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ON A MOMENTUM-SPACE RENORMALIZATION SCHEME, PRESERVING THE CLASSICAL STRUCTURE I:

Subtraction Operators

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A B S T R A C T

The usual momentum-space renormalization scheme, which subtracts ultraviolet divergencies by the use of Taylor operators, is modified in such a way that the whole classical structure, such as equations of motion, the form of the classical current and its conservation, is preserved. We are able to eliminate all anomalies and anisotropies in theories not involving fermions, except the ones related to the trace of the energy-momentum tensor. This first of a series of papers presents the mathematical background and some examples, like the equivalence theorem.

I. Introduction

The quantization of models with space-time dependent internal gauge symmetries has always been a nontrivial problem due to the necessity of renormalization. Since the exhibition of such a symmetry is one of the most important and desirable properties of such a theory, one would like to construct its quantized version, which maintains this classical structure. Yet it is usually not easy to set up a renormalization scheme, which respects the classical symmetry.

In this case there are essentially two ways to proceed[1]. One may stick to a preestablished subtraction scheme, like Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ)[2],which uses Taylor operators around zero external momenta. This has the advantage of easy use, but is in no way adapted to any particular dinamical structure. This straightjacket forces one to modify the classical Lagrangian in order to guarantee the validity of the desired Ward-Takahashi identities, which the subtraction scheme has utterly destroyed. The ensuing complications are not easy to handle[3].

On the other hand one may want to preserve the classical structure as much as possible and adapt one's subtraction scheme to the symmetry at hand. This has the only disadvantage, that the explicit calculation of a particular Feynman graph may be more laborious.

In this paper we introduce a modified version of the soft momentum space subtraction procedure[4][5]with the aim of eliminating all unnecessary anomalies, i.e. extra terms which

are not present in the classical version of the equations of motion and which appear only because of a bookkeeping, which is not suited to the particular problem. This is true, except for the trace of the energy-momentum tensor $\theta^{\mu}_{\mu}(x)$, which is not soft, and when fermions are present as will be pointed out in sect. V.

In the following series of papers we will apply this technique to a non-linear σ-model and its zero mass limit, to the abelian Higgs-Kibble model in 't Hoft's gauge and to non-abelian models.

In section II we give some more motivation for what will be done in sections III and IV, which contain the construction of subtraction operators for one line (one-loop graphs) and their extension to any graph via a forest formula, which classifies graphs according to internal lines instead of vertices. In sect. V we give some examples including the equivalence theorem. Explicit calculations are relegated to the appendix.

II. Motivation

We want to convince the reader that the production of yet another subtraction scheme in order to remove ultraviolet (UV) divergences is very desirable indeed.

As alluded to in the introduction the renormalization necessary in order to remove UV divergences has to be sufficiently "soft" so as not to destroy the classical structure of the theory, which we want to maintain, since this automatically preserves all the symmetries present on the classical level.

The usual "hard" BPHZ[2] subtractive renormalization employs Taylor operators in the external momenta p_i . UV divergences of any Feynman graph are removed by subtracting Taylor terms around $p_i = 0$. The combinatorics is disentangled by Zimmermann's forest formula. When this scheme is used to study equations of motion and conservation laws for composite operators, one discovers the ocurrence of oversubtracted and anisotropic normal products, which have no classical counterpart and are called anomalies.

The purpose of this paper is to introduce a renormalization scheme, which avoids all known anomalies in perturbation theory, when no fermions are present, except when θ^μ_μ is involved. One step in this direction has been done by the introduction of the so called "soft" quantization(4), where one employs Taylor oeprators also in mass-type parameters m_j, besides the external momenta p_i. Were it not for the existence of infrared (IR) divergencies, a soft quantization scheme, using Taylor operators around $p_i=0$, $m_j=0$ would indeed fulfill our requirements. This follows from the following two rules valid for any composite object $N_\delta[\mathcal{O}(x)]$, subtracted around $p_i=0$, $m_j=0$ with degree δ:

$$\partial_\mu N_\delta[\mathcal{O}(x)] = N_{\delta+1}[\partial_\mu \mathcal{O}(x)] \qquad (2.1)$$

$$m^2 N_\delta[\mathcal{O}(x)] = N_{\delta+2}[m^2 \mathcal{O}(x)] \qquad (2.2)$$

Equation (2.2) is not valid in the hard subtraction scheme and in equations of motion one obtains for example objects like $N_4(m^2A^2) = m^2N_4(A^2)$ and $N_4(A^2)$ is an oversubtracted normal product, since $N_2(A^2)$ is already finite. These oversubtracted objects lead to anomalies and we have to avoid them. Our aim is

thus the introduction of a mass-type parameter μ, serving as an IR cut off. This idea has already been used in ref. 6 to derive a Weinberg type(7) homogeneous renormalization group equation and in ref. 8 to discuss the Higgs phenomenon in an abelian theory in the Lorentz gauge. However eq. (2.2) is not satisfied by the schemes of ref. 6 and the trick used by the authors of ref.8 is not immediately extensible to more complex situations, like the non abelian case.

In the present paper on the other hand, we preserve both eq. (2.1) and (2.2). All normal products to be introduced will be minimally subtracted and the index δ will be dropped henceforth.

In order to achieve this goal, one has to take advantage of the fact, that a certain number of subtractions can be done at $m^2=0$, without running foul of IR divergencies. How many can be done is determined by the phase-space d^4k and the amount of derivatives being hooked onto a certain line. To keep track of this all important fact, one has to generalize first the type of subtraction operators used in the past. This will be done in sect. III. Furthermore one also has to modify the usual forest formula and classify graphs according to lines, and not vertices. This is done in sect. IV.

III. Construction of Subtraction Operators

The purpose of this section is the construction of subtraction operators $\tau^{(n)}, n=0,1,2,\ldots$. They will be determined by the requirement, that the application of $(1-\tau^{(n)})$ to the one-loop integrands:

$$\prod_{i=1}^{\ell} \frac{1}{[(P_i+k)^2-m^2]^{k_i}} \quad , \quad k_i = 1,2,3,\ldots ; \ \ell = 1,2,3,\ldots \tag{3.1}$$

lowers their superficial degree of divergence by (n+1) units.

Although for $\ell=3, k_1=k_2=k_3=1$ for example, one has already a convergent integral, in non-renormalizable theories like the non linear σ-model[9], such more-than-minimun subtractions are necessary. They appear, when the integrand under consideration occurs as a subgraph in a graph G of degree of divergence $D_G = n$.

