Gauge Models in Modified Triplectic Quantization

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The modified triplectic quantization is applied to several well-known gauge models: the Freedman-Townsend model of non-abelian antisymmetric tensor fields, W_2-gravity, and 2D gravity with dynamical torsion. For these models we obtain explicit solutions of those generating equations that determine the quantum action and the gauge-fixing functional. Using these solutions, we construct the vacuum functional, determine the $Sp(2)$–invariant effective actions and obtain the corresponding transformations of extended BRST symmetry.

1. Introduction

In recent years the development of covariant quantization rules for general gauge theories on the basis of extended BRST symmetry has become increasingly popular [1] – [11].

The realization of the principle of extended BRST symmetry, combining BRST [12] and anti-BRST [13] transformations, naturally unifies the treatment of auxiliary variables that serve to parametrize the gauge in the functional integral and those that enter the quantum action determined by the corresponding generating equations. Basically, the above tendency manifests itself in enlarging the configuration space of the quantum action with auxiliary gauge-fixing variables (see, e.g., [1, 2, 3]). Recently, however, it has been strengthened by extending the concept of generating equations also to the case of introducing the gauge [2, 6].

The method of $Sp(2)$ covariant quantization [1] was one of the first to provide a realization of the extended BRST symmetry for general gauge theories, i.e., theories of any stage of reducibility with a closed or open algebra of gauge transformations. The complete configuration space ϕ^A of a gauge theory, considered in this approach, is constructed by the rules of the BV quantization [14] and consists of the initial classical fields supplemented by the pyramids of auxiliary variables, i.e., ghosts, antighosts and Lagrange multipliers, according to the corresponding stage of reducibility. Even though these auxiliary variables originally [14] play different roles in the construction of the quantum theory, their consideration within the $Sp(2)$ covariant formalism allows to achieve a remarkable uniformity

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of description. Namely, in the framework of the $Sp(2)$ covariant approach, the pyramids of ghosts are combined with the corresponding pyramids of antighosts and Lagrange multipliers into irreducible representations of the group $Sp(2)$, which form completely symmetric $Sp(2)$ tensors and enter the quantum theory on equal footing in terms of both the quantum action and the gauge fixing functional. The quantum action of the $Sp(2)$ covariant formalism depends on an extended set of variables, including, besides the fields ϕ^A, also the sets of antifields ϕ^*_{Aa} and $\bar{\phi}_A$. Note that in the case of linear dependence of the quantum action on ϕ^*_{Aa} and $\bar{\phi}_A$ they may be interpreted as sources of extended BRST transformations and sources of mixed BRST and anti-BRST transformations, respectively.

In [3], a consistent superfield formulation of the $Sp(2)$ covariant rules was proposed. This approach allows to combine all the variables of the $Sp(2)$ covariant formalism, namely, the fields and antifields $(\phi^A, \phi^*_{Aa}, \bar{\phi}_A)$ that enter the quantum action, the auxiliary variables (π^{Aa}, λ^A) that serve to parametrize the gauge and, finally, the sources J_A to the fields ϕ^A, into superfields $\Phi^A = \phi^A + \pi^{Aa}\theta_a + \frac{1}{2}\lambda^A\theta_a\theta^a$ and superantifields (supersources) $\bar{\Phi}_A = \bar{\phi}_A - \theta^a\phi^*_{Aa} - \frac{1}{2}\theta_a\theta^a J_A$ defined on a superspace with two scalar Grassmann coordinates. The quantum action $S(\Phi^A, \bar{\Phi}_A)$ of that theory is defined as a functional of superfields and superantifields, which makes it possible to realize the transformations of extended BRST symmetry in terms of supertranslations along the Grassmann coordinates.

Moreover, in a recent paper [4] the superspace approach has been extended by considering not only the (sub)group of translations but instead to consider the full group of conformal transformations on that superspace of two Grassmann coordinates. The generators of this conformal group span an algebra being isomorphic to the superalgebra $sl(1,2)$. In that approach it is possible to consider massive gauge theories by introducing mass-dependent BRST and antiBRST operators which are related to translations coupled (with a factor m^2) to special conformal transformations. Furthermore, the $Sp(2)$–symmetry – including ghost number conservation – and the symmetry which underlies the "new ghost number" conservation are realized as (symplectic) rotations and dilatations, respectively.

In the framework of the triplectic quantization [2] another modification of the $Sp(2)$ covariant approach was proposed, based on a different extension of the configuration space of the quantum action. Namely, it was suggested to consider the auxiliary fields π^{Aa} as variables anticanonically conjugated to the antifields $\bar{\phi}_A$ with the corresponding redefinition of the extended antibrackets [1] which appear in the generating equations for the quantum action. Another feature of the triplectic formalism is that the gauge-fixed part of the action in the functional integral is determined by generating equations formally similar to the equations that describe the quantum action. The entire set of variables necessary for the construction of the vacuum functional in the triplectic formalism coincides with the corresponding set of the $Sp(2)$ covariant approach and is composed by the fields (ϕ^A, ϕ^*_{Aa}) and $(\pi^{Aa}, \bar{\phi}_A)$ anticanonically conjugated to each other in the sense of modified antibrackets, as well as by the remaining auxiliary fields λ^A that serve to parametrize the gauge-fixing functional.

In the recent paper [6] a modification of the triplectic formalism was proposed whose parts being essential for the present consideration will be briefly reviewed. While retaining the space of variables of the triplectic formalism and accepting the idea of imposing generating equations on both the quantum action and the gauge-fixing functional, it was suggested to modify the system of these equations, as well as to adjust the definition of the vacuum functional, in order to *ensure the correct boundary conditions for the quantum action*,

$$S|_{\Phi^*=\bar{\phi}=\hbar=0} = S_0.$$

This allows to take into account in a direct manner the information contained in the classical action. It also implies that the classical action of a theory satisfies (in the limit

$\hbar \to 0$) the generating equations for the quantum action $S(\phi, \phi^*_a, \pi^a, \bar{\phi})$,

$$\frac{1}{2}(S,S)^a + V^a S = i\hbar \Delta^a S, \tag{1.1}$$

in complete analogy with earlier quantization schemes, and in contrast to the original triplectic formalism [2]. The gauge fixing functional $X(\phi, \phi^*_a, \pi^a, \bar{\phi}; \lambda)$, of the modified triplectic formalism satisfies similar generating equations,

$$\frac{1}{2}(X,X)^a - U^a X = i\hbar \Delta^a X. \tag{1.2}$$

The above systems of generating equations are expressed in terms of the differential operators

$$\begin{aligned}
(F,G)^a &= \left(\frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi^*_{Aa}} + \varepsilon^{ab} \frac{\delta F}{\delta \pi^{Aa}} \frac{\delta G}{\delta \bar{\phi}_A} \right) - (F \leftrightarrow G)(-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)}, \\
\Delta^a &= (-1)^{\varepsilon_A} \frac{\delta_l}{\delta \phi^A} \frac{\delta}{\delta \phi^*_{Aa}} + (-1)^{\varepsilon_A + 1} \varepsilon^{ab} \frac{\delta_l}{\delta \pi^{Ab}} \frac{\delta}{\delta \bar{\phi}_A}, \\
V^a &= \varepsilon^{ab} \phi^*_{Ab} \frac{\delta}{\delta \bar{\phi}_A}, \qquad U^a \;=\; (-1)^{\varepsilon_A + 1} \pi^{Aa} \frac{\delta_l}{\delta \phi^A},
\end{aligned}$$

where the derivatives with respect to the antifields are taken from the left, and ε^{ab} is the antisymmetric tensor with the normalization $\varepsilon^{12} = 1$. The operators V^a and U^a are closely related to operators which were introduced earlier in the framework of the superfield formalism [3], and which have a clear geometrical meaning as generators of supertranslations in superspace.

