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Abstract

In this work we study the possibility to improve the signal to noise (S/N) ratio for the Auger Fluorescence Detector (*FD*) through the increase of the external radius of the corrector ring.

1 Introduction

In an earlier note [1] we showed, using a ray-tracing program [2], the advantages of a corrector ring (annulus) in order to increase the effective light collection area (A_{eff}) and, consequently, the S/N ratio. We also showed that it was necessary to have the same focal distance for the corrector ring and for the open diaphragm inside the corrector ring. This necessity led us to a particular profile curve for the annulus that optimized (minimized) the spot size for a null incidence angle. However, this particular curve provokes a high dependence for the spot size with the incidence angle.

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In this work we study the possibility to have a further increase in the external radius of the corrector ring and, as a consequence, increase the A_{eff}, improving the S/N ratio for the FD one more time. In order to reach this goal we modified the criterion of optimization to the spot size used in ref.[1]. We asked ourselves if a null incidence angle is the optimal angle to calculate the parameter A (eq.2 in ref.[1]) that would give us the "best" profile curve of the corrector ring in order to have a minimum spot size on the focal surface of the FD. The answer is not. We found a new profile curve for the corrector ring, in which the spot size does not depend on the incidence angle in a strong way. Therefore, we will have a more homogeneous detector. Another advantage is that this new profile curve allows us to extend the external radius of the corrector ring maintaining the resolution of the FD optical system, since the spot size is small when we compare it with the results from the previous work [1], considering the same diaphragm aperture radius. Consequently, we can improve the S/N ratio of the FD.

This work is divided in the following way: in section 2 we present the criterion for the optimization of the FD corrector ring. Section 3 contains our simulations and results. In the section 4 we make some considerations about the reflection and absorption effects by the FD annulus. We put our conclusions in section 5.

2 Criterion of Optimization to the *FD* Corrector Ring

In order to solve the focal distance problem of the corrector ring in the ref.[1], we modified the parameter A (eq.2 in ref.[1]) by simulations in such way that we had a corrector ring with the same focal distance to the inside opened region. In principle, we found the "optimal" profile curve to the annulus. We made our simulations to the new parameter A, considering a null incidence angle for the rays of light. We simulated this optical system for several incidence angles and we noted that most rays of light are concentrated at the center of spot size, for small incidence angles. However, the annulus produce a relatively extended figure for greater incidence angles. In this case, the spot size produced by the corrector ring is approximately equal to the spot size formed by the internal region. Therefore, any increase into the external radius of the annulus will damage the optical resolution of the FD and, in this way, the external radius with the size equal to $1,0m$ seems to be the maximum limit for the corrector with the following parameters:

- focal distance $(R_f) = 1.743m$;
- curvature radius of the mirror $(R_c) = 3.4m$.

We can imagine that the elongation of the spot for large incidence angles is provoked by the rays of light that pass through the lens and emerge with an angle that differs from the "ideal" direction or because the track of light between the lens and the mirror depends on the position that the light cross the lens. However, the elongation of the spot is approximately symmetric when we do not consider the shadow of the PMT's. It is difficult to imagine that these two effects above can explain this elongation. It appears that there is a "virtual lens" that follow the incident light direction, as it shown in the fig.1, but with a small deformation in its profile. The deformation would be provoked because the distance r in the "real lens", on the center of the diaphragm, is seen as $r\cos\theta$ in the "virtual lens", when the light cross $\overline{AB}$ segment (fig. 1). Therefore, the difference in the position is $r(1-\cos\theta)$. For example, this difference is approximately $6cm$ for $r = 1.0m$ and $\theta = 20^o$. This effect is symmetric as we can see in the fig. 1. The small asymmetries that appear in the spot are probably caused by the two other effects cited above.

Therefore, an "ideal" annulus for a null incidence angle does not guarantee that it is the better lens for the optical system, considering incidence angles until 20^o. In this way, there must exist curves for the corrector ring that does not degrade so fast the optical resolution for large incidence angles. Following, we will discuss an algorithm to find a lens profile that is capable to take into account large incidence angle without a considerable damage to the optical resolution of the system. This algorithm will allow us to study the possibility of increasing the external radius of the corrector ring.

By the method used in the earlier notes [1]–[3], we found the parameters of the optical system (focal distance, lens parameters, etc) simulating rays of light that reach the diaphragm region and then we propagate these rays until at the focal surface. However, we can use the rays of light reversibility principle to make these rays reach the annulus from the focal surface and to verify what is the profile curve of the corrector ring that will give us parallel rays at the corresponding direction for the chosen focal point. The normal surface of the lens can be found using the Snell's law to refraction, written in a vectorial form (eq.16, appendix A):

$$n\left[\vec{i}-\left(\vec{i}\cdot\vec{n}\right)\vec{n}\right] = n'\left[\vec{r}-\left(\vec{r}\cdot\vec{n}\right)\right] \rightarrow$$
$$\rightarrow n\vec{i}-n'\vec{r} = \left[n\left(\vec{i}\cdot\vec{n}\right)-n'\left(\vec{r}\cdot\vec{n}\right)\right]\vec{n} \ , \tag{1}$$

where $\vec{i}$, $\vec{r}$ and $\vec{n}$ are unit vectors that give us the direction of the incident ray, the direction of the refracted ray and the direction of the normal vector of the surface that separates the two media, respectively. The parameter n is the refraction index in the medium where the incident ray propagates and n' is the refraction index in the medium of the refracted ray.

Therefore, the factor inside the square bracket is not null too. In this way, the normal vector can be written as

$$\frac{n\vec{i}-n'\vec{r}}{\mid n\vec{i}-n'\vec{r}\mid} = \vec{n} \ . \tag{2}$$

If we consider that the thickness of the lens will not be greater than about $2cm$, the effects introduced by the variation of the thickness can be practically ignored, since the angle that a ray of light form with the symmetry axis of the lens will not exceed 14^o.