Furthermore our operators $\tau^{(n)}$ have to produce routing-independent integrals, e.g. the integral over d^4k of $[(p+k)^2-m^2]^{-1}$ has to be p-independent. It must thus be possible to write the p-dependent terms to be subtracted in the form $\partial/\partial k_\alpha f(p,k,m,\mu)$. It is then easy to show that

$$\frac{\partial}{\partial \alpha}\left\{ \int d^4k \left[\frac{1}{(\alpha p+k)^2-m^2} - \text{subtractions} \right] \right\}$$

can be converted into a vanishing surface integral.

We will begin by the simpler case, where in expression (3.1) $\ell = 1$, i.e.

$$\frac{1}{(p+k)^2-m^2} \tag{3.2}$$

This integrand will be referred to as zero-starred loop.

Consider first the Taylor expansion of the zero-starred loop around $p_\mu = 0$, $m^2 = 0$:

$$\begin{aligned}\frac{1}{(p+k)^2-m^2} &= \frac{1}{k^2} + \\ &+ \frac{m^2}{k^4} + \frac{m^4}{k^6} + \cdots \\ &- 2p\cdot k\left\{\frac{1}{k^4} + \frac{2m^4}{k^6} + \cdots\right\} \\ &- p^2\left\{\frac{1}{k^4} + \frac{m^2}{k^6} + \cdots\right\} - 4(p\cdot k)^2\left\{\frac{1}{k^4} + \frac{6m^2}{k^6} + \cdots\right\} + \cdots = \\ &= \sum_{j\le n} N_j^n(r)\,(p^2)^{\frac{n-j}{2}}\,(p\cdot k)^j\,(m^2)^r\,(k^2)^{-\left[1+r+\frac{n+j}{2}\right]}\end{aligned} \tag{3.3}$$

The numbers $N_j^n(r)$ are defined by this expansion.

To avoid IR divergencies the above expansion has to be modified. To each term arising from $\partial^s/\partial p_\alpha^s . \partial^r/\partial(m^2)^r$ will correspond a new term, which ceases to be a derivative although we will still denote them by $\partial_p^s\, \partial_{m^2}^r$ for convenience.

The term m^2/k^4 will be replaced by

$$(m^2+\lambda\mu^2)\left[\frac{\rho}{k^2(k^2-\mu^2)} + \frac{1-\rho}{(k^2-\mu^2)^2}\right] \tag{3.4}$$

when μ^2 is put equal to zero we obtain m^2/k^2. This guarantees that the correct amount is subtracted off the unsubtracted μ-independent integrand. This will always be observed in the following. The constants ρ and λ will be fixed to ensure the appropriate lowering of the superficial degree of divergence. Applying the same procedure to all terms, except $1/k^2$ and noting that we may multiply different powers of $(p.k)$ and $1/(k^2-\mu^2)$ by different polynomials in μ^2, we obtain the following definition:

$$\begin{aligned}
\tau^{(n)}\frac{1}{(p+k)^2-m^2} &\equiv \frac{1}{k^2} + \\
&+ (m^2+\lambda_{11}^{(0)}\mu^2)\Big[\frac{\rho_{11}^{(0)}}{k^2(k^2-\mu^2)}+\frac{1-\rho_{11}^{(0)}}{(k^2-\mu^2)^2}\Big] + \\
&+ (m^4+\lambda_{21}^{(0)}m^2\mu^2+\lambda_{22}^{(0)}\mu^4)\Big[\frac{\rho_{21}^{(0)}}{k^2(k^2-\mu^2)^2}+\frac{1-\rho_{21}^{(0)}}{(k^2-\mu^2)^3}\Big]+\dots \\
&- 2p\cdot k\Big\{\Big[\frac{\rho_{01}^{(1)}}{k^4}+\frac{\rho_{02}^{(1)}}{k^2(k^2-\mu^2)}+\frac{1-\rho_{01}^{(1)}-\rho_{02}^{(1)}}{(k^2-\mu^2)^2}\Big]+\dots\Big\}+\dots \\
&= \sum_{j\le n} N_j^n(r)\,(p^2)^{\frac{n-j}{2}}\,(p\cdot k)^j\ \mathcal{F}\Big\{P_{rj}^{(u)};\ \rho_{r,s}^{(j)};\ \frac{a+j}{2};\ r+\frac{n-1}{2}\Big\}
\end{aligned} \tag{3.5}$$

where $P_{rj}^{(u)}(\lambda) = (m^2)^r + \lambda_{r1}^{(j)}(m^2)^{r-1}\mu^2 + \dots + \lambda_{rr}^{(j)}(\mu^2)^r$

and $\mathcal{F}\Big\{P_{rj}^{(u)};\ \rho_{rs}^{(j)};\ \frac{a+j}{2};\ r+\frac{n-1}{2}\Big\}$

indicates the sum of terms in square brackets containing the powers of k^2 and $(k^2-\mu^2)$, ranging from $(k^2)^{-\frac{a+j}{2}}(k^2-\mu^2)^{-(r+\frac{n-1}{2})}$ to $(k^2-\mu^2)^{-(1+r+\frac{n-1}{2})}$, where each term is multiplied by a polynomial $P_{rj}^{(u)}$ [9] labeled by u and an independent constant ρ, except the last term, which receives $(1-\sum\rho)$;

$a = \left\{\begin{matrix} 3 \\ 2 \end{matrix}\ \text{for}\ \begin{matrix} j = \text{odd} \\ j = \text{even} \end{matrix}\right\}$.

This is the most general form for $\tau^{(n)}$ consistent with IR convergence and making use of only one subtraction point μ^2.

The countably infinite number of constants ρ and λ parametrizes all the arbitraryness of non-renormalizable theories. The requirement, that we obtain equations of motion of the type

$$N[A^n\partial^2A] = m^2 N[A^{n+1}] + \dots \tag{3.6}$$

will determine the way $A^n\partial^2A$ has to be subtracted, but will not fix any of the constants ρ or λ. We may for example single out one particular theory, by setting all coefficients ρ equal to zero, except the one multiplying the highest power of $(k^2)^{-1}$. This means, that we stay as close as possible to a scheme, where all the subtractions are done at $m^2=0$. We may also say, that in this case the subtraction terms display the slowest increase in μ^2 possible. The coefficients λ corresponding to this case are calculated below[10].

One may also set all coefficients ρ equal to zero. This gives a Taylor subtraction scheme around $p=0$, $m^2=\mu^2$, which will be used in the appendix.

We consider now the various cases.

i) $n=j=0$. Pure $\partial_m^r 2$ terms; $r=0,1,2,\dots$.

