Instituto de Física
Universidade de São Paulo

On Modifications of the Sp(2) Covariant Superfield Quantization

Gitman D.M.,[a]; Moshin P.Yu.[a,b]

[a] Instituto de Física, Universidade de São Paulo, São Paulo, Brasil

[b] Tomsk State Pedagogical University, 634041 Tomsk, Russia

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Abstract

We modify the superfield formalism of $Sp(2)$ covariant quantization to realize a superalgebra of generating operators isomorphic to the massless limit of the corresponding superalgebra of the $osp(1,2)$ covariant quantization. The modified formalism ensures the compatibility of the superalgebra of generating operators with extended BRST symmetry without imposing restrictions eliminating superfield components from the quantum action. The formalism coincides with the $Sp(2)$ covariant superfield scheme and with the massless limit of the $osp(1,2)$ covariant quantization in particular cases of gauge-fixing and solutions of the master equations.

1. Introduction

The superfield formulation [1] of the $Sp(2)$ covariant quantization rules [2] was the first to realize a superspace description of extended BRST symmetry [3, 4] for general gauge theories. Different modifications of the formalism [1] have been considered in [5, 6, 7]. In [5], the arbitrariness of [1] was analyzed in the context of extending the gauge-fixing procedure. In [6, 7], two modifications of [1] were proposed to realize an additional requirement of $Sp(2)$ invariance of the quantum action, originally implemented within the $osp(1,2)$ covariant formulation [8] of the $Sp(2)$ covariant scheme [2].

The ambiguity of the superfield formulation [1] analyzed in [5] is related to a freedom in constructing the generating operators of antibrackets with given algebraic properties. In [5], it was shown that this freedom is fixed uniquely by the requirement of extended BRST symmetry realized in terms of superspace translations. In fact, this unique realization of generating operators is the one implemented by [1].

Two other modifications [6, 7] are related to constructing an extended set of generating operators to realize a superalgebra containing an additional mass parameter and isomorphic to the $osp(1,2)$ superalgebra [9], as required by the quantization scheme [8]. The formulations [6, 7] are not free from difficulties. In [6], there remains the problem of inconsistency [7] between the realization of generating operators and the extended BRST symmetry in terms of supertranslations. In [7], this problem is solved at the cost of eliminating some superfield components from the quantum action, which means that the extended BRST symmetry in [7] is not completely controlled by the master equations.

In [7], it was conjectured that a satisfactory formalism of $osp(1,2)$ covariant superfield quantization should contain the $Sp(2)$ covariant superfield scheme [1] in the massless limit, which is suggested by the relation between the original $osp(1,2)$ and $Sp(2)$ covariant schemes [2, 8], with allowance for the results of [5].

To advance in the solution of this problem, we demonstrate the existence of a superfield scheme with the properties of such a massless limit. Namely, we propose a superfield scheme based on a set of generating operators that form a superalgebra isomorphic to the massless limit of the superalgebra realized in the $osp(1,2)$ covariant scheme [8]. The choice of generating operators is consistent with the form of extended BRST symmetry in terms of supertranslations, without imposing restrictions eliminating superfield components. The formalism contains the original $Sp(2)$ covariant superfield scheme [1] and the massless limit of the $osp(1,2)$ covariant scheme [8].

The paper is organized as follows. In Section 2, we introduce the main definitions. In Section 3, we formulate the quantization rules. In Section 4, we discuss the relation of the proposed formalism to the quantization schemes [1, 8]. In Section 5, we summarize the results and make concluding remarks.

We use the notation adopted in [1, 8]. Derivatives with respect to (super)sources and antifields are taken from the left, and those with respect to (super)fields, from the right. Left derivatives with respect to (super)fields are labeled by the subscript "l". Integration over superfields is understood as integration over their components.

2. Main Definitions

Consider a superspace (x^μ, θ^a), where x^μ are space-time coordinates, and θ^a is an $Sp(2)$ doublet of anticommuting coordinates. Notice that any function $f(\theta)$ has a component representation,

$$f(\theta) = f_0 + \theta^a f_a + \theta^2 f_3, \quad \theta^2 \equiv \frac{1}{2}\theta_a\theta^a,$$

and an integral representation,

$$f(\theta) = \int d^2\theta' \, \delta(\theta' - \theta) f(\theta'), \quad \delta(\theta' - \theta) = (\theta' - \theta)^2,$$

where raising and lowering the $Sp(2)$ indices is performed by the rule $\theta^a = \varepsilon^{ab}\theta_b$, $\theta_a = \varepsilon_{ab}\theta^b$, with ε^{ab} being a constant antisymmetric tensor, $\varepsilon^{12} = 1$, and integration over θ^a is given by

$$\int d^2\theta = 0, \quad \int d^2\theta \, \theta^a = 0, \quad \int d^2\theta \, \theta^a\theta^b = \varepsilon^{ab}.$$

In particular, for any function $f(\theta)$ we have

$$\int d^2\theta \, \frac{\partial f(\theta)}{\partial \theta^a} = 0,$$

which implies the property of integration by parts

$$\int d^2\theta \, \frac{\partial f(\theta)}{\partial \theta^a} g(\theta) = -\int d^2\theta (-1)^{\varepsilon(f)} f(\theta) \frac{\partial g(\theta)}{\partial \theta^a}, \tag{1}$$

where derivatives with respect to θ^a are taken from the left.