Given the quantum action S and the gauge-fixing functional X, the vacuum functional $Z \equiv Z(J = 0)$ in the framework of the modified triplectic quantization [6] is defined by

$$Z = \int d\phi \, d\phi^* d\pi \, d\bar{\phi} \, d\lambda \exp \left\{ \frac{i}{\hbar} \left(S + X + \phi^*_{Aa} \pi^{Aa} \right) \right\}. \tag{1.3}$$

Let us note that the following choice of the gauge fixing functional $X = X(\phi, \pi^a, \bar{\phi}; \lambda)$,

$$X = \left(\bar{\phi}_A - \frac{\delta F}{\delta \phi^A} \right) \lambda^A - \frac{1}{2} \epsilon_{ab} U^a U^b F, \qquad F = F(\phi), \tag{1.4}$$

solves eqs. (1.2) (with ΔX being identical zero). Then, the integrand of eq. (1.3) is invariant under the following transformations (cf. Ref. [6])

$$\begin{aligned}
\delta \phi^a &= - \left(\frac{\delta S}{\delta \phi^*_{Aa}} - \pi^{Aa} \right) \mu_a, \\
\delta \phi^*_{Aa} &= \mu_a \left(\frac{\delta S}{\delta \phi^A} + \frac{\delta^2 F}{\delta \phi^A \delta \phi^B} \lambda^B + (-1)^{\epsilon_A} \frac{1}{2} \epsilon_{bc} \pi^{Bb} \frac{\delta^3 F}{\delta \phi^A \delta \phi^B \delta \phi^C} \phi^{Cc} \right), \\
\delta \pi^{Aa} &= \epsilon^{ab} \left(\frac{\delta S}{\delta \bar{\phi}_A} - \lambda^A \right) \mu_b, \\
\delta \bar{\phi}_A &= \mu_a \epsilon^{ab} \left(\frac{\delta S}{\delta \pi^{Ab}} + \phi^*_{Ab} \right) + \mu_a \frac{\delta^2 F}{\delta \phi^A \delta \phi^B} \pi^{Ba}, \\
\delta \lambda^A &= 0.
\end{aligned} \tag{1.5}$$

Here, μ_a is a doublet of anticommuting constant parameters. If, in addition, $S = S(\phi, \phi^*, \bar{\phi})$ is assumed not to depend on π^a then, obviously, Eq. (1.1) reduces and the vacuum functional (1.3) coincides with that of the $Sp(2)$–covariant formalism.

The aim of this paper is to apply the above reviewed prescriptions of the modified triplectic formalism for quantizing several gauge theory models.

In Section 2, we consider the model of antisymmetric tensor field suggested by Freedman and Townsend [15]. The Freedman–Townsend (FT) model is an abelian gauge theory of first stage reducibility. The corresponding complete configuration space is constructed by the rules of the $Sp(2)$ covariant formalism [1] for reducible gauge theories. In the case of the FT model, the generating equations (1.1) that determine the quantum action in the framework of the modified triplectic formalism can be solved exactly, which allows one to obtain the exact form of the vacuum functional in terms of the effective $Sp(2)$–invariant action S_{eff} and the transformations of extended BRST symmetry.

In Sections 3 and 4 we consider the gauge models of W_2-gravity [16] and of two-dimensional gravity with dynamical torsion [17], respectively. Both these models are examples of irreducible gauge theories with a closed algebra, and their configuration spaces are constructed by the rules of the $Sp(2)$ covariant quantization for irreducible theories. In order to obtain closed solutions of the above generating equations that determine the quantum action in the case of these gauge models, one has to introduce some regularization. Unfortunately, the regularization which reduces all terms containing $\delta(0)$ to zero (see, e.g., [1] cannot be used here since both models under consideration are strictly two-dimensional and, therefore, it is not possible to use dimensional regularization which is equivalent to that procedure. Instead, one could use Pauli-Villars regularization as has been done in Ref. [18] in the case of W_3–gravity. As an intermediate step we consider here only the tree approximation, i.e. we determine proper solutions of the classical master equations. With them we obtaine a closed form of the vacuum functional and the corresponding transformations of extended BRST symmetry as well as the related effective action which depends on the fields only.

2. Freedman–Townsend Model

The theory of a non-abelian antisymmetric field $H^p_{\mu\nu}$, suggested by Freedman and Townsend [15], is described (in the first order formalism) by the action [3]

$$S_0(A^p_\mu, H^p_{\mu\nu}) = \int d^4x \left(-\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma} F^p_{\mu\nu} H^p_{\rho\sigma} + \frac{1}{2} A^p_\mu A^{p\mu} \right), \tag{2.1}$$

where A^p_μ is an (auxiliary) gauge field with the strength $F^p_{\mu\nu} = \partial_\mu A^p_\nu - \partial_\nu A^p_\mu + f^{pqr} A^q_\mu A^r_\nu$ (the coupling constant is absorbed into the structure coefficients f^{pqr}), the Levi-Civita tensor $\varepsilon^{\mu\nu\rho\sigma}$ is normalized as $\varepsilon^{0123} = 1$. Eliminating the auxiliary field A^p_μ through its field equations leads to the more complicated action of the second order formalism [15].

The action (2.1) is invariant under the gauge transformations

$$\delta A^p_\mu = 0, \quad \delta H^p_{\mu\nu} = \mathcal{D}^{pq}_\mu \xi^q_\nu - \mathcal{D}^{pq}_\nu \xi^q_\mu \equiv \mathcal{R}^{pq}_{\mu\nu\alpha} \xi^{q\alpha}, \tag{2.2}$$

where ξ^p_μ are arbitrary parameters, and $\mathcal{D}^{pq}_\mu$ is the covariant derivative with the gauge field A^p_μ ($\mathcal{D}^{pq}_\mu = \delta^{pq}\partial_\mu + f^{prq} A^r_\mu$).

The gauge transformations (2.2) form an abelian algebra with the generators $R^{pq}_{\mu\nu\alpha}$. These gauge transformations are not all independent, i.e. for $\xi^p_\nu = \mathcal{D}^{qp}_\nu \xi^p$ they vanish

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[3] We denoted the antisymmetric tensor field contrary to the usual conventions by H in order to avoid confusion with the auxiliary fields to be introduced below.

on-shell. Therefore, at the extremals of the action (2.1) the generators $\mathcal{R}^{pq}_{\mu\nu\alpha}$ have zero modes $\mathcal{Z}^{pq}_{\mu} \equiv \mathcal{D}^{pq}_{\mu}$, namely,

$$\mathcal{R}^{pq}_{1\mu\nu} \equiv \mathcal{R}^{pr}_{\mu\nu\alpha}\mathcal{Z}^{rq\alpha} = \varepsilon_{\mu\nu\alpha\beta}f^{prq}\frac{\delta S_0}{\delta H^r_{\alpha\beta}}\,, \tag{2.3}$$

which are linearly independent. According to the generally accepted terminology, the model (2.1), (2.2) and (2.3) is an abelian gauge theory of first stage reducibility.

Note that the gauge structure of the FT model [15] is similar to that of the Witten string [19]. The FT model also has been proved to be a convenient conceptual laboratory for the study of the S-matrix unitarity in the framework of covariant quantization [23]. There, it was shown that the application of the BV quantization rules to the model leads to a physically unitary theory being equivalent to a non-linear σ-model in $d = 4$ dimensions [15]. Note also that various aspects of the quantization of the FT model in the framework of standard BRST symmetry have been discussed in Refs. [20, 24].

Now, let us consider the reducible gauge model (2.1), (2.2) and (2.3) in the framework of the modified triplectic quantization.

To this end, we first introduce the complete configuration space ϕ^A, which is constructed according to the standard prescriptions of the $Sp(2)$ covariant formalism [1] for reducible gauge theories. Namely, the space of the variables ϕ^A consists of the initial classical fields $A^{p\mu}$ and $H^{p\mu\nu}$, supplemented, firstly, by $Sp(2)$ doublets of Faddeev–Popov ghosts, C^{pa}_{μ}, introduced according to the gauge parameters ξ^p_μ in eq. (2.2), secondly, by additional sets of first-stage ghost fields, C^{pab}, being symmetric $Sp(2)$ tensors, introduced according to the gauge parameters ξ^p for the generators $R^{pq}_{1\mu\nu}$ in eq. (2.3), and, finally, by sets of auxiliary fields (Lagrange multipliers) B^p_μ, corresponding to the gauge parameters ξ^p_μ, and the first-stage $Sp(2)$ doublets B^{pa}, corresponding to the parameters ξ^p.

