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ABSTRACT. Ultrasonic attenuation measurements were made on the high-field superconducting alloys Ti-11 at.% Nb, Nb-26 at.% Hf, and Nb-41 at.% Ti. These measurements were carried out at liquid helium temperatures, in magnetic fields up to 140 kOe., using shear and longitudinal waves, with frequencies between 5 and 35 MHz. The data are compared with the predictions of a phenomenological model for the magnetic field variation of the ultrasonic attenuation in the mixed state. According to this model, the attenuation in the mixed state is related to the motion of the flux lines and is greatly affected by the strength of the pinning forces which inhibit this motion. It is found that there is good agreement between theory and experiment.

Introduction

Studies of the ultrasonic attenuation in the mixed state of type II superconductors have centered on two classes of materials. In their review article, Gottlieb et al.[1] describe work in very pure materials, with low critical fields H_{c2}, where the most important features of the attenuation in the mixed state are related to the influence of the magnetic field on the superconducting energy gap. High-field superconductors, with large normal resistivities, have been studied by Shapira and Neuringer[2,3] in terms of a phenomenological model where the dominant effect on the attenuation is due to the magnetic field through magnetohydrodynamic interactions with the electrons. Our purpose here is to present further results for high-field superconductors with large normal resistivities.

For the materials under discussion, the most important physical properties are the normal state resistivity and the magnitude of the critical field H_{c2}. For metals with an electron mean free path which is short compared to the ultrasonic wavelength, the normal attenuation at zero magnetic field, α_n, is inversely proportional to the electrical resistivity. Thus, in high-field superconductors, where the normal resistivity is several orders of magnitude larger than that in typical pure type-II superconductors, we expect α_n to be very much smaller than for the pure materials. Consequently, effects such as those described in ref. 1, which are proportional to α_n, should be quite small. On the other hand, the effects of a magnetic field on the ultrasonic attenuation are quite large, both in the mixed state and at fields above H_{c2}. For example, we estimate [4] that for 15 MHz shear waves in Nb-26 at.% Hf, α_n is of the order of 10^{-4}dB/cm. However, in the same material, the observed change in the attenuation near H_{c2} (~50 kOe.) is ~0.2 dB/cm at 4.2 K, which is accounted for, quantitatively, in terms of the magnetohydrodynamic effects to be described below.

In previous works,[2,3] it has been shown that the change in the ultrasonic attenuation with magnetic field can be understood in terms of the Alpher-Rubin theory,[5] which was originally derived for impure metals in the normal state. It was necessary to introduce an effective ac resistivity in the mixed state, as proposed by Gittleman and Rosenblum.[6] Since this model has been discussed in detail previously,[2,3] we will limit our discussion to the most important results of the theory. In this work, we will be concerned only with the effect of the magnetic field on the attenuation and will neglect α_n.

Theoretical Considerations

A. Attenuation in the Normal State ($H > H_{c2}$)

It has been known for some time in magnetohydrodynamics that the magnetic induction and elastic strain may be interdependent, for certain sound modes and field directions, when the sound wavelength λ is large compared with the classical skin depth δ. Because of the eddy currents set up in the material, the total flux through any loop which is fixed everywhere to move with the particle motion is a constant. Thus the magnetic flux lines are pinned to the medium by the eddy currents and the sound propagation is accompanied by a compression or shearing of the flux lines. Theoretical treatments of this phenomenon predict an attenuation, which, for shear waves propagating parallel to the magnetic field direction and longitudinal waves perpendicular to the field, is given by[2,3]

$$\alpha = \frac{\omega\mu H^2}{8\pi d V^3}\left(\frac{\beta}{1+\beta^2}\right) \text{ (neper)} \tag{1}$$

$$\beta = \omega c^2 \rho_n / 4\pi\mu V^2 = 2\pi^2(\delta/\lambda)^2 \tag{2}$$

where ρ_n is the normal electrical resistivity, d is the density, μ the permeability and V the zero field sound velocity. This expression for the magnetic field dependent attenuation, and the corresponding expression for the velocity shift, have been studied extensively experimentally.[7,8,9] Agreement between theory and experiment has been shown to be very good for $\beta < 1$, and somewhat less good for $\beta > 1$.[7]

B. Attenuation in the Mixed State

For a high-field superconductor in the mixed state, we assume that the magnetic field