We now introduce a set of superfields $\Phi^A(\theta)$, $\varepsilon(\Phi^A) = \varepsilon_A$, with the boundary condition

$$\Phi^A(\theta)\Big|_{\theta=0} = \phi^A,$$

and a set of supersources $\bar{\Phi}_A(\theta)$ of the same Grassmann parity, $\varepsilon(\bar{\Phi}_A) = \varepsilon_A$. The structure [2] of the complete configuration space ϕ^A for a general gauge theory of L-stage reducibility is given by

$$\phi^A = (A^i, B^{\alpha_s|a_1 \cdots a_s}, C^{\alpha_s|a_0 \cdots a_s}), \quad s = 0, \ldots, L, \tag{2}$$

where A^i are the initial classical fields, while $B^{\alpha_s|a_1 \cdots a_s}$, $C^{\alpha_s|a_0 \cdots a_s}$ are the pyramids of auxiliary and (anti)ghost fields, being completely symmetric $Sp(2)$ tensors of rank s and $s+1$, respectively.

For arbitrary functionals $F = F(\Phi,\bar\Phi)$, $G = G(\Phi,\bar\Phi)$, we define the superbracket operations $(\ ,\ )^a$ and $\{\ ,\ \}_\alpha$

$$(F,G)^a = \int d^2\theta \left\{ \frac{\delta F}{\delta\Phi^A(\theta)} \frac{\partial}{\partial\theta_a} \frac{\delta G}{\delta\bar\Phi_A(\theta)} (-1)^{\varepsilon_A+1} - (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)} (F \leftrightarrow G) \right\},$$

$$\{F,G\}_\alpha = -\int d^2\theta \Bigg\{ (\sigma_\alpha)_B{}^A \left[\frac{\partial^2}{\partial\theta^2} \left(\frac{\delta F}{\delta\Phi^A(\theta)} \right) \theta^2 - \frac{\partial^2}{\partial\theta^2} \left(\frac{\delta F}{\delta\Phi^A(\theta)} \theta^2 \right) \right] \frac{\delta G}{\delta\bar\Phi_B(\theta)}$$

$$+ \frac{\partial^2}{\partial\theta^2} \left(\frac{\delta F}{\delta\Phi^A(\theta)} \theta_b (\sigma_\alpha)^b{}_a + \frac{\delta F}{\delta\Phi^B(\theta)} \theta_a (\sigma_\alpha)^B{}_A \right) \theta^a \frac{\delta G}{\delta\bar\Phi_A(\theta)} + (-1)^{\varepsilon(F)\varepsilon(G)} (F \leftrightarrow G) \Bigg\}, \quad (3)$$

where

$$\frac{\partial^2}{\partial\theta^2} \equiv \frac{1}{2} \varepsilon^{ab} \frac{\partial}{\partial\theta^b} \frac{\partial}{\partial\theta^a}\,.$$

Notice the properties of derivatives

$$\frac{\delta_l \Phi^A(\theta)}{\delta\Phi^B(\theta')} = \frac{\delta\Phi^A(\theta)}{\delta\Phi^B(\theta')} = \delta(\theta'-\theta)\delta^A_B, \quad \frac{\delta\bar\Phi_A(\theta)}{\delta\bar\Phi_B(\theta')} = \delta(\theta'-\theta)\delta^B_A.$$

In (3), the matrices $(\sigma_\alpha)_A{}^B \equiv -(\sigma_\alpha)^B{}_A$, with the indices (2), are given by

$$(\sigma_\alpha)^B{}_A = (\sigma_\alpha)^b{}_a (P_\pm)^{Ba}_{Ab}. \quad (4)$$

Here, $(\sigma_\alpha)^b{}_a$, with $\alpha = (0,+,-)$, stands for a set of matrices which possess the properties

$$(\sigma_\alpha)^b{}_a = -(\sigma_\alpha)_a{}^b, \quad (\sigma_\alpha)^{ab} = \varepsilon^{ac}(\sigma_\alpha)_c{}^b = (\sigma_\alpha)^a{}_c \varepsilon^{cb} = \varepsilon^{ac}(\sigma_\alpha)_{cd}\,\varepsilon^{db}, \quad (\sigma_\alpha)^{ab} = (\sigma_\alpha)^{ba},$$

