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Equilibrium and stability properties of a coupled two-component Bose-Einstein condensate

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ABSTRACT

The equilibrium and stability properties of a coupled two-component BEC is studied using a variational method and the one-dimensional model of Williams and collaborators. The variational parameters are the population fraction, translation and scaling transformation of the condensate densities, assumed to have a Gaussian shape. We study the equilibrium and stability properties as a function of the strength of the laser field and the traps displacement. We find many branches of equilibrium configurations, with a host of critical points. In all cases, the signature of the onset of criticality is the collapse of a normal mode which is a linear combination of the out of phase translation and an in phase breathing oscillation of the condensate densities. Our calculations also indicate that we have symmetry breaking effects when the traps are not displaced.

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I. INTRODUCTION

One of the most interesting achievements in the field of boson condensation was the experimental observation of binary mixture of trapped condensates [1-4]. The species in the mixture can be atoms with different F spin orientations [3-4] or simply different hyperfine states of the same atom [1-2]. In the last case we can have inter-conversion between the components through the coupling of the atoms with an external laser field [5].

In the case of uncoupled components many effects have been predicted theoretically and determined experimentally such as spatial phase separation [6], stability properties of the equilibrium states [6-7] and symmetry breaking effects [6,8]. These aspects of binary condensate mixtures have been treated in the literature using various methods such as the Thomas-Fermi (TF) approximation [8-9] and numerical solutions of the appropriate coupled Gross-Pitaevskii (GP) equations [10].

In this paper we use the variational method [11-13] to study the equilibrium properties of a coupled two-component BEC. In our exploratory calculation we use the one-dimensional model of reference [14] and take as variational parameters the population fraction, translation and scaling transformation of the equilibrium state densities, all assumed to have a Gaussian shape. We consider the intensity of the interaction between the condensates equal and the detuning is put equal to zero, leading to equal equilibrium population fraction. In our calculation we investigate the behavior of the spatial phase separation as a function of the trap displacement and the strength of the laser field. The coupling with the laser field have a stabilizer effect in the process of phase separation, opposite to the one coming from the trap displacement.Our paper is organized as follows:In section 2 we show briefly how to use the variational method to study the equilibrium and stability properties of a binary mixture of condensates.Specifically,we derive the general form of the equations of motion and develop a scheme to find the normal modes.In section 3 we present our numerical results and discuss its physical significance.A summary of our numerical results is presented in section 4.

II. VARIATIONAL STUDY OF THE EQUILIBRIUM AND STABILITY PROPERTIES OF A BINARY MIXTURE OF CONDENSATES.

A. The variational approach

The starting point of our discussion is the action [15]

$$S = \int dz\, dt \left[\sum_j i\hbar \psi_j^*(z,t) \dot{\psi}_j(z,t) - \mathcal{E}(z,t) \right] \tag{1}$$

where $\psi_j(z,t)$, $j = a, b$ are the condensate wave function of each component in the mixture and $\mathcal{E}(z,t)$ is the energy density,

$$\mathcal{E}(z,t) = \sum_j \psi_j^* \left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + V_{trap}^j \right] \psi_j + \frac{1}{2}\sum_{k,j} \lambda_{kj} |\psi_j|^2 |\psi_k|^2 + \Omega \left[\psi_a^* \psi_b + \psi_b^* \psi_a \right] \tag{2}$$

In the above expression $V_{trap}^j(z)$, $j = a, b$ are the trapping potential of each component

$$V_{trap}^j = \frac{m}{2}\omega_z^2 (z + \gamma_j z_0)^2 \tag{3}$$

where $\gamma_a = -1$ and $\gamma_b = 1$, z_0 is the trap displacement, $\lambda_{aa} = \lambda_{bb}$ and λ_{ab} are the strength of the intraspecies and interspecies interactions, respectively, and the last term comes from the coupling with the laser field responsible by the interspecies tunnelling, with Ω the Rabi frequency.

