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Probes of Supersymmetry**

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Higgs-mediated leptonic decays of B_s and B_d mesons as probes of supersymmetry

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Abstract

If $\tan \beta$ is large, down-type quark mass matrices and Yukawa couplings cannot be simultaneously diagonalized, and flavour violating couplings of the neutral Higgs bosons are induced at the 1-loop level. These couplings lead to Higgs-mediated contributions to the decays $B_s \rightarrow \mu^+ \mu^-$ and $B_d \rightarrow \tau^+ \tau^-$, at a level that might be of interest for the current Tevatron run, or possibly, at B -factories. We evaluate the branching ratios for these decays within the framework of minimal gravity-, gauge- and anomaly-mediated SUSY breaking models, and also in $SU(5)$ supergravity models with non-universal gaugino mass parameters at the GUT scale. We find that the contribution from gluino loops, which seems to have been left out in recent phenomenological analyses, is significant. We explore how the branching fraction varies in these models, emphasizing parameter regions consistent with other observations.

I. INTRODUCTION

Supersymmetry provides a promising way to stabilize the electroweak scale if superpartners are lighter than $\mathcal{O}(1)$ TeV [1]. All SUSY theories [2] necessarily contain many scalar fields, resulting in multiple potential sources of flavour violation. Indeed, in constructing phenomenologically viable models, care has to be taken to make sure that flavour violation is sufficiently suppressed. Within the framework of the Minimal Supersymmetric Standard Model (MSSM) with conserved R -parity, this is ensured (at tree level) by requiring that the matter supermultiplets with weak isospin $T_3 = 1/2$ couple only to the Higgs superfield \hat{h}_u , while those with $T_3 = -1/2$ couple just to the Higgs superfield \hat{h}_d .

At the one loop level, however, a coupling of \hat{h}_u to down type fermions is induced. This induced coupling leads to a new contribution [3], proportional to $\langle h_u \rangle$, to the down type fermion mass matrix. Although this contribution is suppressed by a loop factor relative to the tree-level contribution, this suppression is partially compensated if the ratio $\frac{\langle h_u \rangle}{\langle h_d \rangle} \equiv \tan \beta$ is sufficiently large. As a result, down type Yukawa coupling matrices and down type quark mass matrices are no longer diagonalized by the same transformation, and flavour violating couplings of neutral Higgs scalars h, H and A emerge. Of course, in the limit of large m_A , the Higgs sector becomes equivalent to the Standard Model (SM) Higgs sector with a light Higgs boson $h \simeq h_{SM}$, and the effects of flavour violation decouple from the low energy theory. The interesting feature is that the flavour-violating couplings of h, H and A , *do not decouple for large superparticle mass parameters*: being dimensionless, these couplings depend only on ratios of these mass parameters¹, and so, remain finite even for very large values of SUSY mass parameters.

As pointed out by many authors [4–11], this flavour violating neutral Higgs boson coupling results in a potentially observable branching fraction for the decay $B_s \rightarrow \mu^+ \mu^-$ mediated by the neutral Higgs bosons, h, H and A . If $\tan \beta$ is sufficiently large the most important contribution to the amplitude for this decay then scales as $\tan^3 \beta$: two of these factors arise via the down type quark and lepton Yukawa couplings which are each $\propto 1/\cos \beta \simeq \tan \beta$, while the last factor arises because the offset between the down quark Yukawa coupling matrices and the mass matrices discussed above also increases with $\tan \beta$.

Within the SM, the branching fraction for this decay is $\sim 3.4 \times 10^{-9}$ [12]. This is very far from its current experimental upper limit of 2.6×10^{-6} obtained by the CDF collaboration [13]. Within the SUSY framework, however, this branching fraction can be enhanced by a very large factor and the SUSY contribution may dominate its SM counterpart if $\tan \beta$ is sufficiently large. Indeed, the CDF bound on this branching fraction already constrains some portions of the SUSY parameter space [9,8,14]. It is, therefore, reasonable to expect that the data from the Fermilab Main Injector will probe significant ranges of SUSY parameters. Moreover, because the SUSY amplitude does not decouple for large sparticle masses, it is possible that the CDF or DØ experiments may detect new physics via a measurement of $B(B_s \rightarrow \mu^+ \mu^-)$ even without direct detection of sparticles at the Tevatron.

Most of the analyses [4,6,7,10] of $B_s \rightarrow \mu^+ \mu^-$ have been performed within the MSSM

¹These not only include sparticle masses, but also the superpotential parameter μ and also the soft SUSY breaking A -parameters.

framework. Since sparticle mass matrices as well as soft SUSY breaking scalar coupling matrices both include intrinsic sources of flavour violation, strictly speaking the framework is too broad to be predictive.² Even with assumptions that set tree-level flavour violating effects to zero, the model still has too many free parameters to correlate the branching fraction for $B_s \rightarrow \mu^+ \mu^-$ decay with other observables.

This situation is in sharp contrast to that in highly predictive scenarios such as the minimal supergravity (mSUGRA) model [15], the gauge-mediated SUSY breaking (GMSB) model [16], or the minimal anomaly-mediated SUSY breaking (mAMSB) model [17] in which much of SUSY phenomenology is analysed these days. In each of these scenarios, SUSY is assumed to be dynamically broken in a “hidden sector”, comprised of fields that interact with SM particles (and their superpartners) only via gravity. It is the mechanism by which SUSY breaking is communicated to the superpartners of SM particles that distinguishes these frameworks from one another. These models are all completely specified by just a handful of parameters, usually defined at a scale, much higher than the weak scale, where the physics is simple.

The main goal of this paper is to examine the prospects for observing the decay $B_s \rightarrow \mu^+ \mu^-$ in Tevatron experiments within these framework of well-studied supersymmetry models. We also consider gravity-mediated SUSY breaking models based on $SU(5)$ where the order parameter for SUSY breaking also breaks the GUT symmetry [18], leading to non-universal GUT scale gaugino mass parameters [19]. With an integrated luminosity of 2 fb^{-1} , experiments at the Tevatron are expected to be sensitive to a branching fraction below $\sim 10^{-7}$. With a still bigger data sample (that is expected to accumulate before the Large Hadron Collider (LHC) begins operation) the sensitivity should be even greater.

Of course, while detection of a signal would point to new physics, it would not necessarily establish this new physics to be supersymmetry. Within these specific scenarios, however, it is possible to predict other signals that might also be simultaneously present if an observed $B_s \rightarrow \mu^+ \mu^-$ signal is to be attributed to any particular supersymmetric framework. It is only in the past year that there have been studies [8,9] of this signal within the mSUGRA framework. A corresponding study [11] of this signal within the gauge-mediated and anomaly-mediated SUSY breaking scenario also appeared while this paper was in preparation. As far as we are aware, there is no study of this decay in models with non-universal gaugino masses. As discussed in the next section we have improved upon the existing phenomenological analyses, which consider flavour violating effects just from chargino loops, in that we also include effects from gluino loops which we find to be substantial.

The Higgs-mediated mechanism for the μ -pair decay of B_s mesons also allows the decay $B_s \rightarrow \tau^+ \tau^-$. In fact, since the leptons couple via their Yukawa interactions, the branching ratio for the latter decays would be expected to be enhanced by $(m_\tau/m_\mu)^2$. Unfortunately, identification of τ pair decays of B_s would be very difficult in Tevatron experiments because their invariant mass cannot be accurately reconstructed. Moreover, B_s pair production is not kinematically accessible at the B -factories currently in operation at SLAC or KEK. The BELLE and BABAR experiments at these facilities have already accumulated $\sim 70\text{M}$

²It would, of course, be possible to obtain constraints on flavour-mixing elements of the squark mass matrix or A -parameters.

B_d mesons, and this sample is expected to increase to $\sim 500 - 1000M B_d$ s in a few years. These considerations motivated us to also examine the branching fraction for $B_d \rightarrow \tau^+\tau^-$. Although this decay is suppressed by Kobayashi-Maskawa (KM) matrix elements relative to corresponding decays of B_s , this is offset by the larger τ Yukawa coupling to the Higgs bosons. We are not aware of any studies of the sensitivity of BELLE or BABAR to $B_d \rightarrow \tau^+\tau^-$ decays. We will, therefore, confine ourselves to mapping out the branching fraction for this decay without making any representation of its detectability in experiments at B -factories.

The remainder of this paper is organized as follows. In Sec. II we describe our computation of the amplitude for the decay, focussing on the improvements that we have made, and on the approximations that we have used to simplify the analysis. Sec. III contains our main results. Here we analyse the branching fractions for the decays $B_s \rightarrow \mu^+\mu^-$ and $B_d \rightarrow \tau^+\tau^-$ within the mSUGRA, mGMSB and mAMSB models, as well as in $SU(5)$ supergravity models with non-universal GUT scale gaugino masses. We also comment on other observables (with emphasis on the other flavour-violating decay $b \rightarrow s\gamma$) for parameter regions where $B(B_s \rightarrow \mu^+\mu^-)$ may be observable at the Tevatron. We conclude in Sec. IV with a summary of our results.

II. HIGGS-MEDIATED LEPTONIC DECAYS OF $B_{s,d}$ MESONS

At tree level, the down (up) type quarks couple to the Higgs field h_d^0 (h_u^0) via the Yukawa coupling matrix \mathbf{f}_d (\mathbf{f}_u). Flavour violation in Higgs field interactions occurs because couplings of the field h_u^0 to down type quarks is induced at the one loop level. There are two distinct sets of SUSY sources of this coupling at the 1-loop level [5]. Down type quarks can couple to h_u^0 via

1. a chargino up-type squark loop, with the Higgs field attaching to the squarks via the $\mathbf{A}\mathbf{f}_u$ elements of the soft SUSY breaking trilinear scalar coupling matrix, or
2. via a gluino down-type squark loop, with the Higgs field attaching to the squark via supersymmetric interactions proportional to the matrix $\mu\mathbf{f}_d$. We ignore analogous contributions from neutralino loops; see however, Ref. [10].

The gluino loop, at first sight, appears to give no flavour violation. In the flavour basis, gluino interactions are flavour diagonal, so that the Higgs-squark-squark coupling is the only source of flavour violation. Since the flavour structure of this coupling is proportional to the down quark Yukawa matrix \mathbf{f}_d (*i.e.* the same as the tree-level Yukawa matrix) it does not lead to flavour violation. This reasoning is incorrect because the down-squark matrices need not be flavour diagonal. In other words, the gluino contribution to flavour violation *depends on the a priori unspecified structure of the down squark mass matrices*. The flavour-violating coupling from the chargino contribution above depends on the *essentially unknown* matrix $\mathbf{A}\mathbf{f}_u$. These considerations make it clear why it is difficult to make predictions of the flavour violating branching fractions without resort to specific models, although as we said it is possible to constrain certain (flavour-mixing) matrix elements in a more general analysis. We will, in the following, focus on the four models introduced in Sec. I. The reader should, however, keep in mind that our results for the branching fraction are specific to these

models, and that simple modifications to the flavour structure in the sparticle sector could lead to a very different answer.

It is instructive to analyse the flavour structure of the gluino-induced flavour violation. Following Ref. [5], we will work in the quark basis where the up type Yukawa coupling matrix is diagonal, and the down quark Yukawa coupling matrix has the form, $\mathbf{f}_d = \mathbf{D}\mathbf{V}_{\text{KM}}^\dagger$, where \mathbf{D} is the matrix obtained by diagonalizing \mathbf{f}_d and \mathbf{V}_{KM} is the Kobayashi-Maskawa matrix (uncorrected for SUSY contributions). In the mass basis $[d'_{Li}, d'_{Rj}]$ for quarks, the flavour structure of the gluino contribution can be written as,

$$(Z_R)_{jl}(Z_R^\dagger \mathbf{D} Z_L)_{lk}(Z_L)_{ki} C_0(m_{\tilde{d}_{Lk}}^2, m_{\tilde{d}_{Rl}}^2, m_{\tilde{g}}^2), \quad (1)$$

where $m_{\tilde{d}_{Lk}}^2$ ($m_{\tilde{d}_{Rl}}^2$) is the k th (l th) eigenvalue of the squared mass matrix for left (right) type squarks, Z_L (Z_R) is a unitary matrix that transforms the superpartners of d'_L (d'_R) to the basis in which the left (right) down squark squared mass matrix is diagonal and C_0 is a loop function defined by,

$$C_0(x, y, z) = \frac{1}{y^2 - z^2} \left[\frac{y^2}{x^2 - y^2} \ln\left(\frac{y^2}{x^2}\right) - \frac{z^2}{x^2 - z^2} \ln\left(\frac{z^2}{x^2}\right) \right]. \quad (2)$$

The following features are worth noting:

- If the different types of left and right down squarks have common masses m_L and m_R (not necessarily equal), the function C_0 will be independent of k and l and the flavour violating contribution from (1) vanishes.
- In a wide class of models the left (and, separately, the right) squark mass squared matrices are proportional to the unit matrix at some scale, and only Yukawa interactions distinguish between flavours.³ Then, the matrix \mathbf{m}_{dR}^2 is diagonal in the basis of superpartners of d'_R , and $Z_R = I$. This may be seen by noting that the 1-loop renormalization group equation for the singlet down squark mass matrix depends only on the down type Yukawa coupling matrix. The same is not the case for the doublet squarks since the renormalization group equation now depends also on the up type Yukawa couplings. In all such models, gluino-induced flavour violation depends only upon the mass splittings in the left squark sector. Note that all the models that we consider all fall in this category.

