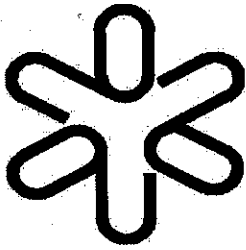# Instituto de Física
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Transverse Isotropy in Identical Particle Scattering

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Abstract

It is pointed out that the cross section for the scattering of identical charged bosons is isotropic over a broad angular range around 90° when the Sommerfeld parameter has a critical value, which depends exclusively on the spin of the particle. A discussion of systems where this phenomenon can be observed is presented.

The scattering of identical particles is a routine exercise in quantum mechanics and its discussion can be found in most text books on the subject [1]. However, in recent years, the rapid oscillation seen in the angular distribution of the Mott elastic scattering of identical charged particles such as nuclei, was utilized to test models such as QCD [2,3] and to discuss small deviations from the Coulomb force law owing to QED-related corrections such as vacuum polarization [4].

In the present work we point out a hitherto unknown feature of the Mott scattering cross-section for bosons, namely an almost isotropic angular distribution over a very wide angular range when the Sommerfeld parameter attains a critical value determined entirely by the spin, s, of the particles viz. $\eta_C = (3s + 2)^{1/2}$. For the purpose of completeness we first give a short account of the theory of scattering of identical particles. We then turn to

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the derivation of η_C and apply to several boson-boson scattering systems. We also briefly discuss the fermion-fermion case.

If in a scattering process the projectile and target particles are identical, when one of them reaches a detector the experiment cannot tell if this particle is the projectile or the target. On the other hand, momentum conservation guarantees that whenever a particle emerges in one direction, the other emerges in the opposite orientation, in the CM frame of reference. Therefore, the amplitude for scattering at the orientations **r** and −**r** will be mixed in some way. In Quantum mechanics, the total wave function for pairs of identical particles with integer spins, i.e. two bosons, must be symmetric with respect to the exchange of these particles, while in the case of particles with half-integer spin, i.e. two fermions, it must be anti-symmetric.

The situation is simple for spinless bosons or in collisions where the projectile and the target are polarized so that their spins are aligned. In such cases, the wave function in the spin space is always symmetric and projectile-target exchange reduces to reflection of the relative vector position **r**. For spherically symmetric potentials, there is axial symmetry and space reflection corresponds to the transformation $\theta \to \pi - \theta$. The elastic cross section is then given by

$$\sigma_{\pm}(\theta) = |f(\theta) \pm f(\pi - \theta)|^2 , \qquad (1)$$

where $f(\theta)$ is the scattering amplitude for discernible particles of the same mass under the same potential $V(r)$. The $+(-)$ sign in eq.(1) applies when the particles involved are bosons (fermions).

Eq.(1) may be rewritten in the form

$$\sigma_{\pm}(\theta) = \sigma_{inc}(\theta) \pm \sigma_{int}(\theta) , \qquad (2)$$

with

$$\sigma_{inc}(\theta) = |f(\theta)|^2 + |f(\pi - \theta)|^2 \qquad (3)$$

and

$$\sigma_{int}(\theta) = 2\text{Re}\left\{f^*(\theta) f(\pi - \theta)\right\} . \tag{4}$$

The first term in eq.(2) is the incoherent sum of the contributions to the cross sections arising from projectile and target, if they were distinguishable. While this term is independent of the particle statistics, the sign of the second term is responsible for the difference in the expressions for the cross section of bosons and fermions. This interference term has no classical analogue.

The situation is more complicated in the case of unpolarized spins. In this case, the cross section mixes different parities as the spins couple to produce symmetric or anti-symmetric states in the spin space. However, taking the proper average over spin orientations one obtains the simple formula

$$\sigma_{\pm}(\theta) = \sigma_{inc}(\theta) \pm \frac{\sigma_{int}(\theta)}{2s + 1} . \tag{5}$$

where s is the spin of the particle in units of $\hbar$. Eq.(5) indicates that the relevance of the interference term decreases with the spin value, vanishing in the classical limit $s \to \infty$.