Imposing that the action (1) is stationary with respect to a variation of $\psi_j(z,t)$ subject only to the normalization constrain $\sum_j |\psi_j(z,t)|^2 = N$, leads to the coupled time-dependent Gross-Pitaevskii equations for the condensate wave functions [15],

$$i\hbar \frac{\partial \psi_a}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + V_{trap}^{(a)} + \lambda_{aa}|\psi_a|^2 + \lambda_{ab}|\psi_b|^2 \right)\psi_a + \Omega\psi_b \tag{4}$$

$$i\hbar \frac{\partial \psi_b}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + V_{trap}^{(b)} + \lambda_{ab}|\psi_a|^2 + \lambda_{bb}|\psi_b|^2 \right)\psi_b + \Omega\psi_a \tag{5}$$

To get the equations for the stationary states, we look for solutions of equations (4) and (5) of the form $\psi_j(z,t) = e^{-i\frac{\mu t}{\hbar}}\psi_j(z)$, which gives rise to the time-independent GP equations:

$$\mu\psi_a = \left(-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + V_{trap}^{(a)} + \lambda_{aa}|\psi_a|^2 + \lambda_{ab}|\psi_b|^2 \right)\psi_a + \Omega\psi_b \tag{6}$$

$$\mu\psi_b = \left(-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + V_{trap}^{(b)} + \lambda_{ab}|\psi_a|^2 + \lambda_{bb}|\psi_b|^2 \right)\psi_b + \Omega\psi_a \tag{7}$$

We can view the two condensate wave functions as components of a spinor of "quasi-spin" equal to $\frac{1}{2}$ [16] which leads immediately to the property that the stationary equations are

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invariant under a transformation which is the product of a space reflection, P_z, and an atom exchange, P_{ex}, where $P_{ex} = i \exp{-i\frac{\pi}{2}\sigma_x}$. Therefore, as $(P_z P_{ex})^2 = P_z P_{ex}$, we can classify the stationary states as even (gerade) and odd (umgerade) under this transformation. In the even class $\psi_a^g(z) = \psi_b^g(-z)$, whereas in the odd class $\psi_a^u(z) = -\psi_b^u(-z)$. The next step is to solve numerically the coupled stationary equations for the condensate wave functions to map the solutions as a function of Ω and z_0. However, before embarking on this task, we found it worthwhile to adopt a simpler approach, the variational method. In the variational method the search for stationary states reduces to finding the stationary points in a finite dimensional energy surface which is much simpler than the corresponding search in an infinite dimensional space, when we solve exactly the coupled GP equations. Besides, the variational solutions can be used as an initial guess in the numerical solution of the coupled GP equations [10].

In the variational approach we parametrize the time dependence of the condensate wave function through a set of $2d$ parameters [12-13] which we denote by $\mathbf{w} = (w_1, w_2, \ldots, w_{2d})$,

$$\psi_j(z, t) = \psi_j(z, \mathbf{w}(t)) \tag{8}$$

When we replace the condensate wave functions parametrized as in equation (8), in equation (1), the action reduces to a "classical" action, whose variation leads to Hamilton-type equations of motion in terms of these parameters[18]

$$\sum_l \Gamma_{kl}(\mathbf{w}) \, \dot{w}_l = \frac{\partial E}{\partial w_k}(\mathbf{w}) \, , \tag{9}$$

where $E(\mathbf{w})$ is the spatial integral of the energy density (2), with the condensate wave functions parametrized as in equation (8)

$$E(\mathbf{w}) = \int dz \mathcal{E}(z, \mathbf{w}) \tag{10}$$

and the antisymmetric matrix $\Gamma_{kl}(\mathbf{w})$ is given by

$$\Gamma_{kl}(\mathbf{w}) = -2 \operatorname{Im} \sum_j \int dz \frac{\partial \psi_j^*}{\partial w_k}(z, \mathbf{w}) \frac{\partial \psi_j}{\partial w_l}(z, \mathbf{w}) \tag{11}$$

