equations (5) and (8) are the same.

To construct the above mentioned generalized Furry picture for both QED and QED- Ω one has to find special sets of classified solutions of the equation (2), namely, two complete and orthonormal sets of solution: $\{\pm\psi_{\{n\}}(x)\}$, which describes particles (+) and antiparticles (-) in the initial time instant ($x^0 \rightarrow -\infty$), and $\{\pm\psi_{\{n\}}(x)\}$, which describes particles (+) and antiparticles (-) in the final time instant ($x^0 \rightarrow +\infty$). According to the general approach [6], which can be easily adapted to QED- Ω , such solutions obey the following asymptotic conditions

$$\begin{aligned} H_{o.p.}(x^0) \zeta \psi_{\{n\}}(x) &= \zeta \varepsilon \zeta \psi_{\{n\}}(x), \quad \text{sgn } \zeta \varepsilon = \zeta, \quad x^0 \rightarrow -\infty, \\ H_{o.p.}(x^0) \zeta \psi_{\{l\}}(x) &= \zeta \varepsilon \zeta \psi_{\{l\}}(x), \quad \text{sgn } \zeta \varepsilon = \zeta, \quad x^0 \rightarrow +\infty, \quad \zeta = \pm, \end{aligned} \quad (9)$$

where $\zeta, \{n\}$ and $\zeta, \{l\}$ are complete sets of quantum numbers which characterize solutions ${}_{\zeta}\psi_{\{n\}}(x)$ and ${}_{\zeta}\psi_{\{l\}}(x)$ respectively, $H_{o.p.}(x^0) = \gamma^0(M\Omega - \gamma^i P_i)$ is one-particle Dirac Hamiltonian; ${}^+\varepsilon, {}^+\varepsilon$ are particle quasi-energies and $|{}^-\varepsilon|$ and $|{}^-\varepsilon|$ are antiparticles quasi-energies. All the information about the processes of particles scattering and creation by an external field (in zeroth order with respect to the radiative corrections) can be extracted from the decomposition coefficients, which form the matrices $G(\zeta|\zeta')$,

$${}_{\zeta}\psi(x) = {}^+\psi(x)G({}^+|\zeta) + {}^-\psi(x)G({}^-|\zeta). \quad (10)$$

The matrices $G(\zeta|\zeta')$ obey the following relations,

$$\begin{aligned} G(\zeta|{}^+)G(\zeta|{}^+)^{\dagger} + G(\zeta|{}^-)G(\zeta|{}^-)^{\dagger} &= \mathbf{I}, \\ G({}^+|{}^+)G({}^-|{}^+)^{\dagger} + G({}^+|{}^-)G({}^-|{}^-)^{\dagger} &= 0, \end{aligned} \quad (11)$$

where \mathbf{I} is the identity matrix. All GF in the formalism may be also constructed with the sets $\{\pm\psi_{\{n\}}(x)\}$ and $\{\pm\psi_{\{n\}}(x)\}$. Below we are going to present such solutions.

The functions $\phi(x)$ can be written in the following form:

$$\phi_{p_3 p_1 n \xi r}(x) = \phi_{p_3 n \xi r}(x_{\parallel}) \phi_{p_1 n r}(x_{\perp}) v_{\xi r}, \quad (12)$$

where $x_{\perp}^{\mu} = (0, x^1, x^2, 0)$, $x_{\parallel}^{\mu} = (x^0, 0, 0, x^3)$; $\{p_3, p_1, n, \xi, r\}$ is a complete set of quantum numbers. Among them p_3 and p_1 are momenta of the continuous spectrum, n is an integer quantum number, $\xi = \pm 1$ and $r = \pm 1$ are spin quantum numbers; $v_{\xi r}$ are some constant orthonormal spinors, $v_{\xi r}^{\dagger} v_{\xi r'} = \delta_{rr'}$. The eq.(8) allows one to subject these spinors to some supplementary conditions,

$$\begin{aligned} \Xi v_{\xi r} &= \xi v_{\xi r}, \quad \Xi = \gamma^0(qE\gamma^3 - bM)/\rho, \quad \rho = \sqrt{(qE)^2 + (bM)^2}, \\ R v_{\xi r} &= r v_{\xi r}, \quad R = \text{sgn}(qH) i \gamma^1 \gamma^2. \end{aligned} \quad (13)$$

If $H \neq 0$, the function $\phi_{p_1nr}(x_\perp)$ has the form

$$\phi_{p_1nr}(x_\perp) = \left(\frac{\sqrt{|qH|}}{2^{n+1}\pi^{\frac{3}{2}}n!} \right)^{1/2} \exp \left\{ -ip_1x^1 - \frac{X^2}{2} \right\} \mathcal{H}_n(X), \quad X = \sqrt{|qH|} \left(x^2 + \frac{p^1}{qH} \right),$$

where $\mathcal{H}_n(x)$ are Hermite polynomials with integer $n = 0, 1, \dots$. If $H = 0$, the discrete quantum number n has to be replaced by the momentum p_2 , and the corresponding function has the form $\phi_{p_1nr}(x_\perp) = (2\pi)^{-1} \exp \{-i(p_1x^1 + p_2x^2)\}$. Let us present the function $\phi(x_\parallel)$ as follows

$$\phi_{p_3n\xi r}(x_\parallel) = (2\pi)^{-1/2} e^{-ip_3x^3} \phi_{p_3n\xi r}(x^0), \quad \phi_{p_3n\xi r}(x^0) = \phi_{p_3n\xi r}(x^0, p_z)|_{p_z=0} \quad (14)$$

where $\phi_{p_3n\xi r}(x^0, p_z)$ is a solution of equation

$$\left[\left(i \frac{\partial}{\partial \tilde{\eta}} \right)^2 - (p_z - \rho \tilde{\eta})^2 - \rho \lambda - i \rho \xi \right] \phi_{p_3n\xi r}(x^0, p_z) = 0, \quad (15)$$

with $\tilde{\eta} = x^0 - \rho^{-2} q E p_3$, $\rho \lambda = p_3^2 (bM/\rho)^2 + \omega + a^2 M^2$,

$$\omega = \begin{cases} |qH|(2n+1-r), & n = 0, 1, \dots, H \neq 0 \\ p_1^2 + p_2^2, & H = 0 \end{cases}$$

One can form two complete sets $\{\pm \phi_{p_3n\xi r}(x^0, p_z)\}$ and $\{\pm \phi_{p_3n\xi r}(x^0, p_z)\}$ of the solutions of equation (15) using the functions

$$\begin{aligned} \mp \phi_{p_3n\xi r}(x^0, p_z) &= C_\xi D_{\nu-\xi/2}[\pm(1-i)\tau], \quad \tau = \frac{1}{\sqrt{\rho}}(\rho \tilde{\eta} - p_z), \\ \pm \phi_{p_3n\xi r}(x^0, p_z) &= C'_\xi D_{-\nu-1+\xi/2}[\pm(1+i)\tau], \quad \nu = \frac{i\lambda}{2} - \frac{1}{2}. \end{aligned} \quad (16)$$

Similar solutions were first presented in [22]. Then, solutions of equation (8) $\phi(x)$ can be constructed as follows,

$$\begin{aligned} \pm \phi_{p_3p_1n\xi r}(x) &= \pm \phi_{p_3p_1n\xi r}(x, p_z)|_{p_z=0}, \\ \pm \phi_{p_3p_1n\xi r}(x, p_z) &= (2\pi)^{-1/2} e^{-ip_3x^3} \pm \phi_{p_3n\xi r}(x^0, p_z) \phi_{p_1nr}(x_\perp) v_{\xi r}, \end{aligned} \quad (17)$$

and in the same form with (\pm) indices above.

One can verify that the solutions of the Dirac equation with different ξ , namely, $(\mathcal{P}_\mu \gamma^\mu + M\Omega)_{\pm} \phi_{p_3, p_1, n, +1, r}(x)$ and $(\mathcal{P}_\mu \gamma^\mu + M\Omega)_{\pm} \phi_{p_3, p_1, n, -1, r}(x)$, or $(\mathcal{P}_\mu \gamma^\mu + M\Omega)_{\pm} \phi_{p_3, p_1, n, +1, r}(x)$ and $(\mathcal{P}_\mu \gamma^\mu + M\Omega)_{\pm} \phi_{p_3, p_1, n, -1, r}(x)$ are linearly dependent for each sign "+" or "-". Thus, to construct the complete sets we may use only the following sets of solutions:

$$\pm \psi_{p_3 p_1 n r}(x) = (\mathcal{P}_\mu \gamma^\mu + M\Omega)_{\pm} \phi_{p_3, p_1, n, +1, r}(x), \quad (18)$$

$$\pm \psi_{p_3 p_1 n r}(x) = (\mathcal{P}_\mu \gamma^\mu + M\Omega)_{\pm} \phi_{p_3, p_1, n, +1, r}(x). \quad (19)$$

Choosing the coefficients C and C' in (16) as follows: $C_{+1} = (2\rho)^{-1/2} \exp(-\pi\lambda/8)$ and $C'_{+1} = (\rho\lambda)^{-1/2} \exp(-\pi\lambda/8)$, one gets two complete sets $\{\pm \psi_{p_3 p_1 n r}(x)\}$ and $\{\pm \psi_{p_3 p_1 n r}(x)\}$ of orthonormalized solutions of the equation (2). These solutions are classified as particles (+) and antiparticles (-) at $x^0 \rightarrow \pm\infty$ according to the asymptotic forms of the corresponding quasienergies, ${}_{\zeta}\varepsilon = \zeta\rho|x^0|$ and ${}_{\zeta}\varepsilon = \zeta\rho|x^0|$ (see [21] for additional arguments advocating such a classification). It matches with classification [22] of similar solutions in QED.