The cross sections of eq.(5) are symmetric with respect to $\theta = 90^\circ$ and their particular shape depends on several factors such as the statistics of the colliding particles, their interaction and the bombarding energy. A particularly interesting situation is the Coulomb scattering of bosons, where σ_+ is known as the Mott cross section and denoted σ_{Mott}. In this case, we have the analytical expressions

$$\sigma_{inc}(\theta) = \frac{a^2}{4} \left[\frac{1}{\sin^4(\theta/2)} + \frac{1}{\cos^4(\theta/2)} \right] \tag{6}$$

and

$$\sigma_{int}(\theta) = \frac{a^2}{4} \left[\frac{2}{\sin^2(\theta/2) \cos^2(\theta/2)} \cos\left(2\eta \ln(\tan^{-1}(\theta/2))\right) \right] . \tag{7}$$

where η is the Sommerfeld parameter,

$$\eta = \frac{q^2}{\hbar v} \tag{8}$$

and a is half the distance of closest approach in a head-on collision.

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$$a = \frac{q^2}{2E}. \tag{9}$$

Above, q is the charge of each of the two collision partners, E is the bombarding energy in the center of mass (CM) reference frame and v is the corresponding velocity of the relative motion. In the present case, σ_{inc} exhibits a minimum at $\theta = 90°$, with the value $\sigma_{inc}(\theta = 90°) = 2a^2$, with an energy-independent shape. On the other hand, σ_{int} has always a maximum at this angle, with the same value $2a^2$. However, its shape depends on the collision energy through the Sommerfeld parameter η. The behavior of σ_{Mott} in the vicinity of $\theta = 90°$ results from a competition between these two opposing trends. For small η values, σ_{int} is a slowly varying function of θ. The shape of σ_{Mott} is then dominated by that of σ_{inc} and it presents a minimum at $\theta = 90°$. For large η, the opposite situation takes place and σ_{Mott} has a maximum at $\theta = 90°$. An interesting situation occurs at the critical value of the Sommerfeld parameter, η_C, where the cross section goes through this transition. The value of η_C is obtained from the condition

$$\left[\frac{d^2 \sigma_{Mott}(\theta)}{d\theta^2} \right]_{\theta=90°} = 0. \tag{10}$$

Using eqs.(6) and (7), we obtain

$$\left[\frac{d^2 \sigma_{Mott}(\theta)}{d\theta^2} \right]_{\theta=90°} = 16a^2 \left[\frac{1 - 2\eta^2}{2s + 1} + 3 \right]$$

and according to eq.(10) we get

$$\eta_C = \sqrt{3s + 2}. \tag{11}$$

In figure 1, we show cross sections normalized to the value of the Rutherford cross section at $90°$, $\sigma_{Ruth}(90°) = a^2$, for collisions of identical bosons with spins $s = 0$ and $s = 1$. In each case, the calculations were performed at η_C. I.e., $\eta = \sqrt{2}$ for $s = 0$ and $\eta = \sqrt{5}$ for $s = 1$. Also shown for comparison is the incoherent cross section of eq.(6), normalized in the same way. Clearly, $\sigma_{inc}(90°)/a^2 = 2$ and $\sigma_b(90°)/a^2 = 4$ as shown in the figure. The striking feature of the figure is the flatness of σ_{Mott} over a very wide angular region around

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$\theta = 90°$. It is essentially constant for $60° < \theta < 120°$, for $s = 0$, and $70° < \theta < 110°$, for $s = 1$. This 'transverse isotropy' (TI) is universal as the only relevant parameter which enters the discussion is the Sommerfeld parameter. In principle, it could be observed in atomic or nuclear systems at the appropriate energy. However, as we shall show below, the most appropriate case to investigate the above 'transverse isotropy' is that of low-energy scattering of light identical nuclei, such us d-d or $\alpha - \alpha$.

Investigating a physical system which shows TI could shed light on several small effects related to QED and possibly to QCD, as well as to atomic effects in nuclear scattering. Also the assumed pure bosonic nature of the multifermionic cluster could be nicely examined by a careful analysis of data taken at η_C.