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TABLE I. PHYSICAL PROPERTIES OF THE SPECIMENS

Nominal Composition	Density at 4.14 K $d(g/cm^3)$	d.c. resistivity [a] $\rho_n(\mu\Omega\text{-cm})$	Shear velocity [b] (10^5 cm/sec)	Longitudinal velocity [b] (10^5 cm/sec)	$H^*_{c2}(0)$ (kOe.)	T_c (K)
Ti-11 at.% Nb	5.04	56	2.53	5.36	57.8	5.50
Nb-26 at.% Hf	10.1	34.4	1.92	4.03	68.6	9.30
Nb-41 at.% Ti	6.16	36	1.93	—	180	9.75

a) Measured at 4.14 K and at $H > H_{c2}$

b) Measured at 4.14 K

variation of the attenuation can be described in terms of the Alpher-Rubin theory, however, with a complex ac resistivity such as that used by Rosenblum and Gittleman [6] in place of the normal resistivity. Writing an equation of motion for a flux line, one may obtain an expression for this complex ac resistivity:

$$\rho = \rho_1 + i\rho_2 \tag{3a}$$

$$\rho_1 = B\rho_n / H^*_{c2}(0)\ (1+r^2) \tag{3b}$$

$$\rho_2 = rB\rho_n / H^*_{c2}(0)\ (1+r^2) \tag{3c}$$

where

$$r = \frac{K}{\eta\omega} \equiv \frac{\omega_o}{\omega} \tag{4}$$

In these equations, K is the pinning force constant per unit length, η the viscosity coefficient, given by

$$\eta = \phi_o H^*_{c2}(0) / \rho_n c^2 \tag{5}$$

where $\phi_o = hc/2e$ is the flux quantum, and $H^*_{c2}(0)$ is the Ginzburg-Landau-Abrikosov-Gor'kov (GLAG) upper critical field at zero temperature. The frequency ω_o, at which the pinning force has the same magnitude as the viscous force, is called the depinning frequency. At fields $H \gg H_{c1}$, the magnetic induction is very nearly equal to H and we have

$$\rho_1 = \rho_f / (1+r^2) \tag{6a}$$

$$\rho_2 = r\rho_f / (1+r^2) \tag{6b}$$

where

$$\rho_f = \rho_n H / H^*_{c2}(0) \tag{7}$$

is the dc flow resistivity. In this case, one can show that the attenuation in the mixed state, for $H \gg H_{c1}$, is given by

$$\alpha = \left(\frac{\omega\mu H^2}{8\pi d V^3}\right) \frac{\beta_f(1+r^2)}{\left[(1+r^2+\beta_f)^2+\beta_f^2\right]} \tag{8}$$

where

$$\beta_f = \beta H / H^*_{c2}(0) = \omega c^2 \rho_f / 4\pi\mu V^2 \tag{9}$$

There are a few limiting cases in which Eq.(8) simplifies considerably. When ω tends to zero, or when the depinning frequency tends to infinity, the attenuation tends to zero. Thus, in the limit of very strong pinning forces, the attenuation in the mixed state at $H \gg H_{c1}$, does not vary with H. On the other hand, when the pinning forces are weak, i.e. $r \ll 1$, Eq.(8) reduces to Eq.(1) with β_f replacing β. That is, in the case of very weak pinning forces, the magnetic field variation in the mixed state can be obtained from that in the normal state by replacing the normal resistivity ρ_n by the flow resistivity ρ_f. We note, finally, that while the flow resistivity ρ_f is always smaller than the normal resistivity ρ_n, the attenuation calculated using Eq.(1) with $\rho = \rho_f$ may be larger than that calculated with $\rho = \rho_n$. This may happen when the parameter β in the normal state is greater than one. As a result, the attenuation in the mixed state may be larger, over a certain field interval, than it would have been had the material been normal.

To calculate ρ_f it is necessary to know the GLAG upper critical field at zero temperature $H^*_{c2}(0)$. We have determined this quantity from the following relation:

$$H^*_{c2}(0) = -0.69\ T_c (dH_{c2}/dT)_{T = T_c} \tag{10}$$

Finally, it is desirable to estimate the depinning frequency ω_o which plays a central role in the discussion of the ultrasonic attenuation in the mixed state. Assuming that the maximum value of the pinning force is equal to the Lorentz force on a flux line at the critical current density J_c, we have, for $H \gg H_{c1}$,[2]

$$\omega_o \simeq 2\pi J_c \rho_n c H^{1/2} / \phi_o^{1/2} H^*_{c2}(0) \tag{11}$$

This equation should be regarded as a means of obtaining a rough estimate of the frequency ω_o.