According to the above discussion the solutions (18) and (19) of the Dirac equation may serve in an intense gravitational background, $a^2 M/b \ll 1$, and in the other cases mentioned after Eq. (8). Satisfying the Cauchy conditions one can see the solutions (18) are valid with $x^0 < 0$, $|x^0| \gg a/b$ and the solutions (19) are valid with $x^0 > 0$, $|x^0| \gg a/b$ for any intensity of the background.

To find the matrices $G(\zeta|\zeta')$ defined by (10), it is convenient to use an asymptotic form of the solutions. Using (8) and (13) we get

$$G(\zeta|\zeta')_{ll'} = \delta_{l,l'} g(\zeta|\zeta'), \quad l = (p_3, p_1, n, r), \quad l' = (p_3', p_1', n', r'), \quad (20)$$

where

$$g(\zeta|\zeta') = \zeta \phi_{p_3, n, +1, r}^* (x^0, p_z) i \overleftrightarrow{\partial}_0 (i\partial_0 - \rho\tilde{\eta}) \zeta' \phi_{p_3, n, +1, r} (x^0, p_z) . \quad (21)$$

III. GREEN FUNCTIONS

Let us start with out-in GF which is the causal propagator

$$S^c(x, x') = c_v^{-1} i \langle 0, out | T\psi(x)\bar{\psi}(x') | 0, in \rangle, \quad c_v = \langle 0, out | 0, in \rangle . \quad (22)$$

Here $\psi(x)$ is quantum spinor field satisfying the Dirac equation (2), $|0, in \rangle$ and $|0, out \rangle$ are the initial and the final vacuum, and c_v is the vacuum-to-vacuum transition amplitude.

The propagator $S^c(x, x')$ obeys the equation

$$(\mathcal{P}_\mu \gamma^\mu - M\Omega) S^c(x, x') = -\delta^{(4)}(x - x') . \quad (23)$$

Another important singular function is the commutation function $S(x, x') = i [\psi(x), \bar{\psi}(x')]_+$. It obeys the homogeneous Dirac equation (2) and the initial condition $S(x, x')|_{x_0=x'_0} = i\gamma^0 \delta(\mathbf{x} - \mathbf{x}')$. Besides, the following GF are studied [6–8]:

$$\begin{aligned} S_{in}^c(x, x') &= i \langle 0, in | T\psi(x)\bar{\psi}(x') | 0, in \rangle, & S_{in}^{\bar{c}}(x, x') &= i \langle 0, in | \psi(x)\bar{\psi}(x') T | 0, in \rangle, \\ S_{in}^-(x, x') &= i \langle 0, in | \psi(x)\bar{\psi}(x') | 0, in \rangle, & S_{in}^+(x, x') &= i \langle 0, in | \bar{\psi}(x')\psi(x) | 0, in \rangle, \\ S_{out}^c(x, x') &= i \langle 0, out | T\psi(x)\bar{\psi}(x') | 0, out \rangle . \end{aligned} \quad (24)$$

Here the T -product acts on both sides: it orders the field operators to the right of its and antiorders them to the left. The functions S_{in}^c and S_{out}^c obey the equation (23), S^\mp satisfy the equation (2), and $S_{in}^{\bar{c}}$ obeys the equation $(\mathcal{P}_\mu \gamma^\mu - M\Omega) S_{in}^{\bar{c}}(x, x') = \delta^{(4)}(x - x')$.

One can express the GF via the solutions (18) and (19) [6–8]:

$$\begin{aligned}
S^c(x, x') &= \theta(x_0 - x'_0) S^-(x, x') - \theta(x'_0 - x_0) S^+(x, x'), \\
S(x, x') &= S^-(x, x') + S^+(x, x'), \\
S_{in}^c(x, x') &= \theta(x_0 - x'_0) S_{in}^-(x, x') - \theta(x'_0 - x_0) S_{in}^+(x, x'), \\
S_{in}^{\bar{c}}(x, x') &= \theta(x'_0 - x_0) S_{in}^-(x, x') - \theta(x_0 - x'_0) S_{in}^+(x, x'), \\
S_{out}^c(x, x') &= \theta(x_0 - x'_0) S_{out}^-(x, x') - \theta(x'_0 - x_0) S_{out}^+(x, x'), \tag{25}
\end{aligned}$$

where

$$\begin{aligned}
S^-(x, x') &= i \int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} {}^+ \psi_{p_3 p_1 nr}(x) g({}^+ | {}^+)^{-1} {}^+ \bar{\psi}_{p_3 p_1 nr}(x'), \\
S^+(x, x') &= i \int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} -\psi_{p_3 p_1 nr}(x) [g({}^- | {}^-)^{-1}]^* -\bar{\psi}_{p_3 p_1 nr}(x'), \\
S_{in}^{\mp}(x, x') &= i \int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} \pm \psi_{p_3 p_1 nr}(x) {}_{\pm} \bar{\psi}_{p_3 p_1 nr}(x'), \\
S_{out}^{\mp}(x, x') &= i \int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} \pm \psi_{p_3 p_1 nr}(x) \pm \bar{\psi}_{p_3 p_1 nr}(x'). \tag{26}
\end{aligned}$$

\sum_{nr} means the summation over all discrete quantum numbers n, r (and the integration over the continuous p_2 if $H = 0$). Using the relations between GF and between the matrices $G(\zeta|\zeta')$ one can present the functions S^{\mp} , S_{in}^{\mp} and S_{out}^{\mp} as follows

$$\begin{aligned}
\pm S^{\mp}(x, x') &= S^c(x, x') \pm \theta(\mp(x_0 - x'_0)) S(x, x'), \\
\pm S_{in}^{\mp}(x, x') &= S_{in}^c(x, x') \pm \theta(\mp(x_0 - x'_0)) S(x, x'), \\
\pm S_{out}^{\mp}(x, x') &= S_{out}^c(x, x') \pm \theta(\mp(x_0 - x'_0)) S(x, x'), \\
S_{in}^c(x, x') &= S^c(x, x') - S^a(x, x'), \quad S_{out}^c(x, x') = S^c(x, x') - S^p(x, x'), \\
S^a(x, x') &= -i \int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} -\psi_{p_3 p_1 nr}(x) [g({}^+ | {}^-) g({}^- | {}^-)^{-1}]^{\dagger} + \bar{\psi}_{p_3 p_1 nr}(x'), \\
S^p(x, x') &= i \int_{-\infty}^{+\infty} dp_3 dp_1 \sum_{nr} {}^+ \psi_{p_3 p_1 nr}(x) [g({}^+ | {}^+)^{-1} g({}^+ | {}^-)] - \bar{\psi}_{p_3 p_1 nr}(x'). \tag{27}
\end{aligned}$$

Let us consider the functions S^\pm and $S^{a,p}$. The coefficients (21) do not depend on p_z , thus, one can present the functions S^\mp and $S^{a,p}$ in the following convenient form

$$S^{\mp,a,p}(x, x') = \int_{-\infty}^{+\infty} dz \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} e^{-ip_3 y^3} S_Q^{\mp,a,p}, \quad y_\mu = x_\mu - x'_\mu, \quad (28)$$

where

$$\begin{aligned} S_Q^- &= i \int_{-\infty}^{+\infty} dp_z dp_1 \sum_{nr} {}^+ \psi_{p_3 p_1 nr}(\tilde{\eta}, x_\perp, z, p_z) g(|^+)^{-1} {}^+ \bar{\psi}_{p_3 p_1 nr}(\tilde{\eta}', x'_\perp, z', p_z), \\ S_Q^+ &= i \int_{-\infty}^{+\infty} dp_z dp_1 \sum_{nr} -\psi_{p_3 p_1 nr}(\tilde{\eta}, x_\perp, z, p_z) [g(|^-)^{-1}]^* {}^- \bar{\psi}_{p_3 p_1 nr}(\tilde{\eta}', x'_\perp, z', p_z), \\ S_Q^a &= -i \int_{-\infty}^{+\infty} dp_z dp_1 \sum_{nr} -\psi_{p_3 p_1 nr}(\tilde{\eta}, x_\perp, z, p_z) [g(|^+)^{-1} g(|^-)^{-1}]^\dagger {}^+ \bar{\psi}_{p_3 p_1 nr}(\tilde{\eta}', x'_\perp, z', p_z), \\ S_Q^p &= i \int_{-\infty}^{+\infty} dp_z dp_1 \sum_{nr} {}^+ \psi_{p_3 p_1 nr}(\tilde{\eta}, x_\perp, z, p_z) [g(|^+)^{-1} g(|^+)^{-1}] {}^- \bar{\psi}_{p_3 p_1 nr}(\tilde{\eta}', x'_\perp, z', p_z), \end{aligned} \quad (29)$$

and

$$\begin{aligned} \pm \psi_{p_3 p_1 nr}(\tilde{\eta}, x_\perp, z, p_z) &= (\gamma^0 i \partial_0 + \gamma^3 (p_3 - q E x^0) + \gamma_\perp (i \partial - q A) + M \Omega)_\pm \phi_{p_3 p_1 nr}, \\ \pm \phi_{p_3 p_1 nr} &= \pm \phi_{p_3 nr}(\tilde{\eta}, z, p_z) \phi_{p_1 nr}(x_\perp) v_{+1,r}, \quad \pm \phi_{p_3 nr}(\tilde{\eta}, z, p_z) = \frac{1}{\sqrt{2\pi}} e^{-ip_z z} \pm \phi_{p_3, n, +1, r}(x^0, p_z), \end{aligned} \quad (30)$$

and in the same form with (\pm) indices above. Within the same approximation one can rewrite the functions $S_Q^{\mp,a,p}$ as follows:

$$S_Q^{\mp,a,p} = (\gamma^0 i \partial_0 + \gamma^3 (p_3 - q E x^0) + \gamma_\perp (i \partial - q A) + M \Omega) \Delta_Q^{\mp,a,p}, \quad (31)$$

where the functions $\Delta_Q^{\mp,a,p}$ obey the equation

$$\left[\left(i \frac{\partial}{\partial \tilde{\eta}} \right)^2 - \left(i \frac{\partial}{\partial z} - \rho \tilde{\eta} \right)^2 + \mathcal{P}_\perp^2 - \frac{p_3^2 (b M^2)}{\rho^2} - i \rho \Xi - \frac{q}{2} F_{\mu\nu}^\perp \sigma^{\mu\nu} \right] \Delta_Q^{\mp,a,p} = 0.$$