At this stage, it is important to investigate the optimal conditions for observation of TI. The effective forces between identical nuclei or ionized atoms are composed of the long range Coulomb part plus a shorter range nuclear or Van der Waals force. Since the above discussion was based on a pure Coulomb force, it is important to seek the physical conditions that allows the neglect of the short range forces. Calling E_C the collision energy corresponding to η_C, the above condition corresponds to the requirement that E_C be sufficiently below the Coulomb barrier V_B, the outermost maximum in the effective potential for $l = 0$. In the collision of identical particles of charge q and mass M, the critical collision energy E_C is given by

$$E_C = \frac{Mq^4}{4\hbar^2(3s+2)}.$$

If one approximates V_B by the coulomb potential at the barrier radius R_B, namely

$$V_B = \frac{q^2}{R_B}, \tag{12}$$

the condition $E_C < V_B$ yields

$$\frac{Mq^2 R_B}{\hbar^2(3s+2)} < 1. \tag{13}$$

Since the barrier radius in atomic collisions is very large, the above condition cannot be satisfied. We then consider nuclear collisions. In the nuclear physics one usually takes

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$R_B = 1.4 \ (M/m_0)^{1/3}$ fm, where m_0 is the nucleon mass. For light nuclei one can assume equal number of protons and neutrons and set $M/m_0 = 2Z$, where Z is the atomic number. Eq.(13) then reduces to

$$Z^{10/3} < 25.4 \times (2s + 1).\qquad(14)$$

It can immediately be checked that the above condition is only satisfied for α−particles, in the case of $s = 0$, and for $d - d$ and $^6Li - ^6Li$ collisions, in the case of $s = 1$ (if one relaxes the condition of equal numbers of protons and neutrons, a couple of additional nuclei can be included in this set). The Mott cross section at $\theta = 90°$ at for collisions with the critical bombarding energy, E_C, is then given by the simple expression

$$\sigma_{Mott}(\theta = 90°) = \frac{\sigma_0}{Z^6} \ (3s + 2)^2 \ ,\qquad(15)$$

with $\sigma_0 = 33.7$ barn.

In table I, we give the relevant quantities in the three above mentioned cases. It is clear from this table that the transverse isotropy is an observable phenomenon, although experimentally difficult in the case of $d - d$. An important question to address now is the sensitivity of our result to the uncertainty of the energy of the beam. Since a slight change in η form η_C might wash out the TI, it is interesting to assess the range of η−values around η_C which can still tolerate a meaningfull study of the phenomenon. Figure 2 shows the way the shape of the Mott cross section changes as η is varied from η_C by 5%. This uncertainty in the η−value corresponds to about 10 % uncertainty in the collision energy, which is certainly attainable in existing accelerators. It is clear from figure 2 that the transverse isotropy should be visible within a few per cent energy resolutions.

Although we have restricted our discussion so far to $E_C < V_B$, we emphasize that the TI may come up at higher energies. However, in this case the details of the short-range interaction becomes important and the discussion becomes model- and system-dependent. We also point out that no transverse isotropy is expected for Coulomb collisions of identical fermions, since in this case both σ_{inc} and σ_{int} have minima at $\theta = 90°$. However, the situation

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may be different when the short range interaction dominates. We have briefly looked into this question by examining the extreme case of hard sphere scattering with no Coulomb interaction. The physical parameter that characterizes the collision process is kR with R being the sum of the radii of the two colliding particles. Our preliminary results indeed show a TI for identical fermions with the rather large spin values $s \geq 9/2$. The critical value of kR is the order of 2.5, which is of the same order of magnitude as the value we found for bosons (1.5). More details of the present work with extensions to other potentials will be published elsewhere [5].

REFERENCES

[1] L.D. Landau and E.M. Lifshitz, 'Quantum Mechanics: Non Relativistic Theory', Pergamon Press, Oxford, 1997

[2] M.S. Hussein, C.L. Lima, M.P. Pato and C.A. Bertulani, Phys. Rev. Lett. **65**, 839 (1990)

[3] A.C.C. Villari et al., Phys. Rev. Lett. **71**, 2551 (1993)

[4] W.Lynch et al., Phys. Rev. Lett. **48**, 979 (1982); D.Vetterli et al. Phys. Rev. Lett. **62**,1453 (1989)

[5] L.F. Canto, R. Donangelo and M.S. Hussein, in preparation.

Tables

- Table I: The relevant quantities associated with the d-d, $^6Li - ^6Li$ and $\alpha - \alpha$ collisions, at the critical value of the Sommerfeld parameter.