It follows from equation (9) that the equilibrium configurations are determined by the equations

$$\frac{\partial E}{\partial w_k}(\mathbf{w}^0) = 0 \, , \qquad k = 1, 2, \ldots, 2d \, . \tag{12}$$

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B. Normal Modes

To investigate the stability of the equilibrium configurations, we calculate the energies of the normal modes. They are stable if the energies are real and positive and unstable if one of the energies is complex. To find the normal modes, we linearize the equations of motion in the neighborhood of an equilibrium configuration, leading to:

$$\sum_l \Gamma_{kl} \dot{\bar{w}}_l(t) = \sum_l H_{kl} \bar{w}_l \ . \tag{13}$$

In this equation $\bar{w}$ are the displacements from equilibrium $w = w^0 + \bar{w}$ and H_{kl}, Γ_{kl} are, respectively, the Hessian and the matrix $\Gamma(w)$ evaluated at the equilibrium configuration, that is

$$H_{kl} \equiv H_{kl}(w^0) = \frac{\partial^2 E}{\partial w_k \partial w_l}(w^0) \tag{14}$$

and

$$\Gamma_{kl} = \Gamma_{kl}(w^0) \tag{15}$$

To proceed, we divide the $2d$ parameters into two groups, $w = (q_1, q_2, \ldots, q_d, p_1, p_2, \ldots, p_d) = (q, p)$, of coordinates q and momenta p. Schematically, the reasoning behind this splitting is that the amplitude of the condensate wave function, whose square is the condensate density, depends only on the coordinates while its phase, whose gradient is the velocity field, depends on the momenta and the coordinates.

In terms of the coordinates and the momenta the equations of motion (13), read

$$\Gamma \begin{pmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \end{pmatrix} = H \begin{pmatrix} \bar{q} \\ \bar{p} \end{pmatrix} \ . \tag{16}$$

where Γ and H are, respectively, antisymmetric and symmetric matrices which can be written in terms of four $d \times d$ blocks

$$\Gamma = \begin{pmatrix} \Gamma_{qq} & \Gamma_{qp} \\ \Gamma_{pq} & \Gamma_{pp} \end{pmatrix} \ , \quad H = \begin{pmatrix} H_{qq} & H_{qp} \\ H_{pq} & H_{pp} \end{pmatrix} \tag{17}$$

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where, for example, $\mathbf{\Gamma}_{qq}$ and $\mathbf{H}_{qq}$ are $d \times d$ matrices whose elements are given by equations (11) and (14), where the derivatives are with respect to the coordinates, with an analogous definition for the other $d \times d$ matrices.

In our method to find the normal modes, we try to stay as close as possible to the one adopted in the standard case [17]. To begin with, we should find a transformation to a set of new coordinates and momenta

$$\begin{pmatrix} \bar{\mathbf{q}} \\ \tilde{\mathbf{p}} \end{pmatrix} = \mathbf{T}^{-1} \begin{pmatrix} \mathbf{Q} \\ \mathbf{P} \end{pmatrix} \quad . \tag{18}$$

such that

$$\mathbf{\Gamma}^{-1} \mathbf{H} \mathbf{T}^{-1} = \mathbf{T}^{-1} \begin{pmatrix} \mathbf{O} & \mathbf{\Lambda} \\ -\mathbf{\Lambda} & \mathbf{O} \end{pmatrix} \tag{19}$$

where $\mathbf{\Lambda}$ is a diagonal $d \times d$ matrix whose diagonal elements are the normal mode energies.