They may be easily expressed via the solutions (30) according to the Eqs. (29). The functions $\Delta_Q^{\mp,a,p}$ are just the GF of the squared Dirac equation in electromagnetic background [10,11],

where $\tilde{\eta}$ is the time, z is the coordinate along the electric field, the mass $m_Q^2 = p_3^2(bM)^2/\rho^2$, the potential of the electromagnetic field is $A_z = \tilde{\eta}\rho/q$, and the spin term $qE\gamma^0\gamma^3$ is changed to $\rho\Xi$. In what follows we are going to use the following representations [11]:

$$\begin{aligned}\pm\Delta_Q^\mp &= \Delta_Q^c \pm \theta(\mp y_0)\Delta_Q, \\ \Delta_Q^c &= \int_{\Gamma_c} f_Q ds, \quad \Delta_Q = \text{sgn}(y_0) \int_{\Gamma_c - \Gamma_2 - \Gamma_1} f_Q ds, \\ \Delta_Q^a &= \int_{\Gamma_a} f_Q ds + \theta(z' - z) \int_{\Gamma_3 + \Gamma_2 - \Gamma_a} f_Q ds, \\ \Delta_Q^p &= \int_{\Gamma_a} f_Q ds + \theta(z - z') \int_{\Gamma_3 + \Gamma_2 - \Gamma_a} f_Q ds,\end{aligned}\tag{32}$$

where $\theta(0) = 1/2$, the contours of the integration are indicated on the Fig.1, and

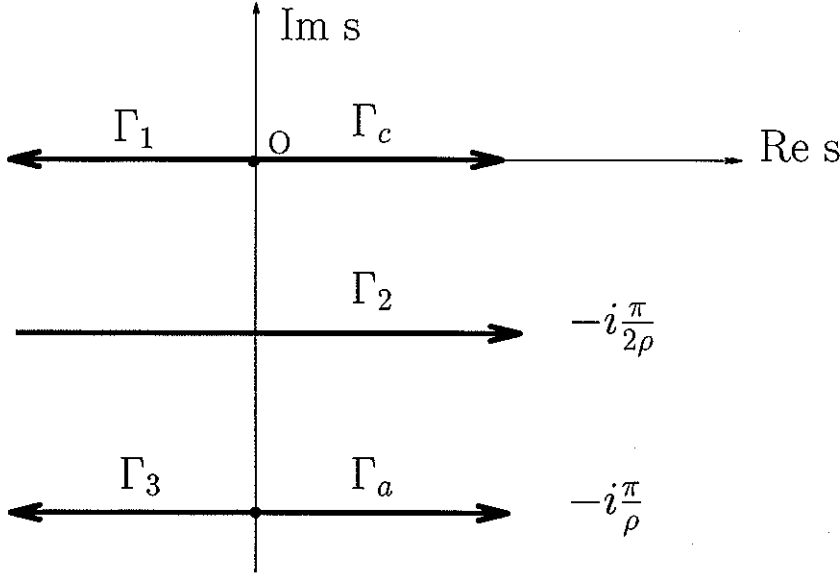


FIG. 1. Contours of integration $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_c, \Gamma_a$

$$\begin{aligned}f_Q &= e^{s\rho\Xi} \exp\left(-\frac{i}{2}qF_{\mu\nu}^\perp \sigma^{\mu\nu} s\right) f_Q^{(0)}, \\ f_Q^{(0)} &= \exp\left\{-iq \int_{x'}^x A_\mu^\perp dx^\mu\right\} f_Q^\parallel(z - z') f_\perp, \\ f_\perp &= (4\pi)^{-2} \frac{qH}{\sin(qHs)} \exp\left\{-\frac{i}{4}y_\perp qF \coth(qFs)y_\perp\right\},\end{aligned}\tag{33}$$

$$f_Q^{\parallel}(z) = \frac{\rho}{\sinh(\rho s)} \exp \left\{ -i\frac{\rho}{2}(\tilde{\eta} + \tilde{\eta}')z - im_Q^2 s + i\frac{\rho}{4} [z^2 - (\tilde{\eta} - \tilde{\eta}')^2] \coth(\rho s) \right\}.$$

One can calculate in (28) all Gaussian integrals over p_3 and z . Thus, one gets

$$S^{(\dots)}(x, x') = (\gamma^\mu \mathcal{P}_\mu + M\Omega) \Delta^{(\dots)}(x, x'), \quad (34)$$

$$\begin{aligned} \Delta^c(x, x') &= \int_{\Gamma_c} f(x, x', s) ds, \quad \Delta(x, x') = \text{sgn}(y_0) \int_{\Gamma} f(x, x', s) ds, \\ \Delta^a(x, x') &= -\Delta^{(1)}(x, x') - \Delta^{(2)}(x, x'), \\ \Delta^p(x, x') &= -\Delta^{(1)}(x, x') + \Delta^{(2)}(x, x'), \\ \Delta^{(1)}(x, x') &= -\frac{1}{2} \int_{\Gamma_3 + \Gamma_2 + \Gamma_a} f(x, x', s) ds, \\ \Delta^{(2)}(x, x') &= \int_{\Gamma_3 + \Gamma_2 - \Gamma_a} f_r(x, x', s) ds, \end{aligned} \quad (35)$$

where $f_r(x, x', s) = \pi^{-1/2}/2 \gamma(1/2, \alpha) f(x, x', s)$, $\gamma(1/2, \alpha)$ is the incomplete gamma-function, and $\alpha = e^{-i\pi/2} (4s(bM)^2\omega)^{-1} [(x_0 + x'_0)s(bM)^2 + qEy^3]^2$. Here

$$\begin{aligned} f(x, x', s) &= \exp \left(\rho \Xi s - i\frac{q}{2} \sigma^{\mu\nu} F_{\mu\nu}^\perp s \right) f^{(0)}(x, x', s), \quad (36) \\ f^{(0)}(x, x', s) &= \exp \left\{ -iq \int_{x'}^x A_\mu dx^\mu \right\} f_{\parallel} f_{\perp}, \\ f_{\parallel} &= \frac{\rho}{\sinh(\rho s)\omega^{1/2}} \exp \left\{ i\frac{qE}{2} (x_0 + x'_0) y^3 - i\frac{\rho}{4} (x_0 - x'_0)^2 \coth(\rho s) \right. \\ &\quad \left. - i(aM)^2 s + i\frac{\rho}{4\omega} y_3^2 \coth(\rho s) - \frac{i}{4\omega} \left[(bM)^2 s (x_0 + x'_0)^2 + 2qEy^3 (x_0 + x'_0) \right] \right\}, \end{aligned}$$

where $\omega = s \coth(\rho s)(bM)^2/\rho + (qE)^2/\rho^2$. One can see that

$$i\frac{d}{ds} f(x, x', s) = \left(M^2\Omega^2 - \mathcal{P}^2 + \frac{q}{2} \sigma^{\mu\nu} F_{\mu\nu} - ibM\gamma^0 \right) f(x, x', s), \quad (37)$$

$$\lim_{s \rightarrow +0} f(x, x', s) = i\delta^{(4)}(x - x') \quad (38)$$

Thus, $f(x, x', s)$ is the Fock-Schwinger function [2,24] of the QED- Ω theory, and s is the Fock-Schwinger proper-time in this case. The contour $\Gamma_c - \Gamma_2 - \Gamma_1$ in representation (35) for Δ transformed into Γ (see Fig.2) after the integration over p_D and z . The result is consistent with the general expression for the commutation function obtained in [25]. The function $f^{(0)}(x, x', s)$ which has appeared in (36) coincides with the Fock-Schwinger function of the scalar case. Due to that we can widely use results presented in [1].

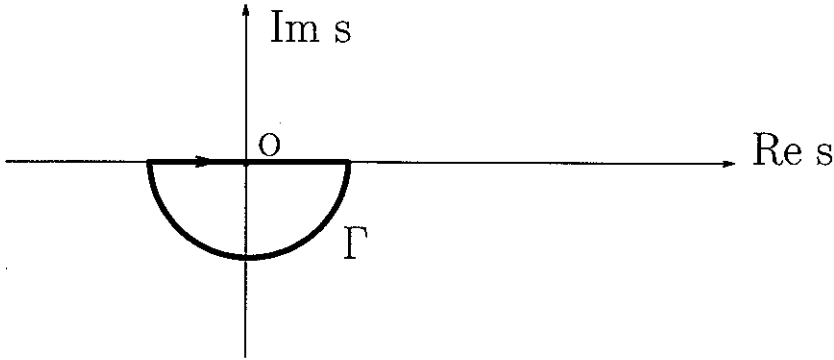


FIG. 2. Contour of integration Γ

If $b \neq 0$, then the function $f(x, x', s)$ has three singular points on the complex region between contours $\Gamma_c - \Gamma_1$ and $\Gamma_a - \Gamma_3$, which are distributed at the imaginary axis: $\rho s_0 = 0$, $\rho s_1 = -i\pi$ and $\rho s_2 = -ic_2$. The latter point is connected with zero value of the function ω . We get an equation for c_2 from the condition $\omega = 0$, $c_2 \tan(c_2 - \pi/2) - (qE/(bM))^2 = 0$, where $\pi/2 < c_2 < \pi$. The position of this point depends on the ratio $qE/(bM)$, e.g. $c_2 \rightarrow \pi$ as $bM/(qE) \rightarrow 0$ and $c_2 \rightarrow \pi/2$ as $qE/(bM) \rightarrow 0$. Notice, in the case $E = 0$ it is convenient to put $c_2 = \pi/2 + 0$ since the contour Γ_2 is also passed above the singular point s_2 . If $b = 0$, then $\omega = 1$, and the function $f(x, x', s)$ has only two singular points s_0 and s_1 on the above mentioned complex region. In this degenerate case we have known representation [11].