System	s	E_C(keV)	V_B(keV)	$\sigma_{Mott}(90^o)$ (barn)
d + d	1	5.0	400	135
$^6Li + ^6Li$	1	1200	2500	1.17
$\alpha + \alpha$	0	400	1260	2.3

Figure Captions

- Figure 1: The Mott cross sections for collisions of identical bosons at the critical value of the Sommerfeld parameter. Results are shown in the case of bosons with spin 0 (solid line) and spin 1 (dashed line), to which corresponds respectively $\eta_C = \sqrt{2}$ and $\eta_C = \sqrt{5}$. For comparison, the incoherent part of the cross section is also shown (dot-dashed line).

- Figure 2: Sensitivity of the transverse isotropy as η deviates from η_C by 5 %. The results are shown in the case of $s = 0$.
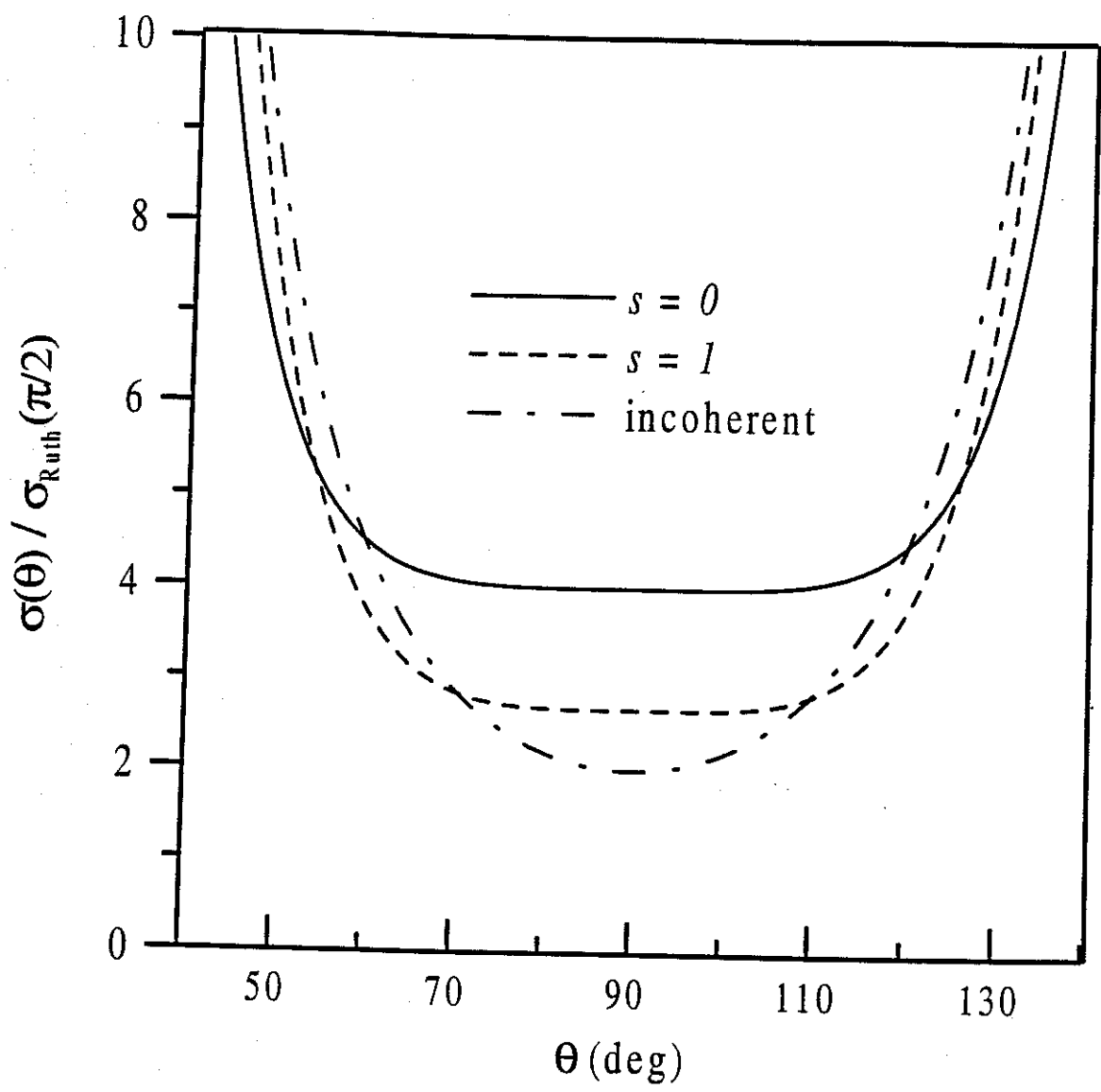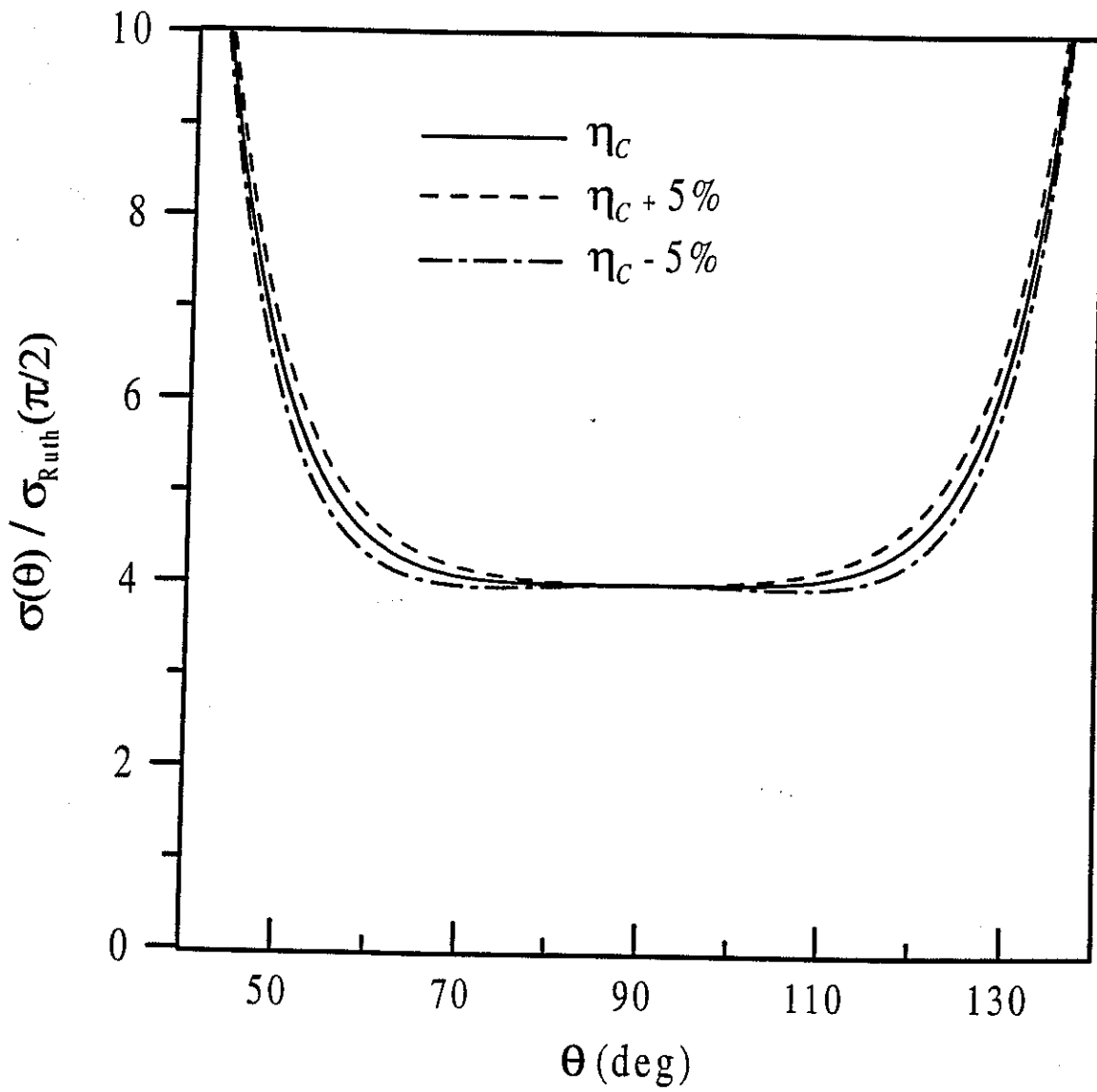
Figure 1

Figure 2