A. Flavour-violating Higgs boson interactions

In our analysis we follow the approach in Ref. [5] and develop an effective Lagrangian to describe the induced flavour violation in the neutral Higgs boson sector. We will assume that squarks with the same quantum numbers have a common mass at some energy scale,

³Operationally, this means that the squark mass matrices have the form, $\mathbf{m}_{\text{dL}}^2 = \bar{m}_L^2 [1 + c_L \mathbf{f}_d^\dagger \mathbf{f}_d + c'_L \mathbf{f}_u^\dagger \mathbf{f}_u]$ and $\mathbf{m}_{\text{dR}}^2 = \bar{m}_R^2 [1 + c_R \mathbf{f}_d \mathbf{f}_d^\dagger]$, with c_L, c'_L and c_R as constants.

and further that superpotential Yukawa interactions are the sole source of flavour violation in the remainder of this analysis. We take the $SU(3)$ gaugino mass parameter to be positive by convention: other gaugino mass parameters may have either sign. As in Ref. [5], we work in the basis where \mathbf{f}_u is diagonal and neglect masses for d and s quarks. Neglecting terms that are second order in the Wolfenstein parameter λ , the effective couplings of down quarks to the Higgs fields of the MSSM can be written as,⁴

$$-\mathcal{L}_{eff} = \overline{D}_R \mathbf{f}_D Q_L h_d + \overline{D}_R \mathbf{f}_D [a_g \mathcal{M}^g + a_u \mathcal{M}^u \mathbf{f}_U^\dagger \mathbf{f}_U + a_w \mathcal{M}^w] Q_L h_u^* + h.c. , \quad (3)$$

with

$$a_g = -\frac{2\alpha_s}{3\pi} \mu m_{\tilde{g}}, \quad a_u = -\frac{1}{16\pi^2} \mu A_t, \quad a_w = \frac{g^2}{16\pi^2} \mu M_2 , \quad (4)$$

and

$$\begin{aligned} \mathcal{M}^g &= \text{diag}(C_{0,1}^{g,i}, C_{0,2}^{g,i}, C_{0,3}^{g,i}), \quad C_{0,j}^{g,i} \equiv C_0(m_{\tilde{g}}, m_{\tilde{d}_{i,R}}, m_{\tilde{d}_{j,L}}), \\ \mathcal{M}^u &= \text{diag}(0, 0, C_{0,3}^u), \quad C_{0,3}^u \equiv C_0(\mu, m_{\tilde{t}_R}, m_{\tilde{t}_L}), \\ \mathcal{M}^w &= \text{diag}(C_{0,1}^w, C_{0,2}^w, C_{0,3}^w), \quad C_{0,j}^w \equiv C_0(M_2, \mu, m_{\tilde{u}_{j,L}}). \end{aligned} \quad (5)$$

The loop function $C_0(x, y, z)$ is given in Eq. (2). Note that the right squark index in the matrix \mathcal{M}^g in (3) is fixed by the corresponding quark; matrix multiplication is implied with the left squark index of the “diagonal matrix” \mathcal{M}^g .

The second term of (3) contains the loop-induced coupling of h_u^0 to down quarks. The first entry in this term comes from the gluino loop, while the last two entries arise due to chargino loops. We see immediately that if $\tilde{Q}_{j,L}$ have a common mass, there is no flavour violation from the gluino term since it then has exactly the same flavour structure as the tree-level term. The term depending on \mathcal{M}^u is proportional to the squared top Yukawa coupling, and arises from the Higgsino components of the chargino, with the fermion chirality flip proportional to the Higgsino mass μ . The last term in (3), which comes from gaugino-higgsino mixing and again vanishes if left type squarks have a common mass. It is, generally speaking, much smaller than the other two terms.

The flavour changing coupling between second and third generation down quarks is given by,

$$\begin{aligned} \mathcal{L}_{FCNC} &= \frac{\bar{f}_b}{\sin \beta} V_{tb}^* V_{ts} \chi_{FC} \bar{b}_R s_L (\cos \beta h_u^* - \sin \beta h_d) \\ &= \frac{\bar{f}_b}{\sqrt{2} \sin \beta} V_{tb}^* V_{ts} \chi_{FC} \bar{b}_R s_L [\cos(\beta + \alpha) h - \sin(\beta + \alpha) H - iA], \end{aligned} \quad (6)$$

where α is the mixing angle in the neutral scalar Higgs boson sector, \bar{f}_b is the “physical b Yukawa coupling” defined by [5],

$$\bar{f}_b = f_b [1 + (a_g C_{0,3}^{g,3} + a_w C_{0,3}^w + a_u C_{0,3}^u f_t^2) \tan \beta], \quad (7)$$

⁴This is the analogue of Eq. (7) of Ref. [5].

and

$$\chi_{FC} = -\frac{[a_g(C_{0,3}^{g,3} - C_{0,2}^{g,3}) + a_w(C_{0,3}^w - C_{0,2}^w) + a_u C_{0,3}^u f_t^2] \tan \beta}{[1 + (a_g C_{0,2}^{g,3} + a_w C_{0,2}^w) \tan \beta][1 + (a_g C_{0,3}^{g,3} + a_w C_{0,3}^w + a_u C_{0,3}^u f_t^2) \tan \beta]}. \quad (8)$$

We have checked that if we multiply (7) by v_d , we recover the SUSY correction to the b quark mass as given by Pierce *et al.* [20]. For degenerate squarks, we recover the expression for χ_{FC} in Ref. [5] by setting $a_w = 0$ in the denominator of (8).

The flavour violating couplings between first and third generation down squarks are obtained by obvious substitutions. If squarks of the first and second generations are degenerate (as is the case in many models, including all models we consider in this paper) we can replace s_L by d_L without changing χ_{FC} . Therefore, for the B_d calculation we only have to replace $V_{ts} \rightarrow V_{td}$ in Eq. (6), keeping everything else the same.

Eq. (8) ignores intra-generation squark mixing. Assuming that this is significant only for third generation squarks, we find the modified result,

$$\chi_{FC} = -\frac{N}{D} \tan \beta, \quad (9)$$

with

$$\begin{aligned} N &= a_g \left[C_0(m_{\tilde{g}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}) - s_b^2 C_0(m_{\tilde{g}}, m_{\tilde{b}_1}, m_{\tilde{s}_L}) - c_b^2 C_0(m_{\tilde{g}}, m_{\tilde{b}_2}, m_{\tilde{s}_L}) \right] + a_u f_t^2 C_0(\mu, m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\ &\quad + a_w \left[c_t^2 C_0(M_2, \mu, m_{\tilde{t}_1}) + s_t^2 C_0(M_2, \mu, m_{\tilde{t}_2}) - C_0(M_2, \mu, m_{\tilde{e}_L}) \right], \\ D &= \left\{ 1 + a_g \tan \beta \left[s_b^2 C_0(m_{\tilde{g}}, m_{\tilde{b}_1}, m_{\tilde{s}_L}) - c_b^2 C_0(m_{\tilde{g}}, m_{\tilde{b}_2}, m_{\tilde{s}_L}) \right] + a_w \tan \beta C_0(M_2, \mu, m_{\tilde{e}_L}) \right\} \\ &\quad \times \left\{ 1 + a_g \tan \beta C_0(m_{\tilde{g}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}) + a_w \tan \beta \left[c_t^2 C_0(M_2, \mu, m_{\tilde{t}_1}) + s_t^2 C_0(M_2, \mu, m_{\tilde{t}_2}) \right] \right. \\ &\quad \left. + a_u f_t^2 \tan \beta C_0(\mu, m_{\tilde{t}_1}, m_{\tilde{t}_2}) \right\}, \end{aligned} \quad (10)$$

where $s_{b,t} \equiv \sin \theta_{b,t}$, $c_{b,t} \equiv \cos \theta_{b,t}$, and $\theta_{b,t}$ are the bottom and top squark mixing angles, respectively.

We should note that our calculation includes only terms that are most enhanced by powers of $\tan \beta$. Our calculation is, therefore, valid only when $\tan \beta \gtrsim 25 - 30$. We will subsequently see that for smaller values of $\tan \beta$ the branching fraction of interest is too low to be of interest at the Tevatron, so that this is not a serious handicap for the purpose of our analysis. We remark that in the loop functions we have assumed that the chargino masses are well approximated by $|M_2|$ and $|\mu|$.

Finally, the leptonic couplings of the neutral Higgs bosons h , H and A of the MSSM that are needed to complete the evaluation of Higgs boson mediated leptonic decays of B_s or B_d are given by,

$$\mathcal{L}_{H\ell\bar{\ell}} = -\frac{gm_\ell}{2M_W \cos \beta} [h \sin \alpha + H \cos \alpha] \bar{\ell}\ell + \frac{igm_\ell \tan \beta}{2M_W} \bar{\ell} \gamma_5 \ell A. \quad (11)$$

B. The branching ratio for $B_{s,d} \rightarrow \ell^+ \ell^-$ decays

The effective Hamiltonian [12] for $B_{d'} \rightarrow \ell^+ \ell^-$, $d' = s, d$, is given by

$$H = \frac{G_F}{\sqrt{2}} V_{td'}^* V_{tb} [c_{10} \mathcal{O}_{10} + c_{Q_1} Q_1 + c_{Q_2} Q_2] + h.c. ,$$

with the relevant operators being given by,

$$\mathcal{O}_{10} = \frac{e^2}{4\pi^2} \bar{d}'_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \gamma_5 \ell, \quad Q_1 = -\frac{e^2}{4\pi^2} \bar{d}'_L b_R \bar{\ell} \ell, \quad Q_2 = -\frac{e^2}{4\pi^2} \bar{d}'_L b_R \bar{\ell} \gamma_5 \ell. \quad (12)$$

The coefficient C_{10} that encapsulates the SM contribution is given by [12],

$$c_{10} = -\frac{Y(x_t)}{s_W^2} \approx -4.2. \quad (13)$$

The function $Y(x_t)$ that appears here is defined in Ref. [12]. The SUSY contributions can be obtained by applying the matching condition between the amplitude given by the effective Hamiltonian and the one obtained using (6) and (11). We find,

$$\begin{aligned} c_{Q_1} &= \frac{2\pi}{\alpha} \chi_{FC} \frac{m_b m_\ell}{\cos^2 \beta \sin^2 \beta} \left(\frac{\cos(\beta + \alpha) \sin \alpha}{m_h^2} - \frac{\sin(\beta + \alpha) \cos \alpha}{m_H^2} \right), \\ c_{Q_2} &= \frac{2\pi}{\alpha} \chi_{FC} \frac{m_b m_\ell}{\cos^2 \beta} \frac{1}{m_A^2}. \end{aligned} \quad (14)$$

Note that as $m_A \rightarrow \infty$, $\alpha + \beta \rightarrow \frac{\pi}{2}$ and $m_H \simeq m_A$. This then implies $C_{Q_1} \simeq -C_{Q_2}$, an observation that will prove useful later.

Finally, using the hadronic matrix elements,

$$\begin{aligned} \langle 0 | \bar{b} \gamma^\mu \gamma_5 d'(x) | B_{d'}(P) \rangle &= i f_{B_{d'}} P^\mu e^{-iP \cdot x}, \\ \langle 0 | \bar{b} \gamma_5 d'(x) | B_{d'}(P) \rangle &= -i f_{B_{d'}} \frac{m_{B_{d'}}^2}{m_b + m_{d'}} e^{-iP \cdot x}, \end{aligned}$$

we can write the branching ratio for the decay $B_{d'} \rightarrow \ell^+ \ell^-$ as,

$$\begin{aligned} B(B_{d'} \rightarrow \ell^+ \ell^-) &= \frac{G_F^2 \alpha^2 m_{B_{d'}}^3 \tau_{B_{d'}} f_{B_{d'}}^2}{64\pi^3} |V_{tb}^* V_{td'}|^2 \sqrt{1 - \frac{4m_\ell^2}{m_{B_{d'}}^2}} \\ &\times \left[\left(1 - \frac{4m_\ell^2}{m_{B_{d'}}^2} \right) \left| \frac{m_{B_{d'}}}{m_b + m_{d'}} c_{Q_1} \right|^2 + \left| \frac{2m_\ell}{m_{B_{d'}}} c_{10} - \frac{m_{B_{d'}}}{m_b + m_{d'}} c_{Q_2} \right|^2 \right] \end{aligned} \quad (15)$$

C. Comparison with other studies

A complete calculation of the 1-loop chargino-induced flavour violating MSSM Higgs boson couplings, along with the corresponding result for the coefficients C_{Q_1} and C_{Q_2} in (14), may be found in Ref. [7]. In comparison, we have only retained leading terms in

$\tan \beta$, so that our simplified calculation is valid only if $\tan \beta \gtrsim 25 - 30$. We have checked that our result for the contribution to the coefficients $C_{Q,1,2}$ from the chargino graphs indeed reduces to (5.13) of Ref. [7], with $m_{\tilde{W}_{1,2}} = |M_2|$ and $|\mu|$. The results of Ref. [7] have been used for the recent phenomenological analysis [8,9] within the mSUGRA framework.⁵ Our calculations improve upon these in that we include contributions from the gluino-mediated flavour violating contributions (which we will see are significant) first discussed by Babu and Kolda [5]. We found it difficult to tell whether or not these contributions were included in the recent phenomenological analysis of the mGMSB and mAMSB scenarios by Baek *et al.* [11]: they do not give any formulae nor do they discuss how their calculation was performed.