From the requirement that the superficial degree of divergence be lowered, in the future to be referred to as requiring UV convergence, we get

$$P_{or}(\lambda_{rs}^{(0)}) = m^2(m^2-\mu^2)^{r-1} \tag{3.7}$$

Notice that the different tensors $p_{\mu_i}\dots p_{\mu_j}$ in eq. (3.5) do not get mixed in requiring UV convergence, but that one gets equations coupling each of the above tensors with no mass derivatives terms to terms with extra derivatives $\partial_m^r 2$, $r=1,2,\dots$

ii) $\partial_p^n \; \partial_{m^2}^r$ terms; n=1,3,5... , odd; r=0,1,2,.....

All terms are trivially of the form. $\partial/\partial_{k\alpha}\, f(p,k,m^2,\mu^2)$. The index "u" takes only one value and the relevant part of expression.(3.5) reduces to

$$N_j^n(r)\, P_{jr}(\lambda_{rs}^{(j)})\,(p^2)^{\frac{n-j}{2}}\,(p\cdot k)^j\,(k^2)^{-\frac{3+j}{2}}\,(k^2-\mu^2)^{-(r+\frac{n-1}{2})} \tag{3.8}$$

Equating now to zero the coefficients of an expansion of expression (3.8) in powers of μ^2/k^2 in order to obtain UV convergence, we obtain the following sistem of equations

$$\sum_{i=0}^{s-1} \lambda_{r-i,s-i}^{(j)}\,(-1)^i\, N_j^n(r-i) \begin{pmatrix} -[\frac{n-1}{2}+r-i] \\ i \end{pmatrix} = \tag{3.9}$$

$$= (-1)^{s+1}\, N_j^n(r-s) \begin{pmatrix} -[\frac{n-1}{2}+r-s] \\ s \end{pmatrix}$$

with n=1,3,5,....

r=0,1,2,...; $1 \leqslant s \leqslant r$.

The above sistem has the solution

$$\lambda_{r,s}^{(j)} = \frac{N_j^n(r-s)}{N_j^n(r)} \begin{pmatrix} -[\frac{n-1}{2}+r-s] \\ s \end{pmatrix} \tag{3.10}$$

Equation (3.10) together with expression (3.8) defines the zero star operators $\tau^{(n)}$ for n odd and $\neq 0$.

iii) $\partial_p^n \; \partial_{m^2}^r$ terms; n=2,4,6,......., even; r=0,1,2,...

Because of the tensor structure of these terms, they are in general not of the form $\partial/\partial k_\alpha\, f(p,k,m,\mu)$. We will guarantee routing independence by taking the derivative $p_\alpha\, \partial/\partial_{k_\alpha}$ of the expression (3.8) with $n \to n-1$. The index "u" runs now over two values; we obtain the expression

$$\frac{1}{n}(p^2)^{\frac{n-j}{2}}(p\cdot k)^j \left\{ \frac{j\, N_{j-1}^{n-1}(r)\, P_{rs}^{(1)}(\lambda_{rs}^{(j-1)}) + (j+1)\, N_{j+1}^{n-1}(r)\, P_{rs}^{(2)}(\lambda_{rs}^{(j+1)})}{(k^2)^{j/2+1}\,(k^2-\mu^2)^{n/2+r}} \right.$$

$$\left. + \frac{(n+2r)\, N_{j-1}^{n-1}(r)\, P_{rs}^{(1)}(\lambda_{rs}^{(j-1)})}{(k^2)^{j/2}\,(k^2-\mu^2)^{n/2+r+1}} \right\} \tag{3.11}$$

where we used the convention:

$N_j^n(r) = 0$, if j is outside the range $1 \leqslant j \leqslant n$.

Imposing UV convergence leads to the following sistem of equations:

$$\sum_{i=0}^{s-1} (-1)^i \left\{ N_{j-1}^{n-1}(r-i)\,(n+j+2r)\,\lambda_{r-i,s-i}^{(j-1)} + N_{j+1}^{n-1}(r-i)\,(j+1)\,\lambda_{r-i,s-i}^{(j+1)} \right\} \times \tag{3.12}$$

$$\times \begin{pmatrix} -[\frac{n}{2}+r-i] \\ i \end{pmatrix} = (-1)^{s+1} \left\{ N_{j-1}^{n-1}(r-n)\,(n+j+2r) + N_{j+1}^{n-1}(r-n)\,(j+1) \right\} \begin{pmatrix} -[\frac{n}{2}+r-s] \\ s \end{pmatrix}$$

with n = 2,4,6,

r = 0,1,2,3, $1 \leqslant s \leqslant r$.

The sistem (3.12) has the solution

$$\lambda_{r,s}^{(j)} = \frac{N_j^{n-1}(r-s)}{N_j^{n-1}(r)} \begin{pmatrix} -[\frac{n}{2}+r-s] \\ s \end{pmatrix} \tag{3.13}$$

This concludes the determination of $\tau^{(n)}$ for the zero-star loop.[11]

We now have to extend this result to the case where $\ell \neq 1$, $k_i \neq 1$. Instead of writing long formulas, we will just say how to proceed.

The pure $\partial^r_{m^2}$ derivatives are easily dealt with. The polynomial P_{or} is just

$$\frac{\partial^k}{\partial (m^2)^k} \left\{ m^2 (m^2 - \mu^2)^{r-1} \right\} \tag{3.14}$$

where $k = \sum_{i=1}^{\ell} k_i$.

For the $\partial^n_p \; \partial^r_{m^2}$ terms, notice that eq. (3.10) is obtained from expression (3.8) by forming the ratio

$$\frac{N^n_j(r-s)}{N^n_j(r)}$$

where these numbers are defined by the Taylor expansion about $p_\mu=0$ and $m^2=0$ of the integrand to be subtracted and the numbers going into the binomial coefficient are read off the power of $(k^2-\mu^2)$. This also holds for eqs. (3.13) and (3.11).

When we consider a general one loop integrand of the form (3.1), all that changes are the numbers $N^n_j(r)$, which are replaced by Xabc...(r) say and the power to which $(k^2-\mu^2)$ is elevated. Call it y. The coefficients $\lambda^{(abc)}$ are then given by

$$\lambda^{(abc\ldots)}_{rs} = \frac{X_{abc\ldots}(r-s)}{X_{abc\ldots}(r)} \binom{-[y-s]}{s} . \tag{3.15}$$

This finishes the prescription of how to subtract all possible one-loop integrands, when there are no polynomials in the momenta attached to the vertices.

IV. Definition of the Subtracted Integrand

Let G be a Feynman diagram. The momentum ℓ_{ji} that flows through a line L_i of G can always be decomposed into two parts, one that is a linear combination of the loop (integration) momenta and another one that is a linear combination of the external momenta, i.e.

$$\ell_{ji} = k_j + p_{ji} \tag{4.1}$$

There are in general several lines with momenta that differ only in the external momentum part and, for subtraction purpose, we will group such lines together. With this in mind the unsubtracted Feynman integrand associated with G is defined as

$$I_G = \prod_j B_j \tag{4.2}$$

where B_j has the form

$$B_j = P_j(\ell_{ji}, k_j)\,\Delta_F(\ell_{j1}) \ldots\ldots \Delta_F(\ell_{j\alpha(j)}) , \tag{4.3}$$

where $P_j(\ell_{ji}, k_j)$ is a polynomial in ℓ_{ji} and k_j and $\Delta_F(\ell_{ji}, m^2)$ is the free propagator associated with the line L_i.