$$(\sigma_\alpha)_a{}^a = (\sigma_\alpha)^a{}_a = 0, \quad \varepsilon^{ad}\delta^b_c + \varepsilon^{bd}\delta^a_c = -(\sigma^\alpha)^{ab}(\sigma_\alpha)^d{}_c \quad (5)$$

and form the algebra $sl(2)$

$$\sigma_\alpha\sigma_\beta = g_{\alpha\beta} + \tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\sigma^\gamma, \quad \sigma^\alpha = g^{\alpha\beta}\sigma_\beta, \quad \mathrm{Tr}(\sigma_\alpha\sigma_\beta) = 2g_{\alpha\beta},$$

$$g^{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}, \quad g^{\alpha\gamma} g_{\gamma\beta} = \delta^\alpha_\beta,$$

with $\epsilon_{\alpha\beta\gamma}$ being an antisymmetric tensor, $\epsilon_{0+-} = 1$.

In (4), the matrices $(P_\pm)^{Ba}_{Ab}$ are given by

$$(P_\mp)^{Ba}_{Ab} = (P_\pm)^{Ba}_{Ab} - (P_\pm)^B_A \delta^a_b + \delta^B_A \delta^a_b, \quad (P_\pm)^B_A = \delta^b_a (P_\pm)^{Ba}_{Ab},$$

where

$$(P_+)^{Ba}_{Ab} = \begin{cases} \delta^i_j \delta^a_b & A = i, B = j, \\ \delta^{\beta_s}_{\alpha_s}(s+1) S^{b_1\cdots b_s a}_{a_1\cdots a_s b} & A = \alpha_s|a_1\cdots a_s,\ B = \beta_s|b_1\cdots b_s, \\ \delta^{\beta_s}_{\alpha_s}(s+2) S^{b_0\cdots b_s a}_{a_0\cdots a_s b} & A = \alpha_s|a_0\cdots a_s,\ B = \beta_s|b_0\cdots b_s, \\ 0 & \text{otherwise.} \end{cases}$$

Here, $S^{b_0\cdots b_s a}_{a_0\cdots a_s b}$ is a symmetrizer (X^a being independent bosonic variables)

$$S^{b_0\cdots b_s a}_{a_0\cdots a_s b} \equiv \frac{1}{(s+2)!} \frac{\partial}{\partial X^{a_0}} \cdots \frac{\partial}{\partial X^{a_s}} \frac{\partial}{\partial X^b} X^a X^{b_s} \cdots X^{b_0},$$

with the properties

$$S^{b_0\cdots b_s a}_{a_0\cdots a_s b} = \frac{1}{s+2}\left(\sum_{r=0}^{s}\delta^{b_r}_{a_0}S^{b_0\cdots b_{r-1}b_{r+1}\cdots b_s a}_{a_1\cdots a_s b} + \frac{1}{s+1}\sum_{r=0}^{s}\delta^{a}_{a_0}\delta^{b_r}_{b}S^{b_0\cdots b_{r-1}b_{r+1}\cdots b_s}_{a_1\cdots a_s}\right),$$
$$S^{b_0\cdots b_s}_{a_0\cdots a_s} = \frac{1}{s+1}\sum_{r=0}^{s}\delta^{b_r}_{a_0}S^{b_0\cdots b_{r-1}b_{r+1}\cdots b_s}_{a_1\cdots a_s}.$$

From the above definitions follow the properties [8]

$$(P_\mp)^{Ab}_{Cd}(P_\pm)^{Cd}_{Ba} = 0, \quad \varepsilon^{ad}(P_\pm)^{Bb}_{Ad} + \varepsilon^{bd}(P_\pm)^{Ba}_{Ad} = -(\sigma^\alpha)^{ab}(\sigma_\alpha)^B{}_A,$$
$$\varepsilon^{ad}(P_\pm)^{Bb}_{Ac} + \varepsilon^{bd}(P_\pm)^{Ba}_{Ac} - (\sigma^\alpha)^{ab}(\sigma_\alpha)^e{}_c(P_\mp)^{Bd}_{Ae} = -(\sigma^\alpha)^{ab}((\sigma_\alpha)^d{}_c\delta^B_A + \delta^d_c(\sigma_\alpha)^B{}_A).$$