The fields ϕ^A of the complete configuration space take values in the adjoint representation of a non-abelian gauge group (in the following the index $p = 1, \dots, N$ will be omitted)

$$\phi^A = (A^\mu,\, H^{\mu\nu};\; B^\mu,\, B^a;\; C^{\mu a},\, C^{ab}).$$

The Grassmann parities of the fields ϕ^A are given by

$$\varepsilon(A^\mu) = \varepsilon(H^{\mu\nu}) = \varepsilon(B^\mu) = \varepsilon(C^{ab}) = 0, \qquad \varepsilon(B^a) = \varepsilon(C^{\mu a}) = 1.$$

In accordance with the quantization rules [6], the set of the fields ϕ^A is supplemented by corresponding sets of variables ϕ^*_{Aa}, π^{Aa} and $\bar{\phi}_A$

$$\begin{aligned}
\phi^*_{Aa} &= (A^*_{\mu a}, H^*_{\mu\nu a};\; B^*_{\mu a}, B^*_{a|b};\; C^*_{\mu a|b}, C^*_{a|bc}),\\
\pi^{Aa} &= (\pi^{\mu a}_{(A)}, \pi^{\mu\nu a}_{(H)};\; \pi^{\mu a}_{(B)}, \pi^{a|b}_{(B)};\; \pi^{\mu a|b}_{(C)}, \pi^{a|bc}_{(C)}),\\
\bar{\phi}_A &= (\bar{A}_\mu, \bar{H}_{\mu\nu};\; \bar{B}_\mu, \bar{B}_a;\; \bar{C}_{\mu a}, \bar{C}_{ab}),
\end{aligned}$$

as well as by the auxiliary variables λ^A

$$\lambda^A = (\lambda^\mu_{(A)},\, \lambda^{\mu\nu}_{(H)};\; \lambda^\mu_{(B)},\, \lambda^a_{(B)};\; \lambda^{\mu a}_{(C)},\, \lambda^{ab}_{(C)}),$$

with the following Grassmann parities:

$$\varepsilon(\phi^*_{Aa}) = \varepsilon(\pi^{Aa}) = \varepsilon(\phi^A) + 1, \quad \varepsilon(\bar{\phi}_A) = \varepsilon(\lambda^A) = \varepsilon(\phi^A). \tag{2.4}$$

The attribution of ghost numbers to the fields and auxiliary variables can be made by the rule that $a = 1$ and $a = 2$ bears ghost number $+1$ and -1, respectively, for *upper* indices, as well as -1 and $+1$, respectively, for *lower* indices. The "external" index a on the variables ϕ^*_{Aa} and π^{Aa} is independent from the (symmetrized) "internal" ones and, therefore, separated by a vertical stroke $''|''$.

A proper solution of the generating equations (1.1) for the model in question can be found in a closed form as follows:

$$\begin{aligned} S = &\int d^4x \left(-\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}H_{\rho\sigma} + \frac{1}{2}A_\mu A^\mu\right) \\ &+ \int d^4x \left\{ H^*_{\mu\nu a}(\mathcal{D}^\mu C^{\nu a} - \mathcal{D}^\nu C^{\mu a}) - \varepsilon^{ab}C^*_{\mu a|b}B^\mu + \bar{H}_{\mu\nu}(\mathcal{D}^\mu B^\nu - \mathcal{D}^\nu B^\mu) \right. \\ &\qquad + C^*_{\mu a|b}\mathcal{D}^\mu C^{ab} - \varepsilon^{ab}C^*_{a|bc}B^c - \frac{1}{2}B^*_{\mu a}\mathcal{D}^\mu B^a + \bar{C}_{\mu a}\mathcal{D}^\mu B^a \\ &\qquad \left. + \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(H^*_{\mu\nu a} \wedge H^*_{\rho\sigma b})C^{ab} - \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(H^*_{\mu\nu a} \wedge \bar{H}_{\rho\sigma})B^a \right\}, \end{aligned} \tag{2.5}$$

where, after having omitted the gauge indices we use the notation $A^pB^p = AB$, $\mathcal{D}_\mu B = \partial_\mu B + A_\mu \wedge B$, $(A \wedge B)^p = f^{pqr}A^qB^r$. Note, that the antifields related to H also appear bilinear in that action. This action, after an appropriate renaming of the various variables, coincides with the corresponding action of [25] (see eq. (31) there).

An minimal admissible solution of the generating equation (1.2) for the gauge-fixing functional X can be represented as

$$\begin{aligned} X \;=\; &\int d^4x \Big\{ (\bar{H}_{\mu\nu} + \frac{\alpha}{2}H_{\mu\nu})\lambda^{\mu\nu}_{(H)} + \left(\bar{C}_{\mu a} - \beta\varepsilon_{ab}C^b_\mu\right)\lambda^{\mu a}_{(C)} \\ &\qquad + \frac{\alpha}{4}\varepsilon_{ab}\pi^{a}_{(H)\mu\nu}\pi^{\mu\nu b}_{(H)} - \frac{\beta}{2}\varepsilon_{ab}\varepsilon_{cd}\pi_{(C)\mu}{}^{a|c}\pi^{\mu b|d}_{(C)} \Big\}, \end{aligned} \tag{2.6}$$

where F has been chosen as $F(H_{\mu\nu}, C_{\mu a}) = -\frac{1}{2}\alpha H_{\mu\nu}H^{\mu\nu} + \beta\epsilon_{ab}C^b_\mu C^{a\mu}$ with α and β being constant parameters.

Now, substituting the solutions of S, eq. (2.5), and X, eq. (2.6), into eq. (1.3), we obtain the corresponding vacuum functional Z, with the integrand according to (1.5) being invariant under the following symmetry transformations

$$\begin{aligned} \delta A^\alpha &= \pi^{\alpha a}_{(A)}\mu_a, \\ \delta H^{\alpha\beta} &= \pi^{\alpha\beta a}_{(H)}\mu_a - \left(\mathcal{D}^\alpha C^{\beta a} - \mathcal{D}^\beta C^{\alpha a} + \varepsilon^{\alpha\beta\gamma\delta}H^*_{\gamma\delta b} \wedge C^{ab} - \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}\bar{H}_{\gamma\delta} \wedge B^a\right)\mu_a, \\ \delta B^\alpha &= \pi^{\alpha a}_{(B)}\mu_a + \frac{1}{2}\mathcal{D}^\alpha B^a\mu_a, \\ \delta B^a &= \pi^{b|a}_{(B)}\mu_b, \\ \delta C^{\alpha a} &= \pi^{\alpha b|a}_{(C)}\mu_b - \left(\varepsilon^{ab}B^\alpha + \mathcal{D}^\alpha C^{ab}\right)\mu_b, \\ \delta C^{ab} &= \pi^{c|ab}_{(C)}\mu_c + \frac{1}{2}\varepsilon^{c\{a}B^{b\}}\mu_c, \\ \delta A^*_{\alpha a} &= \mu_a\Big(A_\alpha - \frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}\mathcal{D}^\beta H^{\gamma\delta} - 2H^*_{\alpha\beta b} \wedge C^{\beta b} - 2\bar{H}_{\alpha\beta} \wedge B^\beta \\ &\qquad - C^*_{\alpha c|b} \wedge C^{cb} + \left(\frac{1}{2}B^*_{\alpha b} - \bar{C}_{\alpha b}\right) \wedge B^b\Big), \\ \delta H^*_{\alpha\beta a} &= -\mu_a\left(\frac{1}{4}\varepsilon_{\alpha\beta\gamma\delta}F^{\gamma\delta} + \frac{\alpha}{2}\lambda_{(H)\alpha\beta}\right), \\ \delta B^*_{\alpha a} &= \mu_a\left(2\mathcal{D}^\beta\bar{H}_{\alpha\beta} - \varepsilon^{cb}C^*_{\alpha c|b}\right), \end{aligned}$$