The p,m and μ dependence of the polynomials

$p_j(\ell_{ji},k_j)$, the latter arising when we consider forests with more than one element, is to be treated as follows: when applying $\tau_\gamma^{\delta(\gamma)}$ to I_G, every external momentum p_γ in the numerator of I_G is not acted upon by $\tau_\gamma^{\delta(\gamma)}$, but $\delta(\gamma)$ is lowered by one unit; correspondingly the monomial $(m^2)^i(\mu^2)^{n-i}$ lowers $\delta(\gamma)$ by n units. Furthermore, in eqs. (4.2) and (4.3), we assume that some specification was given of how momentum factors are associated with the lines of G. The momenta for subgraphs are chosen arbitrarily but within an admissible set in the sense of Zimmermann[11].

Let $\mathcal{F}$ indicate the set of all forests of G, i.e., sets of non-overlapping, superficially divergent subgraphs of G. The subtracted integrand is given then by

$$R_G = \sum_{\mathcal{U}\in\mathcal{F}} \prod_{\gamma\in\mathcal{U}} (-\tau_\gamma^{\delta(\gamma)})\, I_G \tag{4.4}$$

where $\delta(\gamma)$ is equal to $\text{degree}_{(k,m,p)} I_\gamma$ + 4 x (number of loops). The subtraction operator τ^δ acts on G in the following way

$$\tau^{\delta(G)} I_G = \sum_{n=0}^{\delta(G)} \sum_{\substack{\{S_i\} \\ \Sigma S_i = n}} \prod P_{s_j}^{B_j}\, \partial^{s_j} B_j \tag{4.5}$$

where the $P_{s_j}^{B_j}$ in eq. (4.5) are determined by the construction of the previous section and so satisfy

$$\text{degree}_{k_j} (1-\tau^\delta)\, B_j \leq \text{degree}(B_j) - (\delta+1) \tag{4.6}$$

The generalization of (4.5) for subgraphs is immediate with the replacement of B_j by B_j^γ.

We should stress, that under the name subtraction operator τ^δ, we understand one and only one of the possible operators constructed in the previous section and labeled by the constants ρ and λ. That is, once a certain set of ρ's and λ's has been chosen, we do stick to them.

One remark is also necessary with relation to the distribution of momentum factors through the lines of G. A loop momentum factor decresases the IR divergencies coming from soft subtractions (i.e. those made directly at zero external momentum and mass) and so can be used to make more subtractions at $m^2=0$. However, with exception of bilinear vertices, there is an ambiguity due to momentum conservation depending to which line the momentum factors are to be associated. In the non subtracted integrand this is solved by some previous definition and in the case of loop momentum factors generated from subtractions, we, as general rule will not use them to increase the number of soft subtractions (i.e. those momenta are hard under the point of view of subtractions).

One important result of this section is

$$\deg_k (1-\tau^{\delta(G)}) I_G < -4 \times n^{\underline{o}} \text{ of loops in } G \tag{4.7}$$

We will prove (4.7) only is the case that I_G consists of two pieces. The proof for more general I_G's is a straightforward generalization of this case. Let $I_G = B_1 B_2$, then

$$R_G = B_1 B_2 - \sum_{n=0}^{\delta(G)} \sum_{\substack{S_1,S_2 \\ S_1+S_2=n}} P_{S_1}^{B_1} P_{S_2}^{B_2}\, \partial^{S_1} B_1\, \partial^{S_2} B_2$$

Define

$$R'_G \equiv (1-\tau)B_1\cdot(1-\tau)B_2 = B_1 B_2 - [\sum P_{s_1}^{B_1}\partial^{s_1}B_1]B_2 - $$
$$- [\sum P_{s_2}^{B_2}\partial^{s_2}B_2]B_1 + (\sum P_{s_1}^{B_1}\partial^{s_1}B_1)(\sum P_{s_2}^{B_2}\partial^{s_2}B_2) \tag{4.8}$$

which certainly satisfies (4.7). Now

$$R'_G = R_G - [\sum P_{s_1}^{B_1}\partial^{s_1}B_1][B_2 - \sum P_{s_2}^{B_2}\partial^{s_2}B_2] -$$
$$- [\sum P_{s_2}^{B_2}\partial^{s_2}B_2][B_1 - \sum P_{s_1}^{B_1}\partial^{s_1}B_1] + \tag{4.9}$$
$$+ \sum_{\substack{s_1,s_2 < N \\ s_1+s_2 \geq N+1}} P_{s_1}^{B_1} P_{s_2}^{B_2} \partial^{s_1}B_1 \partial^{s_2}B_2$$

where we used

$$\sum_{n=0}^{N} \sum_{\substack{\alpha_1,\alpha_2 \\ \alpha_1+\alpha_2=n}} a^{\alpha_1} b^{\alpha_2} = \sum_{\alpha_1,\alpha_2}^{N} a^{\alpha_1} b^{\alpha_2} - \sum_{\substack{\alpha_1,\alpha_2 \leq N \\ \alpha_1+\alpha_2 \geq N+1}} a^{\alpha_1} b^{\alpha_2}$$

The result follows trivially from eq. (4.9).

The operator τ defined above is a subtraction operator in the sense of ref.4. (Besides eq. (4.7) the other requirements of ref.4 are easily verified.). Thus the convergence proof for the forest formula can be applied and the definition (4.4) is

indeed a finite part prescription for the integrand I_G.

The renormalization scheme defined above has several interesting properties as equation of motion, rules for differentiation, which will be used, to derive, in theories not involving fermion fields, Ward identities not involving θ^μ_μ free from anomalies. This will be discussed in the next section together with the general derivation of equations of motion.

