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Angular dependence of synchrotron radiation intensity

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Abstract

The detailed analysis of angular dependence of the synchrotron radiation (SR) is presented. In particular, we analyze the angular dependence of the integral SR-intensity and peculiarities of the angular dependence of the first harmonics SR. Studying spectral SR-intensities, we have discovered their unexpected angular behavior, completely different from that of the integral SR-intensity. Namely, for any given synchrotron frequency, maxima of the spectral SR-intensities recede from the orbit plane with increasing particle energy. Thus, in contrast with the integral SR-intensity, the spectral ones have the tendency to deconcentrate themselves on the orbit plane.

1 Introduction

At present the theory of synchrotron radiation (SR) is well developed and its predictions are in good agreement with experiment [1, 2, 3, 4]. We recall that the SR is created by charged particles, which are moving with velocities v along circles of radius R in an uniform magnetic field H,

$$R = \frac{\beta E}{eH} = \frac{m_0 c^2}{eH}\sqrt{\gamma^2 - 1}\,,\ \beta = \frac{v}{c}\,,\ \ \gamma = (1-\beta^2)^{-1/2} = \frac{E}{m_0 c^2} \gg 1\,. \quad (1)$$

Here E is the particle energy, e is the charge, and m_0 the rest mass. The radiation frequencies $\omega_\nu = \nu\omega_0\,,\ \nu = 1, 2, ...,$ are multiples of the synchrotron frequency $\omega_0 = ceH/E$. The spectral SR-intensity (SR-intencity for a fixed radiation frequency) has maximum for harmonics with $\nu \sim \gamma^3$. Two limiting cases, the non-relativistic ($\beta \ll 1,\ E \simeq m_0c^2$) and the relativistic limits ($\beta \sim$

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1, $E \gg m_0c^2$), are of particular interest. In the non-relativistic case, only the first harmonic $\omega_1 = \omega$ is effectively emitted. The SR-intensity has a maximum in the direction of the magnetic field. In the relativistic case, the integral SR-intensity (spectral SR-intensity summed over the spectrum) is concentrated in the orbit plane within a small angle $\Delta\theta \sim 1/\gamma \ll 1$. Thus, as the electron energy increases, the integral SR-intensity tends to be concentrated in the orbit plane. Any polarization component of the integral SR-intensity has the same behavior. These results were first derived in the framework of classical theory. Consideration in the framework of quantum theory does not change essentially results of the classical analysis, since quantum corrections are small [1, 2, 3, 4].

However, one ought to say that the analysis of angular dependence of the spectral and the integral SR-intensities was not done before in detail. Recently this work was done by us, and in the present article we present results of such an analysis. In Sect. II we analyze in detail angular dependence of the integral SR-intensity. In Sect. III. we study peculiarities of the angular dependence of the first harmonics SR. Studying spectral SR- intensities (see Sect.IV), we have discovered their unexpected angular behavior, completely different from that of the integral SR-intensity. Namely, one can see that for any given synchrotron frequency, maxima of the spectral SR-intensities recede from the orbit plane with increasing particle energy. There exist limiting angles (at $\beta \to 1$) for the maxima, which depend on the synchrotron frequency. Thus, in contrast with the integral SR-intensity, the spectral ones have the tendency to deconcentrate themselves on the orbit plane. The analysis is done in the framework of classical theory, but as was already mentioned above, quantum corrections cannot change the results essentially.

2 Angular dependence of integral SR- intensity

In the SR theory one introduces polarization components W_i , $i = 0, \pm1, 2, 3$ of the integral SR-intensity [1, 2, 3, 4]. Here $W_{\pm1}$ are the integral SR-intensities of the right $(+1)$ and the left (-1) circular polarization components respectively, whereas W_2 and W_3 are the so called "σ" and "π" linear polarization components. The total integral SR-intensity W_0 is defined as $W_0 = W_1 + W_{-1} = W_\sigma + W_\pi$. In the framework of the classical theory of SR one can find:

$$
\begin{aligned}
&W_i = V_0\Phi_i(\beta), \;\; V_0 = \frac{ce^2\beta^4}{R^2} = \frac{e^4H^2\beta^2(1-\beta^2)}{m_0^2c^3}\,, \\
&\Phi_i(\beta) = \int_0^\pi F_i(\beta,\theta)\sin\theta d\theta, \;\; F_i(\beta,\theta) = \sum_{\nu=1}^{\infty} f_i(\nu,\beta;\theta)\,, \\
&f_0(\nu,\beta;\theta) = f_{-1}(\nu,\beta;\theta) + f_1(\nu,\beta;\theta) = f_2(\nu,\beta;\theta) + f_3(\nu,\beta;\theta)\,. \qquad (2)
\end{aligned}
$$