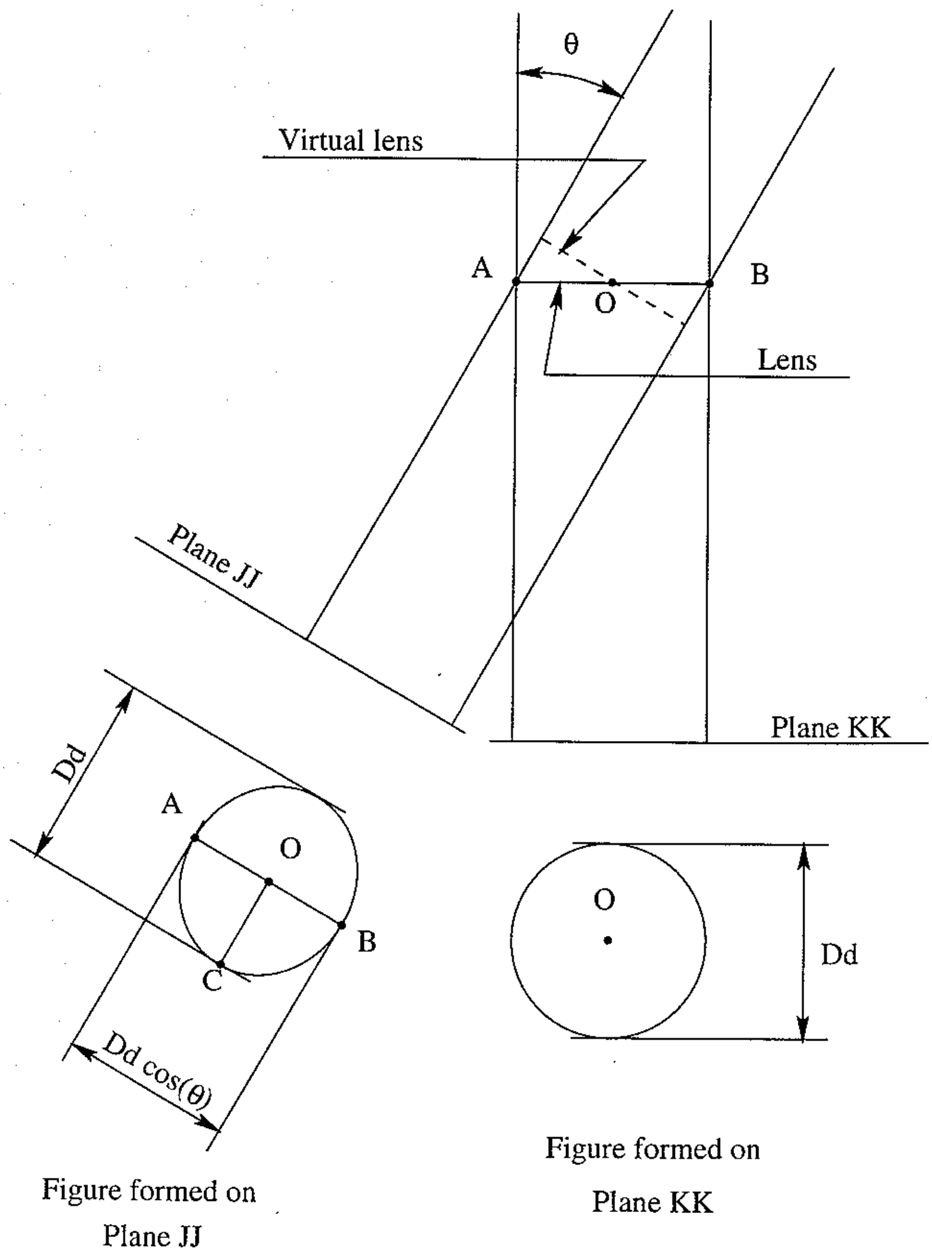

Figure 1: Observation of a circumference by a not null angle with respect to the normal vector that contains this circumference.

Therefore, the error that we commit in the position over the lens is given by the maximum value about $2cm \times \tan(14^o) \approx 5mm$ and, as we can see, it is small when compared with the differences that appear when we consider larger incidence angles.

When we impose the condition that the lens has to be symmetric, we have that the normal vector to the surface of the lens must be contained at the plane formed by the symmetry axis and the considered point. In this way, we can write the normal vector as

$$\hat{n} = a\hat{n}_\perp + b\hat{n}_z \quad , \tag{3}$$

where $\hat{n}_z$ and $\hat{n}_\perp$ are unit vectors in the direction of the symmetry axis of the optical system and parallel to the diaphragm plane, respectively, pointing out from the diaphragm center to the considered point (fig.2). Therefore, in order to define the normal vector, it is enough to determine the value of only one component. The other component can be determined from the normalization factor.

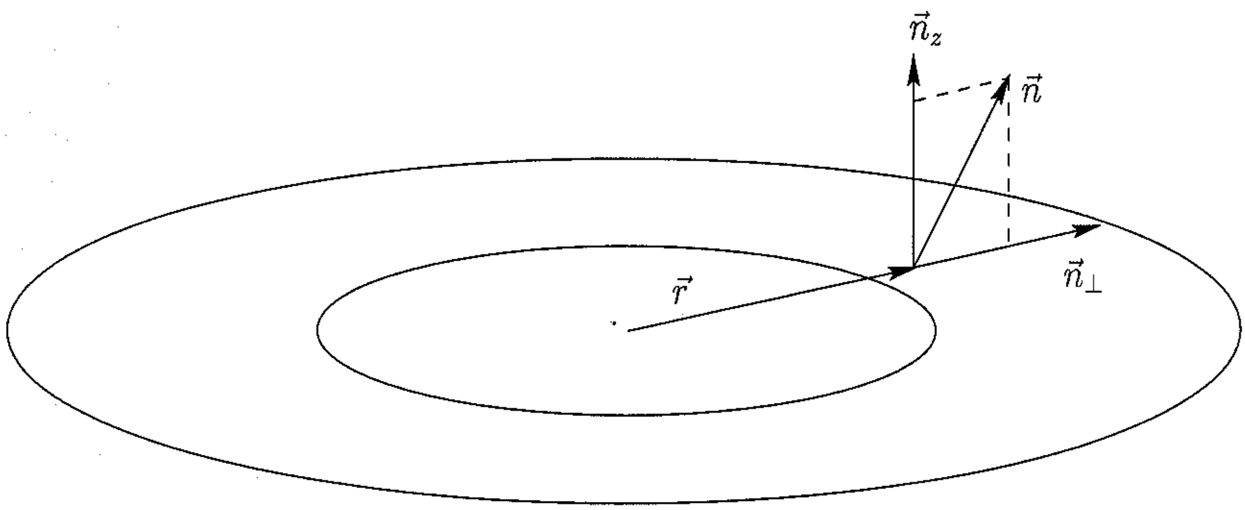

Figure 2: The normal vector components over the lens.