In terms of the new coordinates and momenta, the equations of motion, (16), reduce to

$$\begin{pmatrix} \dot{\mathbf{Q}} \\ \dot{\mathbf{P}} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{\Lambda} \\ -\mathbf{\Lambda} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{Q} \\ \mathbf{P} \end{pmatrix} \quad . \tag{20}$$

leading to

$$\dot{Q}_k = \Lambda_k P_k \quad , \dot{P}_k = -\Lambda_k Q_k \tag{21}$$

The equations (18)-(19) define the transformation to the normal modes and our scheme to find the matrix $\mathbf{T}^{-1}$ is a straightforward generalization of the standard case [17]. The starting point is to solve the eigenvalue problem

$$\mathbf{\Xi}^{-1} \mathbf{H} V^{(k)} = \Lambda_k V^{(k)} \tag{22}$$

where $\mathbf{\Xi}$ is the hermitian and antisymmetric matrix $\mathbf{\Xi} = -i\mathbf{\Gamma}$

As in the standard case, this eigenvalue problem has two properties: (i) if $V^{(k)}$ is an eigenvector with eigenvalue Λ_k, then $V^{(k)*}$ is also an eigenvector with eigenvalue $-\Lambda_k^*$. (ii) The eigenvectors with different eigenvalues are $\mathbf{\Xi}$ orthogonal, that is $V^{(l)+}\mathbf{\Xi}V^{(k)} = 0$, if $\Lambda_l \neq \Lambda_k^*$.

If one of the eigenvalues of equation (22) is complex, the system is unstable. If all the eigenvalues are real it is stable and we can find the transformation to the normal modes as follows. We define a matrix $\mathbf{S}^{-1}$, whose first d columns are the $2d$ components of the d eigenvectors with real positive eigenvalues and the next d columns, the $2d$ components of the corresponding eigenvectors with real negative energies.

In terms of $\mathbf{S}^{-1}$, the eigenvalue equations (22) reads

$$\Xi^{-1}\mathbf{H}\mathbf{S}^{-1} = \mathbf{S}^{-1}\begin{pmatrix} \Lambda & 0 \\ 0 & -\Lambda \end{pmatrix} \tag{23}$$

and the orthogonality of the eigenvectors gives that

$$\mathbf{S}^{-1\dagger}\Xi\mathbf{S}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{24}$$

The matrix $\mathbf{T}^{-1}$ is related to $\mathbf{S}^{-1}$ by

$$\mathbf{T}^{-1} = \mathbf{S}^{-1}\mathbf{U} \tag{25}$$

where $\mathbf{U}$ is the unitary matrix

$$\mathbf{U} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \tag{26}$$

To summarize, we state the main steps in our procedure: (i) First we determine the equilibrium configurations by solving the $2d$ equations (12). (ii) Next, for each equilibrium configuration, we solve the eigenvalue equation (22). (iii) When the configuration is stable, we construct the matrix $\mathbf{S}^{-1}$ from the eigenvectors as indicated above and $\mathbf{T}^{-1}$ as shown in equation (25). When the system oscillates in a normal mode only one pair (Q_k, P_k) is different from zero and equation (18) gives the time evolution of the system in this case.

C. Gaussian ansatz

Now that we have established the general framework of our calculations, we turn to our specific application where the variational parameters are related to the population fraction,

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translation and breathing shape oscillation of the condensate densities. Thus, the condensate wave functions are written as [18]

$$\psi_j(z, \mathbf{w}) = e^{iF_j(z, \mathbf{w})} A_j(z, \mathbf{w}) \tag{27}$$

where the amplitude of the condensate wave function is parametrized as

$$A_j(z, \mathbf{w}) = \sqrt{Nn_j} \frac{1}{\sqrt{\pi^{1/2} q_{2j}}} e^{-\frac{(z - q_{1j})^2}{2q_{2j}^2}} \tag{28}$$

and the phase as

$$F_j(z, \mathbf{w}) = p_{1j}(z - q_{1j}) + \frac{p_{2j}}{2q_{2j}}(z - q_{1j})^2 + \theta_j \tag{29}$$

The parameters introduced in the above expression have the following physical interpretation: n_j is the population fraction of each condensate,

$$Nn_j = \int |\psi_j(z, \mathbf{w})|^2 dz \,, \tag{30}$$

q_{1j} is the center of mass of the spatial distribution of each condensate,

$$Nn_j q_{1j} = \int z |\psi_j(z, \mathbf{w})|^2 dz \,, \tag{31}$$

and $q_{2j}/\sqrt{2}$ is the width of the spatial distribution of each condensate

$$Nn_j \frac{q_{2j}^2}{2} = \int (z - q_{1j})^2 |\psi_j(z, \mathbf{w})|^2 dz \,, \tag{32}$$