Here θ is the angle between the z-axis and the radiation direction. The sum over ν is just the sum over the spectrum, such that the expressions inside the sum represent spectral distributions. The functions $f_i(\nu,\beta;\theta)$ have the form:

$$f_{\mp 1}(\nu,\beta;\theta)=\frac{\nu^2}{2}\left[J'_\nu(z)\mp\frac{\cos\theta}{\beta\sin\theta}J_\nu(z)\right]^2,\quad z=\nu\beta\sin\theta\,,$$

$$f_2(\nu,\beta;\theta)=\nu^2 {J'_\nu}^2(z),\quad f_3(\nu,\beta;\theta)=\frac{\nu^2\cos^2\theta}{\beta^2\sin^2\theta}J_\nu^2(z)\,. \tag{3}$$

Here $J_\nu(x)$ are Bessel functions of integer indices. The following simple properties hold true:

$$f_k(\nu,\beta;\theta)=f_k(\nu,\beta;\pi-\theta),\quad k=0,2,3;\quad f_{-1}(\nu,\beta;\theta)=f_1(\nu,\beta;\pi-\theta)\,. \tag{4}$$

Thus, it is enough to study the functions $f_k(\nu,\beta;\theta)$, $k=0,2,3$, at the interval $0\le\theta\le\pi/2$ only, and between the functions $f_{\pm 1}$ it is enough to study f_1 only.

Exact analytic expressions for the functions $F_k(\beta,\theta)$, $k=0,2,3$ were already known [1, 2, 3, 4]:

$$F_2(\beta,\theta)=\frac{7-3\varepsilon}{16\varepsilon^{5/2}},\quad \varepsilon=1-\beta^2\sin^2\theta,\quad \frac{1}{\gamma^2}\le\varepsilon<1\,,$$

$$F_3(\beta,\theta)=\frac{(\gamma^2\varepsilon-1)(5-\varepsilon)}{16(\gamma^2-1)\varepsilon^{7/2}},\quad F_0(\beta,\theta)=\frac{(3-4\gamma^2)\varepsilon^2+6(2\gamma^2-1)\varepsilon-5}{16(\gamma^2-1)\varepsilon^{7/2}}\,. \tag{5}$$

Expressions for the functions $F_{\pm 1}$ can be found in the form:

$$F_{\pm 1}(\beta,\theta)=\frac{1}{2}F_0(\beta,\theta)\pm\Psi(\beta\sin\theta)\cos\theta\,,\quad \Psi(x)=\frac{1}{2x}\frac{d}{dx}\sum_{\nu=1}^{\infty}\nu J_\nu^2(\nu x)\,. \tag{6}$$

One can find that for any fixed β all the functions $F_i(\beta,\theta)$ have an extremum at $\theta=0$. Moreover, the extremal values of these functions do not depend on β,

$$F_{-1}(\beta,0)=0,\ 2F_0(\beta,0)=2F_1(\beta,0)=4F_2(\beta,0)=4F_3(\beta,0)=1\,. \tag{7}$$

The point $\theta=\pi/2$ provides an extemum for the functions F_k , $k=0,2,3$ only. Here we have:

$$F_0(\beta,\pi/2)=F_2(\beta,\pi/2)=2F_\pm(\beta,\pi/2)=\frac{1}{16}\gamma^3(7\gamma^2-3),\ F_3(\beta,\pi/2)=0\,. \tag{8}$$

Therefore, for F_3 the point $\theta=\pi/2$ is an absolute minimum. For any fixed β the function $F_2(\beta,\theta)$ is a monotonically increasing function of θ on the interval $0\le\theta\le\pi/2$. Thus, $\theta=0$ is an absolute minimum and $\theta=\pi/2$ is an absolute maximum of this function. The maximum of the function F_2 increases as E^5 with increasing particle energy E.