3 Simulations and Results

If we simulate rays of light from the focal point that corresponds to a null incidence angle and to 20^o in a such way that the rays reach the lens passing by the points of the segments $\overline{OA}$, $\overline{OB}$ and $\overline{OC}$ in the fig.1, we obtain the values of the scalar product $n_r = \vec{n} \cdot \vec{n}_\perp$, as we can see in the fig.3. In this figure, the curves represented by the dashed lines (segment $\overline{OA}$) and the dotted lines (segment $\overline{OB}$) are practically coincident. It apparently confirms our suppositions above for radius smaller than $1.0m$ at least. The two effects earlier cited (deviation of the light from the "optimal" direction or the greater distance that the light follows between the lens and the mirror) begin to be considerable for the aperture radius greater than $1.0m$. For the continuous line, identified by the number one (segment $\overline{OC}$), and the dashed-dotted line (for a null incidence angle), we saw that the curves are almost coincident. This is expected because the form of the lens profile is not changed over this segment for the different incidence angles. In this simulation, the refraction index of the lens is equal to 1.5.

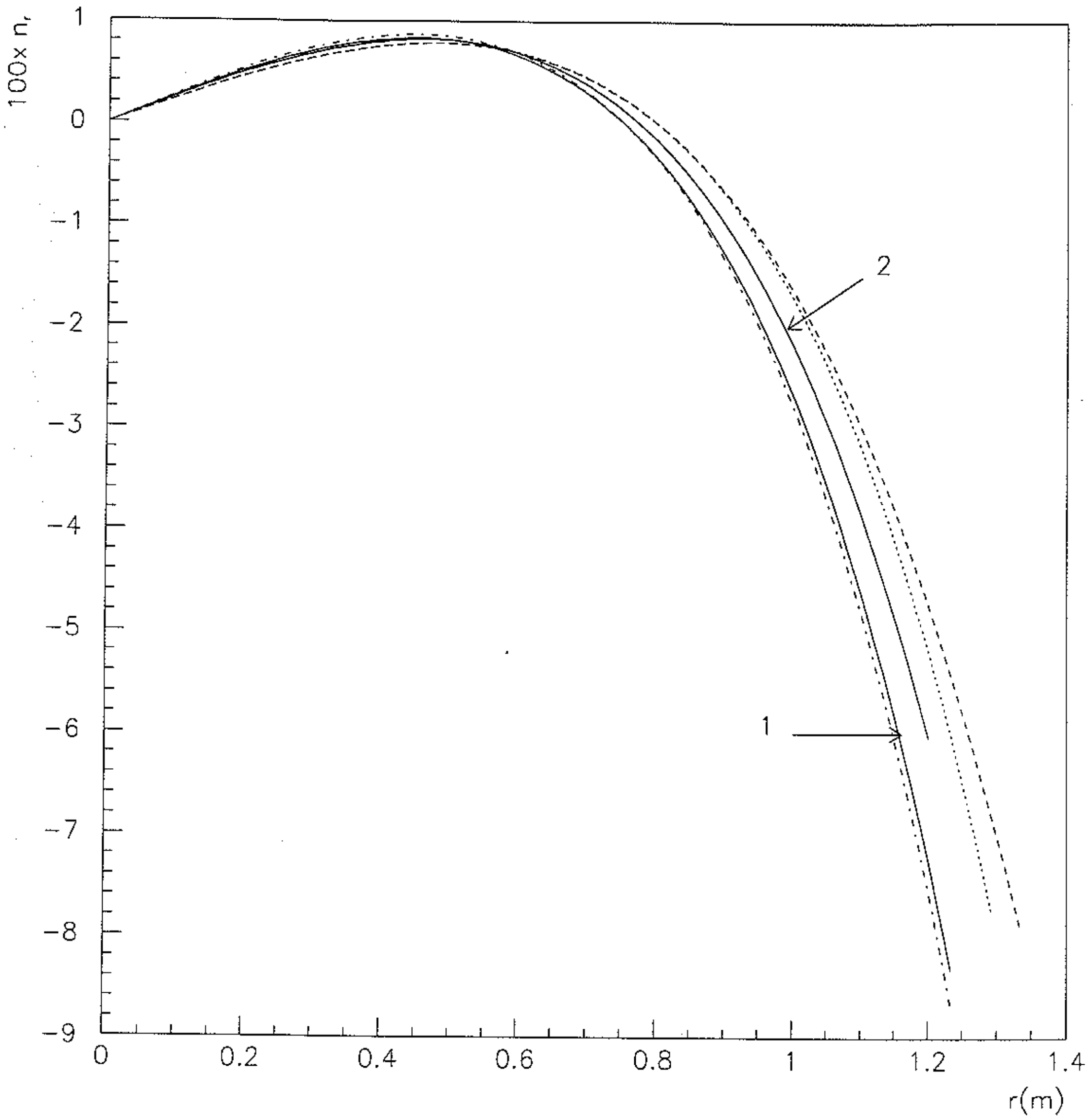

Figure 3: Projection of the normal vector to the lens surface over its plane as a function of the distance from the diaphragm center. The continuous line, identified by 1, is the result of this projection over the diaphragm region between the points O and C (see fig.1), the dashed line is the projection over the region between the points O and A and the dot line between the points O and B, for a focal point that corresponds to an incidence angle equal to 20^o. The dashed-dot line is result of this projection to a focal point that corresponds to an incidence angle equal to 0^o. The continuous line, identified by 2, is the result minimizing the function 4.

We saw too that it is impossible to have a good optical resolution for all incidence angles. The question here is to determine the curve that will give us an optical resolution that does not degrade so fast with the incidence angle. Intuitively, the better case will be between the continuous curve and the dotted curve, since if we consider the dashed line only, the spot size can become very large for a null incidence angle. On the other hand, if we choose the dashed-dotted line we know already that the spot size grows fast for large incidence angles.