While we were preparing this paper, Ref. [10] appeared. This study includes a complete calculation of gluino and also analogous neutralino-induced contributions to flavour violating Higgs boson couplings, but again within the framework of the MSSM with specific assumptions about sfermion mass matrices. It is pointed out that it is possible to find special regions of MSSM parameter space where due to large cancellations between gluino and chargino contributions, neutralino diagrams (which are usually small) can no longer be neglected. Indeed, in this case even the ratio of the branching fractions $B(B_d \rightarrow \mu^+ \mu^-)/B(B_s \rightarrow \mu^+ \mu^-)$ may be different from that expected from ratios of Kobayashi-Maskawa matrix elements.

III. $B(B_{s,d} \rightarrow \ell^+ \ell^-)$ IN SUSY MODELS: RESULTS

We are now ready to proceed with the evaluation of $B(B_{s,d} \rightarrow \ell^+ \ell^-)$ in the various SUSY models discussed above. For a given point in the parameter space of these models, we use the program ISAJET v 7.63 [21] to evaluate the corresponding MSSM parameters that we need for all phenomenological analysis, including the computation of this branching ratio. This then allows us to assess the ranges of model parameters where signals from $B(B_{s,d} \rightarrow \ell^+ \ell^-)$ decays may provide evidence of deviation from the SM. A clear advantage of this approach is that for any *given model*, it is also possible to compute SUSY contributions to other observables; *e.g.* $B(b \rightarrow s \gamma)$ or $g_\mu - 2$ that have already been constrained by experiment [22,23]. Moreover, constraints from the non-observation of any sparticles [24] or Higgs bosons [25] which yield lower limits, $m_{\tilde{W}_1} \geq 103$ GeV, $m_{\tilde{e}} \geq 100$ GeV, $m_{\tilde{\tau}_1} \geq 76$ GeV, $m_h \geq 113$ GeV and $m_A \geq 100$ GeV, on their masses can be readily incorporated.⁶

ISAJET 7.63 includes several improvements over previous versions. From our point of view, the most important of these is the improvement of the bottom Yukawa coupling that enters the computation of m_A . We see from (14) and (15) that the value of m_A plays an important role in the determination of the branching ratio of interest. There are regions

⁵The earlier studies [4,6] were performed within the MSSM for particular choices of parameters.

⁶Although the exact limits on sparticle masses depend somewhat on the model, and also on where we are in parameter space, the limits indicated here are applicable for a wide class of models. The lower limit of 114 GeV on the mass of the SM Higgs boson has to be translated into the limit on m_h . Except when A is light, h is essentially the SM Higgs boson and this limit applies essentially without modification.

of parameter space where, because of cancellations, m_A may be considerably smaller than other sparticle masses. Especially for these parameter ranges, the improved computation of the Yukawa coupling plays a crucial role.

The numerical values of various meson masses, lifetimes, decay constants and Kobayashi-Maskawa mixing matrix elements that we use as inputs to our analysis are,

B_s mesons:

$$m_{B_s} = 5.3696 \text{ GeV}, \quad f_{B_s} = 0.250 \text{ GeV}, \quad \tau_{B_s} = 1.493 \text{ ps}$$

B_d mesons:

$$m_{B_d} = 5.2794 \text{ GeV}, \quad f_{B_d} = 0.208 \text{ GeV}, \quad \tau_{B_d} = 1.548 \text{ ps}$$

Kobayashi-Maskawa matrix elements:

$$|V_{tb}| = 0.999, \quad |V_{ts}| = 0.039, \quad |V_{td}| = 0.009$$

A. Minimal supergravity model (mSUGRA)

SUSY is assumed to be broken in a hidden sector consisting of fields that interact with usual particles and their superpartners only via gravity. SUSY breaking is communicated to the visible sector via gravitational interactions, and soft SUSY breaking sparticle masses and couplings are generated. Without further assumptions, the scalar masses can be arbitrary leading to flavor changing processes in conflict with experiment.

Within the mSUGRA grand unified framework [15], it is assumed that at some high scale (frequently taken to be $\sim M_{GUT}$) all scalar fields have a common SUSY breaking mass m_0 , all gauginos have a mass $m_{1/2}$, and all soft SUSY breaking scalar trilinear couplings have a common value A_0 . Electroweak symmetry breaking is assumed to occur radiatively. This fixes the magnitude superpotential parameter μ . The soft SUSY breaking bilinear Higgs boson mass parameter can be eliminated in favour of $\tan \beta$, so that the model is completely specified by the parameter set:

$$m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu). \quad (16)$$

The weak scale SUSY parameters that enter the computation of sparticle masses and couplings required for phenomenological analyses can be obtained via renormalization group evolution between the scale of grand unification and the weak scale.

We use the program ISAJET to evaluate these MSSM parameters, and via these the various masses and mixing angles that enter the flavour violating coupling χ_{FC} given by Eq. (10). The required partial width can then be computed using Eq. (15).

In Fig. 1 we show the dependence of $B(B_s \rightarrow \mu^+ \mu^-)$ and $B(B_d \rightarrow \tau^+ \tau^-)$ on *a*) $\tan \beta$ for $A_0 = 0$, and *b*) A_0 for $\tan \beta = 40$. We fix $m_0 = m_{1/2} = 300 \text{ GeV}$, and illustrate the results for both positive (solid) and negative (dashed) values of μ . Values of $\tan \beta$ ($|A_0|$) larger than the corresponding value denoted by squares on the curves are where a sparticle or Higgs boson mass (in this case, it is always m_h) falls below its experimental bound. The uppermost set of the dashed and solid lines corresponds to the branching fraction for the

decay $b \rightarrow s\gamma$, the scale for which is given on the right hand axis. Our purpose in showing this figure is to understand the behaviour of the leptonic decays of B_d and B_s , so that the fact that $B(b \rightarrow s\gamma)$ appears to be outside the allowed range should not bother the reader. Several features are worth noting.

1. Since we have neglected contributions from neutralino loops [10], the branching ratio for $B_d \rightarrow \tau^+\tau^-$ decays is larger than that for $B_s \rightarrow \mu^+\mu^-$ decays by just a constant factor fixed by SM parameters.
2. Focussing for the moment on the low $\tan\beta$ end of the $B(B_s \rightarrow \mu^+\mu^-)$ curves, we see that the solid and dashed lines lie on either side of the SM value, reflecting the fact that the sign of the SUSY contribution flips with the sign of μ , and its interference with the SM contribution goes from destructive for $\mu < 0$ to constructive for positive values of μ . Of course, this becomes irrelevant for very large values of $\tan\beta$ where the SUSY contribution dominates.
3. A striking feature in both frames is the very sharp rise in the branching ratio for $\mu < 0$. Naively, we expect the SUSY contribution to behave as $\tan^6\beta/m_A^4$.⁷ The dashed curves in frame *a*) indeed roughly show this scaling behaviour as long as we stay away from the lower range of $\tan\beta$ values where interference with the SM amplitude is significant. But this is somewhat fortituous because the dashed curves in frame *b*) rise much faster than would be naively expected. This is because it is also very important to take into account differences in other MSSM parameters, most notably A_t and μ that enter via a_u and a_g in (3). In general, the dashed curves are steeper than the solid curves because, for $\mu < 0$, m_A decreases sharply as $\tan\beta$ increases: there is no corresponding decrease of positive values of μ .
4. It is interesting to see that both $B(B_s \rightarrow \mu^+\mu^-)$ and $B(B_d \rightarrow \tau^+\tau^-)$ can be potentially very large (and even exceed the current upper limit) for values of parameters where there are no *direct* signals for SUSY or Higgs bosons. This region of parameters is also excluded by the experimental value of the inclusive branching ratio $B(b \rightarrow s\gamma)$ [22]. It should be understood though that our purpose in showing this is pedagogical.
5. We have also examined the case with $m_0 = 1$ TeV and other parameters as before, except that the allowed range of A_0 is much larger. For positive values of μ and $A_0 = 0$, $B(B_s \rightarrow \mu^+\mu^-)$ varies between $(4 - 100) \times 10^{-9}$. For $\mu < 0$, this branching fraction is close to or smaller than the SM prediction, except when $46 \lesssim \tan\beta \lesssim 48$ when the branching fraction shoots up by over two orders of magnitude. For $\tan\beta = 40$ as in Fig. 1*b* and $\mu > 0$, the corresponding dependence on A_0 is qualitatively similar to that the figure. For negative values of μ the branching fraction is close to its SM value for $|A_0| \lesssim 1.2m_0$, but rapidly shoots up to beyond 10^{-7} when $|A_0|$ becomes large. As in the $m_0 = 300$ GeV case, the rapid rise is driven by the rapid drop in m_A .

⁷Recall that the contribution from h exchange is very small as long as h is a SM-like Higgs, and further, that $m_H \sim m_A$ in the same limit.

Having used this pedagogical illustration to obtain an idea of how and why the branching fractions vary with parameters, we now turn to an exploration of these branching ratios in mSUGRA parameter space. Because the sparticle spectrum is most sensitive to m_0 and $m_{1/2}$, the $m_0 - m_{1/2}$ plane provides a convenient way of displaying the results, as illustrated in Fig. 2 for $A_0 = 0$ and *a*) $\mu > 0, \tan\beta = 50$ and *b*) $\mu < 0, \tan\beta = 45$. The dark-shaded regions are excluded by theoretical constraints: charge-breaking minima or lack of electroweak symmetry breaking. In the slant-hatched region, \tilde{Z}_1 is not the lightest supersymmetric particle (LSP). The region covered by open circles is excluded by lower limits on sparticle masses or on m_A . Below the dot-dashed line labelled h , m_h is smaller than 113 GeV: we show this separately because this limit is modified if $m_A \lesssim 150$ GeV. Our main results are the contours of $B(B_s \rightarrow \mu^+\mu^-)$ (solid) and $B(B_d \rightarrow \tau^+\tau^-)$ (dashed) within the mSUGRA framework. The contours are labelled by the values of the corresponding branching fractions. In frame *a*), the innermost dashed contour corresponds to a branching fraction of 10^{-6} . From frame *a*) we see that even in the region allowed by all these constraints $B(B_s \rightarrow \mu^+\mu^-)$ ($B(B_d \rightarrow \tau^+\tau^-)$) may be as large as 10^{-7} (10^{-6}), while from frame *b*) the corresponding branching fraction may be considerably larger. Also shown are contours of fixed branching fraction for the decay $b \rightarrow s\gamma$ computed using the calculation of Ref. [26] with improvements described in Ref. [14]. Below the contours labelled $b \rightarrow s\gamma$ in frame *a*) (frame *b*)) the branching fraction is smaller (larger) than 2×10^{-4} (5×10^{-4}), and appear to be comfortably outside our assessment $2.16 \times 10^{-4} \lesssim B(b \rightarrow s\gamma) \lesssim 4.34 \times 10^{-4}$, [14] for the allowed experimental range [22] of this branching fraction. We caution the reader that the SUSY contribution, for large values of $\tan\beta$, may have considerable theoretical uncertainty, so that this “constraint” should be interpreted with some care.⁸

The dotted curves are contours of $B(B_s \rightarrow \mu^+\mu^-) = 10^{-8}$, obtained by calculating the branching ratio by retaining just the chargino-loop in evaluating the SUSY contribution to the decay *i.e.* if a_g in (3) is set to zero. We remind the reader that it is just this contribution has been included in recent analyses [8,9]. We have checked that the dotted contour in frame *a*) is in good agreement with the corresponding contour in Fig. 3 of Ref. [8]. This provides quantitative confirmation of the validity of our approximations, relative to the complete calculation of Ref. [7]. More importantly, the difference between the dotted contour and the corresponding solid contour highlights the importance of retaining the effect of gluino loops whose contribution appears to interfere destructively with the chargino contribution. Thus, earlier conclusions [8,9] about the SUSY reach of Tevatron experiments may be over-optimistic.