Similarly to Ref. [1] one may demonstrate that the function $\Delta^{(2)}$ from (35) can be pre-

sented via proper-time integrals with the Fock-Schwinger kernel $f(x, x', s)$. Then, using (37), one can verify that the representations (35) for $\Delta^{(1)}$ and $\Delta^{(2)}$ obey the equation (8). Thus, all the Δ -functions considered here, excluding those marked by the index "c", are solutions of the equation (8). The important difference between the functions Δ^c , $\Delta^{(1)}$ and Δ , $\Delta^{(2)}$ is that the first ones are symmetric under simultaneous change of sign in x_0 , x'_0 , x_3 , x'_3 and the second ones change sign in this case.

Using the kernel (36) one can express out-in effective action in the QED- Ω theory,

$$\Gamma_{out-in} = \frac{1}{2} \int dx \int_0^\infty s^{-1} f(x, x, s) ds. \quad (39)$$

IV. VACUUM INSTABILITY, MEAN VALUES OF CURRENT, AND ENERGY MOMENTUM TENSOR. DISCUSSION

Using the exact solutions and the GF constructed above, one may calculate different physical effects having both local and global character. The proper-time representation of the GF gives us a possibility to study all mean values of current and energy momentum tensor in the same manner.

All the information about the processes of particles creation, annihilation, and scattering in an external field (without radiative corrections) can be extracted from the matrices $G(\zeta|\zeta')$ (10). These matrices define a canonical transformation between in and out creation and annihilation operators in the generalized Furry representation [6,8],

$$a^\dagger(out) = a^\dagger(in)G(+|+) + b(in)G(-|+), \quad b(out) = a^\dagger(in)G(+|^-) + b(in)G(-|^-). \quad (40)$$

Here $a_i^\dagger(in)$, $b_i^\dagger(in)$, $a_i(in)$, $b_i(in)$ are creation and annihilation operators of in-particles and antiparticles respectively and $a_i^\dagger(out)$, $b_i^\dagger(out)$, $a_i(out)$, $b_i(out)$ are ones of out-particles and

antiparticles, l are possible quantum numbers (in our case it $l = p_1, p_3, n, r$). For example, the mean differential number of particles created (which are also equal to the number of pairs created) by the external field from the in-vacuum $|0, in \rangle$ is

$$N_l = \langle 0, in | a_l^\dagger(out) a_l(out) | 0, in \rangle = |g(-|+)|^2, \quad (41)$$

(for a review of gravitational particles creation, see [27,28]). The standard space coordinate volume regularization was used to get the latter formula, so that $\delta(p_j - p'_j) \rightarrow \delta_{p_j, p'_j}$. The probability for a vacuum to remain a vacuum is

$$P_v = |c_v|^2 = \exp \left\{ \sum_l \ln(1 - N_l) \right\}. \quad (42)$$

Similar to the electric field case [21] we get

$$N_l = e^{-\pi\lambda}, \quad \text{if } \sqrt{\rho}T \gg 1, \quad \text{and } \sqrt{\rho}T \gg \lambda, \quad \text{and } \rho^2T \gg |qEp_3|, \quad (43)$$

where λ is defined in (15). The latter conditions take place with large enough time for action of the electric-like field, $T = x_{out}^0 - x_{in}^0$. The result (43) coincides with one obtained in [19], and for $b = 0$ it coincides with one obtained in [21]. Evidently the creation process is a coherent effect of both the electromagnetic and gravitational fields. If the condition $p_3^2(bM)^2/\rho^3 \ll 1$ takes place (the gravitational field is in a sense weaker than the electric one), the p_3 dependence on N_l is similar to the case $b = 0$. Thus, one can estimate that $\int dp_3 = (qE)^{-1}\rho^2T$ [21]. Then the particle creation per unit of time may be calculated similar to [18]. In strong enough gravitational background the time dependence of the effect is nonlinear and needs to be studied specially.

To get the total number N of particles created one has to sum over the quantum numbers l . The sum over the momenta can be easily transformed into an integral. Thus, if $b = 0$ one

gets result presented in [21]. If $b \neq 0$, the total number of pairs created per space coordinate volume has the form

$$\tilde{n}^{cr} = \frac{\sum_l N_l}{\int d\mathbf{x}} = \frac{\beta(1) \rho^{3/2}}{4\pi^2 bM} \exp \left\{ -\pi \frac{(aM)^2}{\rho} \right\}, \quad (44)$$

where $\beta(n) = qH \coth(n\pi qH/\rho)$. The observable number density of the created pairs in the asymptotic region $x_0 = x_0^{out} \rightarrow \infty$ is given by the expression $n^{cr} = \tilde{n}^{cr}/\Omega^3(x_0)$. If the electromagnetic field is absent and $a = 0$, these results coincide with ones in [20]. In case $b \rightarrow 0$ the expression (44) is growing infinitely. In this case the particles are created in main by the electric field, whereas the parameter b plays a role of “cut-off” factor, which eliminates creation of particles with extremely high momenta along the electric field. It is seen from the expression (43). Thus, the limit $b \rightarrow 0$ corresponds to the case of the electric field which acts for infinite time. Then the number of particles created is proportional to the time of the field action. As was already remarked above, in this case $\int dp_3 = (qE)^{-1} \rho^2 T$. Then, the parameter b may be understood as $(\sqrt{\rho})(TM)^{-1}$. The vacuum-to-vacuum transition probability can be calculated, using formula (42). Thus, we get an analog of the well-known Schwinger formula [2] in the case under consideration,

$$P_v = \exp \left\{ -\mu \tilde{n}^{cr} \int d\mathbf{x} \right\}, \quad \mu = \sum_{n=0}^{\infty} \frac{\beta(n+1)}{(n+1)^{3/2} \beta(1)} \exp \left\{ -n\pi \frac{(aM)^2}{\rho} \right\}. \quad (45)$$

Now we are going to discuss vacuum matrix elements of current and of metric energy-momentum tensor (EMT) of spinor field. Making the conformal transformation the current in FRW universe may be presented in the following form

$$J_\mu = \Omega^{-2}(x^0) j_\mu, \quad j_\mu = \frac{q}{2} [\bar{\psi}(x), \gamma_\mu \psi(x)], \quad (46)$$

where j_μ is QED- Ω current of the spinor field $\psi(x)$. The latter obeys the Dirac equation (2). The EMT of spinor field in FRW universe may be presented as

$$\begin{aligned} \tau_{\mu\nu} &= \Omega^{-2}(x^0)T_{\mu\nu}, \quad T_{\mu\nu} = \frac{1}{2} (T_{\mu\nu}^{can} + T_{\nu\mu}^{can}), \quad (47) \\ T_{\mu\nu}^{can} &= \frac{1}{4} \left\{ [\bar{\psi}(x), \gamma_\mu \mathcal{P}_\nu \psi(x)] + [\mathcal{P}_\nu^* \bar{\psi}(x), \gamma_\mu \psi(x)] \right\}, \end{aligned}$$

where $T_{\mu\nu}$ is EMT in QED- Ω . In the case of unstable vacuum one needs to calculate three types of matrix elements [8], depending on the problem in question:

$$\langle j_\mu \rangle^c = \langle 0, out | j_\mu | 0, in \rangle c_v^{-1}, \quad \langle T_{\mu\nu} \rangle^c = \langle 0, out | T_{\mu\nu} | 0, in \rangle c_v^{-1}, \quad (48)$$

$$\langle j_\mu \rangle^{in} = \langle 0, in | j_\mu | 0, in \rangle, \quad \langle T_{\mu\nu} \rangle^{in} = \langle 0, in | T_{\mu\nu} | 0, in \rangle, \quad (49)$$

$$\langle j_\mu \rangle^{out} = \langle 0, out | j_\mu | 0, out \rangle, \quad \langle T_{\mu\nu} \rangle^{out} = \langle 0, out | T_{\mu\nu} | 0, out \rangle, \quad (50)$$