For $\gamma\le\gamma_0^{(1)}$, $(\beta\le\beta_0^{(1)})$,

$$\gamma_0^{(1)}=\sqrt{7/6}\approx 1.0801,\quad \beta_0^{(1)}=1/\sqrt{7}\approx 0.378\,, \tag{9}$$

F_0 and F_1 are monotonically decreasing functions of θ (F_0 on the interval $0\le\theta\le\pi/2$ and F_1 on the interval $0\le\theta\le\pi$). Thus, at $\theta=0$ these functions have an absolute maximum. The functions F_0 and F_1 have their absolute minima at $\theta = \pi/2$ and $\theta = \pi$ respectively. Besides, $F_1(\beta, \pi) = 0$. For $\gamma_0^{(1)} < \gamma < \gamma_0^{(2)}$, $(\beta_0^{(1)} < \beta < \beta_0^{(2)})$,

$$\gamma_0^{(2)} = \frac{\sqrt{3}+3\sqrt{2}}{5} \approx 1.1949\,, \quad \beta_0^{(2)} = \sqrt{\frac{2}{3}(\sqrt{6}-2)} \approx 0.5474\,, \tag{10}$$

the points $\theta = 0,\ \pi/2$ are minima for F_0, and the point $\theta = \theta_0(\beta)$,

$$\sin^2\theta_0(\beta) = \frac{6\gamma^2(1-3\gamma^2)+2\gamma^2\sqrt{15(15\gamma^4-22\gamma^2+9)}}{3(4\gamma^2-3)(\gamma^2-1)}, \quad 0 < \theta_0(\beta) < \pi/2\,, \tag{11}$$

provides a maximum for F_0. For $\gamma_0^{(2)} < \gamma$, $(\beta_0^{(2)} < \beta < 1)$, the function F_0 has an absolute maximum at the point $\theta = \pi/2$.

Denoting via $\theta_0^{(m)}(\beta)$ all the maximum points of F_0, we may write:

$$\theta_0^{(m)}(\beta) = \begin{cases} 0, & \beta \le \beta_0^{(1)} \\ \theta_0(\beta), & \beta_0^{(1)} < \beta < \beta_0^{(2)} \\ \pi/2, & \beta_0^{(2)} \le \beta < 1 \end{cases}\ . \tag{12}$$

The plot of the function $\theta_0^{(m)}(\beta)$ see below:

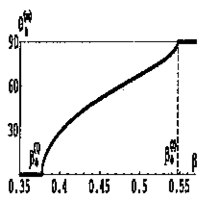

Figure 1: The function $\theta_0^{(m)}(\beta)$.

For any given $\beta \in (\beta_0^{(1)}, 1)$, the function F_1 has its maximum at the point $\theta = \theta_1(\beta)$, $0 < \theta_1(\beta) < \pi/2$. Denoting via $\theta_1^{(m)}(\beta)$ all the maximum points of F_1, we may write:

$$\theta_1^{(m)}(\beta) = \begin{cases} 0, & \beta \le \beta_0^{(1)} \\ \theta_1(\beta), & \beta_0^{(1)} < \beta < 1 \end{cases}\ . \tag{13}$$

At the moment, there is no analytical expression for $\theta_1(\beta)$ similar to (11) for $\theta_0(\beta)$. However, one can see that the function $\theta_1(\beta)$ is a monotonically increasing function of $\beta \in \left[\beta_0^{(1)}, 1\right]$. For $\beta \to 1$ there is an asymptotic form

$$\theta_1(\beta) \approx \pi/2 - a_1/\gamma\,, \tag{14}$$

where $\alpha_1 \approx 0.2672$ is a root of the equation (see [1])

$$5\pi a_1(5+12a_1^2)\sqrt{3} + 64(5a_1^2-1)\sqrt{1+a_1^2} = 0\,. \tag{15}$$

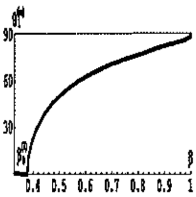

Figure 2: The function $\theta_1^{(m)}(\beta)$

The plot of the function $\theta_1^{(m)}(\beta)$ see below:

For $\beta \leq \beta_3$ $(\gamma \leq \gamma_3)$,

$$\beta_3 = \frac{2}{\sqrt{15}} \approx 0.5164\,, \quad \gamma_3 = \sqrt{\frac{15}{11}} \approx 1.1678\;, \tag{16}$$

F_3 is a monotonically decreasing function on the interval $0 \leq \theta \leq \pi/2$. The point $\theta = 0$ provides the absolute maximum for this function. For $1 > \beta > \beta_3$, $(\gamma > \gamma_3)$, the points $\theta = 0$ and $\theta = \theta_3(\beta)$ provide the minimum and the maximum respectively for F_3 ,

$$\sin^2\theta_3(\beta) = \frac{\sqrt{5(125\gamma^4 - 34\gamma^2 + 5)} - 19\gamma^2 - 5}{6(\gamma^2 - 1)}\,, \quad 0 < \theta_3(\beta) < \pi/2\,. \tag{17}$$