The renormalized amplitudes has the factorization property: If G_1 and G_2 overlapp in only one point or are disjoint then

$$R_{G_1 \cup G_2} = R_{G_1} \cdot R_{G_2} \tag{4.10}$$

The proof consists in verifying the equality

$$\tau^{\delta_1} I_{G_1} \cdot \tau^{\delta_2} I_{G_2} = \tau^{\delta_1+\delta_2}[I_{G_1}\tau^{\delta_2} I_{G_2} + I_{G_2}\tau^{\delta_1} I_{G_1}]$$

V. Equations of Motion, Equivalence Theorem and Current Conservation.

Let us consider the theory described by the effective Lagrangian density

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \mathcal{L}_{int} . \tag{5.1}$$

Green functions are defined by the Gell-Mann Low formula

$$\langle T A_1(x_1)\dots A_n(x_n)\rangle = \text{F.P.}\,\overset{(0)}{\langle} T A_1^{(0)}(x_1)\dots A_n^{(0)}(x_n)\exp i\int \mathcal{L}_{int}^{(0)}(x)\,d^4x\rangle^{(0)} \qquad (5.2)$$

where the finite part (F.P) prescription is the same as described in section IV. Note that there is no Wick ordering in eq. (5.2). The equation of motion

$$(\partial_x^2 + m^2)\langle T A_i(x)\mathrm{X}\rangle = -\langle T N\Big[\partial_\mu \frac{\delta \mathcal{L}_{int}}{\delta\,\partial_\mu A_i} - \frac{\delta \mathcal{L}_{int}}{\delta A_i}\Big](x)\mathrm{X}\rangle + \sum_j \delta_{ij}\,\delta(x-x_j)\langle T\mathrm{X}_{\hat{j}}\rangle \qquad (5.3)$$

with the normal product $N[\mathcal{O}]$ defined by

$$\langle T N[\mathcal{O}]\mathrm{X}\rangle = \text{F.P.}\,\overset{(0)}{\langle} T\,\mathcal{O}^{(0)}(x)\,\mathrm{X}^{(0)}\exp i\int \mathcal{L}_{int}^{(0)}(x)\,d^4x\rangle^{(0)} \qquad (5.4)$$

is easily verified.

In addition to this, it is possible to define normal products that satisfy the classical equation of motion. Thus, in a theory not involving derivative couplings, for example, the equation of motion follows, if the normal product $N[Q\,\partial^2 A]$ (where $Q = A^n$) is defined in the following way. Let U be a forest and $\sigma\varepsilon$ U the smallest graph in U that contains the special line in the normal product. Then the subtraction corresponding to τ acts in the following way

$$\tau^{\delta(\sigma)}\ell_r^2 B_r \prod_{s\neq r} B_s = \sum_{n=0}^{\delta(\sigma)}\sum\left(P_{s_i}^{B_i}\partial^{s_i}B_i\right)_{i\neq r} P_{s_r}^{\mathrm{X}}\partial^{s_r}(B_r\ell^2) \qquad (5.5)$$

where

$$P_{s_r}^{\mathrm{X}}\partial^{s_r}(B_r\ell^2) = \begin{cases} P_{s_r}^{B_r/\Delta_r(\ell)}\,\partial^{s_r}(B_r/\Delta_r(\ell)), & \text{if } \partial^{s_r} \text{ is an operation that does not contain } \partial_m^2\Delta_r(\ell). \\ m^2 P^{B_r}\partial^{s_r}B_r & \text{otherwise (in this case } s_r \text{ ranges only up to } \delta(\sigma)-2) \end{cases}$$

The equation of motion follows by definition. Some care must be taken in the case of derivative coupling.

As an example, we will prove now the equivalence theorem(12), asserting the equality of the on shell Green functions of theories that differ by a local field transformation. Let the effective Lagrangian density be given by

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu A\,\partial^\mu A - \tfrac{1}{2}m^2A^2 + \mathcal{L}_{int}(A);\ \mathcal{L}_{int} = \sum a_n (A)^n \qquad (5.6)$$

and consider

$$\mathcal{L}'_\lambda = \tfrac{1}{2}\partial_\mu A\,\partial^\mu A - \tfrac{1}{2}m^2A^2 - m^2\lambda A V - \tfrac{1}{2}m^2\lambda^2V^2 - \lambda V\partial^2 A - \tfrac{1}{2}\lambda^2 V\partial^2 V + \sum_n \frac{1}{n!}\lambda^n V^n \frac{\delta\mathcal{L}_{int}}{\delta A^n} \qquad (5.7)$$

where the rule to associate momentum factors coming from the vertices in eq. (5.7) is the following

i) The contribution from the vertex $\lambda V\partial^2 A$ is calculated as in eq. (5.5). If there is more than one d'alambertian in the same line, we use only one of them.

ii) The momentum factors coming from the vertex $V\partial^2V$ are not used to increase the number of subtractions made directly at zero.

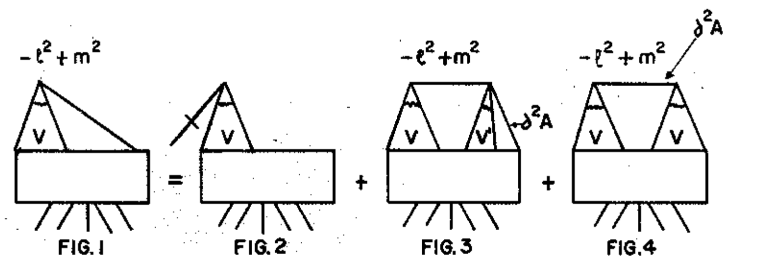
$\mathcal{L}'_\lambda$ is formally obtained from eq. (5.6) after the substitution $A \to A+\lambda V$ and partial integrations in the derivative terms. Let $\langle TX \rangle_\lambda$, $X = \prod A(x_i)$ be the Green functions calculated via eq. (5.7). Then we have

$$\frac{\partial}{\partial\lambda}\langle TX\rangle_\lambda = -\int d^4x \left\{\langle T[V(\partial^2 A + m^2 A + m^2\lambda V + \lambda\partial^2 V)](x) X\rangle\right\} \quad (5.8)$$

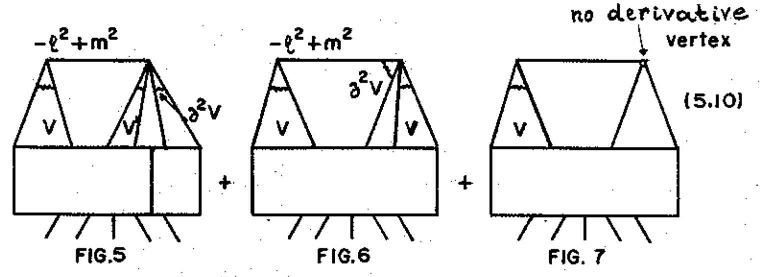
Now

$$\begin{aligned}&\langle T\{V[\partial^2+m^2]A + m^2\lambda V^2 + \lambda V\partial^2 V + m^2\lambda AVV' + \\ &+ m^2\lambda^2 V^2 V' + \lambda VV'\partial^2 A + \lambda^2 VV'\partial^2 V\}(x)\, X\rangle = \\ &= \text{delta terms}\end{aligned} \quad (5.9)$$

