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**Finite temperature gluon self-energy in a class of general temporal gauges**

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# Finite temperature gluon self-energy in a class of general temporal gauges

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The approach which relates thermal Green functions to forward scattering amplitudes of on-shell thermal particles is applied to the calculation of the gluon self-energy, in a class of general temporal gauges. We show to all orders that, unlike the case of general covariant gauges, the exact self-energy of the gluon is transverse at finite temperature. The leading $T^2$ and the sub-leading $\ln(T)$ contributions are obtained for temperatures $T$ high compared with the external momentum. The logarithmic contributions have the same structure as the ultraviolet pole terms which occur at zero temperature.

## I. INTRODUCTION

Thermal gauge field theories in the temporal gauge has been the subject of many investigations both in the imaginary and in the real time formalisms [1–6]. One of the main advantages of the *non-covariant* temporal gauge is that it is physical and effectively ghost-free. At finite temperature, it may be considered a more natural choice, since the Lorentz invariance is already broken by the presence of the heat bath. It is also convenient for calculating the response of the QCD plasma to a chromo-electric field, since in this case only the gluon self-energy is needed (in this gauge, the chromo-electric field depends only linearly on the gauge field $A_\mu^a$) [1,7]. Despite these advantages, explicit calculations of loop corrections to Green functions are known to be more complicated than in covariant gauges. One of the difficulty is associated with the more involved tensorial structure of the propagator (see Eq. (1)). The other more fundamental problem is how to deal with the extra poles at $q \cdot n = 0$ in the gluon propagator, where $q$ is the loop momentum and $n = (n_0, \vec{0})$ $(n_0^2 > 0)$ is the *temporal axial* four-vector.

In the imaginary time formalism, the standard method of calculation of thermal Green functions consists in replacing the Matsubara frequency sum by a *contour integral* in the complex plane [7,8]. Using this approach, all Green functions, at one-loop order, can be written as the sum of the *vacuum part* plus the a *thermal part* which involves the Bose-Einstein (boson loop) or Fermi-Dirac (fermion loop) statistical distributions.

In the case of the temporal gauge, the extra poles at $q \cdot n = 0$ in the gluon propagator demand more care and make the standard *contour integral method* of calculation very involved. In order to deal with this problem, Leibbrandt and Staley [6] have developed a technique for performing perturbative calculations at finite temperature in the temporal gauge (see also reference [3] for a different approach to this problem). Employing a special version of the *Mandelstam-Leibbrandt prescription* [9,10] combined with the *$\zeta$-function method* [11], they have obtained the *complete* $1/T$-expansion for the one-loop self-energy component $\Pi_{00}^{ab}(k_0 = 0, \vec{k})$. Their results show that the leading and sub-leading contributions to the self-energy can be unambiguously determined in the temporal gauge, and are in agreement with the standard *contour integral* method employed in reference [1].

The temperature dependent part of Green functions can be described in terms of *forward scattering amplitudes* of on-shell thermal particles of the thermal medium. This idea was described in reference [12] and has been further elaborated in the *Feynman gauge* and shown to be very useful for determining the partition function in QCD at high temperature [13]. More recently this result has been generalized to the case of *general covariant gauges* [14]. This method has been derived using both the imaginary time formalism and the real time formalism up to two-loop order [15,16]. There is also an interesting relation with the path integral approach [17].

One of the purposes of this work is to extend the forward scattering method to a class of general temporal gauges. The main advantage of this method is that in the high-temperature limit, it is straightforward to obtain the *full tensor structure* of both the leading $T^2$ and the sub-leading logarithmic contributions for temperatures $T$ such that $T \gg k$, where $k$ denotes the external four-momentum. For the leading $T^2$ part we obtain the well known gauge invariant *hard thermal loop* result. The *gauge dependent* sub-leading logarithmic part shares with the previous calculations in general covariant gauges [14] the interesting property of having the same structure as the *ultraviolet pole contributions* which occur at zero temperature. Another purpose of this work is to study, in the axial gauges, the transversality property of the thermal gluon self-energy. As is well known [2,14,18], in general covariant gauges, the exact self-energy of the gluon is not transverse at finite temperature.

In section II we derive, in a class of general temporal gauges, the full tensor structure of the forward scattering amplitude associated with the gluon self-energy. In section III we show that, in contrast to the behavior in general