Using GF, which were found before, one can present these matrix elements as follows:

$$\langle j_\mu \rangle^c = iq \text{tr} \{ \gamma_\mu S^c(x, x) \} = iq \text{tr} \{ \gamma_\mu (\gamma^\kappa \mathcal{P}_\kappa + M\Omega) \Delta^c(x, x') \} \Big|_{x=x'}, \quad (51)$$

$$\begin{aligned} \langle T_{\mu\nu} \rangle^c &= i/4 \text{tr} \left\{ \left(\gamma_\mu (\mathcal{P}_\nu + \mathcal{P}'_\nu) + \gamma_\nu (\mathcal{P}_\mu + \mathcal{P}'_\mu) \right) S^c(x, x') \right\} \Big|_{x=x'} \\ &= i \text{tr} \{ B_{\mu\nu} \Delta^c(x, x') \} \Big|_{x=x'}, \quad (52) \end{aligned}$$

$$\langle j_\mu \rangle^{in} = \langle j_\mu \rangle^c + \langle j_\mu \rangle^{(1)} + \langle j_\mu \rangle^{(2)}, \quad (53)$$

$$\langle j_\mu \rangle^{out} = \langle j_\mu \rangle^c + \langle j_\mu \rangle^{(1)} - \langle j_\mu \rangle^{(2)}, \quad (54)$$

$$\langle T_{\mu\nu} \rangle^{in} = \langle T_{\mu\nu} \rangle^c + \langle T_{\mu\nu} \rangle^{(1)} + \langle T_{\mu\nu} \rangle^{(2)}, \quad (55)$$

$$\langle T_{\mu\nu} \rangle^{out} = \langle T_{\mu\nu} \rangle^c + \langle T_{\mu\nu} \rangle^{(1)} - \langle T_{\mu\nu} \rangle^{(2)}, \quad (56)$$

$$\langle j_\mu \rangle^{(1,2)} = iq \text{tr} \left\{ \gamma_\mu (\gamma^\kappa \mathcal{P}_\kappa + M\Omega) \Delta^{(1,2)}(x, x') \right\} \Big|_{x=x'}, \quad (57)$$

$$\langle T_{\mu\nu} \rangle^{(1,2)} = i \text{tr} \left\{ B_{\mu\nu} \Delta^{(1,2)}(x, x') \right\} \Big|_{x=x'}, \quad (58)$$

$$B_{\mu\nu} = 1/4 \left\{ \gamma_\mu (\mathcal{P}_\nu + \mathcal{P}'_\nu) + \gamma_\nu (\mathcal{P}_\mu + \mathcal{P}'_\mu) \right\} (\gamma^\kappa \mathcal{P}_\kappa + M\Omega),$$

where $\mathcal{P}'_\mu = -i \frac{\partial}{\partial x'^\mu} - q A_\mu(x')$, the GF are given by Eqs. ((35), and the relation $\Delta^c(x, x) = (1/2) [\Delta^-(x, x) - \Delta^+(x, x)]$ is used.

To get convenient forms of $\langle j_\mu \rangle^{in}$ and $\langle T_{\mu\nu} \rangle^{in}$ we may rewrite $\Delta^{(1)}$ as follows:

$$\begin{aligned}\Delta^{(1)}(x, x') &= -\frac{1}{2}\Delta^{\Gamma_2}(x, x') + \Delta^{(3)}(x, x'), \quad \Delta^{\Gamma_2}(x, x') = \int_{\Gamma_2} f(x, x', s)ds, \\ \Delta^{(3)}(x, x') &= -\frac{1}{2}\int_{\Gamma_3+\Gamma_a} f(x, x', s)ds.\end{aligned}\quad (59)$$

The contour Γ_2 does not pass through the singular points of the function $f(x, x', s)$, thus the contributions from Δ^{Γ_2} to EMT and to the current are finite. Displacing the contour Γ_2 to the real axis we may present the difference $\Delta^c - \Delta^{\Gamma_2}$ as

$$\begin{aligned}\Delta^c(x, x') - \Delta^{\Gamma_2}(x, x') &= \Delta^{\bar{c}}(x, x') + \Delta^\Gamma(x, x'), \\ \Delta^{\bar{c}}(x, x') &= \int_{-\infty}^0 f(x, x', s)ds, \quad \Delta^\Gamma(x, x') = \int_\Gamma f(x, x', s)ds,\end{aligned}\quad (60)$$

where $\Delta^{\bar{c}}$ is related to the anticausal GF, $S^{\bar{c}}(x, x') = (\gamma\mathcal{P} + M\Omega)\Delta^{\bar{c}}(x, x')$, and the integral Δ^Γ can be expressed by means of the Δ -function related to the commutation function, $S(x, x') = (\gamma\mathcal{P} + M\Omega)\Delta(x, x')$ [25]: $\Delta^\Gamma(x, x') = \text{sgn}(x_0 - x'_0)\Delta(x, x')$. Taking into account the above definition the space-like part of the limit $x - x' \rightarrow 0$ in the EMT expressions have to be treated either as the limit $x_0 - x'_0 = +0$ or as $x_0 - x'_0 = -0$, respectively. Then, in according to the initial condition for the commutation function we get $\Delta(x, x')|_{x_0=x'_0} = 0$, $\partial_0\Delta(x, x')|_{x_0=x'_0} = \delta(\mathbf{x} - \mathbf{x}')$. Thus, the contributions from Δ into $\langle T_{\mu\nu} \rangle^{in}$ and $\langle j_\mu \rangle^{in}$ are zero. (All contributions from Δ , which might appear as a result of a change of the limit definition, are background independent. They may be eliminated by a renormalization.) It follows from (36) that

$$f(x, x', -s^*) = \gamma^0 f(x', x, s)^\dagger \gamma^0. \quad (61)$$

Changing $s \rightarrow -s$ one can represent integral $\Delta^{\bar{c}}$ from (60) in the form

$$\Delta^{\bar{c}}(x, x') = -\int_0^\infty \gamma^0 f(x', x, s)^\dagger \gamma^0 ds, \quad (62)$$

to get

$$\langle j_\mu \rangle^{in} = \text{Re} \langle j_\mu \rangle^c + \langle j_\mu \rangle^{(2)} + \langle j_\mu \rangle^{(3)}, \quad (63)$$

$$\langle T_{\mu\nu} \rangle^{in} = \text{Re} \langle T_{\mu\nu} \rangle^c + \langle T_{\mu\nu} \rangle^{(2)} + \langle T_{\mu\nu} \rangle^{(3)}, \quad (64)$$

where the terms $\langle j_\mu \rangle^{(3)}$ and $\langle T_{\mu\nu} \rangle^{(3)}$ represent the contributions from $\Delta^{(3)}$:

$$\langle j_\mu \rangle^{(3)} = iq \text{tr} \left\{ \gamma_\mu (\gamma^\kappa \mathcal{P}_\kappa + M\Omega) \Delta^{(3)}(x, x') \right\} \Big|_{x=x'}, \quad (65)$$

$$\langle T_{\mu\nu} \rangle^{(3)} = i \text{tr} \left\{ B_{\mu\nu} \Delta^{(3)}(x, x') \right\} \Big|_{x=x'}. \quad (66)$$

Using the Eq. (61) one can verify that each of the terms $\langle j_\mu \rangle^{(2)}$, $\langle j_\mu \rangle^{(3)}$, $\langle T_{\mu\nu} \rangle^{(2)}$ and $\langle T_{\mu\nu} \rangle^{(3)}$ is real, as it follows from their initial definitions.

One can get similar expressions for the out-out-matrix elements of the current and EMT:

$$\langle j_\mu \rangle^{out} = \text{Re} \langle j_\mu \rangle^c - \langle j_\mu \rangle^{(2)} + \langle j_\mu \rangle^{(3)}, \quad (67)$$

$$\langle T_{\mu\nu} \rangle^{out} = \text{Re} \langle T_{\mu\nu} \rangle^c - \langle T_{\mu\nu} \rangle^{(2)} + \langle T_{\mu\nu} \rangle^{(3)}. \quad (68)$$

All nondiagonal matrix elements of EMT are zero:

$$\langle T_{\mu\nu} \rangle^c = \langle T_{\mu\nu} \rangle^{(2)} = \langle T_{\mu\nu} \rangle^{(3)} = 0, \quad \mu \neq \nu. \quad (69)$$

Using the formulas

$$\begin{aligned} \exp(\rho \Xi s) &= \cosh(\rho s) + \Xi \sinh(\rho s), \\ \exp\left(-i \frac{q}{2} \sigma^{\mu\nu} F_{\mu\nu}^\perp s\right) &= \cos(qHs) + \gamma^2 \gamma^1 \sin(qHs), \end{aligned}$$

one may calculate the traces for the current and EMT:

$$\begin{aligned} \tau(s) &= \text{tr} Y = 4 \cosh(\rho s) \cos(qHs), \quad Y = \left\{ \exp\left(\rho \Xi s - i \frac{q}{2} \sigma^{\mu\nu} F_{\mu\nu}^\perp s\right) \right\}, \\ \text{tr} \{\Xi Y\} &= \tanh(\rho s) \tau(s), \quad \text{tr} \{\gamma^0 \gamma^3 Y\} = \frac{qE}{\rho} \tanh(\rho s) \tau(s), \\ \text{tr} \{\gamma^0 Y\} &= -\frac{bM}{\rho} \tanh(\rho s) \tau(s), \quad \text{tr} \{\gamma^2 \gamma^1 Y\} = -\tan(qHs) \tau(s), \end{aligned}$$

and other traces are zero. Then, the current matrix elements can be found in the forms

$$\langle j_\mu \rangle^c = \int_{\Gamma_c} \alpha_\mu(s) \tau(s) f^{(0)}(x, x', s) ds \Big|_{x=x'} , \quad (70)$$

$$\langle j_\mu \rangle^{(3)} = -\frac{1}{2} \int_{\Gamma_3 + \Gamma_a} \alpha_\mu(s) \tau(s) f^{(0)}(x, x', s) ds \Big|_{x=x'} , \quad (71)$$

$$\langle j_\mu \rangle^{(2)} = \int_{\Gamma_3 + \Gamma_2 - \Gamma_a} \alpha_\mu(s) \tau(s) f_r^{(0)}(x, x', s) ds \Big|_{x=x'} , \quad (72)$$

$$\alpha_\mu(s) = iq\delta_\mu^3 \left[\mathcal{P}_3 + \mathcal{P}_0 \frac{qE}{\rho} \tanh(\rho s) \right] ,$$