The proof of eq. (5.9) is as usual: Consider the vertex $V(\partial^2+m^2)A$. (fig.1) The field $(\partial^2+m^2)A$ in this vertex can be linked either to i) no vertex (this gives the delta terms on the r.h.s. of eq. (5.9) - fig. 2 below); ii) to the vertex $V\partial^2 A$ (figs.3 and 4); to the vertex $V\partial^2 V$ (figs 5 and 6) or to a no derivative vertex (fig.7)



(5.10)



(5.10)

Thus

$$\frac{\partial}{\partial\lambda}\langle TX\rangle_\lambda = $$

$$= \lambda\int d^4x \left\{\langle T[VV'(\partial^2 A + m^2 A + m^2\lambda V + \lambda\partial^2 V)](x)\, X\rangle\right\} \quad (5.11)$$

This process can be repeated as many times as we want so that

$$\frac{\partial}{\partial\lambda}\langle TX\rangle_\lambda\Big|_{\substack{\text{mass}\\ \text{shell}}} = 0 \quad (5.12)$$

which is the desired result. Note that, contrary to the usual proof, here it is not necessary to introduce anisotropic normal products.

In the usual normal product formalism[13] there is the so called differentiation formula which permits to pass a derivative through the normal product symbol increasing its degree by one unity. Here such a formula is also true but should be exercised with some care. Thus

$$\partial_\mu N[A^n \partial_\mu A] = N[A^n \partial^2 A] + N[\partial_\mu A^n \partial^\mu A] \qquad (5.13)$$

is verified but the distribution of momentum factors in the last normal product on the r.h.s. is not arbitrary. If p is the total momentum entering at the vertex $A^n\partial_\mu A$ and k+p is the momentum associated with the field $\partial_\mu A$ in this vertex, then the momentum -k.(k+p) should be associated to the line corresponding to $\partial_\mu A$ in the vertex $\partial_\mu A^n \partial^\mu A$.

The construction of currents in specific examples will be the object of future publications. Using properties like eq. (5.13) we will obtain Ward identities free from anomalies. These may not involve fermions neitherthe dilatation current. We cannot remove anomalies like that of the axial vector current in Q.E.D., since it is associated with the non comutability of the Dirac matrices. As far as the energy momentum tensor is concerned, it is generally impossible to construct a conserved and traceless second order tensor.

Footnotes and References

1) At no stage do we have to introduce regulators. In this sense our approach has no overlap with the very interesting work on dimensional regularization introduced by C.G.Bollini,J.J.Giambiagi and A.Gonzalez Dominguez, Nuovo Cimento 31, 550(1964) and used by G.'t Hooft and M.Veltman, Nuclear Physics B44, 189 (1972).

2) N.N.Bogoliubov and D.W.Shirkov, Introduction to the Theory of Quantized Fields, N.Y. Interscience Publs. 1959; K.Hepp,Commun. Math.Phys. 2,301 (1965); W.Zimmermann, Commun.Math.Phys.15,208(1969).

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4) M.Gomes, J.Lowenstein and W.Zimmerman,Commun.Math.Phys.39,81 (1974).

5) F.Jegerlehner,B.Schroer; Nucl.Phys. B68, 461(1976).

6) M.Gomes and B. Schroer, Phys.Rev. D10,3525 (1974).

7) J.Lowenstein and W.Zimmermann, Nucl.Phys. B 86,77 (1975).

8) See Y.P.M.Lam, Phys.Rev. D7, 2950 (1973) for a conventional treatment of the non-linear σ-model, where IR divergencies forbid one to take the zero mass limit.

9) We have for simplicity of notation not multiplied $\rho^{(0)}_{ij}$ and $1-\rho^{(0)}_{ij}$ by different polynomials in μ^2. For the two cases $\rho^{(0)}_{ij} = 1$ and $\rho^{(0)}_{ij} = 0$ considered explicitly in this paper this is not necessary.

10) We hope to come back for a complete classification of these theories from a physical point of view.

11) W.Zimmermann, Commun.Math.Phys. 15, 208 (1969).

12) Y.P.M.Lam, Phys.Rev. D7, 2943 (1973).

13) See for example J. Lowenstein, Seminars on Renormalization Theory, II,Univ.Maryland.Techn. Report. n° 73-068,1972.

FIGURE CAPTIONS

FIG. 8 - First order contribution to $\langle N[A^2\partial_\mu A](x)\,A(y)\rangle$.

Appendix

i) Let us calculate an example for $\langle N[A^2\partial_\mu A](x)\,A(y)\rangle$ to first order in g in the $g\,A^4$ model.

The relevant graph G is shown in fig. 8:

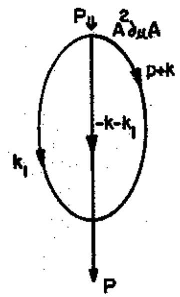

FIG.8

The unsubtracted integrand I_μ is:

$$I_\mu = \frac{(p+k)_\mu}{[(p+k)^2-m^2][(k+k_1)^2-m^2](k_1^2-m^2)} \tag{A1}$$

According to (4.2) we write:

$$B_1 = \frac{(p+k)_\mu}{(p+k)^2-m^2}\,,\quad B_2 = \frac{1}{(k+k_1)^2-m^2}\,,\quad B_3 = \frac{1}{k_1^2-m^2} \tag{A2}$$

Let us use a subtraction scheme, where we try to keep only one subtraction at $m^2=0$ for B_1 and all at $m^2=\mu^2$ for B_2 and B_3.

Consider first the forest G, with $D_G = 3$.

For B_1 we get:

$$1^{st} \text{ subtraction} \longrightarrow \frac{k_\mu}{k^2}$$

$$\partial_m \longrightarrow \frac{m^2 k_\mu}{k^2(k^2-\mu^2)} \tag{A3}$$

$$\partial_p \longrightarrow \frac{p_\mu}{k^2} - \frac{2k_\mu p\cdot k}{k^4} = p\cdot\partial_k \left\{ \frac{k_\mu}{k^2} \right\}$$

(Notice that routing independence forced two subtractions at $m^2 = 0$.)