$$
\begin{aligned}
\delta B^{*}_{a|b} &= \mu_a\Big(\varepsilon^{cd}C^{*}_{c|bd}+\frac{1}{2}\mathcal{D}^{\alpha}B^{*}_{\alpha b}-\mathcal{D}^{\alpha}\bar{C}_{\alpha b}-\frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}(H^{*}_{\alpha\beta b}\wedge\bar{H}_{\gamma\delta})\Big),\\
\delta C^{*}_{\alpha a|b} &= \mu_a\Big(2\mathcal{D}^{\beta}H^{*}_{\alpha\beta b}+\beta\varepsilon_{bc}\lambda^{c}_{(C)\alpha}\Big),\\
\delta C^{*}_{a|bc} &= \mu_a\Big(-\frac{1}{2}\mathcal{D}^{\alpha}C^{*}_{\alpha\{b|c\}}+\frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}(H^{*}_{\alpha\beta\{b}\wedge H^{*}_{\gamma\delta|c\}})\Big),\\
\delta\pi^{\alpha a}_{(A)} &= 0\\
\delta\pi^{\alpha\beta a}_{(H)} &= -\varepsilon^{ab}\lambda^{\alpha\beta}_{(H)}\mu_b+\varepsilon^{ab}\Big(\mathcal{D}^{\alpha}B^{\beta}-\mathcal{D}^{\beta}B^{\alpha}+\frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}(H^{*}_{\delta\gamma c}\wedge B^{c})\Big)\mu_b,\\
\delta\pi^{\alpha a}_{(B)} &= 0\\
\delta\pi^{a|b}_{(B)} &= 0\\
\delta\pi^{\alpha a|b}_{(C)} &= -\varepsilon^{ac}\lambda^{\alpha b}_{(C)}\mu_c+\varepsilon^{ac}\mathcal{D}^{\alpha}B^{b}\mu_c,\\
\delta\pi^{a|bc}_{(C)} &= 0\\
\delta\bar{A}_{\alpha} &= \mu_a\varepsilon^{ab}A^{*}_{\alpha b},\\
\delta\bar{H}_{\alpha\beta} &= \mu_a\Big(\varepsilon^{ab}H^{*}_{\alpha\beta b}-\frac{\alpha}{2}\pi^{a}_{(H)\alpha\beta}\Big),\\
\delta\bar{B}_{\alpha} &= \mu_a\varepsilon^{ab}B^{*}_{\alpha b},\\
\delta\bar{B}_{a} &= \mu_c\varepsilon^{cb}B^{*}_{b|a},\\
\delta\bar{C}_{\alpha a} &= \mu_c\Big(\varepsilon^{cb}C^{*}_{\alpha b|a}+\beta\varepsilon_{ab}\pi^{c|b}_{(C)\alpha}\Big),\\
\delta\bar{C}_{ab} &= \mu_c\varepsilon^{cd}C^{*}_{d|ab}.
\end{aligned}
\tag{2.7}
$$

where symmetrization over $Sp(2)$ indices is taken as $A^{\{ab\}}=A^{ab}+A^{ba}$. Eqs. (2.7) realize the transformations of extended BRST symmetry of the vacuum functional in terms of the anticanonically conjugated pairs of variables $\{\phi^{A},\,\phi^{*}_{Aa}\}$ and $\{\pi^{Aa},\,\bar{\phi}_{A}\}$.

Integrating in eq. (1.3) over the variables ϕ^{*}_{Aa}, π^{Aa}, $\bar{\phi}_{A}$ and λ^{A}, we represent the vacuum functional Z as an integral over the fields ϕ^{A} of the complete configuration space,

$$
Z=\int d\phi\,\Delta\exp\left\{\frac{i}{\hbar}S^{(0)}_{\rm eff}(\phi)\right\},
\tag{2.8}
$$

where

$$
\begin{aligned}
S^{(0)}_{\rm eff} &= \int d^4x\left(-\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}H_{\rho\sigma}+\frac{1}{2}A_{\mu}A^{\mu}\right)\\
&\quad+\int d^4x\left\{\frac{\alpha}{4}G^{a}_{\mu\nu}\mathcal{M}^{-1}_{ab}\mathcal{K}^{b[\mu\nu][\rho\sigma]}_{c}G^{c}_{\rho\sigma}-\frac{\beta}{2}\varepsilon_{ab}\varepsilon_{cd}\Big(\mathcal{D}_{\mu}C^{ac}\Big)\Big(\mathcal{D}^{\mu}C^{bd}\Big)\right\}\\
&\quad+\int d^4x\left(\alpha B_{\mu}\mathcal{D}_{\nu}H^{\nu\mu}+\beta(\varepsilon_{ab}B^{a}\mathcal{D}_{\mu}C^{\mu b}-B_{\mu}B^{\mu})\right),
\end{aligned}
\tag{2.9}
$$

$$
\Delta = \int dH^{*}\exp\left\{\frac{2i}{\alpha\hbar}\int d^4x\,H^{*}_{0ia}\mathcal{M}^{ab}H^{*}_{0jb}\eta^{ij}\right\}.
\tag{2.10}
$$

In eq. (2.9) we have used the following notations:

$$
\mathcal{K}^{a[\mu\nu][\rho\sigma]}_{b}\equiv\frac{1}{2}\{\delta^{a}_{b}(\eta^{\mu\rho}\eta^{\nu\sigma}-\eta^{\mu\sigma}\eta^{\nu\rho})+\alpha\mathcal{C}^{a}_{b}\varepsilon^{\mu\nu\rho\sigma}\},
$$

$$
G^{a}_{\mu\nu}\equiv\Big(\mathcal{D}_{\mu}C^{a}_{\nu}-\mathcal{D}_{\nu}C^{a}_{\mu}\Big)-\frac{\alpha}{4}\varepsilon_{\mu\nu\rho\sigma}\mathcal{B}^{a}H^{\rho\sigma}.
$$

The matrix $\mathcal{M}^{-1}_{ab}$ in (2.10) is the inverse of $\mathcal{M}^{ab}$,

$$\mathcal{M}^{ab} \equiv \varepsilon^{ab} - \alpha^2 \mathcal{C}^a_c \mathcal{C}^b_d \varepsilon^{cd}, \quad \mathcal{M}^{ac}\mathcal{M}^{-1}_{cb} = \delta^a_b;$$

here the matrices $\mathcal{C}^a_b$ and $\mathcal{B}^a$ are defined by $\mathcal{C}^a_b E \equiv \varepsilon_{bc} C^{ac} \wedge E$ and $\mathcal{B}^a E \equiv B^a \wedge E$.

The functional $S^{(0)}_{\rm eff}$ in eq. (2.9) is the tree approximation to the gauge-fixed quantum action of the theory, while the functional Δ in eq. (2.10) can be considered as a contribution to the integration measure.

The integrand in eq. (2.8) is invariant under the following symmetry transformations:

$$\begin{aligned}
\delta A_\mu &= 0,\\
\delta H^{\alpha\beta} &= -\varepsilon^{ab}\mathcal{M}^{-1}_{bc}\mathcal{K}^{c[\alpha\beta][\gamma\delta]}_d G^d_{\gamma\delta}\mu_a,\\
\delta B^\alpha &= \frac{1}{2}\mathcal{D}^\alpha B^a \mu_a,\\
\delta B^a &= 0,\\
\delta C^{\alpha a} &= (\mathcal{D}^\alpha C^{ab} - \varepsilon^{ab} B^\alpha)\mu_b,\\
\delta C^{ab} &= \frac{1}{2} B^{\{a}\varepsilon^{b\}c}\mu_c.
\end{aligned} \tag{2.11}$$

The transformations (2.11) are the (anti)BRST transformations of the $Sp(2)$ invariant action $S^{(0)}_{\rm eff}$ which, together with the integration measure $d\phi\Delta$ in eq. (2.8), is left invariant:

$$\delta(d\phi) = d\phi\, \delta^4(0) \int d^4x \, {\rm Tr}\, \mathcal{W},$$

$$\delta\Delta = -\Delta\delta^4(0) \int d^4x \, {\rm Tr}\, \mathcal{W},$$

$$\delta\left(\exp\left\{\frac{i}{\hbar} S^{(0)}_{\rm eff}\right\}\right) = 0;$$

here the following notations have been used:

$$\mathcal{W} = -3\alpha^2\varepsilon^{ab}(\mathcal{M}_{bc}\mathcal{C}^c_d\mathcal{B}^d)\mu_a\,, \quad {\rm Tr}\,\mathcal{W} \equiv \sum_{p=1}^{N} \mathcal{W}^{pp}.$$

Consequently, eqs. (2.11) realize the transformations of extended BRST symmetry for the vacuum functional (2.8) in terms of the variables ϕ^A of the complete configuration space. As a remarkable fact it appears that these BRST transformations essentially depend on the gauge parameter α. This can be traced back to the fact that the antifields $B^*_{\mu\nu}$ and $\bar{B}_{\mu\nu}$ occure bilinear in the extended action S.