To see how the sensitivity of these experiments varies with $\tan\beta$, we show in Fig. 3

⁸It is also worth emphasizing that unlike constraints from direct searches, constraints from $B(b \rightarrow s\gamma)$ are very sensitive to details of the model. For instance, small amount of flavour mixing (from some unknown physics) in the squark sector could lead to large differences in the predictions for $b \rightarrow s\gamma$, and for that matter, $B_{q'} \rightarrow \ell^+\ell^-$ decays. For this reason, we urge our experimental colleagues to view theorists’ assessment of “excluded regions” (especially when these are excluded due to SUSY loop effects as opposed to direct searches) in the proper perspective. It is logically possible that small deviations from the defining assumptions of a particular framework such as mSUGRA may permit much larger signals without being in conflict with current constraints.

contours of $B(B_s \rightarrow \mu^+\mu^-) = 10^{-7}$ (solid lines) and $B(B_d \rightarrow \tau^+\tau^-) = 10^{-6}$ (dashed lines) for the values of $\tan\beta$ that label the contours. We take $A_0 = 0$ and show results for a) $\mu > 0$ and b) $\mu < 0$. The hatched region corresponds to $\tan\beta = 35$. The lines in frame a) terminate at the corresponding boundaries of the theoretically forbidden regions. As expected, the region over which the branching fraction may be probed at the Tevatron is very sensitive to $\tan\beta$. Even for $\tan\beta = 35$, this channel appears to yield a better SUSY sensitivity than direct searches [27] for an integrated luminosity of $\sim 2 \text{ fb}^{-1}$.

From Fig. 2 and Fig. 3 we note the following:

- The branching fraction under study is significantly larger for negative values of μ . We have traced this to the structure of the denominator of χ_{FC} in Eq. (10). Changing the sign of μ changes the sign of all the a_i , so that a suppression for positive μ changes to an enhancement for $\mu < 0$. Notice that while our denominator factor reduces to that in Ref. [5] in the appropriate limit, it differs from that in Ref. [8] who do not have the a_g term in the second factor.⁹ Quantitatively, this leads to differences of $\mathcal{O}(10\%)$ and do not affect the qualitative conclusions.
- Although $\mu < 0$ is generally thought to be disfavoured by the determination of the muon anomalous magnetic moment by the E821 experiment [23], we advise caution in this regard: in contrast to conventional wisdom [28] a conservative estimate [29] of the theoretical error suggests that there is a region allowed [14] by this constraint, though perhaps in conflict with $B(b \rightarrow s\gamma)$, where $B_s \rightarrow \mu^+\mu^-$ decays may provide the first hint of new physics if $\tan\beta \gtrsim 42$ or so. This region would expand as Tevatron experiments accumulated more data.
- From these figures, we see that a sensitivity of 10^{-6} for $B(B_d \rightarrow \tau^+\tau^-)$ would roughly yield the same reach as an order of magnitude better sensitivity that is expected to be attained at the Tevatron with an integrated luminosity of $\sim 2 \text{ fb}^{-1}$. We do not know whether experiments at B factories will be able to attain this sensitivity.

Up to now, in our scan of mSUGRA parameter space, we have only considered models with $A_0 = 0$. To understand just how large the branching ratio for $B_s \rightarrow \mu^+\mu^-$ can become for other values of A_0 , we have scanned mSUGRA models with m_0 between 100 GeV and $\sqrt{50}m_{1/2}$ and $-3m_0 < A_0 < 3m_0$. In Fig. 4, we show $B(B_s \rightarrow \mu^+\mu^-)$ as a function of $m_{1/2}$, for a) $\tan\beta = 55$, $\mu > 0$, b) $\tan\beta = 45$, $\mu < 0$, and c) $\tan\beta = 40$, $\mu < 0$. Models are accepted only if they satisfy the theoretical and the experimental constraints from direct searches for sparticles and Higgs bosons. For each model, we put a cross (dot) if $B(b \rightarrow s\gamma)$ lies within (outside) $(2 - 5) \times 10^{-4}$. We note the following:

1. Although we have shown just $B(B_s \rightarrow \mu^+\mu^-)$ in this figure, the range of $B(B_d \rightarrow \tau^+\tau^-)$, which differs only by a constant factor can easily be estimated.

⁹In Ref. [7], which is a strict diagrammatic calculation, does not have this denominator. Dedes *et al.* who use the results of Ref. [7], include a denominator correction in their formulae.

2. We see that for $\mu > 0$, in frame *a*) the branching fraction is typically smaller than 10^{-8} , and never exceeds $\sim 3 \times 10^{-8}$ for models loosely satisfying the $b \rightarrow s\gamma$ constraint. But for this constraint, it can be as high as 10^{-7} .
3. Turning to frame *b*) we see that for negative values of μ the branching fraction can be as large as 10^{-5} , but most of these points are excluded by the experimental measurement of $b \rightarrow s\gamma$ because the predicted branching fraction is too large. For large values of $m_{1/2}$, however, $B(B_s \rightarrow \mu^+\mu^-)$ may even exceed 10^{-7} for parameter values allowed by $b \rightarrow s\gamma$ constraints. For the slightly smaller value of $\tan\beta$ shown in frame *c*), we see that these large values of $B(B_s \rightarrow \mu^+\mu^-)$ are possible over a wide range of $m_{1/2}$.
4. In all frames we see that although the branching ratio may be much larger than the SM value for some set of SUSY parameters, it never falls below about half the SM value. Experiments at the LHCb should be sensitive (at the 3σ level) down to this branching fraction [30], so that a non-observation of this decay in these experiments would signal new physics other than the scenarios considered here.¹⁰
5. A striking feature of the figure is the “gap” for small $m_{1/2}$ values in frame *a*). It appears that for positive μ and very large values of $\tan\beta$ the branching ratio in mSUGRA is unlikely to be at the SM value. Of course, when $m_{1/2}$ (and hence m_A) is very large, the branching ratio tends to the SM value. The gap is absent for $\mu < 0$. The existence of this gap can be qualitatively understood by focussing on the chargino contribution (proportional to the a_u term) and recognizing that models with positive μ allow both signs of A_t , while models with negative μ almost always have $A_t < 0$. As a result, the SUSY contribution may interfere constructively or destructively with the SM contribution when $\mu > 0$, while for negative values of μ the interference is almost always destructive. Of course, the interference is important only when the SUSY and SM amplitudes have comparable magnitudes. For the large values of $\tan\beta$ in this figure, this happens only if m_A is large (otherwise the SUSY amplitude is much larger than the SM one). In this case, the coefficients c_{Q_1} and c_{Q_2} in Eq. (15) have the same magnitude.¹¹ In units where the SM contribution in the square parenthesis of Eq. (15) is 1, if we write the c_{Q_2} contribution as $x = -c_{Q_2}m_{B_s}/2c_{10}m_\mu$, the partial width is determined by $F = x^2 + (1+x)^2$. This has a minimum of $1/2$, explaining why the branching fraction does not fall below about half its SM value. Moreover, as long as x is positive and not small (as is the case for small values of $m_{1/2}$) F significantly exceeds the SM value. If $x < 0$, F becomes large only if $|x|$ is very large, thereby accounting for the gap for positive values of μ . For negative values of μ , there is no “positive x branch”, and hence, no gap.

¹⁰Admittedly we have not scanned all parameter space. However, if $\tan\beta$ is small the SUSY contribution is reduced, and we would expect the branching fraction to be closer to its SM value.

¹¹This really follows because of the properties of the MSSM Higgs sector, and so should be true in other models also.

6. Although we have not shown this, the scatter plot for $\tan\beta = 50$, $\mu > 0$ is very similar to frame *a*), except that the “gap” is somewhat less pronounced. In other words, the difference between the distribution of dots and crosses in frames *b*) and *c*) is absent.

B. Minimal gauge-mediated SUSY breaking model (mGMSB)

In these models, SUSY breaking is again assumed to occur in a hidden sector which somehow couples to a set of messenger particles that couple not only to this hidden sector, but also have SM gauge interactions. The coupling to the hidden sector induces SUSY breaking in the messenger sector, which is then conveyed to superpartners of usual particles via SM gauge interactions. Messenger particles are assumed to occur in n_5 complete vector representations of $SU(5)$ with quantum numbers of $SU(2)$ doublets of quarks and leptons. We assume $n_5 \leq 4$ since otherwise gauge couplings do not remain perturbative up to the scale of grand unification. The messenger sector mass scale is characterized by M . The soft SUSY breaking masses for the SUSY partners of SM particles are thus proportional to the strength of their corresponding gauge interactions, so that squarks are heavier than sleptons. Gaugino masses satisfy the usual “grand unification” mass relations, though for very different reasons. Within the minimal version of this framework, the couplings and masses of the sparticles in the observable sector are determined (at the messenger scale M) by the parameter set;

$$\Lambda, M, n_5, \tan\beta, \text{sign}(\mu), C_{grav}. \quad (17)$$

Λ sets the scale of sparticle masses and is the most important of these parameters. The parameter $C_{grav} \geq 1$ and enters only into the partial width for sparticle decays to the gravitino and is irrelevant for our analysis. The model predictions for soft-SUSY breaking parameters at the scale M are evolved to the weak scale using ISAJET and used to compute $B(B_{q'} \rightarrow \ell^+ \ell^-)$ as before.

The novel feature of the GMSB framework is that SUSY breaking can be a relatively low energy phenomenon if the messenger scale is small. It is in this case that the gravitino can be an ultra-light LSP. In such a scenario heavy sparticles cascade decay to the next lightest supersymmetric particle (NLSP) which then decays to the gravitino and an ordinary particle. The NLSP may be the lightest neutralino or the stau.¹² SUSY signatures at the Tevatron [31] are sensitive to the nature of the NLSP. In our considerations we will focus on models with relatively low messenger scales for which the collider phenomenology differs the most from mSUGRA.

An important difference between this framework and the mSUGRA model is that A -parameters (at the scale M) which are generated only at two loops are much smaller than scalar and gaugino masses. The weak scale parameter A_t that is obtained by renormalization group evolution from the messenger scale is typically smaller than in mSUGRA if the messenger scale is not large. We thus expect that a_u that sets the scale of the chargino

¹²For low $\tan\beta$ values not of interest to us, the sleptons of different generations are essentially degenerate leading to the so-called co-NLSP scenario [31].

contributions in (3) is significantly smaller than in the mSUGRA framework. We also expect that the gluino contribution to the loop decay of $B_{q'}$ would also be smaller than in mSUGRA because there is not enough room to run and induce large mass splitting between the different Q_L s by renormalization group evolution if the messenger scale is low. Therefore, we typically expect lower values of $B(B_{q'} \rightarrow \ell^+ \ell^-)$ relative to corresponding results for mSUGRA in these scenarios. This branching fraction would, for the same reasons, increase logarithmically with increasing messenger scale.

Our results for the branching fraction within the GMSB framework are shown in the $\Lambda - \tan \beta$ plane in Fig. 5. Here, we fix $M = 3\Lambda$, $n_5 = 1$ and illustrate the results for both signs of μ . As before, the dark-shaded region is excluded by the theoretical considerations on EWSB discussed previously. The bulge on the left comes from $m_{\tilde{\tau}_1}^2 < 0$, while in the horizontal strip at high $\tan \beta$ that extends to large Λ values, $m_A^2 < 0$. In the slant-shaded region $m_{\tilde{\tau}_1} < 76$ GeV, while in the slivers covered by triangles along the boundary of the horizontal dark-shaded region, $m_A < 100$ GeV. Finally, values of Λ up to $\Lambda \sim 53$ TeV (covered by open circles) are excluded because $m_{\tilde{e}_R} < 100$ GeV, and possibly also by constraints on the chargino mass or Z^0 decay properties [32]. To the left of the dot-dashed contour labelled h , $m_h < 113$ GeV. The branching fraction for the decay $b \rightarrow s\gamma$ is between $(2 - 5) \times 10^{-4}$ over the entire plane in frame *a*), while it is in this range to the right of the corresponding contour in frame *b*). Contours of $B(B_s \rightarrow \mu^+ \mu^-)$ and $B(B_d \rightarrow \tau^+ \tau^-)$ are shown as solid and dashed lines, respectively. These are labelled by the value of the corresponding branching fraction. We see that over almost the entire plane $B(B_s \rightarrow \mu^+ \mu^-)$ ($B(B_d \rightarrow \tau^+ \tau^-)$) is smaller than 10^{-8} (10^{-7}) confirming our earlier expectation that the branching fractions will be smaller than in mSUGRA. It would be difficult to probe these decays at this level at existing facilities. It is only for the largest values of $\tan \beta$ where we approach the region where m_A^2 dives to very small values that these branching fractions might be accessible: however, of this region in frame *b*) is “excluded” by the experimental value of $B(b \rightarrow s\gamma)$. In contrast, direct SUSY searches should be sensitive to Λ values of 118-145 TeV depending on the integrated luminosity that is accumulated [31,33]. Direct searches at the LHC will easily probe the entire plane [34].