$$\partial_m\partial_p \longrightarrow m^2 \left\{ \frac{p_\mu}{k^2(k^2-\mu^2)} - 2k_\mu p\cdot k \left[\frac{1}{k^4(k^2-\mu^2)} + \frac{1}{k^2(k^2-\mu^2)^2} \right] \right\} =$$

$$= m^2 p\cdot\partial_k \left\{ \frac{k_\mu}{k^2(k^2-\mu^2)} \right\}$$

$$\partial_p^2 \longrightarrow -\frac{2p_\mu p\cdot k + p^2 k_\mu}{k^2(k^2-\mu^2)} + \frac{4k_\mu (p\cdot k)^2}{k^2(k^2-\mu^2)^2}$$

$$\partial_p^3 \longrightarrow -\frac{p_\mu p^2}{k^2(k^2-\mu^2)} + \frac{p_\mu (p\cdot k)^2}{3}\left[\frac{4}{k^4(k^2-\mu^2)} + \frac{8}{k^2(k^2-\mu^2)^2}\right] +$$

$$+ \frac{k_\mu p^2 (p\cdot k)}{3}\left[\frac{2}{k^4(k^2-\mu^2)} + \frac{10}{k^2(k^2-\mu^2)^2}\right] -$$

$$-\frac{8}{3}k_\mu (p\cdot k)^3 \left[\frac{1}{k^4(k^2-\mu^2)^2} + \frac{2}{k^2(k^2-\mu^2)^3}\right] =$$

$$= \frac{1}{3} p\cdot\partial_k \left\{ -\frac{2p_\mu p\cdot k + p^2 k_\mu}{k^2(k^2-\mu^2)} + \frac{4k_\mu (p\cdot k)^2}{k^2(k^2-\mu^2)^2} \right\}$$

(Same comment as for ∂p.)

Using (A3) in equ. (4.4) we now obtain

$$\begin{aligned} \tau_G^{(3)} I_\mu = & \left\{ \frac{k_\mu}{k^2} + \frac{m^2 k_\mu}{k^2(k^2-\mu^2)} + p\cdot\partial_k\left(\frac{k_\mu}{k^2}\right) + m^2 p\cdot\partial_k\left(\frac{k_\mu}{k^2(k^2-\mu^2)}\right) + \right. \\ & \left. + \left(1+\frac{p\cdot\partial_k}{3}\right)\left(-\frac{2p_\mu p\cdot k + p^2 k_\mu}{k^2(k^2-\mu^2)} + \frac{4k_\mu (p\cdot k)^2}{k^2(k^2-\mu^2)^2}\right)\right\}\left\{\frac{1}{(k+k_1)^2-\mu^2}\right\}\left\{\frac{1}{k_1^2-\mu^2}\right\} + \\ & + (m^2-\mu^2)\left\{\frac{k_\mu}{k^2} + p\cdot\partial_k\left(\frac{k_\mu}{k^2}\right)\right\}\left\{\frac{1}{(k+k_1)^2-\mu^2}\frac{1}{(k_1^2-\mu^2)^2} + \frac{1}{[(k+k_1)^2-\mu^2]^2}\frac{1}{k_1^2-\mu^2}\right\} \end{aligned} \tag{A4}$$

It is now easy to write down the subtractions, corresponding to the other six forests.

Define the subgraphs γ_1, γ_2 and γ_3 by cutting line B_1, B_2 and B_3 respectively. They will be linearly divergent.

We thus obtain the following subtractions:

$$\tau_{\gamma_1}^{(1)} I_\mu = \frac{(p+k)_\mu}{(p+k)^2-m^2}\left\{\frac{1}{k_1^2-\mu^2} + \frac{2(p+k)\cdot k_1}{(k_1^2-\mu^2)^2}\right\}\frac{1}{k_1^2-\mu^2}$$

$$\begin{aligned} \tau_{G\gamma_1}^{(3)} I_\mu = & \left\{ \frac{k_\mu}{k^2} + \frac{m^2 k_\mu}{k^2(k^2-\mu^2)} + p\cdot\partial_k\left(\frac{k_\mu}{k^2}\right) + m^2 p\cdot\partial_k\left(\frac{k_\mu}{k^2(k^2-\mu^2)}\right) + \right. \\ & \left. + \left(1+\tfrac{1}{3}p\cdot\partial_k\right)\left(-\frac{2p_\mu p\cdot k + p^2 k_\mu}{k^2(k^2-\mu^2)} + \frac{4k_\mu (p\cdot k)^2}{k^2(k^2-\mu^2)^2}\right)\right\}\left\{\frac{1}{(k_1^2-\mu^2)^2} - \frac{2k\cdot k_1}{(k_1^2-\mu^2)^3}\right\} - \\ & - \frac{2p\cdot k_1}{(k_1^2-\mu^2)^3}\left\{(1+p\cdot\partial_k)\left(\frac{k_\mu}{k^2}\right) + m^2(1+p\cdot\partial_k)\left(\frac{k_\mu}{k^2(k^2-\mu^2)}\right) - \right. \end{aligned}$$

$$- \frac{2 p_\mu p \cdot k + p^2 k_\mu}{k^2 (k^2-\mu^2)} + \frac{4 k_\mu (p \cdot k)^2}{k^2 (k^2-\mu^2)^2} \Big\}$$

$$\tau^{(1)}_{\gamma_2} I_\mu = \frac{1}{(k+k_1)^2-m^2} \Big\{ \frac{k_\mu}{k^2} + (p+k+k_1) \cdot \partial_k \Big(\frac{k_\mu}{k^2} \Big) \Big\} \frac{1}{k_1^2-\mu^2}$$

$$\tau^{(3)}_{G\gamma_2} I_\mu = \Big\{ \frac{1}{(k+k_1)^2-\mu^2} + \frac{m^2-\mu^2}{[(k+k_1)^2-\mu^2]^2} \Big\} \Big\{ \frac{k_\mu}{k^2} +$$

$$+ (p+k+k_1) \cdot \partial_k \Big(\frac{k_\mu}{k^2} \Big) \Big\} \frac{1}{k_1^2-\mu^2}$$

(A5)

$$\tau^{(1)}_{\gamma_3} I_\mu = \frac{1}{k_1^2-m^2} \Big\{ \frac{k_\mu}{k^2} + p \cdot \partial_k \Big(\frac{k_\mu}{k^2} \Big) \Big\} \Big\{ \frac{1}{k^2-\mu^2} - \frac{2 k_1 \cdot k}{(k^2-\mu^2)^2} \Big\}$$

$$\tau^{(3)}_{G\gamma_3} I_\mu = \Big\{ \frac{1}{k_1^2-\mu^2} + \frac{m^2-\mu^2}{(k_1^2-\mu^2)^2} \Big\} \Big\{ \frac{k_\mu}{k^2} + p \cdot \partial_k \Big(\frac{k_\mu}{k^2} \Big) \Big\} \cdot$$

$$\cdot \Big\{ \frac{1}{k^2-\mu^2} - \frac{2 k_1 \cdot k}{(k^2-\mu^2)^2} \Big\}$$

ii) Let us now verify the equation satisfied by $N\,[A^2\, \partial^2 A]\,(x)$ for the same graph of fig. 1.