Physical Properties of the Specimens

Experiments were performed on vacuum annealed superconducting alloys with the following nominal compositions: Ti - 11 at.% Nb, Nb - 26 at.% Hf, and Nb - 41 at.% Ti. Resistance measurements were performed using the standard four-wire technique and H_{c2} was identified with the resistive transition field in a manner described previously.[10] The values of the normal resistivity obtained at 4.14 K ($H > H_{c2}$) are listed in the Table I. The experimental uncertainty in the normal resistivity is about 5%. Also listed in Table I are the values of $H^*_{c2}(0)$ obtained from Eq. (10). Transit times for sound velocity calculations were obtained using the pulse-echo

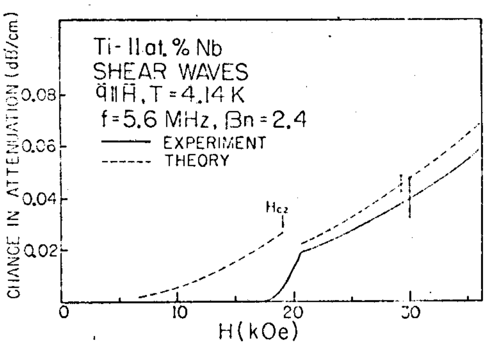

Fig. 1. Magnetic field variation of the attenuation of shear waves in Ti-11 at.% Nb at 4.14 K. The dashed curves were calculated from Eq. (1) with $\rho = \rho_f$ for $H < H_{c2}$ and $\rho = \rho_n$ for $H > H_{c2}$. The bars reflect the proportional uncertainty due to the experimental measurements involved.

overlap method [11] and experimental uncertainties associated with the velocities reflect, principally, uncertainties in sample length.

Results and Discussion

Ultrasonic attenuation measurements were carried out on the superconducting alloys in liquid helium temperatures and in steady magnetic fields up to 140 kOe. The variation of the ultrasonic attenuation with magnetic field was measured using standard pulse techniques. The accuracy of the attenuation measurements is estimated to be about 10%.

In Fig.1, we show the change in attenuation for 5.6 MHz shear waves in Ti-11 at.% Nb. With the parameters of Table I, the theoretical curves were calculated using Eq. (1), with $\rho = \rho_n$ for $H > H_{c2}$ and with $\rho = \rho_f$ for $H < H_{c2}$. We see that there is good agreement between theory and experiment for $H > H_{c2}$. We notice that the attenuation is found to be independent of H for $H < H_{c2}$. In fact, this suggests a case of strong pinning forces. From the previous discussion, we expect that when $\omega_o >> \omega$ the magnetic field variation of the attenuation should be substancially smaller than that predicted by Eq. (1) with $\rho = \rho_f$. This is what is observed.

The change in attenuation for longitudinal waves of various frequencies in Nb-26 at.% Hf is shown in Fig.2. We notice that, for each frequency, there appears a dip in the attenuation for fields just below H_{c2}. Such dips have been observed previously and have been associated with the peak effect in the critical current density. For $\omega \sim \omega_o$, the attenuation is a sensitive function of ω_o. Thus, a sharp increase in J_c corresponds to an increase of the pinning forces and should result in a decrease in the attenuation.

In Fig. 3 we have the change in attenuation for shear waves in Nb-26 at.% Hf. We notice that, for f = 25.1 MHz, the dip in the attenuation for fields just below H_{c2} has disappeared. This might occur for $\omega >> \omega_o$,

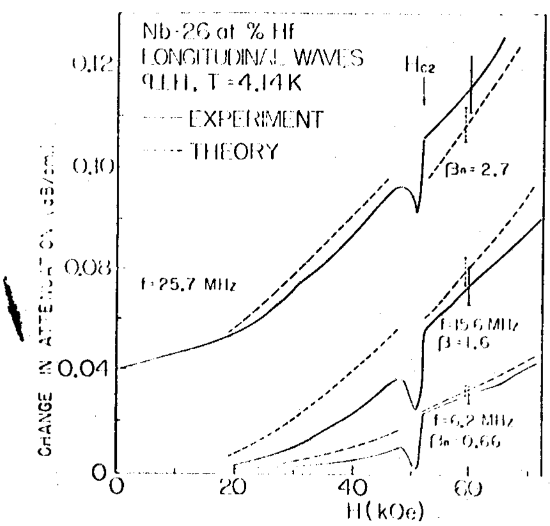

Fig. 2. Magnetic field variation of the attenuation of longitudinal waves in Nb-26 at.% Hf at 4.14 K. The dashed curves were calculated from Eq.(1) with $\rho = \rho_f$ for $H < H_{c2}$ and $\rho = \rho_n$ for $H > H_{c2}$. The curve for 25.7 MHz was shifted upward by 0.04 dB/cm. The bars reflect the proportional uncertainty due to the experimental measurements involved.