Up to now, we have assumed $n_5 = 1$. Since scalar (gaugino) masses scale with $\sqrt{n_5}$ (n_5) we would, expect large sensitivity to n_5 if we show the results in terms of Λ . To avoid a proliferation of figures, in Fig. 6 we show the branching fractions versus $m_{\tilde{g}}$ for $n_5 = 1 - 4$. We have fixed $M = 3\Lambda$ and $\tan \beta = 40$ and illustrated the results for both signs of μ . The curves are terminated at values of $m_{\tilde{g}}$ that violate any of the constraints discussed previously. For very large values of $m_{\tilde{g}}$ the branching fractions approach the SM values because m_A tends to be large. For smaller values of $m_{\tilde{g}}$ the branching fractions (for a given value of $m_{\tilde{g}}$) do depend on n_5 . Nevertheless, it seems that *the range over which these vary is insensitive to the choice of n_5* , so that our general conclusions drawn from parameter scan for $n_5 = 1$ should, broadly speaking, remain unaltered.

C. Minimal Anomaly-Mediated SUSY Breaking Model (AMSB)

Within the supergravity framework, sparticle masses always receive loop contributions originating in the super-Weyl anomaly when SUSY is broken. These loop contributions are generally much smaller than tree level masses. There are, however, classes of models where

these loop contributions may dominate [17]. These include models where there are no SM gauge singlet superfields that can acquire a Planck scale v_{ev} and the usual supergravity contribution to gaugino masses is suppressed by an additional factor $\frac{M_{\text{SUSY}}}{M_P}$ relative to $m_{\frac{3}{2}} = M_{\text{SUSY}}^2/M_P$, or higher dimensional models where the coupling between the observable and hidden sectors is strongly suppressed.

The anomaly-mediated SUSY breaking contributions to gaugino masses are proportional to the corresponding gauge group β -function, and so are non-universal. Likewise, scalar masses and trilinear terms are given in terms of gauge group and Yukawa interaction beta functions. As a result sparticles with the same gauge quantum numbers have a common mass so that flavour changing effects from this sector are naturally small. Slepton squared masses, however, turn out to be negative (tachyonic). A “minimal” fix, that does not upset the resolution of the flavour problem is to assume an additional contribution m_0^2 for all scalars. Since the magnitude of μ is fixed by the observed value of M_Z , the parameter space of the model then consists of

$$m_0, m_{3/2}, \tan\beta \text{ and } \text{sign}(\mu). \quad (18)$$

Weak scale SUSY parameters can now be computed via renormalization group evolution, and $B(B_{q'} \rightarrow \ell^+\ell^-)$ can again be calculated using (15).

For small values of m_0 , the scale of sparticle masses is set by $m_{3/2}$ which cannot be much smaller than about 35 TeV. In Fig. 7 we show the branching ratio for $B_s \rightarrow \mu^+\mu^-$ and $B_d \rightarrow \tau^+\tau^-$ decays versus $\tan\beta$, with $m_{3/2} = 40$ TeV and a) $m_0 = 400$ GeV, and b) $m_0 = 1$ TeV. The solid (dashed) curves correspond to positive (negative) values of μ . As before, the squares mark the experimentally allowed upper limit on $\tan\beta$. Also, shown by the right hand scale in the figure, is the branching fraction for the decay $b \rightarrow s\gamma$.

We see from Fig. 7 that for positive values of μ the $B(B_{q'} \rightarrow \ell^+\ell^-)$ is close to its SM value, and relatively insensitive to the value of $\tan\beta$. For negative values of μ , these branching fractions can indeed become large, again when m_A dives to low values. However, for the range of $\tan\beta$ where $B(b \rightarrow s\gamma) < 5 \times 10^{-4}$, the branching fraction for $B_s \rightarrow \mu^+\mu^-$ decay will be difficult to detect in Tevatron experiments. We have also examined the dependence of these branching fractions on $m_{3/2}$. For positive values of μ , and $\tan\beta = 40 - 60$, they are once again always close to the corresponding SM value, independent of $m_{3/2}$. For negative values of μ , $B(B_s \rightarrow \mu^+\mu^-)$ increases rapidly with $m_{3/2}$, and even for $\tan\beta = 40$ and $m_0 = 1$ TeV can be as large as 10^{-5} ; however, for the range of $m_{3/2}$ where $B(b \rightarrow s\gamma)$ is not too large, the branching fraction can be as large as several times 10^{-8} and may be accessible at the Tevatron, but would require considerable integrated luminosity.

In Fig. 8, we show contours for $B(B_{q'} \rightarrow \ell^+\ell^-)$ in the $m_0 - m_{3/2}$ plane, where we have fixed $\tan\beta = 42$. We show results only for $\mu < 0$ since for positive values of μ the branching fraction appears to be close to the SM one even for very large values of $\tan\beta$. We remind the reader that in the AMSB framework negative μ is also favoured by the result of the E821 experiment [23]. The dark-shaded region is excluded by theoretical constraints and in the open circle region $m_{\tilde{W}_1} < 100$ GeV. Along the line of squares and triangles running diagonally across the figure, m_h or m_A , respectively fall below their experimental bounds. Finally, except in the narrow region covered with dots that follows the boundary of the upper shaded region where $B(b \rightarrow s\gamma)$ is too large, the branching fraction for the decay $b \rightarrow s\gamma$ is in its “allowed range” of $(2 - 5) \times 10^{-4}$. The three solid contours correspond

to $B(B_s \rightarrow \mu^+\mu^-) = 10^{-8}, 5 \times 10^{-8}$ and 10^{-7} , while the dashed contours correspond to $B(B_d \rightarrow \tau^+\tau^-) = 10^{-7}, 10^{-6}$ and 10^{-5} . We see that the branching fractions within the AMSB framework is small over most of the parameter plane. The decay $B_s \rightarrow \mu^+\mu^-$ may be probed in Tevatron experiments only over the limited range where m_A is diving to relatively low values. With a large integrated luminosity, there is a limited parameter range (consistent with other collider constraints) where this decay may be the harbinger of new physics at the Tevatron. With just 10 fb^{-1} of integrated luminosity direct searches for sparticles at the LHC should be sensitive [35] to $m_{3/2} \sim 70 - 90 \text{ TeV}$, depending on the value of m_0 .

For larger values of $\tan\beta$ and $\mu < 0$, the situation is qualitatively similar except that the region excluded by the theoretical constraints expands leaving a yet narrower wedge of “allowed parameters”.

D. $SU(5)$ models with non-universal gaugino masses

Since supergravity is not a renormalizable theory, there is no reason to suppose that the gauge kinetic function $f_{ab} = \delta_{ab}$. Indeed, if the gauge kinetic function develops a SUSY breaking vev that also breaks the GUT symmetry, non-universal gaugino masses result. Expanding the gauge kinetic function as $f_{ab} = \delta_{ab} + \hat{\Phi}_{ab}/M_{\text{Planck}} + \dots$, where the fields $\hat{\Phi}_{ab}$ transform as left handed chiral superfields under supersymmetry transformations, and as the symmetric product of two adjoints under gauge transformations, we parametrize the lowest order contribution to gaugino masses by,

$$\mathcal{L} \supset \int d^2\theta \hat{W}^a \hat{W}^b \frac{\hat{\Phi}_{ab}}{M_{\text{Planck}}} + h.c. \supset \frac{\langle F_{\Phi} \rangle_{ab}}{M_{\text{Planck}}} \lambda^a \lambda^b + \dots, \quad (19)$$

where \hat{W}^a is the superfield that contains the gaugino field λ^a , and F_{Φ} is the auxillary field component of $\hat{\Phi}$ that acquires a SUSY breaking vev . In principle, the chiral superfield $\hat{\Phi}$ which communicates supersymmetry breaking to the gaugino fields can lie in any representation contained in the symmetric product of two adjoints, and so can lead to gaugino mass terms that break the underlying gauge symmetry. We require, of course, that SM gauge symmetry is preserved.

In the context of $SU(5)$ grand unification, F_{Φ} would most generally be a superposition of irreducible representations which appears in the symmetric product of two **24**s,

$$(\mathbf{24} \times \mathbf{24})_{\text{symmetric}} = \mathbf{1} \oplus \mathbf{24} \oplus \mathbf{75} \oplus \mathbf{200}, \quad (20)$$

where only **1** yields universal gaugino masses as in the mSUGRA model. The relations amongst the various GUT scale gaugino masses have been worked out *e.g.* in Ref. [18], and are listed in Table I along with the approximate masses after RGE evolution to $Q \sim M_Z$. Motivated by the measured values of the gauge couplings at LEP, we assume that the vev of the SUSY-preserving scalar component of $\hat{\Phi}$ is negligible. Each of these three non-singlet models is as predictive as the canonical singlet case, and all are compatible with the unification of gauge couplings. Although superpositions are possible, for definiteness we only consider the predictive subset of scenarios where F_{Φ} transforms as one of the irreducible representations of $SU(5)$. The model parameters may then be chosen to be,

$$m_0, M_3^0, A_0, \tan\beta \text{ and } \text{sign}(\mu), \quad (21)$$

where M_i^0 is the $SU(i)$ gaugino mass at scale $Q = M_{GUT}$: M_2^0 and M_1^0 can then be calculated in terms of M_3^0 using Table I. The sparticle masses and mixing angles can then be computed using the *non-universal SUGRA* option in ISAJET. An illustrative example of the spectrum may be found in Ref. [36] where this framework is reviewed. We see from Table I that the pattern of gaugino masses at the weak scale differs quite significantly from mSUGRA expectation. This suggests that the relative contribution of the chargino and gluino-mediated contributions to $B_s \rightarrow \mu^+ \mu^-$ decays, and hence the branching fraction may differ significantly from their values in the mSUGRA framework.

To illustrate this, we show $B(B_s \rightarrow \mu^+ \mu^-)$ as a function of the input GUT scale gluino mass parameter in Fig. 9 for the various models in Table I. We fix $m_0 = 400$ GeV, $A_0 = 0$, $\tan\beta = 40$ and show our results for both signs of μ . The dotted line labelled 1 DR shows the branching fraction within the mSUGRA framework where $\hat{\Phi}$ transforms as a singlet of $SU(5)$, while the solid and dashed lines show the result for the cases where $\hat{\Phi}$ transforms as a **24** or **200** dimensional representation, respectively. The **75** case is theoretically excluded for our choice of parameters. Indeed, we see that the branching fraction is sensitive to the underlying pattern of non-universality.¹³ Especially striking is the dip at $M_3^0 \simeq 200$ GeV in frame b). We have checked that this is due to an almost complete cancellation between the chargino and gluino loops, which as far as we can ascertain is completely accidental. Individually, either the chargino or gluino loop contribution would be much larger than the SM one (m_A is only 175 GeV), but the total SUSY amplitude is comparable to the SM one and the two interfere destructively to yield a branching fraction which is close to its minimum value (recall our discussion at the end of Sec. III A). This case highlights the importance of including the gluino loop, without which the branching fraction would be almost an order of magnitude bigger. We should also mention that contributions from neutralino loops that have been neglected in our analysis may now be significant, in which case $B(B_d \rightarrow \tau^+ \tau^-)$ would not have the canonical ratio with $B(B_s \rightarrow \mu^+ \mu^-)$ [10].

To see how large $B(B_s \rightarrow \mu^+ \mu^-)$ could be in these scenarios, we scanned over the parameter space of these models. We allow m_0 to vary between 100 GeV and $\sqrt{50}M_3^0$ and $-3m_0 < A_0 < 3A_0$. In Fig. 10, where we take $\tan\beta = 55$, we show results for $\mu > 0$ for the three cases of non-universality discussed above. The corresponding result for the mSUGRA case was shown in Fig. 4a. Analogous results for $\mu < 0$ and $\tan\beta = 45$ are shown in Fig. 11 with the corresponding result for the mSUGRA case now in Fig. 4b. Again, points that satisfy other experimental constraints¹⁴ but where $B(b \rightarrow s\gamma)$ falls within (outside) the range $(2-5) \times 10^{-4}$ are denoted by a cross (dot). The following features of these figures are worth noting:

¹³Values of M_3^0 smaller than 200 GeV (300 GeV) are “excluded” for the **200** (**24**) model since these yield $m_h < 113$ GeV.

¹⁴In the case of the **75** and **200** models the mass gap between the chargino and the LSP is small, and LEP constraints on $m_{\tilde{W}_1}$ cited here may be too stringent; see, however, the discussion in Ref. [37].