Note that, taking into account the action (2.9) and the contribution to the integration measure (2.10), the vacuum functional (2.8) obtained for the Freedman–Townsend model leads to the unitarity [21] of the physical S matrix (for discussions of the unitarity problem in the case of this model, see also [23, 20, 22]). For the first time the covariant quantization of the Freedman-Townsend model in the framework of extended BRST invariance has been performed in [25]. However, they used a more complicated, not $Sp(2)$–invariant gauge fixing. This might prevented them for explicitly solving their expression for the fields ϕ only.

3. W_2–gravity

The model of W_2–gravity [16] is described by the action

$$S_0(\varphi, h) = \frac{1}{2\pi}\int d^2z \left(\partial\varphi\bar{\partial}\varphi - h(\partial\varphi)^2\right), \tag{3.1}$$

where φ and h are bosonic classical fields ($\varepsilon(\varphi) = \varepsilon(h) = 0$) defined on a space with complex coordinates, $(z, \bar{z})$, so that $\partial = \partial/\partial z$, $\bar{\partial} = \partial/\partial\bar{z}$.

The action (3.1) is invariant under the gauge transformations

$$\begin{aligned} \delta\varphi &= (\partial\varphi)\xi, \\ \delta h &= \bar{\partial}\xi - h\partial\xi + (\partial h)\xi \end{aligned} \tag{3.2}$$

with the gauge function $\xi(z, \bar{z})$. These transformations form a closed algebra,

$$\begin{aligned} &[\delta_{\xi(1)},\ \delta_{\xi(2)}] = \delta_{\xi(1,2)}\,, \\ &\xi_{(1,2)} = (\partial\xi_{(1)})\xi_{(2)} - (\partial\xi_{(2)})\xi_{(1)}. \end{aligned} \tag{3.3}$$

Note that the quantum properties of W_2–gravity considered within the BV method [14] have been discussed in [26, 27], where also the 1-loop anomaly has been determined. Recently, its quantization has been performed within the triplectic formalism [28].

Now, we consider the gauge model (3.1), (3.2) and (3.3) in the framework of the modified triplectic quantization. First, let us introduce the complete configuration space ϕ^A, whose structure in the case of the model in question is determined by the rules of the $Sp(2)$ formalism for irreducible gauge theories. Thus, the space of the variables ϕ^A is constructed by supplementing the initial space of the fields (φ, h) with the doublet C^a, $\varepsilon(C^a) = 1$, of Faddeev–Popov ghosts, and the Lagrange multiplier B, $\varepsilon(B) = 0$, corresponding to the gauge parameter ξ in eq. (3.2).

The fields ϕ^A of the complete configuration space

$$\phi^A = (\varphi,\ h;\ B,\ C^a)$$

are supplemented by the sets of the variables ϕ^*_{Aa}, π^{Aa} and $\bar{\phi}_A$

$$\begin{aligned} \phi^*_{Aa} &= (\varphi^*_a,\ h^*_a;\ B^*_a,\ C^*_{a|b}), \\ \pi^{Aa} &= (\pi^a_{(\varphi)},\ \pi^a_{(h)};\ \pi^a_{(B)},\ \pi^{a|b}_{(C)}), \\ \bar{\phi}_A &= (\bar{\varphi},\ \bar{h};\ \bar{B},\ \bar{C}_a), \end{aligned}$$

as well as by the additional variables λ^A,

$$\lambda^A = (\lambda_{(\varphi)},\ \lambda_{(h)};\ \lambda_{(B)},\ \lambda^a_{(C)}),$$

with the Grassmann parities given by eq. (2.4).

Now, an action functional is given in closed form that satisfies the generating equations (1.1) – but reduced to the classical action S – in the case of the gauge model (3.1), (3.2) and (3.3):

$$S = \frac{1}{2\pi}\int d^2z \left(\partial\varphi\, \bar{\partial}\varphi - h(\partial\varphi)^2\right)$$

$$
\begin{aligned}
&+\int d^2z\Big\{\varphi^*_a C^a\partial\varphi + h^*_a\left(\bar\partial C^a - h\partial C^a + C^a\partial h\right)\\
&+\Big(\frac{1}{2}B^*_a - \bar C_a\Big)\Big[(C^a\partial B - B\partial C^a) + \frac{1}{6}\varepsilon_{bd}\Big(C^{\{a}(\partial^2 C^{d\}})C^b - C^{\{a}(\partial C^{d\}})\partial C^b\Big)\Big]\\
&-C^*_{a|b}\Big(\varepsilon^{ab}B + \frac{1}{2}C^{\{a}\partial C^{b\}}\Big) + \bar\varphi\Big(B\partial\varphi + \frac{1}{2}\varepsilon_{ab}C^a\partial(C^b\partial\varphi)\Big) + \bar h\Big(\bar\partial B - h\partial B + B\partial h\Big)\\
&+\frac{1}{2}\bar h\varepsilon_{ab}\Big(C^a\partial\left(\bar\partial C^b - h\partial C^b + C^b\partial h\right) + \left(\bar\partial C^b - h\partial C^b + C^b\partial h\right)\partial C^a\Big)\Big\}. \qquad (3.4)
\end{aligned}
$$

Obviously, the application of the differential operator Δ^a leads to terms being proportional to $\delta(0)$. However, since the model is restricted to two dimensions only dimensional regularization is not applicable. Therefore, it is not surprising that the model has an anomaly which can not be compensated by some counterterm.

Furthermore, a solution of the generating equations determining the gauge-fixing functional X can be represented as

$$
\begin{aligned}
X &= \int d^2z\Big\{(\bar\varphi - \alpha\varphi - \beta h)\lambda_{(\varphi)} + (\bar h - \beta\varphi - \gamma h)\lambda_{(h)}\\
&\qquad -\frac{\alpha}{2}\varepsilon_{ab}\pi^a_{(\varphi)}\pi^b_{(\varphi)} - \beta\varepsilon_{ab}\pi^a_{(\varphi)}\pi^b_{(h)} - \frac{\gamma}{2}\varepsilon_{ab}\pi^a_{(h)}\pi^b_{(h)}\Big\}, \qquad (3.5)
\end{aligned}
$$

where F has been chosen now as $F(\varphi,h) = \frac{1}{2}\alpha\varphi^2 + \beta\varphi h + \frac{1}{2}\gamma h^2$ with α, β and γ being constant parameters.