Let us introduce a set of first-order operators V^a, U^a (odd) and V_α, U_α (even),

$$V^a = \int d^2\theta \frac{\partial\bar\Phi_A(\theta)}{\partial\theta_a}\frac{\delta}{\delta\bar\Phi_A(\theta)},$$
$$U^a = \int d^2\theta \frac{\partial\Phi^A(\theta)}{\partial\theta_a}\frac{\delta_l}{\delta\Phi^A(\theta)},$$
$$V_\alpha = \int d^2\theta\left(\bar\Phi_B(\sigma_\alpha)^B{}_A\frac{\delta}{\delta\bar\Phi_A(\theta)} - \frac{\partial^2}{\partial\theta^2}\left(\bar\Phi_A(\theta)\theta_b\right)(\sigma_\alpha)^b{}_a\theta^a\frac{\delta}{\delta\bar\Phi_A(\theta)}\right),$$
$$U_\alpha = \int d^2\theta\left(\Phi^A(\sigma_\alpha)_A{}^B\frac{\delta_l}{\delta\Phi^B(\theta)} + \frac{\partial^2}{\partial\theta^2}\left(\Phi^A(\theta)\theta^a\right)(\sigma_\alpha)_a{}^b\theta_b\frac{\delta_l}{\delta\Phi^A(\theta)}\right). \quad (6)$$

These operators obey a superalgebra with the following non-trivial (anti)commutation relations:

$$[V_\alpha, V_\beta] = \epsilon_{\alpha\beta}{}^\gamma V_\gamma, \quad [V_\alpha, V^a] = V^b(\sigma_\alpha)_b{}^a, \quad \{V^a, V^b\} = 0,$$
$$[U_\alpha, U_\beta] = -\epsilon_{\alpha\beta}{}^\gamma U_\gamma, \quad [U_\alpha, U^a] = -U^b(\sigma_\alpha)_b{}^a, \quad \{U^a, U^b\} = 0. \quad (7)$$

Let us introduce a set of second-order operators Δ^a (odd) and Δ_α (even)

$$\Delta^a = -\int d^2\theta\frac{\delta_l}{\delta\Phi^A(\theta)}\frac{\partial}{\partial\theta_a}\frac{\delta}{\delta\bar\Phi_A(\theta)},$$
$$\Delta_\alpha = (-1)^{\varepsilon_A+1}\int d^2\theta\left\{(\sigma_\alpha)_B{}^A\left[\frac{\partial^2}{\partial\theta^2}\left(\frac{\delta_l}{\delta\Phi^A(\theta)}\right)\theta^2 - \frac{\partial^2}{\partial\theta^2}\left(\frac{\delta_l}{\delta\Phi^A(\theta)}\theta^2\right)\right]\right.$$
$$\left. + \frac{\partial^2}{\partial\theta^2}\left(\frac{\delta_l}{\delta\Phi^B(\theta)}\theta_b(\sigma_\alpha)^b{}_a + \frac{\delta_l}{\delta\Phi^A(\theta)}\theta_a(\sigma_\alpha)^A{}_B\right)\theta^a\right\}\frac{\delta}{\delta\bar\Phi_B(\theta)}. \quad (8)$$

These operators possess the algebraic properties

$$[\Delta_\alpha, \Delta_\beta] = 0, \quad \{\Delta^a, \Delta^b\} = 0, \quad [\Delta_\alpha, \Delta^a] = 0, \quad (9)$$

$$[\Delta_\alpha, V_\beta] + [V_\alpha, \Delta_\beta] = \epsilon_{\alpha\beta}{}^\gamma\Delta_\gamma,$$
$$\{\Delta^a, V^b\} + \{V^a, \Delta^b\} = 0,$$
$$[\Delta_\alpha, V^a] + [V_\alpha, \Delta^a] = \Delta^b(\sigma_\alpha)_b{}^a. \quad (10)$$

From (8) it follows that the action of the operators Δ^a and Δ_α on the product of two functionals defines the superbracket operations (3), namely,

$$\begin{aligned}\Delta_\alpha(FG) &= (\Delta_\alpha F)G + F(\Delta_\alpha G) + \{F,G\}_\alpha,\\ \Delta^a(FG) &= (\Delta^a F)G + F(\Delta^a G)(-1)^{\varepsilon(F)} + (F,G)^a(-1)^{\varepsilon(F)}.\end{aligned} \tag{11}$$

Using the relations (9), (10), (11), one can establish the properties of the superbrackets (3) at the algebraic level [8].

Finally, we introduce the operators

$$\bar{\Delta}^a \equiv \Delta^a + \frac{i}{\hbar}V^a, \quad \bar{\Delta}_\alpha \equiv \Delta_\alpha + \frac{i}{\hbar}V_\alpha.$$

From (7), (9), (10) it follows that these operators obey the superalgebra

$$\begin{aligned}[][\bar{\Delta}_\alpha, \bar{\Delta}_\beta] &= (i/\hbar)\epsilon_{\alpha\beta}{}^\gamma \bar{\Delta}_\gamma,\\ [\bar{\Delta}_\alpha, \bar{\Delta}^a] &= (i/\hbar)\bar{\Delta}^b(\sigma_\alpha)_b{}^a,\\ \{\bar{\Delta}^a, \bar{\Delta}^b\} &= 0,\end{aligned}$$

isomorphic to the massless limit of the superalgebra of generating operators used in the method of $osp(1,2)$-covariant quantization [8].