Denoting via $\theta_3^{(m)}(\beta)$ all the maximum points of F_3 , we may write:

$$\theta_3^{(m)}(\beta) = \begin{cases} 0, & \beta \leq \beta_3 \\ \theta_3(\beta), & \beta_3 < \beta < 1 \end{cases}\;. \tag{18}$$

For $\beta \to 1$ the following asymptotic expression holds true:

$$\theta_3^{(m)} \approx \pi/2 - \frac{1}{\gamma}\sqrt{\frac{2}{5}}. \tag{19}$$

The plot of the function $\theta_3^{(m)}(\beta)$ see below:

3 Angular dependence of spectral SR-intensity

3.1 First harmonic radiation

The angular distribution of SR from the first harmonic $(\nu = 1)$ is distinctly different from that of the higher harmonics $(\nu \geq 2)$. Previously it was known [1, 2, 3, 4] that: a) The first harmonic alone contributes essentially to the radiation in the directions $\theta = 0, \pi$. b) In the nonrelativistic case $(\beta \sim 0)$, the radiation is maximal exactly in these directions.

Figure 5: The threshold values $\gamma_0\nu$, $\gamma_2\nu$ and the limit values $\delta\nu^k$ (in degrees)

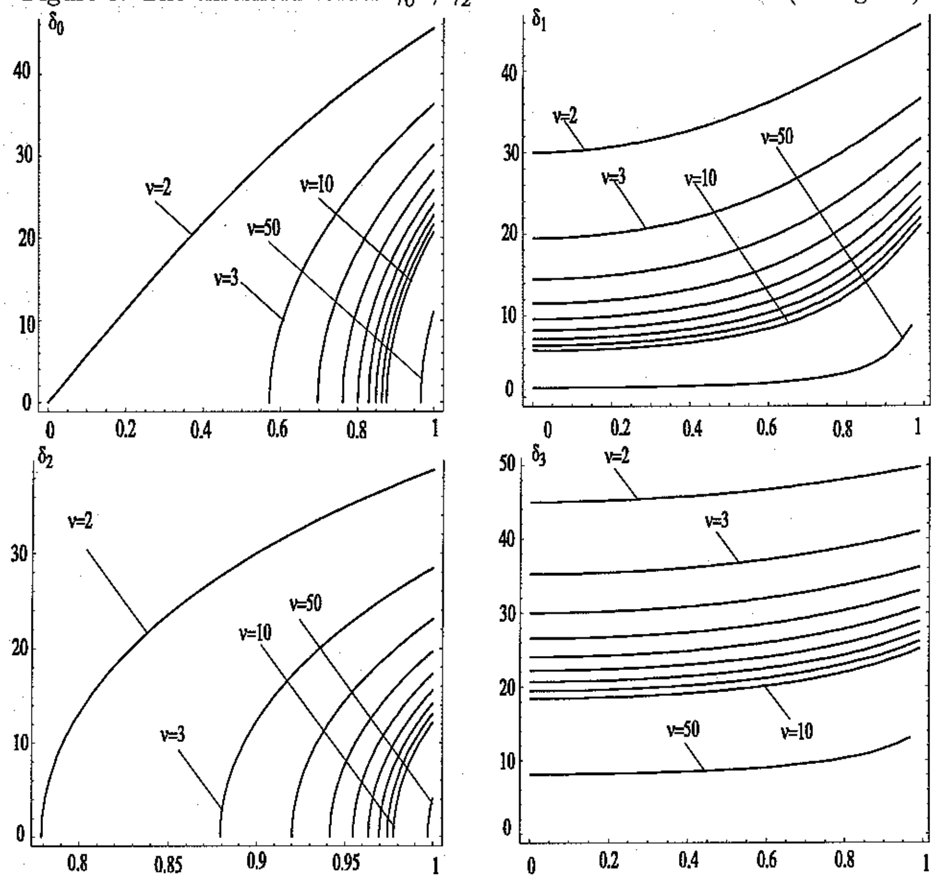

Figure 6: The plots of the functions $\delta_k(\nu, \beta), (k = 0, 1, 2, 3)$ at $\nu = 1 - 10, 50$.