A criterion that it seems to be suitable is to choose n_r in such way that the function

$$f(r,n_r) = max\left[R_s{}^{max}(r,n_r,\theta=0), R_s{}^{max}(r,n_r,\theta=20^o)\right] \quad , \tag{4}$$

be a minimum for each value of r. Here, $R_s{}^{max}(r,n_r,\theta)$ is the maximum distance from the center of the spot that the rays of light reach the focal surface, passing through the lens in a distance r from its center, for a given incidence angle θ. The eq.4 is the maximum value of the spot size between $\theta = 0^o$ and $\theta = 20^o$. Why do we take the maximum value? We are trying to eliminate the strong dependence of the spot size with the incidence angle. Then, it is clear that we take into account the two worst spots.

We obtain the curve identified by 2 in fig.3 and the spot size $[f(r,n_r)]$ in terms of the diaphragm radius in the fig.4, using this criterion. It is important to note that with the external radius of the annulus equal to $1.10m$ the resolution of the corrector ring becomes practically equal to the resolution of the internal region. An increase up to this value will provoke the degradation of the optical resolution of this system. The value of $1.10m$ is the maximum limit to the external radius of the annulus. We will show that this increase will give us a considerable gain in the effective light collection area and, consequently, in the S/N ratio, when we consider the previous works ([1] and [3]).

From the corrector ring profile, $T(r)$, the unit normal vector, $\hat{n}$, of the lens surface can be written as

$$\hat{n} = \frac{(-dT/dr)\ \hat{n}_\perp + \hat{n}_z}{\sqrt{1+(dT/dr)^2}} \quad . \tag{5}$$

As we can see by the fig.3, $\hat{n}\cdot\hat{n}_\perp \ll 1$. This fact allows us to write the profile lens as an approximation:

$$T(r) \simeq T_0 - \int_{r_1}^{r} \hat{n}(r')\cdot\hat{n}_\perp\, dr' \quad , \tag{6}$$

where T_0 is the thickness of the lens at $r = r_1$. Taking $T_0 = 6mm$ and $r_1 = 0$, the plot of the function $T(r)$ is given by the fig.5. It is very similar to the profile of the lens type II [1], at least until around $85cm$. In this way, if we write the profile curve of the lens as

$$T(r) = T_0 + a_1 r^2 + a_2 r^4 + a_3 r^6 \quad . \tag{7}$$

We find by numerical adjustment that the coefficients a_1, a_2 and a_3 are given by

$$a_1 = 1.24612 \times 10^{-2} m^{-1} \quad ;$$
$$a_2 = -8.6476 \times 10^{-3} m^{-3} \quad ;$$

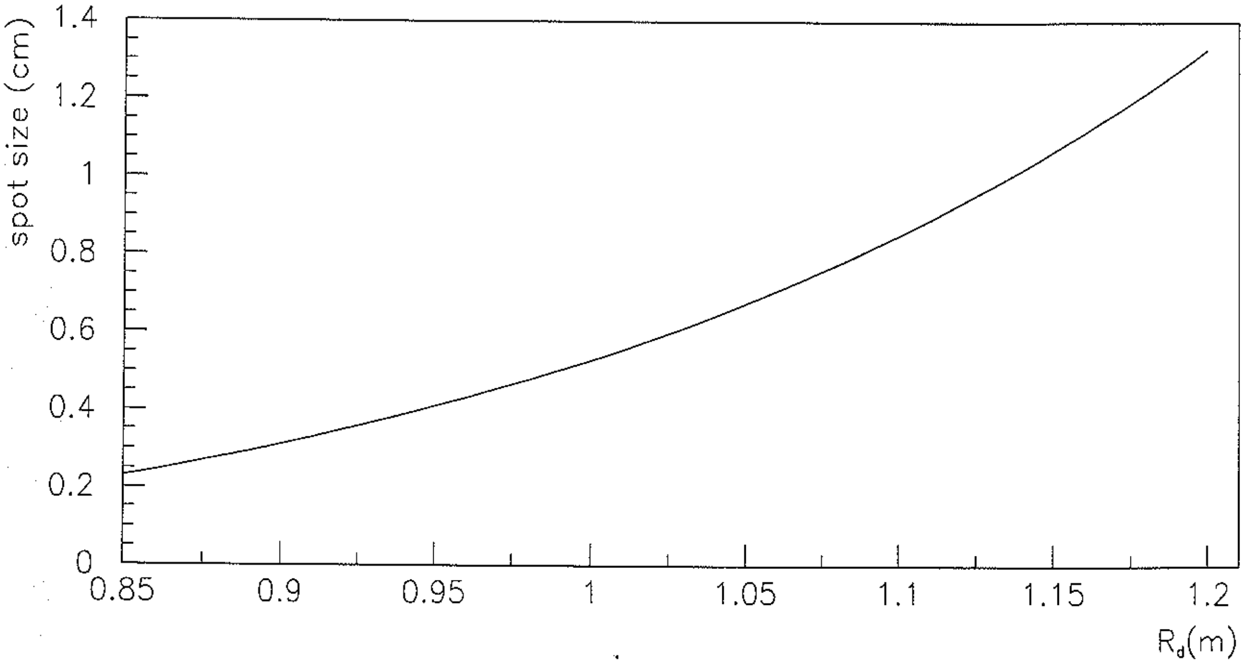

Figure 4: Spot size as a function $[f(r, n_r)]$ of the external radius (R_d) of the corrector ring (the annulus profile curve was estimated using the criterion of eq.4).

$$a_3 = -2.0361 \times 10^{-3} m^{-5} \quad ,$$

considering $85cm < r < 115cm$. Substituting these values in the function of the lens profile, we obtain the figures formed over the focal surface to incidence angles between 0^o and 20^o (fig.6) and for the external radius equal to $1.10m$. The corresponding densities are in the fig.6 too.

Considering this procedure, the optical resolution is worst when we compare it with the optical resolution of fig.7, but not so much. However, the spot size formed by the internal region of the lens is practically equal to the spot size formed by the region of the annulus, in spite of the increase of the diaphragm aperture radius.

When we investigate a fabrication method for the corrector annulus, one question that appear is if the profile annulus can be approximated from some other profile curves and what is the maximum error in this approximation.