3. Quantization Rules

Define the vacuum functional Z as the following path integral:

$$Z = \int d\Phi\, d\bar{\Phi}\, \exp\left[\frac{i}{\hbar}\left(W(\Phi,\bar{\Phi}) - \frac{1}{2}\varepsilon_{ab}U^aU^bF(\Phi) + \bar{\Phi}\Phi\right)\right], \tag{12}$$

where $W = W(\Phi,\bar{\Phi})$ is the quantum action that satisfies the master equations

$$\bar{\Delta}^a \exp\left(\frac{i}{\hbar}W\right) = 0, \tag{13}$$

and the subsidiary conditions

$$\bar{\Delta}_\alpha \exp\left(\frac{i}{\hbar}W\right) = 0, \tag{14}$$

with $\bar{\Delta}^a$ and $\bar{\Delta}_\alpha$ given by (8). Equations (13) and (14) are equivalent to

$$\frac{1}{2}(W,W)^a + V^aW = i\hbar\Delta^aW, \tag{15}$$

$$\frac{1}{2}\{W,W\}_\alpha + V_\alpha W = i\hbar\Delta_\alpha W, \tag{16}$$

where the superbrackets $(\;,\;)^a$, $\{\;,\;\}_\alpha$ and the operators V^a, V_α, Δ^a, Δ_α are defined by (3), (6), (8). The quantum action W is also assumed to be an admissible solution of (15) and (16), which implies the fulfillment of the restriction

$$\int d^2\theta\,\theta^2\left(\frac{\delta W}{\delta\bar{\Phi}_A(\theta)} + \Phi^A(\theta)\right) = 0. \tag{17}$$

In (12), $\bar{\Phi}\Phi$ is a functional of the form

$$\bar{\Phi}\Phi = \int d^2\theta\, \bar{\Phi}_A(\theta)\Phi^A(\theta), \tag{18}$$

while $F(\Phi)$ is a gauge-fixing Boson restricted by the conditions

$$U_\alpha F(\Phi) = 0, \tag{19}$$

where U_α are the operators (6).

An important property of the integrand in (12) is its invariance under the following transformations:

$$\delta\Phi^A(\theta) = \mu_a U^a \Phi^A(\theta), \quad \delta\bar{\Phi}_A(\theta) = \mu_a V^a \bar{\Phi}_A(\theta) + \mu_a (W, \bar{\Phi}_A(\theta))^a, \tag{20}$$

$$\delta\Phi^A(\theta) = \mu^\alpha U_\alpha \Phi^A(\theta), \quad \delta\bar{\Phi}_A(\theta) = \mu^\alpha V_\alpha \bar{\Phi}_A(\theta) + \mu^\alpha \{W, \bar{\Phi}_A(\theta)\}_\alpha, \tag{21}$$

where U^a are operators given by (6), while μ_a and μ^α are constant (anti)commuting parameters, $\varepsilon(\mu_a) = 1$, $\varepsilon(\mu^\alpha) = 0$. The validity of the symmetry transformations (20), (21) follows from the master equations (15), (16) and the conditions (19) for the gauge-fixing Boson, with allowance for integration by parts (1) and the algebraic properties (7).

The transformations (20) realize the extended BRST symmetry, while the transformations (21) express the symmetry related to the $Sp(2)$ invariance of the quantum action. This interpretation is explained in the following section by the relation of the present superfield formalism to the original $Sp(2)$ covariant superfield scheme [1] and the $osp(1,2)$ covariant approach [8]. Note that the admissibility condition (17) is not required for the proof of invariance. As will be shown in the following section, this condition establishes the relation between the present formalism and the quantization schemes [1, 8].

The transformations of extended BRST symmetry (20) permit establishing the independence of the vacuum functional (12) from the choice of the gauge Boson $F(\Phi)$. Indeed, any infinitesimal change $F \to F + \delta F$ can be compensated by a change of variables (20) with the parameters $\mu_a = -(i/2\hbar)\varepsilon_{ab}U^b\delta F(\Phi)$, and therefore $Z_{F+\delta F} = Z_F$, which implies the independence of the S-matrix on the choice of gauge in the proposed formalism.

4. Component Analysis

Let us consider the component representation of the formalism proposed in the previous section in order to establish its relation to the $Sp(2)$ covariant superfield scheme [1] and the $osp(1,2)$ covariant approach [8].