Figure 7: The threshold values $\gamma_0\nu$, $\gamma_2\nu$ and the limit values $\delta\nu^k$ (in degrees)

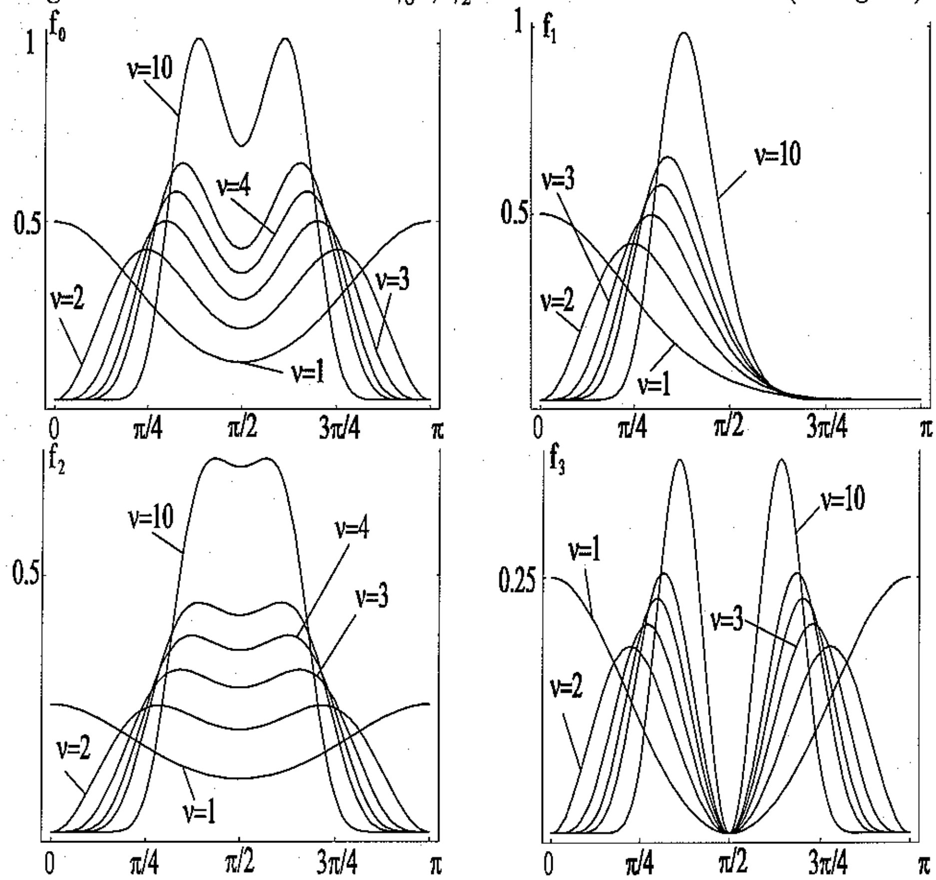

Figure 8: The plots of the functions $f_k(\nu, 1; \theta), k = 0, 1, 2, 3$ at $\nu = 1, 2, 3, 4, 5, 10$.

Table 1: The threshold values $\gamma_0\nu$, $\gamma_2\nu$ and the limit values $\delta\nu^k$ (in degrees)

ν	2	3	4	5	6	7	10	15	20	25
γ_0^ν	1.00	1.22	1.40	1.54	1.67	1.79	2.08	2.46	2.75	3.00
γ_2^ν	1.59	2.10	2.55	2.97	3.36	3.73	4.75	6.25	7.59	8.82
δ_0^ν	45.50	36.22	31.29	28.11	25.84	24.10	20.66	17.48	15.59	14.30
δ_1^ν	45.88	36.83	32.02	28.91	26.68	24.98	21.57	18.39	16.48	15.16
δ_2^ν	38.84	28.44	23.06	19.67	17.30	15.54	12.14	9.20	7.57	6.51
δ_3^ν	49.83	41.09	36.29	33.11	30.80	29.00	25.34	21.83	19.69	18.19
ν	30	35	40	45	50	100	200	300	400	500
γ_0^ν	3.21	3.40	3.58	3.74	3.88	4.98	6.35	7.31	8.07	8.70
γ_2^ν	9.98	11.07	12.10	13.10	14.06	22.38	35.58	46.66	56.54	65.63
δ_0^ν	12.34	12.59	11.98	11.47	11.03	8.60	6.74	5.86	5.31	4.92
δ_1^ν	14.18	13.41	12.77	12.24	11.79	9.24	7.28	6.34	5.75	5.33
δ_2^ν	5.75	5.18	4.74	4.38	4.08	2.56	1.61	1.23	1.01	0.87
δ_3^ν	17.06	16.16	15.43	14.81	14.28	11.26	8.90	7.76	7.04	6.53

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