To answer this question, we observe the curve given by fig.4. If we choose a diaphragm aperture radius of $1.10m$, we have that spot size is approximately equal to $0.75cm$. However, this curve is not valid only to diaphragm aperture radius. It indicates, of course, that if a ray of light passes a distance r from the center of the diaphragm, the ray crosses the focal surface in a distance smaller or equal to distance given by this curve, if the profile curve of the lens is given by the eq.7. However, what we need is only a profile lens that gives us a spot size that does not exceed a resolution choice ($0.75cm$). **Notice that this spot corresponds on the camera position to a spot of** 0.5^o. For example, a ray of light that pass to $1m$ from the center of the lens will reach the focal surface in a

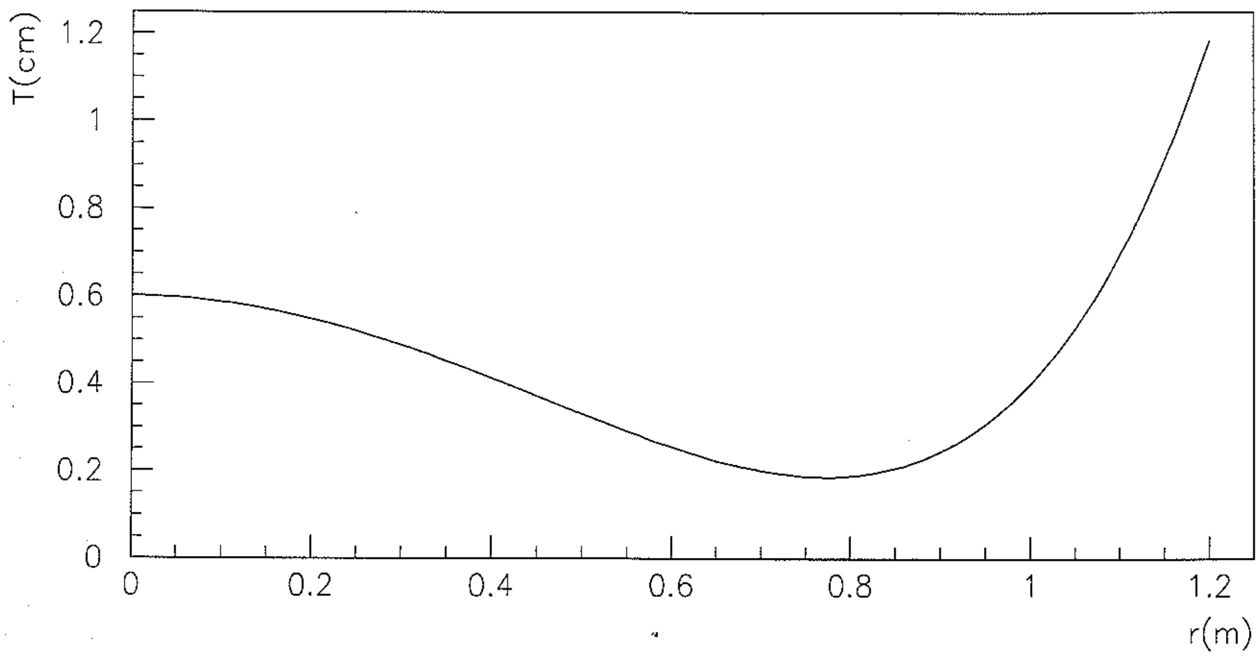

Figure 5: Profile curve for the corrector ring.

distance smaller or equal to $0.5cm$. This indicates that we do not need the normal surface of annulus given by the curve 2 in the fig.3.

Therefore, it is not needed that the function $f(r, n_r)$ be a minimum, but only:

$$f(r, n_r) \leq 0.75cm \quad . \tag{8}$$

The result that we obtain is given in fig.8, where the dashed curves are given by eq.8. Consequently, any profile curve of the lens between these two dashed curves give us approximately the same results for the spot size.

In order to know how much this increase of A_{eff} represents to the FD, we calculate the difference between the area of the diaphragm and the area obscured by the array of photo-tubes that, for a null incidence angle, corresponds to 37% of the diaphragm area with a radius of $85cm$. Table 1 contains the value of A_{eff} for some values of the diaphragm radius, considering the incidence angle between 0^o and 20^o. It is important to note that at this point we did not consider effects of reflection or absorption of the light by the material of the annulus, which will be taken into account in the next section. We put our qualitative analysis in table 1 too.