1. For the **24** model, the branching fraction is always smaller than about 2×10^{-8} at least for the $\tan\beta$ values shown, and would be difficult to probe at the Tevatron. However, in this scenario that Tevatron experiments should see signals in the \cancel{E}_T , $jets + 1\ell + \cancel{E}_T$, and possibly, $jets + Z^0 \rightarrow \ell^+\ell^- + \cancel{E}_T$ channels if $M_3^0 \lesssim 175$ GeV and $m_0 \lesssim 400 - 500$ GeV [37]. For $\mu < 0$ [38] favored by the measured value of the muon anomalous magnetic moment, the branching fraction tends to be within a factor ~ 2 of its SM value.
2. The branching fraction for $B_s \rightarrow \mu^+\mu^-$ decays can be large for the **75** model, regardless of the sign of μ . For positive values of μ favoured by the E821 experiment [23], $B(B_s \rightarrow \mu^+\mu^-)$ can be as large as 10^{-6} , which is just a little over a factor of 2 from its current upper limit. The gap in Fig. 10b is presumably for the same reason as in the mSUGRA case. It is, however, more pronounced because small values of $|A_0|$ are not allowed. For the **75** model, direct SUSY searches at the Tevatron can give observable signals only in the \cancel{E}_T channel, and that too only if M_3^0 and m_0 are rather small. In this case, $m_{\tilde{W}_1}$, $m_{\tilde{Z}_2}$ and $m_{\tilde{Z}_1}$ are all rather close in mass, so that decays of \tilde{W}_1 and \tilde{Z}_2 typically lead to very small visible energy [37].
3. For the **200** model, $B(B_s \rightarrow \mu^+\mu^-)$ can exceed 10^{-7} , even for very large values of M_3^0 for both signs of μ and parameter values consistent with the measured value of $B(b \rightarrow s\gamma)$. As in the **75** case, direct SUSY signals at the Tevatron, for the most part, will be restricted to the \cancel{E}_T channel [37].

IV. SUMMARY

If $\tan\beta$ is large, substantial flavour violating couplings of neutral MSSM Higgs bosons to down type quarks are induced at the 1-loop level via diagrams with squarks and charginos or squarks and gluinos in the loop. These induced interactions, in turn, lead to new Higgs boson mediated SUSY contributions to the amplitude for the decays $B_s \rightarrow \mu^+\mu^-$ and $B_d \rightarrow \tau^+\tau^-$. Depending on model parameters, the branching fraction for the decays may be several orders of magnitude larger than its corresponding SM expectation. Moreover, while the SUSY contribution to the decay decouples when m_A becomes large (so that the Higgs sector of the MSSM reduces to that of the SM with a light Higgs boson), it does not decouple for heavy sparticles.

The CDF experiment has already established that $B(B_s \rightarrow \mu^+\mu^-) \leq 2.6 \times 10^{-6}$, and with the data sample of $\sim 2 \text{ fb}^{-1}$ that is expected to be collected after 2-3 years of Main Injector operation, will be sensitive to branching fractions smaller than about 10^{-7} . The sensitivity will be even greater as Tevatron experiments approach their goal of $\sim 15 \text{ fb}^{-1}$. This then opens up the possibility of discovering new physics at the Tevatron even if sparticles are heavy and their signals from direct production below the level of observability.

These considerations led us to examine predictions for $B(B_s \rightarrow \mu^+\mu^-)$. This is, however, not possible within the context of the generic MSSM since SUSY amplitudes are sensitive to the *a priori* unknown flavour structure of squark mass matrices and trilinear couplings. In other words general predictions (as a function of sparticle masses) are not possible, and one has to resort to specific models. In this paper, we have examined how this branching ratio

within the framework of several popular models of SUSY breaking: the mSUGRA model, the minimal gauge mediated SUSY breaking model, and the anomaly-mediated SUSY breaking model.¹⁵ We also examined the range of this branching ratio in supergravity $SU(5)$ models with non-universal gaugino masses. Specifically, we studied how the $B(B_s \rightarrow \mu^+ \mu^-)$ varies with parameters, and delineated regions of parameter space where it might be observable at the Tevatron. Without making any representation about the sensitivity of BELLE and BABAR, we have also examined the branching fraction for the related decay $B_d \rightarrow \tau^+ \tau^-$. Since the coupling of the Higgs to SM fermions is proportional to the fermion mass, the partial width for this decay is enhanced (relative to that for $B_s \rightarrow \mu^+ \mu^-$) by a factor $(m_\tau/m_\mu)^2$, but reduced by a factor $(|V_{td}|/|V_{ts}|)^2$ that originates in the flavour violating Higgs boson vertex. Our rule of thumb is that these experiments will be competitive with Tevatron experiments if their sensitivity to $B(B_d \rightarrow \tau^+ \tau^-)$ is no more than an order of magnitude worse than the Tevatron sensitivity to $B(B_s \rightarrow \mu^+ \mu^-)$.

Our main results are summarized in Fig. 2–4 for mSUGRA, in Figs. 5 and Fig. 6 for the GMSB model, in Fig. 8 for the AMSB model, and in Fig. 10 and Fig. 11 for $SU(5)$ models with non-universal GUT scale gaugino masses. We will not elaborate on the details here. Generally speaking, in the GMSB and AMSB frameworks, the branching fraction for this decay is significantly smaller than in the mSUGRA model. Within the GMSB and AMSB scenarios, the reach of Tevatron experiments via this decay will be relatively limited. A branching fraction close to 10^{-7} is possible only for regions of parameters where m_A tends to be small due to accidental cancellations. In supergravity models with non-universal gaugino masses, the size of the signal is sensitive to the gaugino mass pattern. Indeed, in the **75** and **200** models the signal can be very large for values of parameters consistent with other constraints. In all these models there are varying regions of parameters that are not in conflict with observations, where $B_s \rightarrow \mu^+ \mu^-$ should be observable at the Tevatron, and where there are no direct sparticle or Higgs boson signals.

The decays $B_s \rightarrow \mu^+ \mu^-$ and $B_d \rightarrow \tau^+ \tau^-$ may also be incisive probes of $SO(10)$ SUSY models with Yukawa coupling unification since these require $\tan \beta$ to be large. These models have recently received considerable attention [41], especially in light of the interpretation of the atmospheric neutrino data [42] as neutrino oscillations originating in a non-trivial flavour structure of the neutrino mass matrix. Since Yukawa coupling unification is possible only in very special regions of parameter space that require extensive scanning to find, we defer the analysis of $B(B_q \rightarrow \ell^+ \ell^-)$ within this framework to a dedicated study of the phenomenology of these models that is currently in progress.

¹⁵We did not separately examine the gaugino-mediated SUSY breaking framework [39]. This is a model based on extra dimensions where usual matter and SUSY breaking fields reside on different spatially separated branes, while gauge fields live in the bulk. As a result, gauginos which directly “feel” the SUSY breaking develop masses at the compactification scale M_c , while matter scalars do not. Renormalization effects then induce scalar masses and A -parameters at $Q = M_{GUT} < M_c$. Although some GUT scale non-universality is induced because of Yukawa couplings, we expect that results in this framework would be qualitatively similar to those in mSUGRA where $m_0, -A_0 \lesssim 0.5m_{1/2}$. For a discussion of the collider phenomenology of these models, see Ref. [40].

Before concluding, we again emphasize that the branching fractions that we have examined are sensitive to details of the model in that small deviations in some of the assumptions (about sfermion flavour structure at the high scale) that define the framework could result in considerable changes in the answer. Our point here is that we should use various “model predictions” only for guidance, or as benchmarks, but keep open the possibility that the real world may be somewhat different. By the same token, accurate determination of flavour violating processes serves as a sensitive probe of flavour structure at very high scales. The observation of processes such as $B_s \rightarrow \mu^+ \mu^-$ at the Tevatron or $B_d \rightarrow \tau^+ \tau^-$ at B -factories is important not only because it would herald new physics, but also because they may serve as probes of flavour physics at scales not directly accessible to experiment.

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TABLES

F_Φ	M_{GUT}			M_Z		
	M_3	M_2	M_1	M_3	M_2	M_1
1	1	1	1	~ 6	~ 2	~ 1
24	2	-3	-1	~ 12	~ -6	~ -1
75	1	3	-5	~ 6	~ 6	~ -5
200	1	2	10	~ 6	~ 4	~ 10

TABLE I. Relative gaugino masses at M_{GUT} and M_Z in the four possible irreducible representations that F_Φ could transform as.

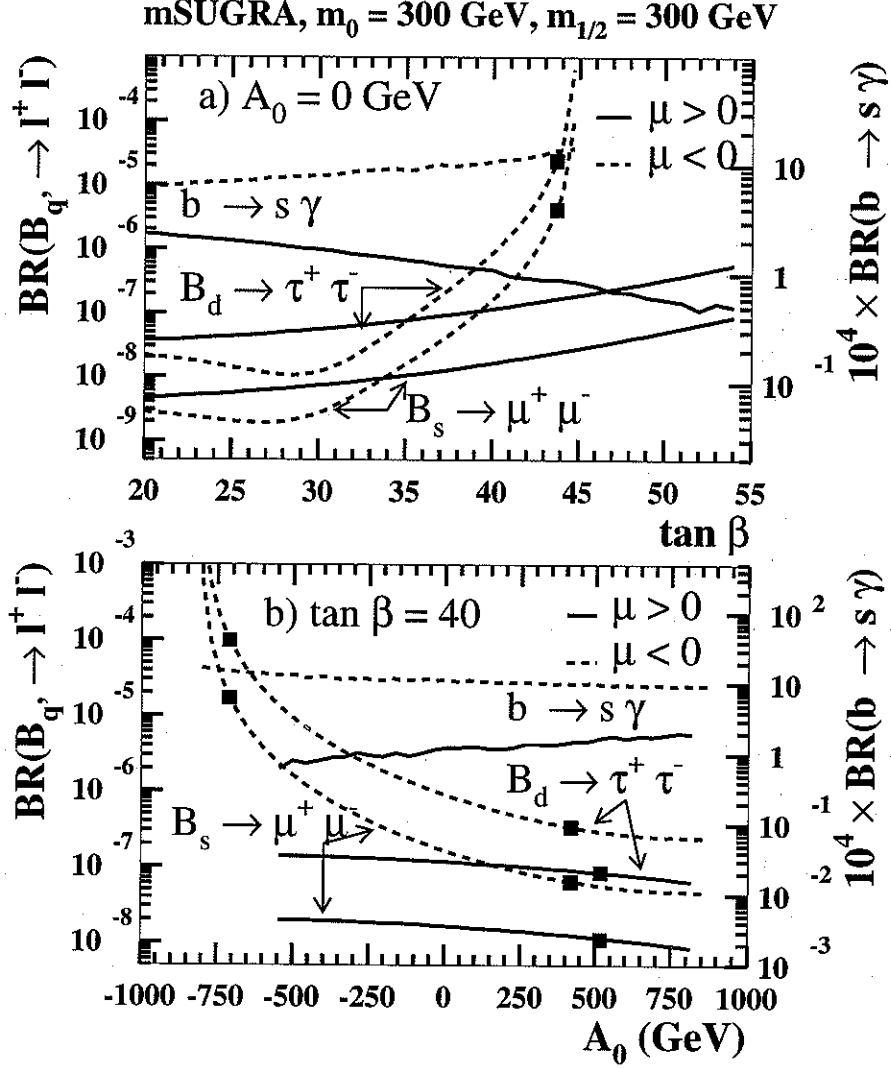


FIG. 1. Branching fractions for the decays $B_s \rightarrow \mu^+ \mu^-$ and $B_d \rightarrow \tau^+ \tau^-$ in the mSUGRA model with $m_0 = 300$ GeV and $m_{1/2} = 300$ GeV, for $\mu > 0$ (solid) and $\mu < 0$ (dashed). In frame a) we show the branching fractions versus $\tan \beta$ for $A_0 = 0$, while in frame b) we show these versus A_0 for $\tan \beta = 40$. Also shown is the branching fraction for the decay $b \rightarrow s \gamma$ which is to be read off using the scale on the right. The squares mark the limits of the experimentally allowed regions as discussed in the text.