The component form of superfields $\Phi^A(\theta)$ and supersources $\bar{\Phi}_A(\theta)$ reads

$$\Phi^A(\theta) = \phi^A + \pi^{Aa}\theta_a + \lambda^A\theta^2,$$

$$\bar{\Phi}_A(\theta) = \bar{\phi}_A - \theta^a\phi^*_{Aa} - \theta^2\eta_A.$$

The components $(\phi^A, \pi^{Aa}, \lambda^A, \bar{\phi}_A, \phi^*_{Aa}, \eta_A)$ are identical with the set of variables required for the construction of the vacuum functional in the quantization schemes [1, 8].

With allowance for the manifest structure of $\Phi^A(\theta)$, $\bar{\Phi}_A(\theta)$, the component representation of the integration measure in (12) is given by

$$d\Phi\, d\bar{\Phi} = d\phi\, d\pi\, d\lambda\, d\bar{\phi}\, d\phi^*\, d\eta, \tag{22}$$

and the functional $\bar{\Phi}\Phi$ in (18) has the form

$$\bar{\Phi}\Phi = \bar{\phi}_A\lambda^A + \phi^*_{Aa}\pi^{Aa} - \eta_A\phi^A. \tag{23}$$

Denote $F(\Phi, \bar{\Phi}) \equiv \tilde{F}(\phi, \pi, \lambda, \bar{\phi}, \phi^*, \eta)$. Then the superbrackets $(\ ,\ )^a$ and $\{\ ,\ \}_\alpha$ in (3) have the following component structure:

$$\begin{aligned}
(F,G)^a &= \frac{\delta\tilde{F}}{\delta\phi^A}\frac{\delta\tilde{G}}{\delta\phi^*_{Aa}} + \varepsilon^{ab}\frac{\delta\tilde{F}}{\delta\pi^{Ab}}\frac{\delta\tilde{G}}{\delta\bar{\phi}_A} - (\tilde{F}\leftrightarrow\tilde{G})\,(-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)},\\
\{F,G\}_\alpha &= (\sigma_\alpha)_B{}^A\left(\frac{\delta\tilde{F}}{\delta\phi^A}\frac{\delta\tilde{G}}{\delta\eta_B} + \frac{\delta\tilde{F}}{\delta\lambda^A}\frac{\delta\tilde{G}}{\delta\bar{\phi}_B}\right) + \left(\frac{\delta\tilde{F}}{\delta\pi^{Ab}}(\sigma_\alpha)^b{}_a + \frac{\delta\tilde{F}}{\delta\pi^{Ba}}(\sigma_\alpha)^B{}_A\right)\frac{\delta\tilde{G}}{\delta\phi^*_{Aa}}\\
&\quad +(\tilde{F}\leftrightarrow\tilde{G})(-1)^{\varepsilon(F)\varepsilon(G)},
\end{aligned}\tag{24}$$

while the second-order operators Δ^a and Δ_α in (8) have the form

$$\begin{aligned}
\Delta^a &= (-1)^{\varepsilon_A}\frac{\delta_l}{\delta\phi^A}\frac{\delta}{\delta\phi^*_{Aa}} + (-1)^{\varepsilon_A+1}\varepsilon^{ab}\frac{\delta_l}{\delta\pi^{Ab}}\frac{\delta}{\delta\bar{\phi}_A},\\
\Delta_\alpha &= (-1)^{\varepsilon_A}(\sigma_\alpha)_B{}^A\left(\frac{\delta_l}{\delta\phi^A}\frac{\delta}{\delta\eta_B} + \frac{\delta_l}{\delta\lambda^A}\frac{\delta}{\delta\bar{\phi}_B}\right)\\
&\quad +(-1)^{\varepsilon_A+1}\left(\frac{\delta_l}{\delta\pi^{Ab}}(\sigma_\alpha)^b{}_a + \frac{\delta_l}{\delta\pi^{Ba}}(\sigma_\alpha)^B{}_A\right)\frac{\delta}{\delta\phi^*_{Aa}}.
\end{aligned}\tag{25}$$

In (6), the first-order operators V^a and V_α have the component representation

$$\begin{aligned}
V^a &= \varepsilon^{ab}\phi^*_{Ab}\frac{\delta}{\delta\bar{\phi}_A} - \eta_A\frac{\delta}{\delta\phi^*_{Aa}},\\
V_\alpha &= \bar{\phi}_B(\sigma_\alpha)^B{}_A\frac{\delta}{\delta\bar{\phi}_A} + \left(\phi^*_{Ab}(\sigma_\alpha)^b{}_a + \phi^*_{Ba}(\sigma_\alpha)^B{}_A\right)\frac{\delta}{\delta\phi^*_{Aa}} + \eta_B(\sigma_\alpha)^B{}_A\frac{\delta}{\delta\eta_A},
\end{aligned}\tag{26}$$