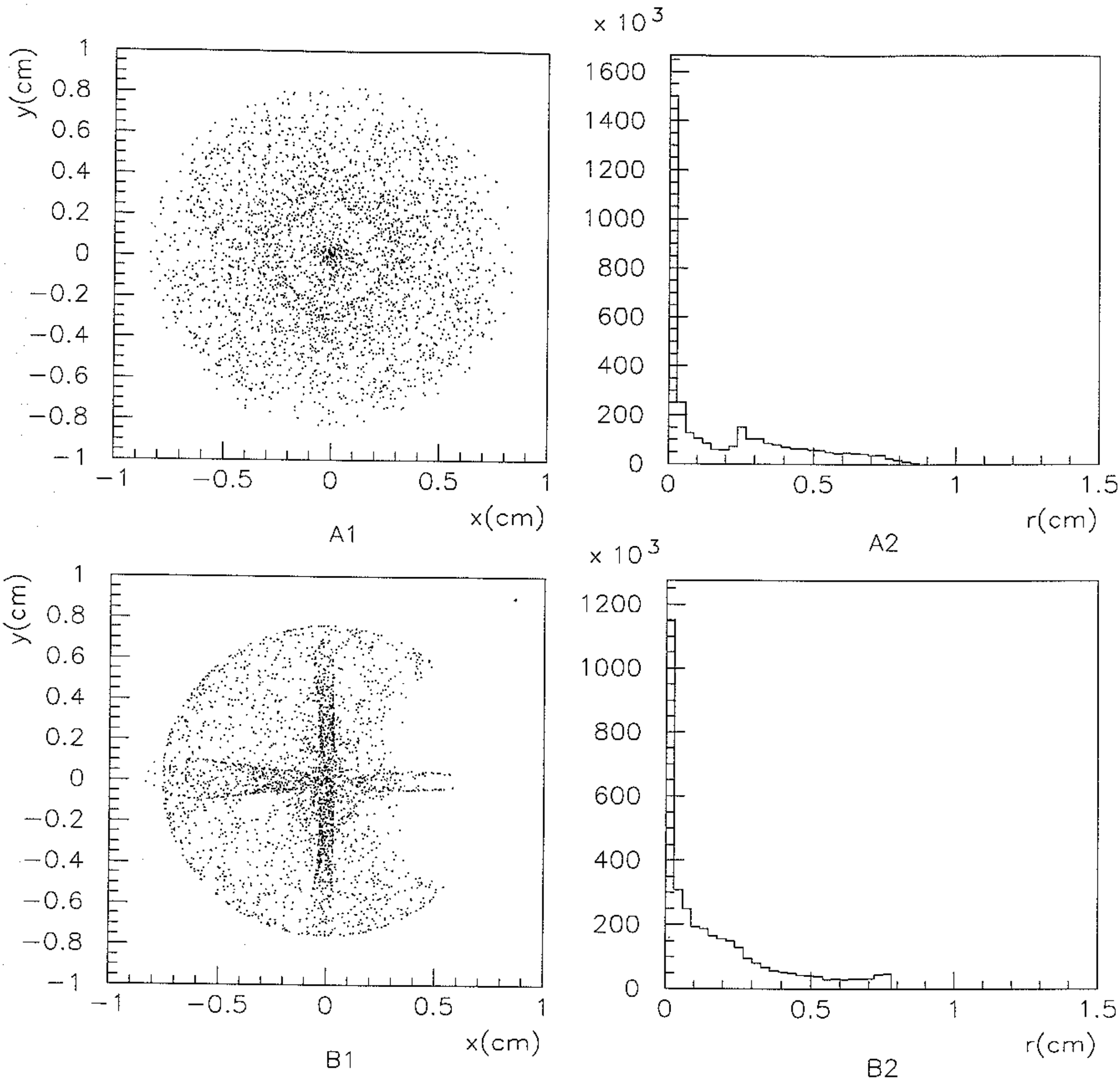

Figure 6: Figure formed over the *FD* focal surface considering the incidence angle between 0^o (A1) and 20^o (B1) and the corresponding densities, for simulation using a corrector ring given by the eq.7.

4 Considerations about the Reflection and Absorption Effects by the *FD* Corrector Ring

One of the well-know problems in manufacturing of the optical system is the dependence between the refraction index of the material used in fabrication lens and the wavelength

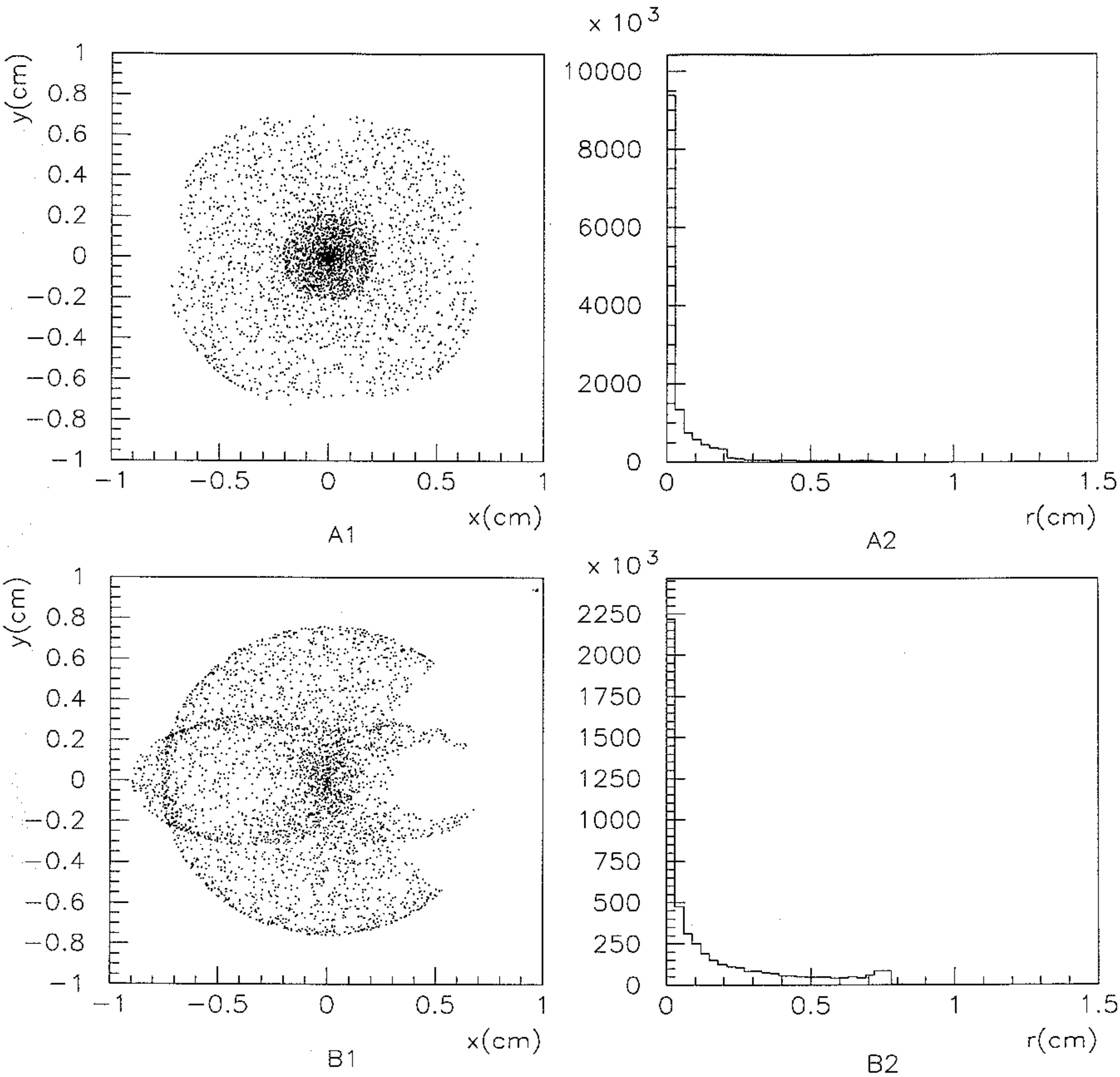

Figure 7: Figure formed over the *FD* focal surface considering the incidence angle between 0^o (A1) and 20^o (B1) and the corresponding densities, for the corrector ring simulated in the ref.[1].