The momenta p_{1j}, p_{2j} and θ_j appear in the phase of the condensate wave functions. The p_{1j} is the center of mass moment of each condensate

$$Nn_j p_{1j} = -i \int \psi_j^*(z, \mathbf{w}) \frac{\partial \psi_j}{\partial z}(z, \mathbf{w}) dz \tag{33}$$

and p_{2j} is connected to the expectation value of the dilatation operator in the center of mass frame of each condensate

$$\frac{1}{2} Nn_j p_{2j} q_{2j} = -\frac{i}{2\hbar} \int \left(\psi_j^*(z, \mathbf{w}) \frac{\partial \psi_j}{\partial z}(z, \mathbf{w}) - \frac{\partial \psi_j^*}{\partial z}(z, \mathbf{w}) \psi_j(z, \mathbf{w}) \right)(z - q_{1j}) dz \,, \tag{34}$$

showing that q_{1j}, p_{1j} is related to the translational degrees of freedom and q_{2j}, p_{2j} to the breathing shape oscillation.

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Since $E(\mathbf{w})$ depends only on the phase difference between the two condensates, the equations of motion (9) give that the total number of particles is conserved, $n_a(t) + n_b(t) = 1$. This reduces the number of degrees of freedom to ten, which we take as the relative phase $\theta = \theta_b - \theta_a$ and the relative population fraction $n = \frac{n_b - n_a}{2}$, in addition to the $q_{1j}, q_{2j}, p_{1j}, p_{2j}, j = a, b$.

III. NUMERICAL RESULTS

To perform the calculations we specify the model parameters . Following reference [14], we consider our system to be ^{87}Rb and take $N = 2.3 \times 10^4$, $\nu_z = 60$Hz which gives $z_{sho} = \sqrt{\hbar/m\omega_z} = 1.4\mu$m and we put all the interaction strength equal to $\lambda_{aa} = \lambda_{bb} = \lambda_{ab} = 17.5$ μm sec$^{-1}\hbar$, in order to reproduce qualitatively the spatial distribution of the condensates shown in FIG.2 of reference [14].

Our first task is to solve the ten equilibrium equations (12). Six of the equations lead immediately to $p_{1j}^0 = p_{2j}^0 = 0, j = a, b$, equal equilibrium population fraction $n^0 = 0$, and $\theta^0 = 0$ or π. The solution with $\theta^0 = 0$ belongs to the even class under $P_z P_{ex}$ and the ones with $\theta^0 = \pi$, to the odd class. We restrict the calculations to the odd class since, as $\Omega > 0$, the lowest energy configuration necessarily belongs to this class. Besides, to be an eigenstate of $P_z P_{ex}$ the equilibrium parameters should obey the conditions $q_{1a}^0 = -q_{1b}^0$ and $q_{2a}^0 = q_{2b}^0$. Therefore, to find the equilibrium configurations we calculate the zeros of four functions $\frac{\partial E}{\partial q_{kj}}(\mathbf{w}^0) = 0$ with $k = 1, 2$ and $j = a, b$ and the parameters restricted as indicated in the above discussion.

We characterize the equilibrium configurations (eqc) by the localization of the center of mass of each component and in Fig.1 we have a graph of the relative distance between the centers of mass as a function of Ω, for appropriately chosen values of z_0. As shown in Fig.1, for $z_0 = 0.23z_{sho}$ we have only one branch of eqc. For small values of $\Omega/\hbar\omega_z$ the condensates are well separated and when the intensity of the laser field increases the overlap between the condensates increases very slowly up to a value of Ω when there is a sharp transition to a mixed phase.