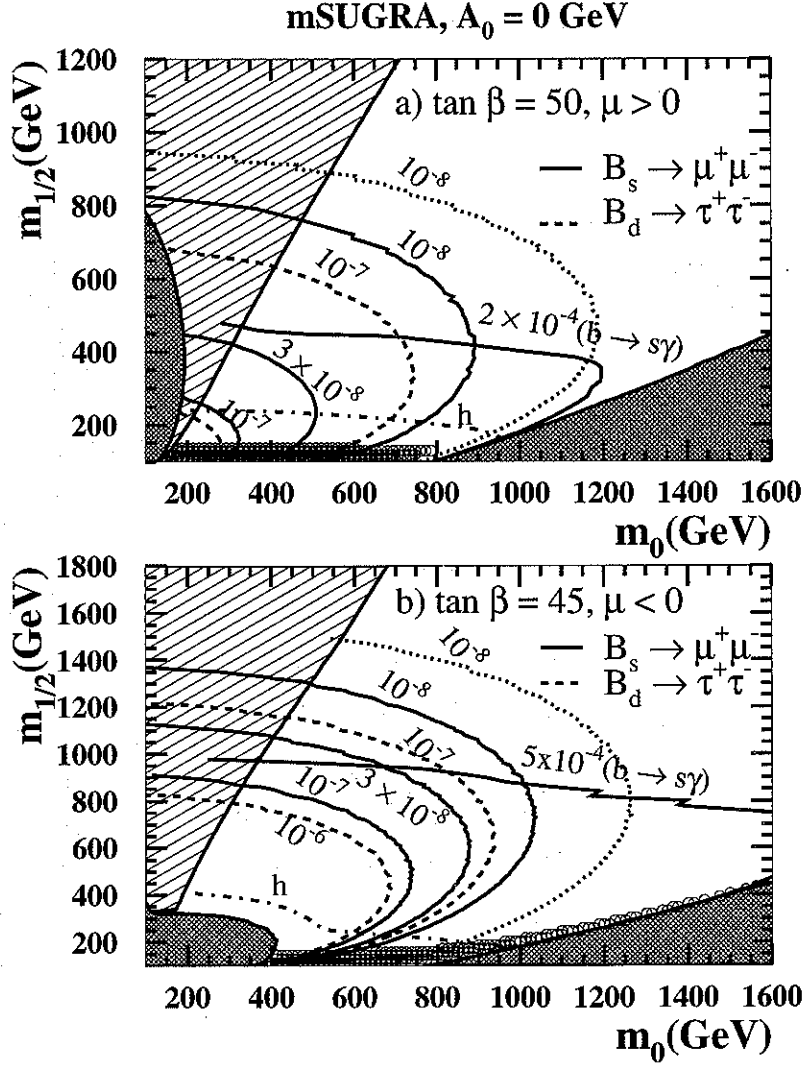


FIG. 2. Contours of constant branching fraction for the decays $B_s \rightarrow \mu^+\mu^-$ (solid) and $B_d \rightarrow \tau^+\tau^-$ (dashed) marked on the curves. The results are shown in the $m_0 - m_{1/2}$ plane of the mSUGRA model with $A_0 = 0$, for a) $\tan \beta = 50, \mu > 0$, and b) $\tan \beta = 45, \mu < 0$. The dotted contour labelled 10^{-8} shows the result for $B(B_s \rightarrow \mu^+\mu^-)$ if the gluino loop contribution discussed in the text is set to zero. The dark-shaded region is excluded by theoretical constraints on EWSB. In the slant-hatched region, \tilde{Z}_1 is not the LSP. The region covered by open circles is excluded by experimental constraints on sparticle masses or on m_A , as discussed in the text. Below the dot-dashed curve labelled h , $m_h < 113$ GeV. In frame a) below the solid curve labelled $b \rightarrow s\gamma$, $B(b \rightarrow s\gamma) < 2 \times 10^{-4}$. Below the corresponding curve in frame b), $B(b \rightarrow s\gamma) > 5 \times 10^{-4}$.

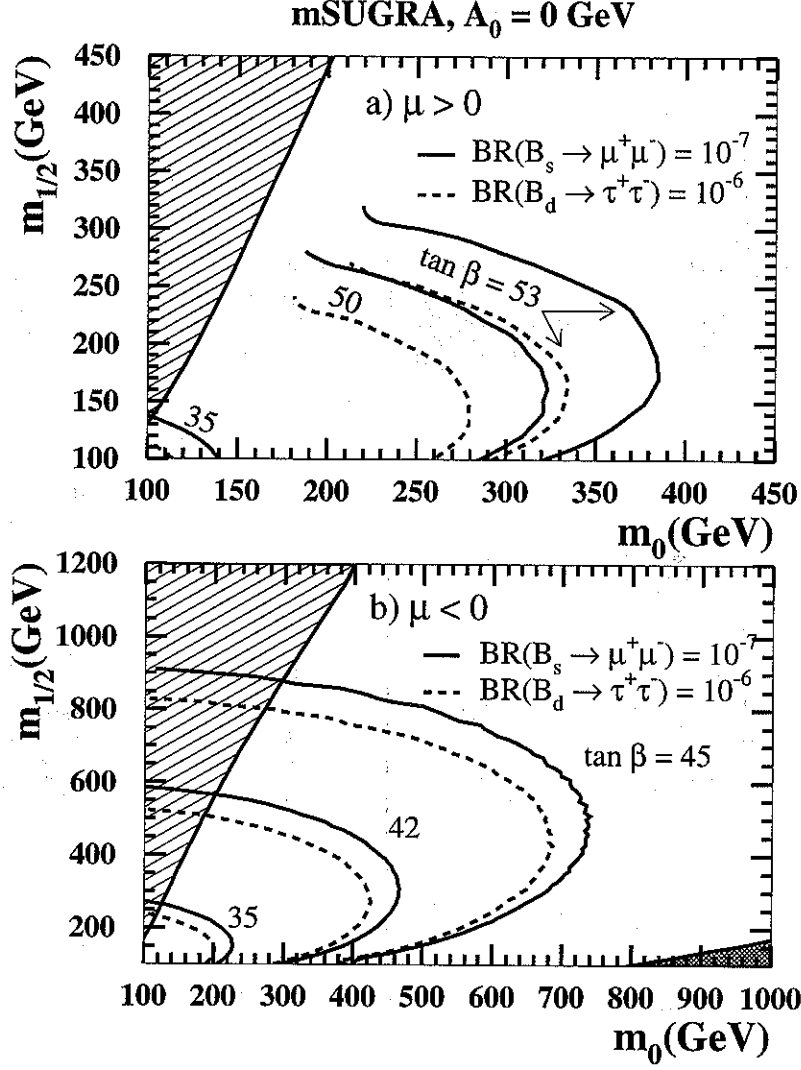


FIG. 3. Contours in the $m_0 - m_{1/2}$ plane of the mSUGRA model with $A_0 = 0$ where the branching fraction for $B_s \rightarrow \mu^+ \mu^-$ ($B_d \rightarrow \tau^+ \tau^-$) is 10^{-7} (10^{-6}) for several values of $\tan \beta$. Shaded regions are as in the previous figure. In frame a) we take $\mu > 0$, while in frame b) we take $\mu < 0$.

mSUGRA, $-3 m_0 < A_0 < 3 m_0$

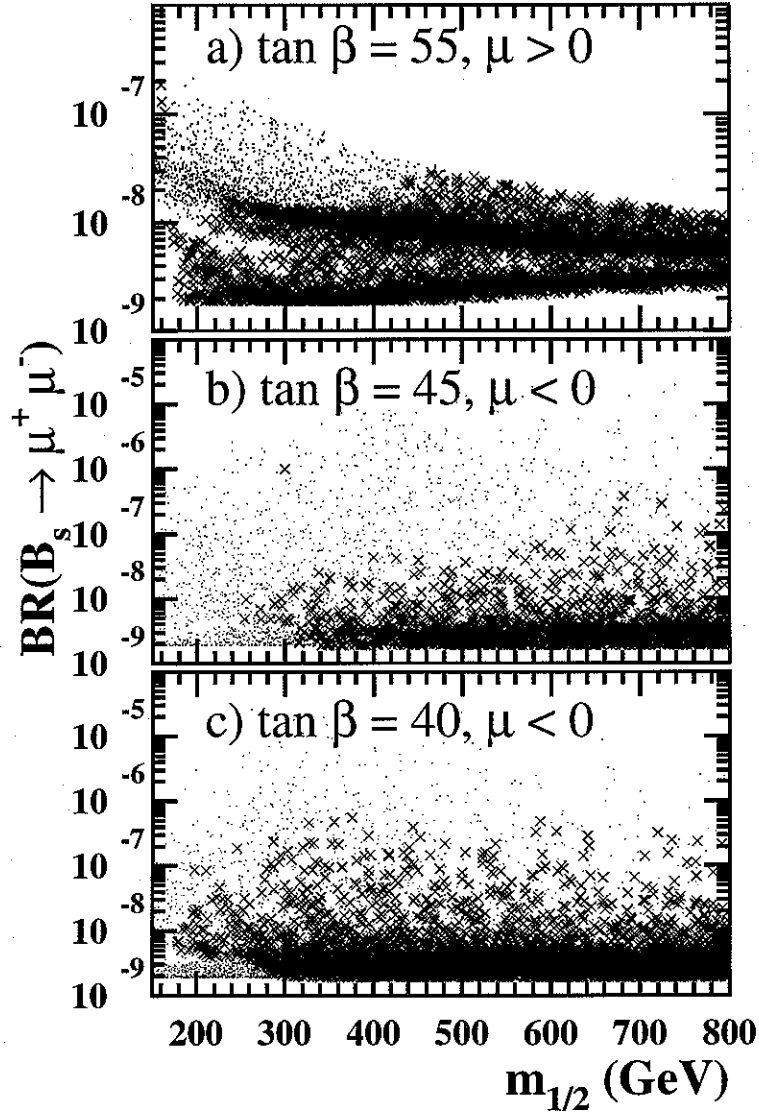


FIG. 4. The branching fraction for the decay $B_s \rightarrow \mu^+ \mu^-$ for a scan of the mSUGRA parameter space over the range mentioned in the text. We show results versus $m_{1/2}$ for a) $\tan \beta = 55, \mu > 0$, b) $\tan \beta = 45, \mu < 0$, and c) $\tan \beta = 40, \mu < 0$. Each cross (dot) denotes a model where $B(b \rightarrow s\gamma)$ lies within (outside) the range $(2 - 5) \times 10^{-4}$.

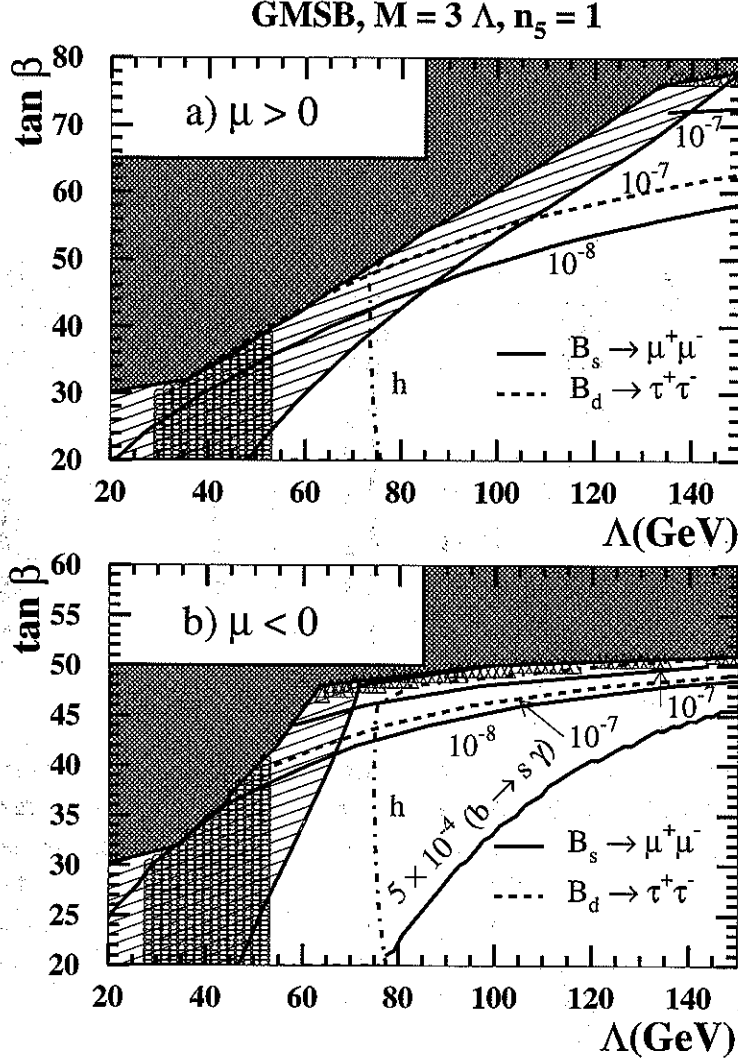


FIG. 5. Contours of constant branching fraction for the decays $B_s \rightarrow \mu^+\mu^-$ (solid) and $B_d \rightarrow \tau^+\tau^-$ (dashed) with values marked on the curves in the $\Lambda - \tan\beta$ plane of the minimal GMSB model. We take the messenger scale $M = 3\Lambda$ and $n_5 = 1$ in this figure and show results for a) $\mu > 0$, and b) $\mu < 0$. In both frames, the dark-shaded region is excluded by theoretical considerations discussed in the text. In the slant-hatched region $m_{\tilde{\tau}_1} < 76$ GeV, in the region covered by open circles $m_{\tilde{\tau}_R} < 100$ GeV, and in the region covered by triangles, $m_A < 100$ GeV. The open-circle region extends all the way to the smallest values of Λ (as does the slant-hatched region) but has been terminated at $\Lambda = 30$ TeV for clarity. To the right of the contour labelled $b \rightarrow s\gamma$ in frame b), $B(b \rightarrow s\gamma)$ is in the range $(2 - 5) \times 10^{-4}$. It is in this range all over the plane in frame a).