of the light (*chromatic aberration*). This dependence affects directly the optical resolution of the system. Usually this problem is solved using two lenses with different materials in such way that the chromatic aberration from one is compensated by the other. Therefore, all the systems present a small chromatic aberration. This procedure can make the optical

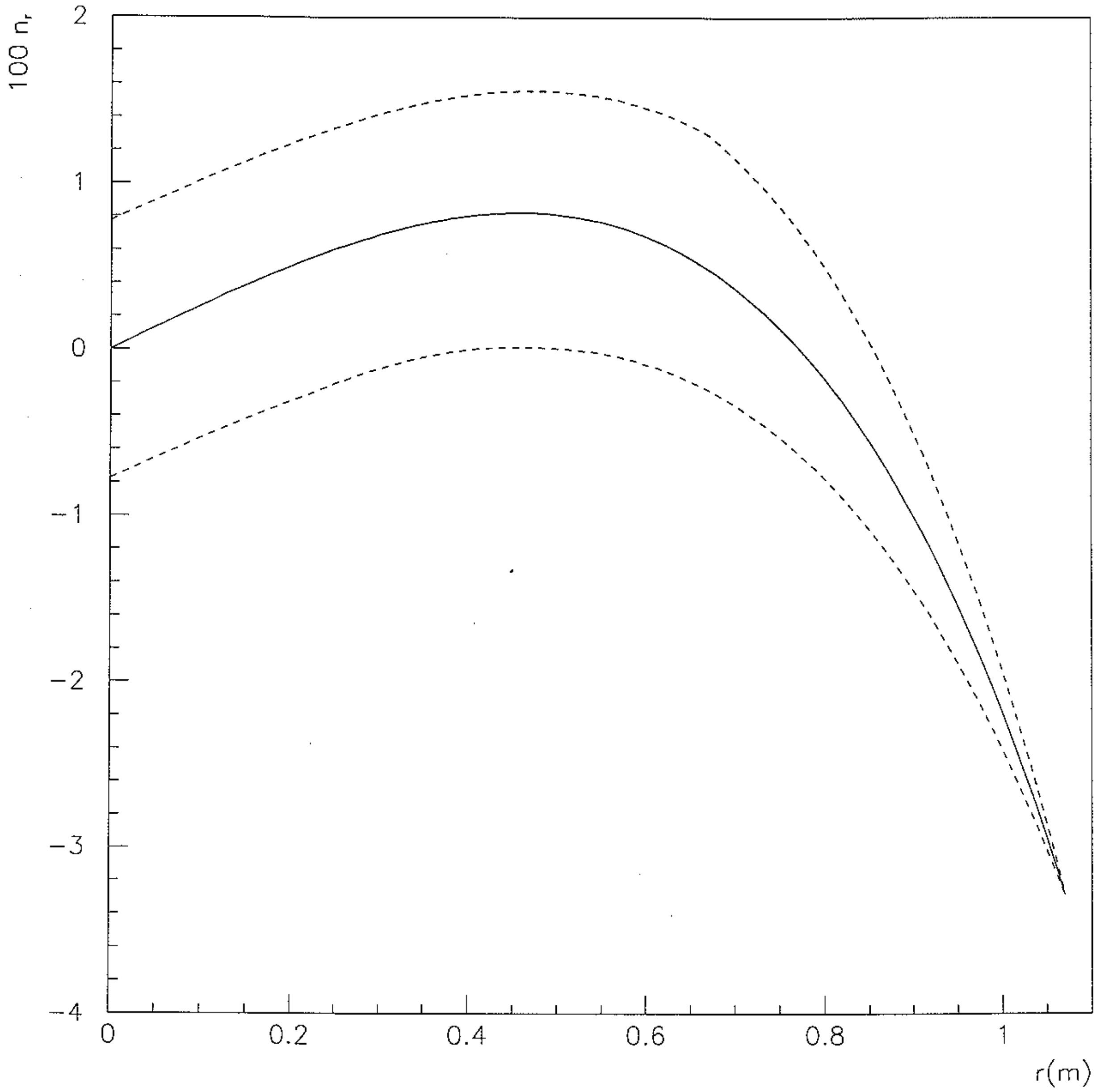

Figure 8: The two dashed lines give the projection of the normal vector to the lens surface over its plane as a function of the distance from the diaphragm center, in such way that the maximum spot size is equal to $7.5mm$ to incidence angle from 0^o to 20^o. The continuous line is the curve identified by the number 2 in the fig.3.

system very expensive. In addition, the optical resolution demanded for the *FD* is small when we compare it with the resolution of the usual telescopes used in astronomy.

Another problem that must be taken into account concerns the internal transmittance

angle(degrees)	$R_d(m)$	0.85	1.00	1.05	1.10	1.15
0	$A_{eff}(m^2)$	1.44	2.34	2.66	3.00	3.35
	R_a	1.	1.62	1.84	2.07	2.32
	${}^{80}R_a$	1.	1.49	1.67	1.86	2.06
20	$A_{eff}(m^2)$	1.58	2.25	2.52	2.82	3.14
	R_a	1.	1.42	1.60	1.79	1.99
	${}^{80}R_a$	1.	1.34	1.48	1.63	1.79

Table 1: Effective light collection area (A_{eff}) for some diaphragm aperture radius with corrector ring. R_a is the ratio between the effective area for these diaphragm aperture radius and the effective area for an diaphragm aperture radius equal to $85.0cm$ without corrector ring. ${}^{80}R_a$ is the same ratio considering the total transmission coefficient of the annulus equal to 80%

coefficient of the lens that depends of two different effects: the first is the light absorption for the material of the lens and the second effect is the partial reflections that occur when the light pass through from a medium to another with a different index of refraction.