When we diminish the value of z_0, Fig.1, the behavior of eqc changes qualitatively. When $\Omega/\hbar\omega_z$ is small the system behaves as in the previous case. However, when $\Omega/\hbar\omega_z$ increases

there is a critical value of Ω, $\Omega_{c<}$, where two branches of eqc appear, one stable, the other unstable. The two stable eqc distinguished by the separation of the centers of mass are called, respectively, *distant* and *near stable* eqc. When we further increase the value of Ω, the relative distance of the centers of mass in the unstable eqc increases and merges with the stable distant eqc at a critical value of Ω, $\Omega_{c>}$, such that at $\Omega > \Omega_{c>}$ one is left with only the near stable eqc, where the condensates are mixed. For smaller values of z_0 we have the same pattern, the values of $\Omega_{c<}$ where we have three branches of eqc and of $\Omega_{c>}$ where we have the merger of the unstable and distant stable eqc diminishing, this effect being less pronounced for the latter.

Also shown in Fig.1 is the graph of the branches of eqc for $z_0 = 0$. We see that already at Ω near zero we have three branches of eqc, where now in the near stable eqc there is complete overlap between the condensates ($\psi_{oa}(z) = \psi_{ob}(z)$). When Ω increases, the totally mixed eqc remains, with a density profile independent of Ω, whereas the distant stable and the unstable eqc approach each other and merge at $\Omega_{c>}$, such that, at $\Omega > \Omega_{c>}$ one is left with only the totally mixed eqc. Our results also show that we have spontaneous symmetry breaking effects at $z_0 = 0$ [6,8]. Indeed, at $z_0 = 0$, our equations are separately invariant by space reflection, P_z, and atom exchange P_{ex}. The totally mixed eqc obey these symmetries separately, whereas the distant stable eqc do not, being invariant only by the product of these transformations.

To find the signature of the onset of criticality, we calculate the normal modes along the branches of eqc. We can group the five normal modes into two sets. In one set there are two normal modes which are a linear combination of an out-of-phase oscillation of the centers of mass of each condensate and an in-phase breathing oscillation of the condensate densities with the center of mass of the mixture and the population fraction at its equilibrium values. In the second set we have three normal modes which are a linear combination of an in-phase oscillation of the centers of mass, out-of-phase breathing oscillation of the condensate densities and particle exchange between the condensates. The splitting of the normal modes into these two groups is a general result since it follows from the invariance of the equations (4-5) under $P_z P_{ex}$, the normal modes of the first group being even under this transformation and the one of the second, odd.

We found that the signature of criticality in all cases is the collapse of a normal mode

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which is an out-of-phase oscillation of center of mass (dipole oscillation) of the condensates and an in-phase breathing oscillation of the condensate densities. In fig.2 we have a graph of the energies of the two normal modes of the first group, which involves the dipole oscillation of the condensates, for $z_0 = 0.08z_{sho}$, which is a case where we found the existence of critical points (see Fig.1).

In Fig.2a and Fig.2b we present a graph of the energies of the two normal modes along the distant and near stable eqc. In the distant eqc we see that there is one normal mode with an almost constant energy, Fig.2a, and other whose energy increases beginning from $\Omega = 0$, reaches a maximum value and starts to decrease,Fig.2b. When Ω approaches the critical value $\Omega_{c>}$, where the distant stable eqc disappears, the energies of the two normal modes approaches each other and at $\Omega = \Omega_{c>}$ one of the energies goes very abruptly to zero. We have a similar behavior in the graph of the normal mode energies along the near stable eqc, Figs.2a-2b. Approaching from above the point where the near stable eqc disappears, the two energies approaches each other and again one the of them goes abruptly to zero at $\Omega = \Omega_{c<}$. This behavior is completely general near a critical point.

A "scar" of this critical behavior is also present for a value of z_0 at the interface between values where we have and we do not have critical points, such as $z_0 = 0.23z_{sho}$ (see Fig.1). We see in Figs.2c-2d that, corresponding to the very narrow range of values where the eqc change from separated to mixed, we have also an abrupt change in the values of the two normal modes energies. In Fig.3a we detached the region of the sharp change and we see that it occurs in a very narrow range of values of Ω and it is a consequence of a strong level repulsion between the two approaching even normal mode energies.