The vacuum functional (1.3) corresponding to the solutions (3.4) and (3.5) of the generating equations that determine the quantum action S and the gauge-fixing functional X is invariant under the following transformations of extended BRST symmetry, expressed (for simplicity) in terms of the derivatives of S:

$$
\begin{aligned}
\delta\phi^A &= \Big(\pi^{Aa} - \frac{\delta S}{\delta\phi^*_{Aa}}\Big)\mu_a,\\
\delta\varphi^*_a &= \mu_a\Big(\frac{\delta S}{\delta\varphi} + \alpha\lambda_{(\varphi)} + \beta\lambda_{(h)}\Big),\\
\delta h^*_a &= \mu_a\Big(\frac{\delta S}{\delta h} + \beta\lambda_{(\varphi)} + \gamma\lambda_{(h)}\Big),\\
\delta B^*_a &= \mu_a\frac{\delta S}{\delta B},\\
\delta C^*_{a|b} &= \mu_a\frac{\delta S}{\delta C^b},\\
\delta\pi^a_{(\varphi)} &= \varepsilon^{ab}\Big(\frac{\delta S}{\delta\bar\varphi} - \lambda_{(\varphi)}\Big)\mu_b,\\
\delta\pi^a_{(h)} &= \varepsilon^{ab}\Big(\frac{\delta S}{\delta\bar h} - \lambda_{(h)}\Big)\mu_b,\\
\delta\pi^a_{(B)} &= 0\\
\delta\pi^{a|b}_{(C)} &= \varepsilon^{ac}\frac{\delta S}{\delta\bar C_b}\mu_c,\\
\delta\bar\varphi &= \mu_a(\varepsilon^{ab}\varphi^*_b + \alpha\pi^a_{(\varphi)} + \beta\pi^a_{(h)}),\\
\delta\bar h &= \mu_a(\varepsilon^{ab}h^*_b + \beta\pi^a_{(\varphi)} + \gamma\pi^a_{(h)}),\\
\delta\bar B &= \mu_a\varepsilon^{ab}B^*_b,\\
\delta\bar C_a &= \mu_a\varepsilon^{bd}C^*_{d|b}. \qquad (3.6)
\end{aligned}
$$

Substituting the solutions (3.4) and (3.5) that determine the action S and the gauge-fixing functional X into eq. (1.3), and integrating out the variables ϕ^*_{Aa}, π^{Aa}, $\bar{\phi}_A$ and λ^A, we obtain the vacuum functional Z as an integral over the fields ϕ^A of the complete configuration space,

$$Z = \int d\phi \, \exp\left\{\frac{i}{\hbar} S_{\mathrm{eff}}(\phi)\right\}; \tag{3.7}$$

here S_{eff} is the gauge-fixed tree approximation of the quantum action

$$\begin{aligned}
S_{\mathrm{eff}} &= \frac{1}{2\pi}\int d^2z \left(\partial\varphi\bar{\partial}\varphi - h(\partial\varphi)^2\right) \\
&+ \int d^2z\Big[(\alpha\varphi+\beta h)B\partial\varphi + (\beta\varphi+\gamma h)\left(\bar{\partial}B - h\partial B + B\partial h\right)\Big] \\
&+\frac{1}{2}\varepsilon_{ab}\int d^2z\Big[\Big(\alpha C^b\partial\varphi + \beta\left(\bar{\partial}C^b - h\partial C^b + C^b\partial h\right)\Big)C^a\partial\varphi - (\alpha\varphi+\beta h)C^a\partial(C^b\partial\varphi) \\
&+\Big(\beta C^b\partial\varphi + \gamma\left(\bar{\partial}C^b - h\partial C^b + C^b\partial h\right)\Big)\left(\bar{\partial}C^a - h\partial C^a + C^a\partial h\right) \\
&-(\beta\varphi+\gamma h)\Big(\left(\bar{\partial}C^b - h\partial C^b + C^b\partial h\right)\partial C^a + C^a\partial\left(\bar{\partial}C^b - h\partial C^b + C^b\partial h\right)\Big)\Big].
\end{aligned} \tag{3.8}$$

The quantum action S_{eff}, eq. (3.8), and the integration measure $d\phi$ in eq. (3.7) are invariant under the following (anti) BRST transformations:

$$\begin{aligned}
\delta\varphi &= C^a\partial\varphi\,\mu_a, \\
\delta h &= \left(\bar{\partial}C^a - h\partial C^a + C^a\partial h\right)\mu_a, \\
\delta B &= \frac{1}{2}\left(C^a\partial B - B\partial C^a\right)\mu_a + \frac{1}{12}\varepsilon_{bd}\Big(C^{\{a}\partial^2 C^{d\}}C^b - C^{\{a}\partial C^{d\}}\partial C^b\Big)\mu_a, \\
\delta C^a &= \Big(\varepsilon^{ab}B - \frac{1}{2}C^{\{a}\partial C^{b\}}\Big)\mu_b.
\end{aligned} \tag{3.9}$$

Thus we conclude that eqs. (3.9) realize the transformations of extended BRST symmetry for the vacuum functional (3.7) in terms of the variables of the complete configuration space. In fact, introducing the action of the (anti)BRST operators s^a onto the fields ϕ^A according to $\delta\phi^A = (s^a\phi^A)\mu_a$ the effective action (3.8) may be rewritten in the following quite simple form,

$$S_{\mathrm{eff}} = S_0 + \frac{1}{2}\varepsilon_{ab}s^b s^a \int d^2z F(\varphi, h), \tag{3.10}$$

i.e., one obtains the usual effective action of the $Sp(2)$–covariant approach.

4. Two-dimensional Gravity with Dynamical Torsion

The theory of two-dimensional gravity with dynamical torsion is described in terms of the zweibein and Lorentz connection $(e^i_\mu, \omega^{ij}_\mu = \varepsilon^{ij}\omega_\mu)$ by the action [17]

$$S_0(e^i_\mu, \omega_\mu) = \int d^2x\, e\Big(\frac{1}{16\alpha}R_{\mu\nu}{}^{ij}R^{\mu\nu}{}_{ij} - \frac{1}{8\beta}T_{\mu\nu}{}^{i}T^{\mu\nu}{}_{i} - \gamma\Big), \tag{4.1}$$

where α, β and γ are constant parameters. In eq. (4.1), the Latin indices are lowered with the help of the Minkowski metric $\eta_{ij} = \mathrm{diag}(+1, -1)$, and the Greek indices, with the help of the metric tensor $g_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\nu$. Besides, the following notations are used:

$$\begin{aligned} e &= \det e^i_\mu\,, \\ R_{\mu\nu}{}^{ij} &= \varepsilon^{ij}\partial_\mu\omega_\nu - (\mu \leftrightarrow \nu), \\ T_{\mu\nu}{}^i &= \partial_\mu e^i_\nu + \varepsilon^{ij}\omega_\mu e_{\nu j} - (\mu \leftrightarrow \nu), \end{aligned}$$

where ϵ^{ij} is a constant antisymmetric tensor, $\epsilon^{01} = -1$.

Note that the model (4.1) is the most general theory of two-dimensional R^2-gravity with independent dynamical torsion that leads to second-order equations of motion for the zweibein and Lorentz connection. Thus, supplementing the action eq. (4.1) by the Einstein–Hilbert term eR would not affect the classical field equations, since in two dimensions it reduces to a trivial total divergence. Originally, the action (4.1) was proposed [29] in the context of bosonic string theory, where it was used to describe the dynamics of string geometry. There, moreover, it was proved that the string with dynamical geometry has no critical dimension. An attrctive feature of the model (4.1) is its complete integrability. The corresponding equations of motion have been studied in conformal [17, 30] as well as in light-cone [31] gauge. It was established that this model also contains solutions with constant curvature and zero torsion, thus incorporating several other two-dimensional gravity models [32] whose actions, however, do not have a purely geometric interpretation.