The absorption of the light by the matter has a high dependence with the wavelength that we want to observe which it is between $300nm$ and $400nm$. It is difficult to find glasses that have good transmittance for all this range and, principally, for the small values of wavelength. However, light with wavelength bellow of $350nm$ is strongly reduced by the Rayleigh scattering in the atmosphere that has a dependence of λ^{-4}. As a consequence, we can increase the lower limit of the wavelength to $350nm$ and, in this case, we find many kinds of glasses and perhaps some plastics that has a internal transmittance coefficient upper to 95% for a thickness of $25mm$.

When the light pass through from a medium to another with a different refraction index, it is practically inevitable that part of the light be reflected, when we do not consider interference effects. Considering the intensity of the light proportional to the square of the electric field magnitude of the incidence light and the two surfaces of the lens parallels, we have that the internal transmittance coefficient, T, is given by

$$T = \left(\frac{E}{E_0}\right)^2 = \left[\frac{4\alpha}{(1+\alpha)^2}\right]^2 \; ; \; \alpha = \sqrt{\frac{\nu\epsilon'}{\nu'\epsilon}\frac{\cos\theta'}{\cos\theta}} \; , \tag{9}$$

where E and E_0 are the magnitude of the electric fields for the transmitted and incident light, respectively. The parameters ϵ and ν are the dielectric constant and the magnetic permeability of the external medium, respectively, and ϵ' and ν' are the same parameters but for the material of the lens. The incidence angle is θ and θ' is the angle between the transmitted light inside the lens and the normal direction to the lens surface. If we suppose that $\nu/\nu_0 \approx 1$ and $\sqrt{\epsilon/\epsilon_0} \approx n/n'$, where n and n' are the refraction indices of the external medium and the material of the lens, we have that the losses by reflections are

$$\vec{r} = \frac{n_1}{n_2}\left(\vec{i} - \vec{n}\cos\theta\right) + \vec{n}\cos\theta' \ . \tag{17}$$

Rewriting eq.(17), we have

$$\begin{aligned} \cos\theta' &= \pm\sqrt{1-\sin^2\theta'} \\ &= \pm\sqrt{1-\left[\left(\frac{n_1}{n_2}\right)\sin\theta\right]^2} \\ &= \pm\sqrt{1-\left[\left(\frac{n_1}{n_2}\right)\vec{s}\right]^2} \ , \end{aligned} \tag{18}$$

where the vector $\vec{s}$ is given by eq.(15). The signal must be choose in such way that the direction of the refracted light satisfies the Snell's law.

Acknowledgments

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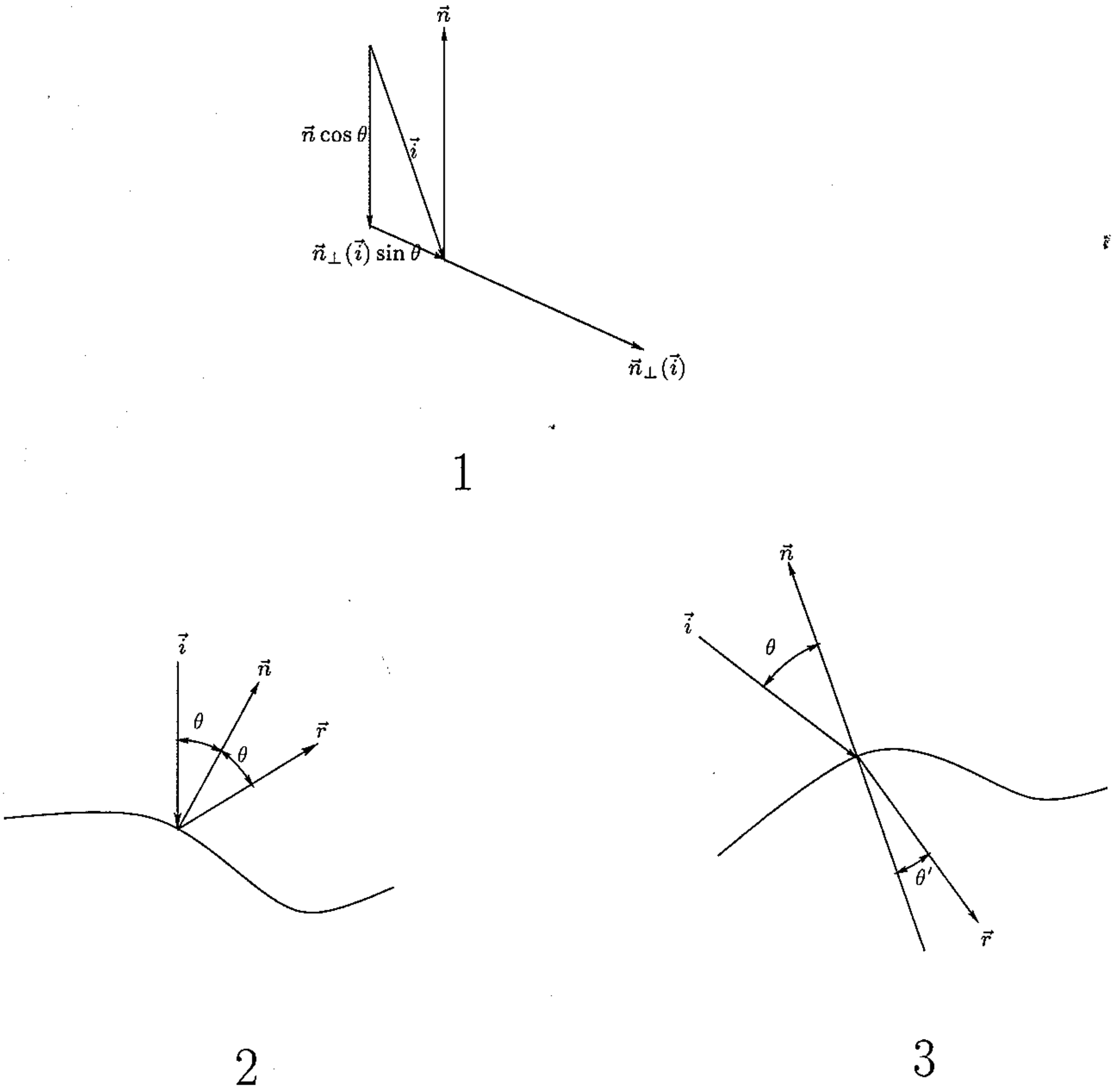

Figure 9: Direction of the incident, reflected and refracted rays of light.