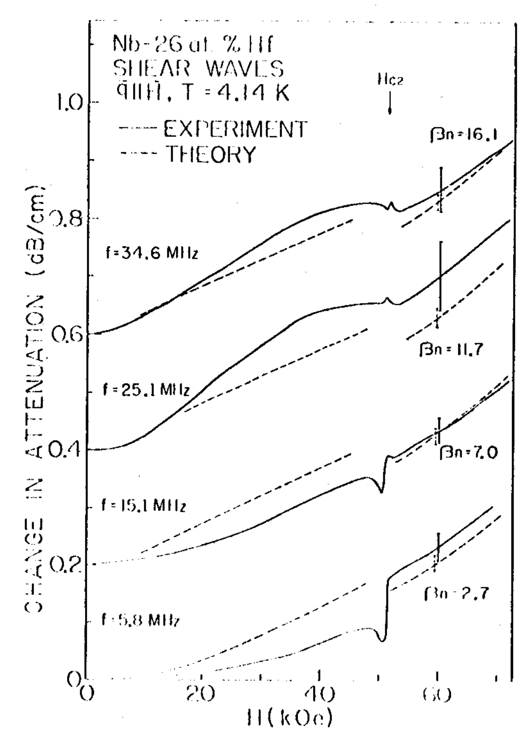

Fig. 3. Magnetic field variation of the attenuation of shear waves in Nb-26 at.% Hf at 4.14 K. The dashed curves were calculated from Eq. (1) with $\rho = \rho_f$ for $H < H_{c2}$ and $\rho = \rho_n$ for $H > H_{c2}$. The curves for 15.1, 25.1 and 34.6 MHz were shifted upward by 0.2, 0.4 and 0.6 dB/cm respectively. The bars reflect the proportional uncertainty due to the experimental measurements involved.

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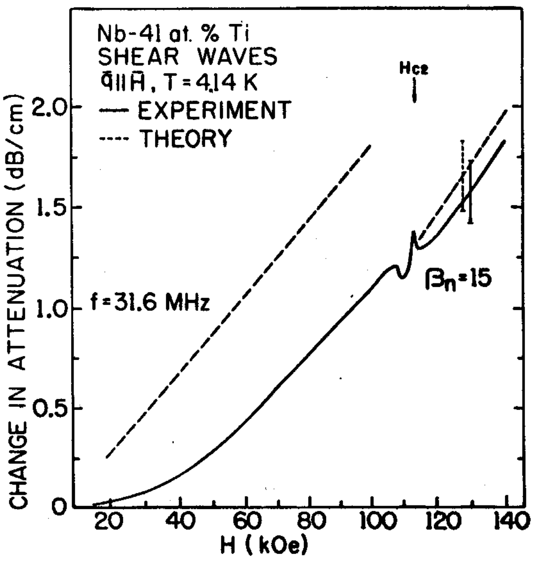

Fig. 4. Magnetic field variation of the attenuation of shear waves in Nb-41 at.% Ti at 4.14 K. The dashed curves were calculated from Eq. (1) with $\rho = \rho_f$ for $H < H_{c2}$ and $\rho = \rho_n$ for $H > H_{c2}$. The bars reflect the proportional uncertainty due to the experimental measurements involved.

the attenuation, in this case, being insensitive to variations in ω_o. We had previously seen that for 25.7 MHz longitudinal waves the decrease in the attenuation was present for fields just below H_{c2}. This suggests that the pinning force constant (and therefore ω_o) for shear waves propagating parallel to the magnetic field is somewhat different from that for longitudinal waves propagating perpendicular to the field. This is not too surprising since the type of distortion which the flux line undergoes is expected to be different in the two cases.

We note, also, in Fig. 3, that for f = 34.6 MHz, and for a certain magnetic field interval, the attenuation in the mixed state is greater than that in the normal state. This, too, is in agreement with our previous comments.

Finally, in Fig. 4, we show the change in attenuation for 31.6 MHz shear waves in Nb-41 at.% Ti. In the mixed state the attenuation is smaller than that predicted by Eq. (1) with $\rho = \rho_f$. Furthermore, a slight decrease in the attenuation occurs just below H_{c2}. This suggests $\omega \overset{<}{\sim} \omega_o$ and that we must use Eq. (8) to calculate the attenuation in the mixed state.

The experimental results presented here suggest that the phenomenological model accounts qualitatively for the ultrasonic attenuation in high-field superconductors. A detailed comparison of the experimental data with Eq. (8) is being prepared and will be published at a later time.

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