The action (4.1) is invariant under local Lorentz rotations of the zweibein e^i_μ, which infinitesimally imples the gauge transformations

$$\delta_\zeta e^i_\mu = \varepsilon^{ij} e_{\mu j}\zeta, \qquad \delta_\zeta\omega_\mu = -\partial_\mu\zeta, \tag{4.2}$$

with the parameter ζ. Similarly, the general coordinate invariance of eq. (4.1) leads to the gauge transformations

$$\delta_\xi e^i_\mu = e^i_\nu\partial_\mu\xi^\nu + (\partial_\nu e^i_\mu)\xi^\nu, \qquad \delta_\xi\omega_\mu = \omega_\nu\partial_\mu\xi^\nu + (\partial_\nu\omega_\mu)\xi^\nu \tag{4.3}$$

with the parameters ξ^μ. The gauge transformations (4.2) and (4.3) form a closed algebra

$$\begin{aligned} [\delta_{\zeta(1)},\, \delta_{\zeta(2)}] &= 0\,, \\ [\delta_{\xi(1)},\, \delta_{\xi(2)}] &= \delta_{\xi(1,2)}\,, \\ [\delta_\zeta,\, \delta_\xi] &= \delta_{\zeta'}\,, \end{aligned} \tag{4.4}$$

where

$$\xi^\mu{}_{(1,2)} = (\partial_\nu\xi^\mu{}_{(1)})\xi^\nu{}_{(2)} - (\partial_\nu\xi^\mu{}_{(2)})\xi^\nu{}_{(1)}\,, \quad \zeta' = (\partial_\mu\zeta)\xi^\mu.$$

Note that in Ref. [33] a gauge model classically equivalent to (4.1), (4.2), (4.3) and (4.4) was proposed by means of artificially adding the Einstein-Hilbert term coupled to an additional scalar field, σeR; however, in this equivalent formulation the algebra of the corresponding gauge transformations closes only on-shell.

The Hamiltonian structure of the gauge symmetries of the original model was studied in Ref. [34], and its canonical quantization, in Ref. [35]. Quantum properties of that theory in the light-cone gauge were discussed in Ref. [36], proving also, despite of the nonpolynomial structure of the theory, its renormalizability.

Now we consider the gauge model (4.1), (4.2), (4.3) and (4.4) in the framework of the modified triplectic formalism [6].

The complete configuration space ϕ^A, constructed by the rules of the $Sp(2)$ covariant quantization of irreducible theories, consists of the initial classical fields (e^i_μ, ω_μ), the doublets of the Faddeev–Popov ghosts (C^a, $C^{\mu a}$) and the Lagrangian multipliers (B, B^μ) introduced according to the number of the gauge parameters in eqs. (4.2) and (4.3), i.e. ζ and ξ^μ, respectively. The Grassmann parities of the fields ϕ^A,

$$\phi^A = (e^i_\mu,\ \omega_\mu;\ B,\ B^\mu;\ C^a,\ C^{\mu a}),$$

is given by

$$\varepsilon(e^i_\mu) = \varepsilon(\omega_\mu) = \varepsilon(B) = \varepsilon(B^\mu) = 0, \qquad \varepsilon(C^a) = \varepsilon(C^{\mu a}) = 1.$$

The fields ϕ^A of the complete configuration space are supplemented by following the sets of the variables ϕ^*_{Aa}, π^{Aa}, $\bar{\phi}_A$ and λ^A

$$\begin{aligned}
\phi^*_{Aa} &= (e^{*\mu}_{ia},\ \omega^{*\mu}_a;\ B^*_a,\ B^*_{\mu a};\ C^*_{a|b},\ C^*_{\mu a|b}),\\
\pi^{Aa} &= (\pi^{ia}_{(e)\mu},\ \pi^a_{(\omega)\mu};\ \pi^a_{(B)},\ \pi^{\mu a}_{(B)};\ \pi^{a|b}_{(C)},\ \pi^{\mu a|b}_{(C)}),\\
\bar{\phi}_A &= (\bar{e}^\mu_i,\ \bar{\omega}^\mu;\ \bar{B},\ \bar{B}_\mu;\ \bar{C}_a,\ \bar{C}_{\mu a}),\\
\lambda^A &= (\lambda^i_{(e)\mu},\ \lambda_{(\omega)\mu};\ \lambda_{(B)},\ \lambda^\mu_{(B)};\ \lambda^a_{(C)},\ \lambda^{\mu a}_{(C)}).
\end{aligned}$$

Again, we are faced with the problem that the model is strictly two-dimensional and, therefore, the regularization by setting $\delta(0) = 0$ will not be applicable. A functional that satisfies the generating equations (1.1) for the classical action S in the case of the gauge model (4.1), (4.2), (4.3) and (4.4) can be found in a closed form as follows:

$$\begin{aligned}
S = &\int d^2x\, e\Big(\frac{1}{16\alpha}R_{\mu\nu}{}^{ij}R^{\mu\nu}{}_{ij} - \frac{1}{8\beta}T_{\mu\nu}{}^{i}T^{\mu\nu}{}_{i} - \gamma\Big)\\
&+\int d^2x\Big\{e^{*\mu}_{ia}\left(\varepsilon^{ij}e_{\mu j}C^a + C^{\lambda a}\partial_\lambda e^i_\mu + e^i_\lambda\partial_\mu C^{\lambda a}\right) + \omega^{*\mu}_a\left(-\partial_\mu C^a + C^{\lambda a}\partial_\lambda\omega_\mu + \omega_\lambda\partial_\mu C^{\lambda a}\right)\\
&+\Big(\frac{1}{2}B^*_a - \bar{C}_a\Big)\Big[(C^{\mu a}\partial_\mu B - B^\mu\partial_\mu C^a) + \frac{1}{6}\varepsilon_{bd}\left(C^{\lambda\{a}\partial_\lambda C^{\mu d\}}\partial_\mu C^b - C^{\mu b}\partial_\mu C^{\lambda\{a}\partial_\lambda C^{d\}}\right)\Big]\\
&+\Big(\frac{1}{2}B^*_{\mu a} - \bar{C}_{\mu a}\Big)\Big[\left(C^{\lambda a}\partial_\lambda B^\mu - B^\lambda\partial_\lambda C^{\mu a}\right) + \frac{1}{6}\varepsilon_{bd}\left(C^{\sigma\{a}\partial_\sigma C^{\lambda d\}}\partial_\lambda C^{\mu b} - C^{\lambda b}\partial_\lambda C^{\sigma\{a}\partial_\sigma C^{\mu d\}}\right)\Big]\\
&-C^*_{a|b}\left(\varepsilon^{ab}B + \frac{1}{2}C^{\mu\{a}\partial_\mu C^{b\}}\right) - C^*_{\mu a|b}\left(\varepsilon^{ab}B^\mu + \frac{1}{2}C^{\lambda\{a}\partial_\lambda C^{\mu b\}}\right)\\
&+\bar{e}^\mu_i\Big[\varepsilon^{ij}Be_{\mu j} + B^\lambda\partial_\lambda e^i_\mu + e^i_\lambda\partial_\mu B^\lambda + \frac{1}{2}\varepsilon_{ab}\Big(\left(e^i_\mu C^b + \varepsilon^{ij}C^{\lambda b}\partial_\lambda e_{\mu j} + \varepsilon^{ij}e_{\lambda j}\partial_\mu C^{\lambda b}\right)C^a\\
&-C^{\lambda a}\partial_\lambda\left(\varepsilon^{ij}e_{\mu j}C^b + C^{\sigma b}\partial_\sigma e^i_\mu + e^i_\sigma\partial_\mu C^{\sigma b}\right) + \left(\varepsilon^{ij}e_{\lambda j}C^b + (\partial_\sigma e^i_\lambda)C^{\sigma b} + e^i_\sigma\partial_\lambda C^{\sigma b}\right)\partial_\mu C^{\lambda a}\Big)\Big]\\
&+\bar{\omega}^\mu\Big[-\partial_\mu B + B^\lambda\partial_\lambda\omega_\mu + \omega_\lambda\partial_\mu B^\lambda - \frac{1}{2}\varepsilon_{ab}\Big(C^{\lambda a}\partial_\lambda\left(C^{\sigma b}\partial_\sigma\omega_\mu + \omega_\sigma\partial_\mu C^{\sigma b} - \partial_\mu C^b\right)\\
&-\left(C^{\sigma b}\partial_\sigma\omega_\lambda + \omega_\sigma\partial_\lambda C^{\sigma b} - \partial_\lambda C^b\right)\partial_\mu C^{\lambda a}\Big)\Big]\Big\}.
\end{aligned} \tag{4.5}$$

A solution of the generating equations determining the gauge-fixing functional X can be chosen as

$$\begin{aligned}
X = &\int d^2x\Big\{\Big(\bar{e}^\mu_i - p\eta^{\mu\nu}\eta_{ij}e^j_\nu\Big)\lambda^i_{(e)\mu} + \Big(\bar{\omega}^\mu - q\eta^{\mu\nu}\omega_\nu\Big)\lambda_{(\omega)\mu}\\
&\qquad -\frac{p}{2}\varepsilon_{ab}\eta_{ij}\eta^{\mu\nu}\pi^{ia}_{(e)\mu}\pi^{jb}_{(e)\nu} - \frac{q}{2}\varepsilon_{ab}\eta^{\mu\nu}\pi^a_{(\omega)\mu}\pi^b_{(\omega)\nu}\Big\},
\end{aligned} \tag{4.6}$$

where $F(e^i_\mu, \omega_\mu) = \frac{1}{2}p\eta^{\mu\nu}\eta_{ij}e^i_\mu e^j_\nu + \frac{1}{2}q\eta^{\mu\nu}\omega_\mu\omega_\nu$ has been used with p, q being constant parameters, and $\eta^{\mu\nu} = \mathrm{diag}(+1, -1)$ is the metric of the two-dimensional Minkowski space.