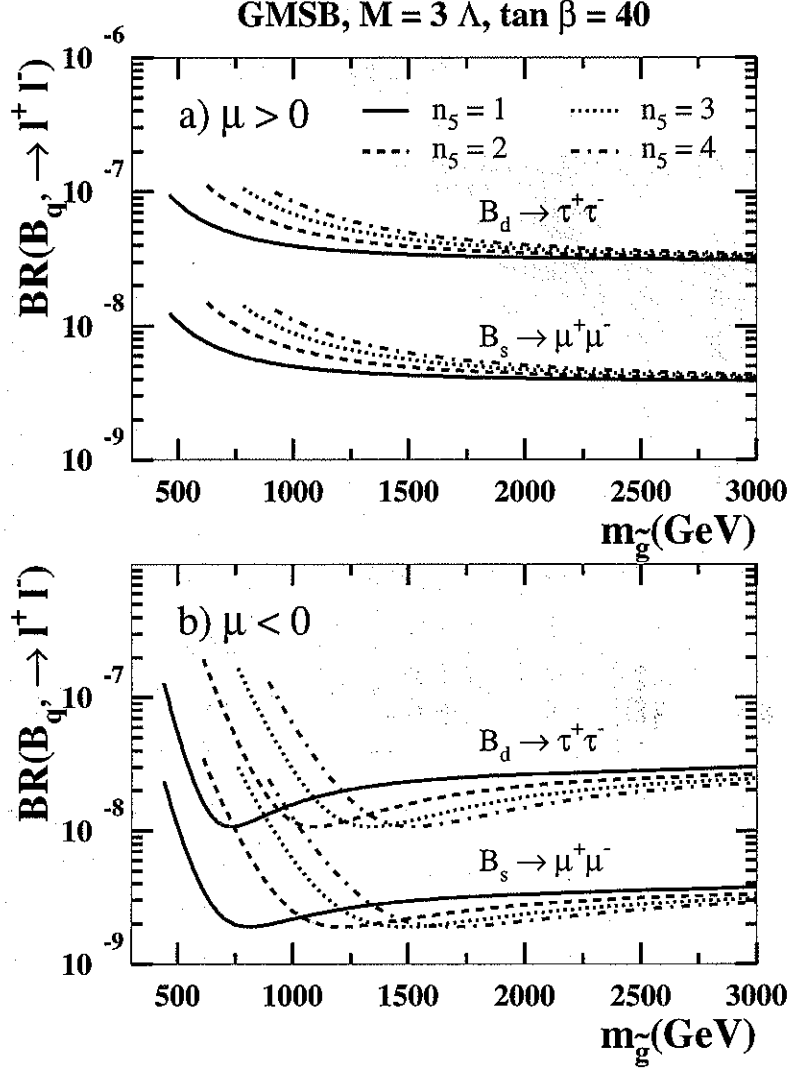


FIG. 6. The dependence of the branching fractions for the decays $B_s \rightarrow \mu^+ \mu^-$ and $B_d \rightarrow \tau^+ \tau^-$ on the parameter n_5 . We plot the branching fractions versus a sparticle mass (we choose $m_{\tilde{g}}$) rather than the theoretical parameter Λ for reasons discussed in the text. We show our results for a) $\mu > 0$, and b) $\mu < 0$. Other parameters are as shown on the figure.

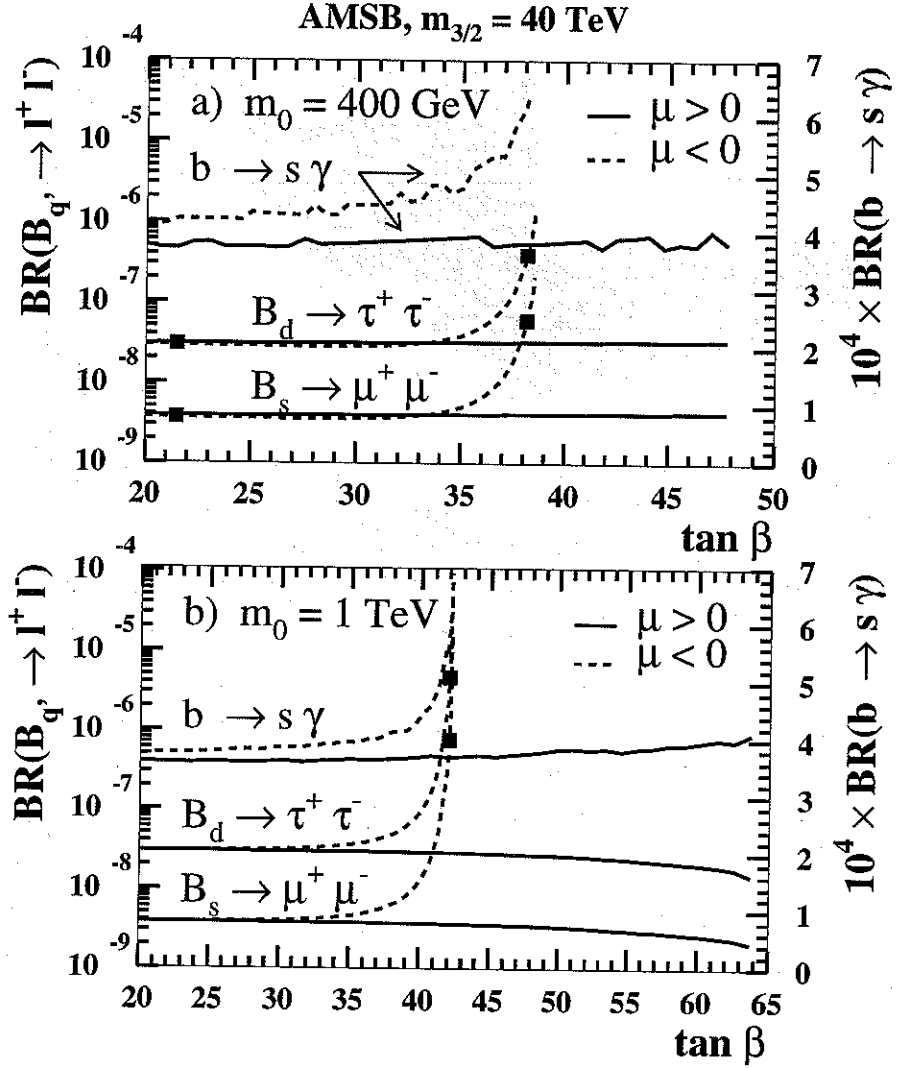


FIG. 7. Branching fractions for the decays $B_s \rightarrow \mu^+ \mu^-$ and $B_d \rightarrow \tau^+ \tau^-$ within the framework of the minimal anomaly-mediated SUSY breaking model. We show the branching fractions versus $\tan \beta$ for a) $m_0 = 400$ GeV and b) $m_0 = 1$ TeV, and other parameters as labelled on the figure. Also shown is the branching fraction for the decay $b \rightarrow s \gamma$ which should be read using the scale on the right.

non-univ, $\tan \beta = 55$, $-3 m_0 < A_0 < 3 m_0$, $\mu > 0$

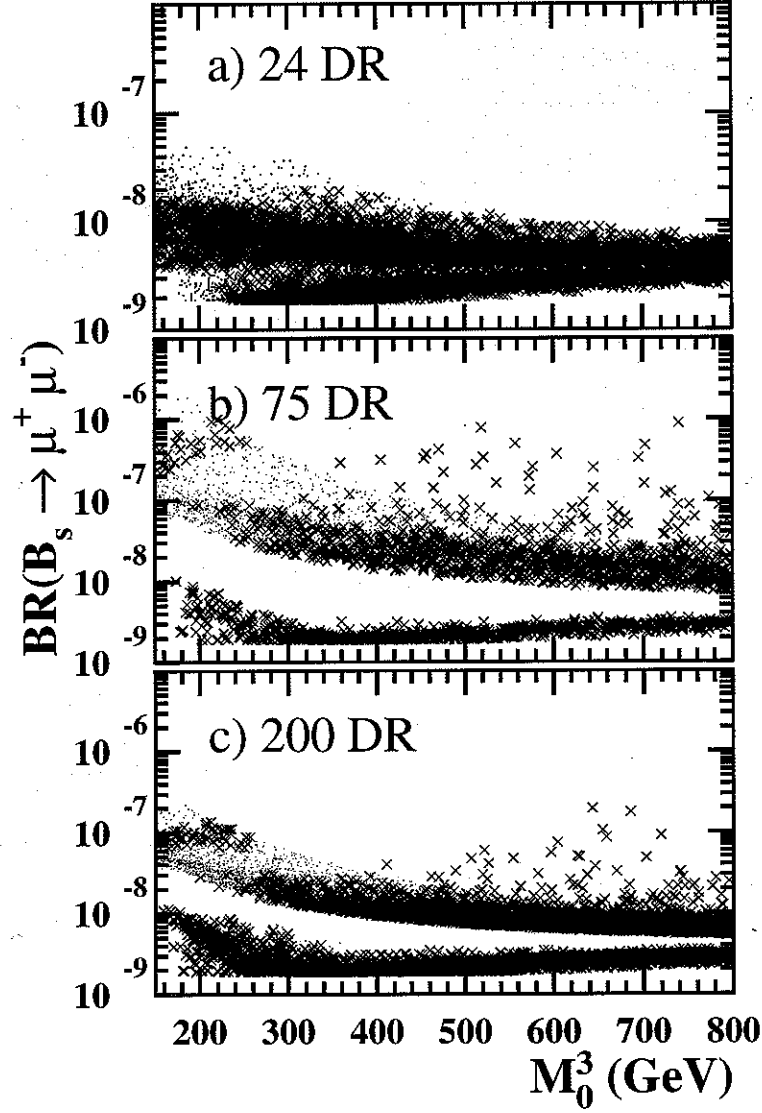


FIG. 10. The branching fraction for the decay $B_s \rightarrow \mu^+ \mu^-$ for a scan of the parameter space of $SU(5)$ supergravity models with non-universal gaugino masses over the range mentioned in the text. We take $\mu > 0$. We show results versus the GUT scale gluino mass parameter M_3^0 for the a) 24 model, b) 75 model, and c) 200 model discussed in the text. Each cross (dot) denotes a model where $B(b \rightarrow s \gamma)$ lies within (outside) the range $(2 - 5) \times 10^{-4}$.

non-univ, $\tan \beta = 45$, $-3 m_0 < A_0 < 3 m_0$, $\mu < 0$

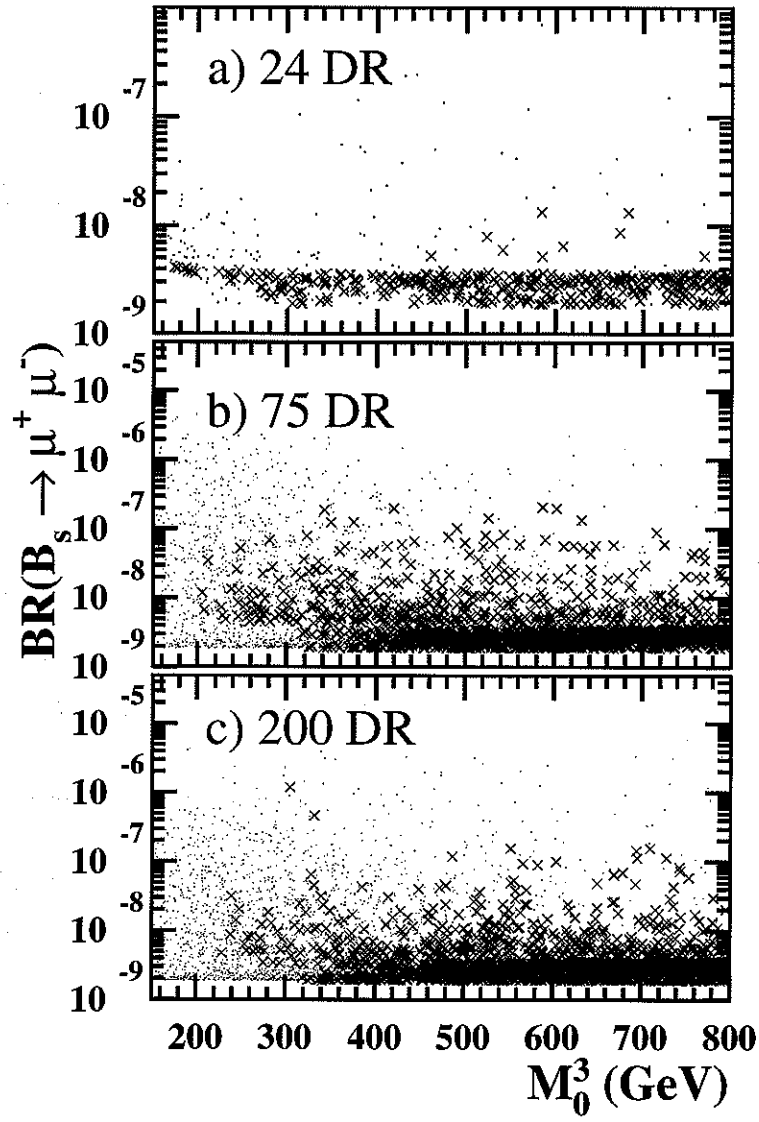


FIG. 11. The same as Fig. 10, except for $\mu < 0$.