while the first-order operators U^a and U_α are given by

$$\begin{aligned}
U^a &= (-1)^{\varepsilon_A}\varepsilon^{ab}\lambda^A\frac{\delta_l}{\delta\pi^{Ab}} - (-1)^{\varepsilon_A}\pi^{Aa}\frac{\delta_l}{\delta\phi^A},\\
U_\alpha &= \phi^B(\sigma_\alpha)_B{}^A\frac{\delta_l}{\delta\phi^A} + \left(\pi^{Ab}(\sigma_\alpha)_b{}^a + \pi^{Ba}(\sigma_\alpha)_B{}^A\right)\frac{\delta_l}{\delta\pi^{Aa}} + \lambda^B(\sigma_\alpha)_B{}^A\frac{\delta_l}{\delta\lambda^A}.
\end{aligned}\tag{27}$$

The component form of the admissibility condition (17)

$$\frac{\delta\tilde{W}}{\delta\eta_A} = \phi^A \tag{28}$$

implies a simplification of the quantum action:

$$\tilde{W} = \mathcal{W}(\phi, \lambda, \pi, \bar{\phi}, \phi^*) + \eta_A\phi^A. \tag{29}$$

To establish the relation between the proposed superfield scheme and the $osp(1,2)$ covariant formalism [8], note, first of all, that the operators U_α and V_α in (26), (27) coincide with the generators of $Sp(2)$ invariance [8]. In particular, equation (19) implies the condition of $Sp(2)$ invariance for the gauge Boson $\tilde{F}(\phi, \pi, \lambda)$.

Let us subject the quantum action $\tilde{W}$ to the conditions

$$\frac{\delta \tilde{W}}{\delta \lambda^A} = \frac{\delta \tilde{W}}{\delta \pi^{Aa}} = 0. \tag{30}$$

reducing the dependence of $\tilde{W}$ to the set of variables $(\phi^A, \bar{\phi}_A, \phi^*_{Aa}, \eta_A)$ parameterizing the quantum action in the $osp(1,2)$ covariant formalism. By virtue of (30) and the component representations (24)–(27), the set of equations (15), (16) becomes identical with the massless limit of the master equations in the $osp(1,2)$ covariant scheme.

Let us now restrict the gauge-fixing Boson to the class of gauges used in the $osp(1,2)$ covariant scheme: $\tilde{F} = \tilde{F}(\phi)$. Then, with allowance for the component form (27) of the operators U_α, the condition of $Sp(2)$ invariance (19) reduces to

$$(\sigma_\alpha)_B{}^A \frac{\delta \tilde{F}}{\delta \phi^A} \phi^B = 0, \tag{31}$$

which, in view of the admissibility condition (28), can be represented as

$$(\sigma_\alpha)_B{}^A \frac{\delta \tilde{F}}{\delta \phi^A} \frac{\delta \tilde{W}}{\delta \eta_B} = 0. \tag{32}$$

Equations (31) and (32) reproduce the whole set of additional restrictions used in the $osp(1,2)$ covariant scheme [8] to provide an $Sp(2)$ invariant gauge-fixing. Note that in [8] the condition (28) arises as a particular case of (31) and (32). Using (28) and the restrictions (30), with allowance for (4), (5), one can transform the subsidiary master equations (16) into the condition of $Sp(2)$ invariance of the quantum action [8]

$$(\sigma_\alpha)_B{}^A \frac{\delta \tilde{W}}{\delta \phi^A} \phi^B + V_\alpha \tilde{W} = 0,$$

which establishes the interpretation of the related symmetry transformations (21).

Let us demonstrate the relation of the vacuum functional (12), given in terms of $\tilde{W} = \tilde{W}(\phi, \bar{\phi}, \phi^*, \eta)$ and $\tilde{F} = \tilde{F}(\phi)$, to the vacuum functional of the $osp(1,2)$ covariant scheme [8]. Using the component form of the operators U^a, given by (27), and integrating out the variables η_A, with allowance for (22), (23), (29), we can represent the vacuum functional (12) in the form

$$Z = \int d\phi \, d\phi^* \, d\pi \, d\bar{\phi} \, d\lambda \, \exp\left[\frac{i}{\hbar}\left(\mathcal{W} + \mathcal{X} + \bar{\phi}_A \lambda^A + \phi^*_{Aa} \pi^{Aa}\right)\right], \tag{33}$$

where the quantum action $\tilde{W} = \mathcal{W} + \eta_A \phi^A$ satisfies (15), (16), (28), and the gauge-fixing term $\mathcal{X}$ is given by