Again, the vacuum functional Z is obtained by substituting the explicit solutions for the quantum action S, eq. (4.5), and the gauge-fixing functional X, eq. (4.6), into the expression into eq. (1.3). The expressions for the symmetry transformations (1.5) will not be given explicitly; their determination is straightforward but the result is quite lenghty.

Performing the integration over the variables ϕ^*_{Aa}, π^{Aa}, $\bar{\phi}_A$ and λ^A, we obtain Z in the form (3.7) with the gauge-fixed effective action S_{eff}

$$\begin{aligned} S_{\mathrm{eff}} =& \int d^2x\, e\Big(\frac{1}{16\alpha}R_{\mu\nu}{}^{ij}R^{\mu\nu}{}_{ij} - \frac{1}{8\beta}T_{\mu\nu}{}^{i}T^{\mu\nu}{}_{i} - \gamma\Big) \\ &+\int d^2x\Big\{q\eta^{\mu\nu}(\partial_\mu\omega_\nu)B + \eta^{\lambda\nu}\Big[p\Big(e_{\lambda i}\partial_\mu e^i_\nu - \partial_\lambda(e_{\mu i}e^i_\nu)\Big) + q\Big(\omega_\lambda\partial_\mu\omega_\nu - \partial_\lambda(\omega_\mu\omega_\nu)\Big)\Big]B^\mu\Big\} \\ &+\frac{1}{2}\varepsilon_{ab}\int d^2x\Big\{p\Big(\eta^{\mu\nu}e^i_{[\lambda}\partial_{\nu]}C^{\lambda a} + \eta^{\lambda\nu}e^i_\nu\partial_\lambda C^{\mu a}\Big)\Big(\varepsilon_{ij}C^b e^j_\mu - \eta_{ij}(C^{\sigma b}\partial_\sigma e^j_\mu + e^j_\sigma\partial_\mu C^{\sigma b})\Big) \\ &-q\Big(\eta^{\mu\nu}\Big(\partial_\nu C^a - \omega_{\{\lambda}\partial_{\nu\}}C^{\lambda a} - \eta^{\lambda\nu}\omega_\nu\partial_\lambda C^{\mu a}\Big)\Big(\partial_\mu C^b - C^{\sigma b}\partial_\sigma\omega_\mu - \omega_\sigma\partial_\mu C^{\sigma b}\Big)\Big\}. \end{aligned} \tag{4.7}$$

The effective action S_{eff} (4.7) and the integration measure $d\phi^A$ in the functional integral are invariant under the following transformations:

$$\begin{aligned} \delta e^i_\sigma &= \Big(\varepsilon^{ij}e_{\sigma j}C^a + C^{\lambda a}\partial_\lambda e^i_\sigma + e^i_\lambda\partial_\sigma C^{\lambda a}\Big)\mu_a, \\ \delta\omega_\sigma &= \Big(-\partial_\sigma C^a + C^{\lambda a}\partial_\lambda\omega_\sigma + \omega_\lambda\partial_\sigma C^{\lambda a}\Big)\mu_a, \\ \delta B &= \frac{1}{2}\Big(C^{\lambda a}\partial_\lambda B - B^\lambda\partial_\lambda C^a + \frac{1}{6}\varepsilon_{bd}\Big(C^{\lambda\{a}\partial_\lambda C^{\kappa d\}}\partial_\kappa C^b - C^{\kappa b}\partial_\kappa(C^{\lambda\{a}\partial_\lambda C^{d\}})\Big)\Big)\mu_a, \\ \delta B^\sigma &= \frac{1}{2}\Big(C^{\lambda a}\partial_\lambda B^\sigma - B^\lambda\partial_\lambda C^{\sigma a} + \frac{1}{6}\varepsilon_{bd}\Big(C^{\lambda\{a}\partial_\lambda C^{\lambda d\}}\partial_\kappa C^{\kappa b} - C^{\lambda b}\partial_\lambda(C^{\kappa\{a}\partial_\kappa C^{\sigma d\}})\Big)\Big)\mu_a, \\ \delta C^a &= \Big(\varepsilon^{ab}B - \frac{1}{2}C^{\lambda\{a}\partial_\lambda C^{b\}}\Big)\mu_b, \\ \delta C^{\sigma a} &= \Big(\varepsilon^{ab}B^\sigma - \frac{1}{2}C^{\lambda\{a}\partial_\lambda C^{\sigma b\}}\Big)\mu_b. \end{aligned} \tag{4.8}$$

Eqs. (4.8) realize the corresponding transformations of extended BRST symmetry for the vacuum functional in terms of the variables ϕ^A of the complete configuration space. Again, the effective action, using the corresponding (anti)BRST operators, can be written more simple as follows:

$$S_{\mathrm{eff}} = S_0 + \frac{1}{2}\varepsilon_{ab}s^b s^a\int d^2z F(e^i_\mu, \omega_\mu), \tag{4.9}$$

which, of course, coincides with the usual $Sp(2)$–invariant action in the $Sp(2)$–covariant approach [1].

5. Conclusion

In this paper we have exemplified the method of modified triplectic quantization [6] on the basis of several gauge theory models. Thus, we have considered the model [15] of non-abelian antisymmetric tensor field (Freedman–Townsend model), the model [16] of W_2-gravity, and the model [17] of two-dimensional gravity with dynamical torsion. For the models in question we have found manifest solutions of the generating equations that determine the quantum action S and the gauge-fixing functional X in the framework of the modified triplectic formalism [6]. In the case of the 2-dimensional models we did not determine possible anomalies which occure if loop corrections are taken into account.

The above solutions are expressed in terms of the variables ϕ^A, ϕ^*_{Aa} and π^{Aa}, $\bar{\phi}_A$ anticanonically conjugated in the sense of the extended antibrackets [2, 6], as well as in terms of the additional variables λ^A that serve to parametrize the gauge-fixing functional X. However, it should be remarked that by the special choice of both the action functional and the gauge fixing functional triplecticity of the formalism in fact is reduced to the usual case of the $Sp(2)$–covariant quantization. Using the solutions for S and X, we have obtained the vacuum functional and explicitly constructed the corresponding transformations [6] of extended BRST symmetry in terms of the anticanonically conjugated variables. Finally, we have obtained manifest $Sp(2)$–symmetric expressions for the effective action $S_{\rm eff}$ that results from integrating out the variables ϕ^*_{Aa}, π^{Aa}, $\bar{\phi}_A$ and λ^A in the functional integral, and have constructed the corresponding transformations (1.5) of extended BRST symmetry in terms of the variables ϕ^A of the complete configuration space. In any case we finally obtained a $Sp(2)$–symmetric action which is invariant under BRST and antiBRST transformations. In the case of irreducible theories we were able to write down the gauge fixing part in a very simple manner. In the case of the first stage reducible FT model the situation occured much more difficult. Especially the dependence of the corresponding (anti)BRST transformation for the physical H–field on the gauge parameter α deserves further study.

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