where the only x^3 components (along the electric field) of the currents differ from zero and vanish in the absence of the electric field, as it presented to be. The nonzero components of $\langle T_{\mu\nu} \rangle^c$ are

$$\langle T_{\mu\nu} \rangle^c = \int_{\Gamma_c} t_\mu(s) \tau(s) f^{(0)}(x, x, s) ds , \quad \text{if } \mu = \nu , \quad (73)$$

$$t_0(s) = -\frac{\rho}{\sinh(2\rho s)} , \quad t_2(s) = t_1(s) = \frac{qH}{\sin(2qHs)} ,$$

$$t_3(s) = \frac{\rho}{2\omega} \coth(\rho s) - \frac{(qE)^2}{2\rho} \tanh(\rho s) + i \left(\frac{qE}{2} (x_0 + x'_0) (1 - \omega^{-1}) \right)^2 ,$$

and nonzero components of $\langle T_{\mu\nu} \rangle^{(2)}$ and $\langle T_{\mu\nu} \rangle^{(3)}$ can be written as follows:

$$\langle T_{\mu\nu} \rangle^{(3)} = -\frac{1}{2} \int_{\Gamma_3 + \Gamma_a} l_\mu(s) \tau(s) f^{(0)}(x, x, s) ds , \quad \text{if } \mu = \nu , \quad (74)$$

$$\langle T_{\mu\nu} \rangle^{(2)} = \int_{\Gamma_3 + \Gamma_2 - \Gamma_a} l_\mu(s) \tau(s) f_r^{(0)}(x, x', s) ds \Big|_{x=x'} , \quad \text{if } \mu = \nu , \quad (75)$$

$$l_1(s) = t_1(s) , \quad l_2(s) = t_2(s) , \quad l_3(s) = i(\mathcal{P}_3)^2 - \frac{(qE)^2}{2\rho} \tanh(\rho s) ,$$

$$l_0(s) = l_1(s) + l_2(s) + l_3(s) + i \left[2 \left(\omega^{-1} b^2 M^2 s x_0 \right)^2 - i \omega^{-1} b^2 M^2 s + M^2 \Omega^2 \right] - \frac{\rho}{2} \tanh(\rho s) .$$

Here, $l_0(s)$ is obtained from $t_0(s)$, using Eq. (37) and taking into account that the $\Delta^{(2)}$ function obeys the equation (8).

The components $\langle j_\mu \rangle^c$ and $\langle T_{\mu\nu} \rangle^c$ are expressed by the causal GF in the well-known Schwinger's form. They are sources of information about local features of the theory.

For the first time it is obtained exactly with respect to the given external background. The components $\langle j_\mu \rangle^{(2,3)}$ and $\langle T_{\mu\nu} \rangle^{(2,3)}$ are only related to global features of the theory and characterize the vacuum instability. Such expressions cannot be calculated in the frame of the perturbation theory with respect to the external background or in the frame of the WKB method. Then, such terms has never been studied. If the parameter $b \neq 0$, the only expression (73) for $\langle T_{\mu\nu} \rangle^c$ has to be regularized and then renormalized. The expression (70) for $\langle j_\mu \rangle^c$ is finite after the regularization lifting. The terms $\langle j_\mu \rangle^{(2,3)}$ and $\langle T_{\mu\nu} \rangle^{(2,3)}$ are also finite. Thus, as one can see from Eqs. (64) and (68), the divergence in $\langle T_{\mu\nu} \rangle^{in}$ and $\langle T_{\mu\nu} \rangle^{out}$ is the same as in $\langle T_{\mu\nu} \rangle^c$. That is consistent with the fact that the ultraviolet divergences have a local nature and result (as in the theory without an external field) from the leading local terms at $s \rightarrow +0$. Nevertheless, as $b \rightarrow 0$ (flat space limit) the terms $\langle j_\mu \rangle^{(2,3)}$ and $\langle T_{\mu\nu} \rangle^{(2,3)}$ have electric field dependent divergences defined by the dimensionless parameter $\beta = bM/qE$. The nature of that is similar to the divergence of \tilde{n}^{cr} for $b \rightarrow 0$. In the case $b = 0$ the density of excited vacuum states is proportional to the time T . After a special regularization with respect to time T [21] all of the terms become finite.

Now we are going to discuss current, energy density and pressure of the created particles. Let us introduce normalized values of current and EMT, which may be easily connected to observable values in some appropriate asymptotic region $x_0 = x_0^{as}$,

$$j_\mu^{cr} = \tilde{j}_\mu^{cr}/\Omega^3(x_0), \quad T_{\mu\nu}^{cr} = \tilde{T}_{\mu\nu}^{cr}/\Omega^3(x_0), \quad (76)$$

where according to the definitions (49) and (50) the corresponding densities of particles created per space-coordinates volume are

$$\tilde{j}_\mu^{cr} = \frac{\int d\mathbf{x} (\langle j_\mu \rangle^{in} - \langle j_\mu \rangle^{out})}{\int d\mathbf{x}}, \quad x_0 = x_0^{as}, \quad (77)$$

$$\tilde{T}_{\mu\nu}^{cr} = \frac{\int d\mathbf{x} (\langle T_{\mu\nu} \rangle^{in} - \langle T_{\mu\nu} \rangle^{out})}{\int d\mathbf{x}}, \quad x_0 = x_0^{as}. \quad (78)$$

Then, using representations (63), (64) and (67), (68) one gets from (77) and (78),

$$\tilde{j}_\mu^{cr} = 2 \langle j_\mu \rangle^{(2)}, \quad \tilde{T}_{\mu\nu}^{cr} = 2 \langle T_{\mu\nu} \rangle^{(2)}, \quad x_0 = x_0^{as}. \quad (79)$$

The component $T_0^{cr\ 0} = \Omega^{-4} \tilde{T}_{00}^{cr}$ is the energy density and $-T_\mu^{cr\ \nu} = \Omega^{-4} \tilde{T}_{\mu\nu}^{cr}$ for $\mu = \nu = 1, 2, 3$ are components of pressure which are measured by the cosmic observer relative to the measured volume. In contrast to that, measured energy and pressure taken per coordinate volume are given by $\check{T}_0^{cr\ 0} = \Omega^{-1} \tilde{T}_{00}^{cr}$, $-\check{T}_\mu^{cr\ \nu} = \Omega^{-1} \tilde{T}_{\mu\nu}^{cr}$, $\mu = \nu = 1, 2, 3$.

We are going to analyze contributions to the quantities (79) at $x_0 = x_0^{as}$. The leading asymptotics in $\langle j_\mu \rangle^{(2)}$ and $\langle T_{\mu\nu} \rangle^{(2)}$ at $x_0 \gg \sqrt{\rho}/(bM)$ are determined by the expressions

$$j_\mu \rangle^{(2)} = \alpha_\mu(s_1) \text{tr} \left\{ \Delta^{(2)}(x, x') \right\} \Big|_{x=x'}, \quad T_{\mu\mu} \rangle^{(2)} = l_\mu(s_1) \text{tr} \left\{ \Delta^{(2)}(x, x') \right\} \Big|_{x=x'}. \quad (80)$$

An asymptotic expression for $\text{tr} \Delta^{(2)}$ with $x \rightarrow x'$ can be found using the method described in the App. A of Ref. [1] for the scalar case. Then, one gets

$$\text{tr} \Delta^{(2)} = \frac{-i\tilde{n}^{cr}}{\rho(x_0 + x'_0)} \exp \left\{ iq\Lambda + \frac{iqE}{2}(x_0 + x'_0)y^3 - \frac{\rho^3(y_3)^2}{4\pi(bM)^2} + y_\perp \frac{qF}{4} \cot(\pi qF/\rho) y_\perp \right\},$$

where \tilde{n}^{cr} is defined in (44). This expression is also valid when $x_0 < 0$, since the function $\Delta^{(2)}$ is odd in x_0 as $x = x'$ (see Sec.III). Thus, one can see when $x_0 \gg \sqrt{\rho}/(bM)$ the leading terms in $\langle j_\mu \rangle^{(2)}$ and $\langle T_{\mu\nu} \rangle^{(2)}$ are

$$\begin{aligned} \langle j_\mu \rangle^{(2)} &= -q^2 E \rho^{-1} \tilde{n}^{cr} \delta_\mu^3; \\ \langle T_{00} \rangle^{(2)} &= \left[\rho^2 x_0^2 + a^2 M^2 + 2qH \sinh^{-1}(2\pi qH/\rho) \right] \frac{\tilde{n}^{cr}}{\rho x_0}, \end{aligned}$$

$$\begin{aligned}
\langle T_{11} \rangle^{(2)} &= \langle T_{22} \rangle^{(2)} = qH \sinh^{-1} (2\pi qH/\rho) \frac{\tilde{n}^{cr}}{\rho x_0}, \\
\langle T_{33} \rangle^{(2)} &= \left[(qEx_0)^2 + \frac{\rho^3}{2\pi(bM)^2} \right] \frac{\tilde{n}^{cr}}{\rho x_0}.
\end{aligned} \tag{81}$$

Comparing our results with the scalar theory case [1], when $x_0 \rightarrow \infty$ we may see the difference only in the factor \tilde{n}^{cr} which enters in the leading terms of $\langle j_\mu \rangle^{(2)}$, $\langle T_{00} \rangle^{(2)}$ and $\langle T_{33} \rangle^{(2)}$.