In Fig.3b we illustrate a generic phenomenon that occurs when $\Omega \to 0$, the appearance of a Goldstone zero energy mode. Indeed, when $\Omega \to 0$, the particle number of each component of the mixture is a conserved quantity and since our theory conserves only the total number of atoms, this violation is translated into the appearance of a zero energy mode. Fig.3b shows how the energy of one of odd normal mode goes to zero for $z_0 = 0.23z_{sho}$.

One question left untouched up to now is the identification of the lowest energy configuration when we have many branches. In our model the answer is that, for small Ω, the distant eqc is always the lowest energy configuration, changing to the near eqc at a higher value of Ω, smaller than $\Omega_{c>}$. However, the energy differences are very small, the equilibrium

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configurations are almost degenerate.

Our conclusions are based in calculations where we took $\lambda_{aa} = \lambda_{bb} = \lambda_{ab}$. It is well known that for homogeneous condensate mixtures, the parameters that controls the mechanism of phase separation is $\lambda_{aa}\lambda_{bb} - \lambda_{ab}^2$ [6]. In coupled mixtures, we add two additional factors which have opposite effects in the mechanism of spatial separation, the trap displacement and the laser coupling field and a point that deserves investigation is how robust are our conclusions when we relax the equal interaction strength condition.

IV. SUMMARY

To summarize, we have studied the equilibrium and stability properties of a coupled two-component BEC, as function of the laser field strength and trap displacement, using the variational method and the one-dimensional model of reference [14]. The laser field has a stabilizer effect in the mechanism of spatial separation of components in the mixture, opposite to the effect of the trap displacement. We found many branches of eqc, with a host of critical points. In all cases the signature of the onset of criticality is the collapse of a normal mode, which is a linear combination of an out-of-phase translation and an in-phase breathing oscillation of the condensate densities.

When the traps are not displaced, we found eqc which exhibits symmetry breaking effects. In principle these eqc with broken symmetry can be reached by, starting at a sufficiently high value of Ω and z_0, adiabatically take the limit $\Omega \to 0$ and $z_0 \to 0$. Taking the limit in the opposite order, we end up in the symmetric eqc (see Fig.1).

Undoubtedly our calculations are simple. However, it unveils a very rich structure in systems of coupled condensates, which should be explored experimentally and theoretically by more complete calculations.

[1] Myatt C J *et al* 1997 *Phys. Rev. Lett.* **78** 586

[2] Hall D S *et al* 1998 *Phys. Rev. Lett.* **81** 1539

[3] Stumper Kurn D M *et al* 1998 *Phys. Rev. Lett.* **80** 2072

[4] Stenger J *et al* 1998 *Nature* **396** 345

[5] Matthews M R *et al* 1999 *Phys. Rev. Lett.* **83** 3358

[6] Timmermans E 1998 *Phys. Rev. Lett.* **81** 5718

[7] Law C K *et al* 1997 *Phys. Rev. Lett.* **79** 3105

[8] Trippenbach M *et al* 2000 J. Phys. B: At. Mol. Opt. Phys.**33** 4017

[9] Ho T -L and Shenoy V B 1996 *Phys. Rev. Lett.***77** 3276

[10] Edwards M *et al* 1996 *Phys. Rev. A***53** R1950

Dalfovo F and Stringari S 1996 *Phys. Rev. A***53** 2477

[11] Baym G and Pethick C J 1996 *Phys. Rev. Lett.* **76** 6

[12] Pérez-Garcia V M *et al* 1996 *Phys. Rev. Lett.* **77** 5320

[13] Pires M O da C and de Passos E J V 2000 J. Phys. B: At. Mol. Opt. Phys.**33** 3929