The unsubtracted integrand I is

$$I = \frac{(p+k)^2}{(p+k)^2-m^2} \; \frac{1}{(k+k_1)^2-m^2} \; \frac{1}{k_1^2-m^2} \qquad \text{(A6)}$$

Instead of (A2) we now have for B_i:

$$B_1 = \frac{(p+k)^2}{(p+k)^2-m^2}, \quad B_2 = \frac{1}{(k+k_1)^2-m^2}, \quad B_3 = \frac{1}{k_1^2-m^2} \qquad \text{(A7)}$$

For the forest G we obtain the subtractions, with $D_G = 4$:

$\underline{B_1}$:

1^{st} subtraction $\longrightarrow$ 1

$\partial_p, \partial_p^2, \partial_p^3, \partial_p^4 \longrightarrow 0$

$\partial_m \longrightarrow \dfrac{m^2}{k^2-\mu^2}$

(we also could have chosen $\frac{m^2}{k^2}$,but let us try to obtain a Taylor - operator around $m^2 = \mu^2$ for $N(A^3)$)

$$\partial_m \partial_p \longrightarrow -\frac{2m^2 p\cdot k}{(k^2-\mu^2)^2}$$

$$\partial_m \partial_p^2 \longrightarrow -m^2\left[\frac{p^2}{(k^2-\mu^2)^2}-\frac{4(p\cdot k)^3}{(k^2-\mu^2)^3}\right] =$$

$$= m^2 (p\cdot\partial_k)^2 \left(\frac{1}{k^2-\mu^2}\right) \qquad \text{(A8)}$$

$$\partial_m^2 \longrightarrow \frac{m^2(m^2-\mu^2)}{(k^2-\mu^2)^2}$$

In order to treat all three lines symmetrically in $N(A^3)$, we use Taylor operators around $m^2=\mu^2$ also for B_2, B_3. This yields

$$\tau_G^{(4)} I = \left[\frac{1}{k_1^2-\mu^2}\, t_{m^2}^{(4)} \frac{1}{(k+k_1)^2-m^2} + \frac{m^2-\mu^2}{(k_1^2-\mu^2)^2}\, t_{m^2}^{(2)} \frac{1}{(k+k_1)^2-m^2} + \right.$$

$$\left. + \frac{(m^2-\mu^2)^3}{(k_1^2-\mu^2)^3} \frac{1}{(k+k_1)^2-\mu^2}\right] + m^2 \left\{ \frac{1}{k^2-\mu^2}\left[\frac{1}{(k+k_1)^2-\mu^2}\, t_{m^2}^{(2)} \frac{1}{k_1^2-m^2} + \right.\right.$$

$$\left. + \frac{m^2-\mu^2}{[(k+k_1)^2-\mu^2]^2} \frac{1}{k_1^2-\mu^2}\right] + \left[-\frac{2p\cdot k}{k^2-\mu^2} + \right. \qquad \text{(A9)}$$

$$\left.\left. + (p\cdot\partial_k)^2 \frac{1}{k^2-\mu^2} + \frac{m^2-\mu^2}{(k^2-\mu^2)^2}\right]\left[\frac{1}{(k+k_1)^2-\mu^2}\frac{1}{k_1^2-\mu^2}\right]\right\}$$

For the other forests we get:

$$\tau_{\gamma_1}^{(2)} I = t_{p+k,m^2}^{(2)}\left(\frac{1}{(k+k_1)^2-m^2}\right)\frac{1}{k_1^2-\mu^2} + \frac{1}{(k+k_1)^2-\mu^2}\frac{1}{k_1^2-\mu^2} +$$

$$+ \frac{m^2}{(p+k)^2-m^2}\frac{1}{k_1^2-\mu^2}\frac{1}{(k+k_1)^2-\mu^2}$$

$$\tau_{G\gamma_1}^{(4)} I = \left(t_{p+k,m^2}^{(2)}\frac{1}{(k+k_1)^2-m^2}\right)\frac{1}{k_1^2-\mu^2} + \frac{1}{(k+k_1)^2-\mu^2}\frac{1}{k_1^2-\mu^2} +$$

$$+ m^2\, t_{p,m^2}^{(2)}\left(\frac{1}{(p+k)^2-m^2}\right)\frac{1}{k_1^2-\mu^2}\frac{1}{(k+k_1)^2-\mu^2} \qquad \text{(A10)}$$

$$\tau_{\gamma_2}^{(2)} I = \frac{1}{(k+k_1)^2-m^2}\left[\frac{1}{k_1^2-\mu^2} + \frac{m^2-\mu^2}{(k_1^2-\mu^2)^2} + m^2\frac{1}{k^2-\mu^2}\frac{1}{k_1^2-\mu^2}\right]$$

$$\tau_{G\gamma_2}^{(4)} I = \frac{1}{k_1^2-\mu^2}\, t_{m^2}^{(4)}\frac{1}{(k+k_1)^2-m^2} + \left[\frac{m^2-\mu^2}{(k_1^2-\mu^2)^2} + \right.$$

$$\left. + m^2\frac{1}{k^2-\mu^2}\frac{1}{k_1^2-\mu^2}\right] t_{m^2}^{(2)}\frac{1}{(k+k_1)^2-m^2}$$

$$\tau_{\gamma_3}^{(2)} I = \frac{1}{k_1^2-m^2}\left[t_{k_1,m^2}^{(2)}\frac{1}{(k+k_1)^2-m^2} + m^2\frac{1}{k_1^2-\mu^2}\frac{1}{k^2-\mu^2}\right]$$

$$\tau_{G\gamma_3}^{(4)} I = \frac{1}{(k+k_1)^2-\mu^2}\, t_{m^2}^{(4)}\frac{1}{k_1^2-m^2} + \left[\frac{m^2-\mu^2}{(k^2-\mu^2)^2} - \frac{2k_1\cdot k}{(k^2-\mu^2)^2} + \right.$$

$$\left. + m^2\frac{1}{(k^2-\mu^2)^2}\right] t_{m^2}^{(2)}\frac{1}{k_1^2-m^2} \quad ,$$

where we have introduced the notation $t^{(n)}_{p,m^2}$ for the Taylor operator around $p = 0$ and $m^2=\mu^2$ up to order n.

All terms with at least one derivative ∂_m on line B_1 contribute to $m^2 < N(A^3)(x)\,A(y) >$ and yield a mass term subtracted with a Taylor operator around $m^2=\mu^2$. The rest contributes to $g < N(A^5)(x)A(y) >$ and gives

$$\left[\left(1-t^{(2)}_{m^2}\right)\frac{1}{k_1^2-m^2}\right]\left[\left(1-t^{(2)}_{k_1,m^2}\right)\frac{1}{(k+k_1)^2-m^2}\right] \qquad \text{(A11)}$$

After integration this gives the desired product structure.