Doubling the expressions (81) according to the Eqs. (79), one gets the mean densities for current and EMT of particles created. It turns out that these quantities are proportional to the density of total number of particles and antiparticles created ($2\tilde{n}^{cr}$) for the infinite time and do not change their structure with increasing of x_0 . The latter means that one can consider all the expressions obtained at any fixed x_0 as asymptotic forms if $x_0 \gg \sqrt{\bar{\rho}}/bM$. In a strong background $a^2 M^2/\rho \leq 1$ and, therefore, such a time has to obey the condition $x_0 \gg a/b$. Thus, in the strong background our asymptotic conformal time x_0 , which is large enough in quantum sense explained, corresponds to the large cosmological time t .

Note, that one can neglect the second term in the brackets of the expression (81) for $\langle T_{33} \rangle^{(2)}$ at $bM/(qE) \leq 1$. Also one can neglect both the term $a^2 M^2$ in the brackets of the expression (81) for $\langle T_{00} \rangle^{(2)}$ in case of strong background $a^2 M^2/\rho \leq 1$ and third term in the same expression if the magnetic field is not strong enough, $qH/\rho \leq 1$. The current density $\tilde{j}_\mu^{cr} = 2 \langle j_\mu \rangle^{(2)}$ does not depend on the asymptotic time. As $b \rightarrow 0$ this expression coincides with the one for flat space, $\langle \tilde{j}_\mu \rangle^{cr} = -2q\tilde{n}^{cr}\delta_\mu^3$. The pressure component along the electric field direction $\tilde{T}_{33}^{cr} = 2 \langle T_{33} \rangle^{(2)}$ is growing with time upon the action of the field. However, if $qE/(bM) \ll 1$ then the asymptotic condition for x_0 is consistent with the fact that the term $(qEx_0)^2$ in the expressions $\langle T_{33} \rangle^{(2)}$ from (81) will not be dominant before large enough time instant x_0 . In this case one can neglect the contribution which

depends on the electric field, if the field is switched off at a fixed time.

The components of the pressure in the directions, which are perpendicular to the electric field, $\tilde{T}_{11}^{cr} = 2 \langle T_{11} \rangle^{(2)}$ and $\tilde{T}_{22}^{cr} = 2 \langle T_{22} \rangle^{(2)}$, decrease when the magnetic field increases. If an electromagnetic field is absent, the pressure is isotropic according to the symmetry of the space-time and to the corresponding symmetry of the vacuum. For $x^0 \rightarrow \infty$ the remaining terms of the measured energy and pressure of the created particles taken per coordinate volume are

$$\tilde{T}_0^{cr\ 0} = \frac{\rho}{b} 2\tilde{n}^{cr}, \quad -\tilde{T}_3^{cr\ 3} = \frac{(qE)^2}{\rho b} 2\tilde{n}^{cr},$$

and, if an electric field is absent the only remaining term is $\tilde{T}_0^{cr\ 0} = 2M\tilde{n}^{cr}$. The last represents the total rest mass per coordinate volume and coincides with the result of Ref. [18].

Another problem is to study a back-reaction of particles created on the electromagnetic field and metrics, or to be more correct, it is better to say about back-reaction effects produced by both of particles created from a vacuum and polarization of an unstable vacuum. To this end one needs to use the expressions $\langle j_\mu \rangle^{in}$ and $\langle T_{\mu\nu} \rangle^{in}$ for all times x_0 . The explicit proper-time expressions found above give us promising tool for accurate analysis of the total back-reaction related to the vacuum instability. In this approach we do not need to select phenomenologically parts come from real particles and from a vacuum polarization, that has been usually done in literature (see, for example, review in [27]). Of course, to find self-consistent solution taking into account the back-reaction one needs numerical estimations (compare with pure electromagnetic case, [29]). Keeping in mind such an application, one needs to get preliminary information about the behavior of the expressions (63) and (64) in time and to select the leading components. To this end let us estimate the above mentioned expressions at characteristic large time, $x_0^2 \gg \rho/(bM)^2$, and at characteristic

small time, $x_0^2 \ll \rho/(bM)^2$.

Neglecting divergent terms in $\langle T_{\mu\nu} \rangle^c$ according to the standard renormalization procedure, one can reduce Eq. (64) to the following finite form

$$\langle T_{\mu\nu} \rangle_{fin}^{in} = \text{Re} \langle T_{\mu\nu} \rangle_{fin}^c + \langle T_{\mu\nu} \rangle^{(2)} + \langle T_{\mu\nu} \rangle^{(3)}. \quad (82)$$

Here $\langle T_{\mu\nu} \rangle_{fin}^c$ is a finite part of $\langle T_{\mu\nu} \rangle^c$. An estimation of $\langle T_{\mu\nu} \rangle_{fin}^c$ can be made only after renormalization. Since we are here interested in to reveal global features of the theory, details of the renormalization problem together with others, related to the renormalization, will be considered in the next paper. To select contributions related to the vacuum instability, we note that the functions Δ^c (35) and $\Delta^{(3)}$ (59) with $x = x'$ are even in x_0 . Thus, the functions $\langle T_{\mu\nu} \rangle^c$ and $\langle T_{\mu\nu} \rangle^{(3)}$ are also even and do not vanish when $x_0 \rightarrow 0$, whereas the functions $\langle j_\mu \rangle^c$ and $\langle j_\mu \rangle^{(3)}$ are odd and vanish in the limit. Moreover, $\langle j_\mu \rangle^c = \langle j_\mu \rangle^{(3)} = 0$ for all x_0 with $b = 0$. The proper-time integral $\Delta^{(2)}$ (35) is odd in x_0 with $x = x'$ and vanishes when $x_0 \rightarrow 0$. Thus, the expression $\langle T_{\mu\nu} \rangle^{(2)}$ is also odd in x_0 and vanishes in this limit. The term $\langle j_\mu \rangle^{(2)}$ is not zero if $E \neq 0$. It is even in x_0 and differs from zero when $x_0 \rightarrow 0$.

The asymptotic behavior of $\langle j_\mu \rangle^{(3)}$ and $\langle T_{\mu\nu} \rangle^{(3)}$ is defined by the asymptotic expression for $\Delta^{(3)}$. Using the method presented in Appendix A of Ref. [1] one can verify that asymptotically

$$\Delta^{(3)}(x, x') = \text{sign}(x_0) \Delta^{(2)}(x, x') \quad , \quad (83)$$

and therefore,

$$\langle j_\mu \rangle^{(3)} = \text{sign}(x_0) \langle j_\mu \rangle^{(2)} \quad , \quad \langle T_{\mu\nu} \rangle^{(3)} = \text{sign}(x_0) \langle T_{\mu\nu} \rangle^{(2)} \quad . \quad (84)$$

The expression (70) does not need to be renormalized, thus, one can easily verify that the relation $\langle j_3 \rangle^c \sim x_0^{-1} \rightarrow 0$ holds asymptotically. Then, the asymptotics of $\langle j_\mu \rangle^{in}$ is

determined by the asymptotic behavior of $\langle j_\mu \rangle^{(2)}$ and $\langle j_\mu \rangle^{(3)}$. If x_0 with the large modulus is negative, $\langle j_\mu \rangle^{in} = \text{Re} \langle j_\mu \rangle^c$ and $\langle T_{\mu\nu} \rangle_{fin}^{in} = \text{Re} \langle T_{\mu\nu} \rangle_{fin}^c$. Thus, in this regime the vacuum instability in the time-dependent background does not affect the expressions. If $\beta = bM/(qE) \ll 1$, the expression $\text{Re} \langle T_{\mu\nu} \rangle_{fin}^c$ remains finite with $b \rightarrow 0$, while the terms $\langle T_{\mu\nu} \rangle^{(2)}$ and $\langle T_{\mu\nu} \rangle^{(3)}$ diverge in the limit since the global effect: the total density of excited vacuum states in an electric field grows infinitely and its mean momentum along the electric field increases. In this case the asymptotics of $\langle T_{\mu\nu} \rangle_{fin}^{in}$ is determined by asymptotic behavior of $\langle T_{\mu\nu} \rangle^{(2)}$ and $\langle T_{\mu\nu} \rangle^{(3)}$, i.e., by the asymptotic global properties of the theory.

Let us estimate $\langle j_\mu \rangle^{in}$ and $\langle T_{\mu\nu} \rangle_{fin}^{in}$ for small x_0 . The terms $\langle j_\mu \rangle^c$ and $\langle j_\mu \rangle^{(3)}$ are odd in x_0 . They vanish at $x_0 \rightarrow 0$. That is why the leading term in (63) is $\langle j_\mu \rangle^{(2)}$. The leading contribution from the latter can be represent as

$$\langle j_\mu \rangle^{(2)} = \alpha_\mu(s_2) \text{tr} \left\{ \Delta^{(2)}(x, x') \right\} \Big|_{x=x'}$$

Then, using the expressions (A1) from the Appendix A, one gets

$$\langle j_\mu \rangle^{(2)} = qn^{(2)} \left[1 + \left(c_2(bM)^2 / (\rho qE) \right)^2 \right] \delta_\mu^3. \quad (85)$$

The expression (85) differs essentially from the asymptotic one (81). That remains true in the quasi-flat metrics ($\beta = bM/(qE) \ll 1$), when the vacuum instability results only from the electric field. In the case of the small time limit the expression $\langle j_\mu \rangle^{(2)} = q\tilde{n}^{cr} \delta_\mu^3$ differs by a sign from the asymptotics (81). The factor \tilde{n}^{cr} is known from the asymptotics. That is natural since value of the time instant x_0 is large enough ($x_0 + T/2 \gg (qE)^{-1/2}$ and $x_0 + T/2 \gg (aM)^2(qE)^{-3/2}$ according to (43)) with respect to the initial time instant ($x_0^{in} = -T/2$). Then the field action time is large enough to obey the stabilization condition

and the density of excited states has already the asymptotic form. However, such x_0 is still less than the asymptotic time for any β . Hence, at the time instant x_0 the vacuum differs essentially from $|0, out\rangle$ vacuum. Technically that means that at the small time, $x_0 \ll \sqrt{qE}/(bM)$, one cannot obtain the normal form of operators by means of the Δ^{out} -function and, consequently, to use only the term $\langle j_\mu \rangle^{(2)}$ for calculation of the mean current of quasiparticles, which are created up to that time. The small time expressions one can use only to calculate the back-reaction. Thus, in the external background under consideration a small time back-reaction to the mean electromagnetic field is not a screening (as it may be expected by analogy with a back-reaction of real particles created) but an induction. Since the small time considered is related to early stages of the Universe, such an observation may be important.