[14] Williams J *et al* 1999 *Phys. Rev. A.* **59** R31

[15] Dalfovo F Giorgini S Pitaevskii L P and Stringari S *Rev. Mod. Phys.* **71** 463

[16] Williams J *et al* 2000 *Phys. Rev. A.* **61** 033612

[17] Blaizot J P and Ripka G 1986, *Quantum Theory of Finite Systems* (Cambridge, MA, MIT Press)

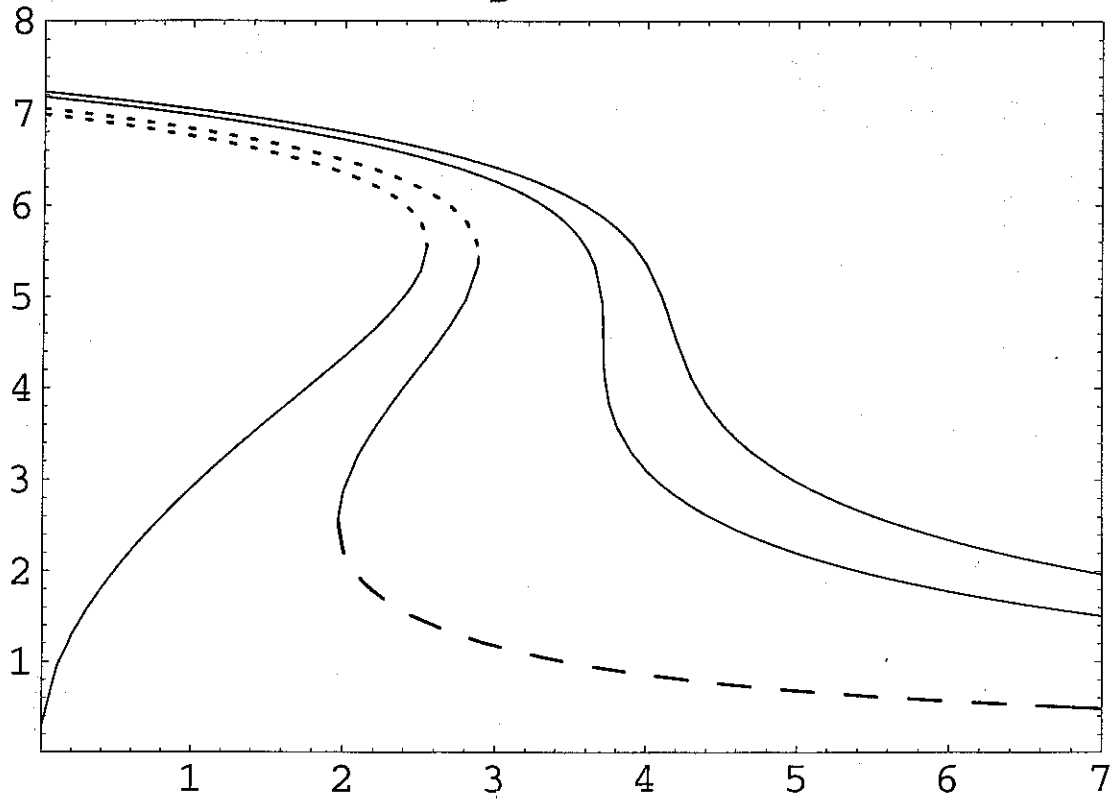[18] Lin C Y de Passos E J V and Lee D S 2000 *Phys. Rev.* **A62** 055603

Figure 1

Figure 1. The plot shows the relative distance of the centers of mass of the two condensates in the eqc, $2q_{1a}$, as function of the laser strength Ω, for fixed values of z_0. Curves from the top correspond to $z_0 = 0.3, 0.23, 0.08, 0$ z_{sho}. The dotted and dashed curves indicate, respectively, the distant and near stable eqc. For $z_0 = 0$ the straight line $2q_{1a} = 0$ correspond to one branch of eqc. The laser strength is expressed in units of $\hbar\omega_z$ and the distance in units of z_{sho}. See text for more details.