$$\mathcal{X} = -\frac{\delta \tilde{F}}{\delta \phi^A} \lambda^A - \frac{1}{2} \varepsilon_{ab} \pi^{Aa} \frac{\delta^2 \tilde{F}}{\delta \phi^A \delta \phi^B} \pi^{Bb},$$

with $\tilde{F}$ subject to (31). On the other hand, the vacuum functional in the massless limit of the $osp(1,2)$ covariant formalism [8] can be represented as

$$Z = \int d\phi \exp\left(\frac{i}{\hbar} S_{\rm eff}\right), \tag{34}$$

$$S_{\rm eff}(\phi) = S_{\rm ext}(\phi, \bar{\phi}, \phi^*, \eta)\Big|_{\bar{\phi}=\phi^*=\eta=0}, \quad \exp\left[(i/\hbar)S_{\rm ext}\right] = \hat{U}(Y)\exp\left[(i/\hbar)S\right].$$

Here, $S = S(\phi, \bar{\phi}, \phi^*, \eta)$ is the quantum action subject to the system of master equations and subsidiary conditions (15), (16), (28) satisfied by $\tilde{W} = \tilde{W}(\phi, \bar{\phi}, \phi^*, \eta)$, and $\hat{U}(Y)$ is an operator of the form

$$\hat{U}(Y) = \exp\left(\frac{\delta Y}{\delta\phi^A}\frac{\delta}{\delta\bar{\phi}_A} + \frac{i\hbar}{2}\varepsilon_{ab}\frac{\delta}{\delta\phi^*_{Aa}}\frac{\delta^2 Y}{\delta\phi^A\delta\phi^B}\frac{\delta}{\delta\phi^*_{Bb}}\right),$$

where $Y = Y(\phi)$ is a gauge-fixing Boson restricted by the same condition (31) of $Sp(2)$ invariance which is imposed on $\tilde{F} = \tilde{F}(\phi)$. To establish the identity between the vacuum functionals (33) and (34), it is sufficient to set $S = \tilde{W}$ and $Y = \tilde{F}$.

Let us now establish the relation of the proposed superfield formalism to the original $Sp(2)$ covariant superfield scheme [1]. First, note that U^a and V^a in (6) are generators of supertranslations [1], which is important for the interpretation of the related symmetry transformations (20). Next, with allowance for $(\ ,\ )^a$ and Δ^a in (3), (8), equations (15) are identical to the master equations of the formalism [1]. Finally, using the admissibility condition (17) and the related form of η-dependence (29), with allowance for (22), (23), we can rewrite the vacuum functional (12) as follows:

$$Z = \int d\Phi\, d\bar{\Phi}\, \rho(\bar{\Phi}) \exp\left[\frac{i}{\hbar}\left(W(\Phi, \bar{\Phi}) - \frac{1}{2}\varepsilon_{ab}U^a U^b F(\Phi) + \bar{\Phi}\Phi\right)\right], \tag{35}$$

where $\rho(\bar{\Phi})$ is an additional integration weight, given by

$$\rho(\bar{\Phi}) = \delta\left(\int d^2\theta\, \bar{\Phi}(\theta)\right) = \delta\,(\eta)\,.$$

The vacuum functional (35) is formally identical with the corresponding functional of the $Sp(2)$ covariant superfield scheme [1]. Namely, the vacuum functional of [1] coincides with (35) in case $W(\Phi, \bar{\Phi})$ satisfies the subsidiary conditions (16), (17), and $F(\Phi)$ is subject to the condition of $Sp(2)$ invariance (19). In view of the invariance of $\rho(\bar{\Phi})$ under the transformations (20), the integrand in (35) remains invariant under (20), which realize the superfield form of extended BRST symmetry [1] in terms of supertranslations.

5. Conclusion

In this paper, we have proposed a modification of the $Sp(2)$ covariant superfield scheme [1] on the basis of a superalgebra of generating operators isomorphic to the massless limit of the corresponding superalgebra of $osp(1,2)$ covariant quantization [8]. The extended BRST symmetry realized in terms of superfield translations is completely controlled by the master equations. An additional admissibility condition reduces the formalism to the original $Sp(2)$ covariant superfield scheme and to the massless limit of the $osp(1,2)$ covariant scheme in particular cases of gauge-fixing and solutions of the master equations. As mentioned in the introduction, the present study is motivated by the problem of constructing a superfield scheme based on a superalgebra of generating operators isomorphic to $osp(1,2)$, and containing the $Sp(2)$ covariant superfield scheme in the massless limit, by analogy with the relation between the original $osp(1,2)$ and $Sp(2)$ covariant methods [2, 8]. In this connection, the present formalism possesses the properties of such a massless limit. The question of existence of a superfield $osp(1,2)$ covariant scheme containing this limit as a particular case is related to the problem of massive extensions [8] of the superalgebra based on the generating operatos realized in the present formalism.

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