Since the function $\langle T_{\mu\nu} \rangle^{(2)}$ is odd in x_0 and vanishes as $x_0 \rightarrow 0$, the leading contribution to (82) is determined by $\text{Re} \langle T_{\mu\nu} \rangle_{fin}^c$ and $\langle T_{\mu\nu} \rangle^{(3)}$. However, if $\beta = bM/(qE) \ll 1$, the situation is more simple in the domain where $\beta^{-2} \gg qEx_0^2$, and the leading contributions come from $\langle T_{\mu\nu} \rangle^{(3)}$ only. That is related to the same global effect, mentioned above in the asymptotic case, since the expression $\text{Re} \langle T_{\mu\nu} \rangle_{fin}^c$ remains finite with $b \rightarrow 0$, while the term $\langle T_{\mu\nu} \rangle^{(3)}$ diverges in the limit. The total density of excited vacuum states grows infinitely and the mean momentum of the states along the electric field increases, as well. Taking into account that the leading contributions in $\langle T_{\mu\nu} \rangle^{(3)}$ result from the integral over a neighborhood of the singular point s_1 , one gets

$$\begin{aligned} \langle T_{00} \rangle^{(3)} = \langle T_{33} \rangle^{(3)} &= \frac{\sqrt{qE}}{\pi\beta} \tilde{n}^{cr} , \\ \langle T_{11} \rangle^{(3)} = \langle T_{22} \rangle^{(3)} &= qH/\sqrt{qE} \sinh^{-1}(2\pi H/E) \tilde{n}^{cr} \beta \ln \beta^{-1} . \end{aligned} \quad (86)$$

We see that leading vacuum polarization effects are in main defined by the total density

of excited vacuum states, which depend on the complete history of a state. Thus, it is a nonlocal contribution.

Considering the small time limit of $\langle T_{\mu\nu} \rangle^{(2)}$ we get

$$\langle T_{\mu\mu} \rangle^{(2)} = l_\mu(s_2) \operatorname{tr} \left\{ \Delta^{(2)}(x, x') \right\} \Big|_{x=x'} ,$$

where $\operatorname{tr}\Delta^{(2)}$ was found in (A1). If $\beta \ll 1$, the result has a simple form:

$$\begin{aligned} \langle T_{00} \rangle^{(2)} &= \langle T_{33} \rangle^{(2)} = qE x_0 \tilde{n}^{cr} , \\ \langle T_{11} \rangle^{(2)} &= \langle T_{22} \rangle^{(2)} = -2qH \sinh^{-1}(2\pi H/E) x_0 \pi \beta^2 \tilde{n}^{cr} . \end{aligned}$$

One can see that expressions for $\langle T_{\mu\nu} \rangle^{(2)}$ at the asymptotic time and at the small time are quite different. The only components $\langle T_{00} \rangle^{(2)} = \langle T_{33} \rangle^{(2)}$ for $\beta \ll 1$ coincide.

The technics developed in the article allows one take into account global effects of both a real particles creation and a polarization on the same footing. The explicit form and the limit estimations for the mean vacuum values of EMT may be used for the calculations of the back reaction from the unstable vacuum to the gravitational background (in particular computing simulation similar to [30], see also [31] and references therein). We see a behaviour of such components in time are quite different, and one needs to take into account characteristic polarization effect. The proper-time representations of the GF may be the necessary step in the study of chiral symmetry breaking in QED and the four-fermion models under the action of gravitational and electromagnetic fields [14]. Such a study may have an immediate important application to EU, for example, through the construction of inflationary Universe where role of inflaton is played by the condensate $\langle \bar{\Psi}\Psi \rangle$. One can also analyse symmetry breaking phenomenon under the combined action of these fields in the Standard Model (using also its gauged NJL form [32]), or GUT in the same way as it has been done in

curved spacetime (without electromagnetic field) [12]. Our methods maybe applied to the study of SUSY NJL model in curved spacetime introduced in [33]. The chiral symmetry breaking under the action of weak gravitational field and constant electromagnetic field has been studied in [34]. Having our results we may study the chiral symmetry breaking for such model in FRW Universe with constant electromagnetic field.

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APPENDIX A: SMALL TIME EXPANSION OF $\Delta^{(2)}$ -FUNCTION

To calculate small time expansion of $\text{tr} \{ \Delta^{(2)} \}$ given by Eq. (35), in the case $x_0^2 \ll \rho/(bM)^2$ and $x \rightarrow x'$, one can use the method presented in App. B of Ref. [1]. The final result is

$$\begin{aligned}
\text{tr} \{ \Delta^{(2)}(x, x') \} &= e^{iq\Lambda} \left\{ \left[i(x_0 + x'_0)c_2(bM)^2/\rho - qEy^3 \right] n^{(2)}/(qE)\varphi_0 \right. \\
&+ \left. i \left[-(x_0 + x'_0)^3 \frac{(bM)^4}{12qE\rho^2} K(2) + (1/2)(x_0 + x'_0)(y_3)^2 qEK(0) \right] \right\} , \tag{A1} \\
\varphi_0 &= \exp \left(i \frac{qE}{2} (x_0 + x'_0)y^3 - \frac{\rho}{4c_2} \left(\frac{qE}{bM} \right)^2 (x_0 - x'_0)^2 - \frac{\rho^3(y_3)^2}{4c_2(bM)^2} \right) \\
&\times \exp \left(\frac{1}{4} y_\perp qF \cot(\pi qF/\rho) y_\perp \right) , \\
K(l) &= - \frac{c_2^{l+1} (\rho/(bM))^2}{c_2^2 + \left(\frac{qE}{bM} \right)^2 + \left(\frac{qE}{bM} \right)^4} \left\{ a^2 M^2/\rho - c_2^{-1}(l - 1/2) - c_2^{-1} \left(\frac{qE}{bM} \right)^2 \right\}
\end{aligned}$$

$$\begin{aligned}
& -c_2 \left(\frac{bM}{qE} \right)^2 + 2qH/\rho \sinh^{-1}(2c_2qH/\rho) + 2c_2^{-1} \frac{\left[1 + \left(\frac{qE}{bM} \right)^2 \right] \left[c_2^2 + \left(\frac{qE}{bM} \right)^4 \right]}{c_2^2 + \left(\frac{qE}{bM} \right)^2 + \left(\frac{qE}{bM} \right)^4} \left. \right\} n^{(2)}, \\
n^{(2)} &= -1/2\tau(s_2)n_{sc}^{(2)}, \\
n_{sc}^{(2)} &= \frac{\sqrt{c_2^2 + \left(\frac{qE}{bM} \right)^4}}{8\pi^{3/2}\sqrt{c_2} \left[c_2^2 + \left(\frac{qE}{bM} \right)^2 + \left(\frac{qE}{bM} \right)^4 \right]} \frac{q^2 HE \rho^{5/2}}{(bM)^3 \sin(c_2qH/\rho)} e^{-c_2 a^2 M^2/\rho},
\end{aligned}$$

If $bM/qE \ll 1$, the coefficients $K(l)$ and $n^{(2)}$ from (A1) have more simple form. Thus, in the case of an intensive electric field ($a^2 M^2/(qE) < 1$, $|H/E| < 1$) we get

$$n^{(2)} = \bar{n}^{cr}, \quad K(l) = -\pi^l \bar{n}^{cr}. \quad (\text{A2})$$

Since the final formula (B4) in App. B of Ref. [1], which describes small time expansion of $\Delta_{sc}^{(2)}$ in scalar case, was written with some misprints, we would like to represent its corrected form here:

$$\begin{aligned}
\Delta_{sc}^{(2)}(x, x') &= e^{iq\Lambda} \left\{ \left[i(x_0 + x'_0)c_2(bM)^2/\rho - qEy^3 \right] (-1/2)n_{sc}^{(2)}/(qE)\varphi_0 \right. \\
&+ \left. i \left[-(x_0 + x'_0)^3 \frac{(bM)^4}{12qE\rho^2} K_{sc}(2) + (1/2)(x_0 + x'_0)(y_3)^2 qE K_{sc}(0) \right] \right\}, \\
K_{sc}(l) &= -\frac{c_2^{l+1} (\rho/(bM))^2}{c_2^2 + \left(\frac{qE}{bM} \right)^2 + \left(\frac{qE}{bM} \right)^4} \left\{ a^2 M^2/\rho - c_2^{-1}(l - 1/2) - c_2^{-1} \left(\frac{qE}{bM} \right)^2 \right. \\
&+ \left. (qH/\rho) \coth(c_2qH/\rho) + 2c_2^{-1} \frac{\left[1 + \left(\frac{qE}{bM} \right)^2 \right] \left[c_2^2 + \left(\frac{qE}{bM} \right)^4 \right]}{c_2^2 + \left(\frac{qE}{bM} \right)^2 + \left(\frac{qE}{bM} \right)^4} \right\} n_{sc}^{(2)}.
\end{aligned}$$

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