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Quasinormal modes of d-dimensional spherical black holes with near extreme cosmological constant

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Quasinormal modes of d-dimensional spherical black holes with near extreme cosmological constant

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We derive an expression for the quasinormal modes of scalar perturbations in near extreme d-dimensional Schwarzschild-de Sitter and Reissner-Nordström-de Sitter black holes. We show that, in the near extreme limit, the dynamics of the scalar field is characterized by a Pöshl-Teller effective potential. The results are qualitatively independent of the spacetime dimension and field mass.

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I. INTRODUCTION

Recent observational results suggests that the universe in large scale is described by an Einstein equation with an (at least effective) cosmological constant. In this context, the dynamics of fields in spacetimes which are not asymptotically flat has taken a new importance. Quasinormal modes are an important part of this dynamics. In general, perturbations in the spacetime exterior of a black hole are followed by oscillations which decay exponentially in time. Roughly speaking, these are quasinormal modes, complex frequency modes which carry information about the background geometry, and are independent of the initial perturbation. The characterization of the quasinormal modes originated from compact objects are of particular relevance in gravitational wave astronomy [1]. Their detection is expected to be realized through gravitational wave observations in the near future.

In asymptotically anti-de Sitter spacetimes, the anti-de Sitter/Conformal field theory (AdS/CFT) correspondence [2] plays a very important role. In this framework, a link is established among the quasinormal frequencies of a test field in AdS black holes and the decay rates in the dual CFT field theory. A perturbation in the thermal states in the strongly coupled CFT corresponds to perturbing the black hole. The first study of the scalar quasinormal modes in AdS space was performed by Chan and Mann [3]. More recently, Horowitz and Hubeny [4] considered the problem of quasinormal modes on the background of Schwarzschild-anti de Sitter black holes in four, five and seven dimensions. Their basic results were confirmed in [5] by direct calculation of the wave functions.

For asymptotically de Sitter spacetimes, similar conjectures have been formulated. Strominger [6] proposed an holographic duality relating quantum gravity on d-dimensional de Sitter space (dS_d) to a Conformal Field Theory residing on the past boundary of dS_d. This dS/CFT correspondence has motivated several works (as e.g. [7]).

A very different and intriguing application of quasinormal modes have been suggested recently. In the context of loop quantum gravity, it has been shown that the area of the event horizon is quantized, but the expression involves a free quantity, the Barbero-Immirzi parameter. Hod, Dreyer and Mothl [8] proposed using the information of the quasinormal frequencies to fix the Barbero-Immirzi parameter in the expression of the quantum of area of the event horizon. Kunstatter used these ideas in [9] to derive the Bekenstein-Hawking entropy spectrum for d-dimensional spherically symmetric black holes. And in [10], Abdalla, Castello-Branco and Lima-Santos proposed an area quantization prescription for the near extreme Schwarzschild-de Sitter and Kerr black holes.

Calculating analytic expressions for the quasinormal frequencies is usually difficult, except in particular situations. One of these cases is the Pöshl-Teller potential [11]. For this potential, many properties have been proved and the frequencies have been calculated [12, 13]. In a recent work [14], Cardoso and Lemos studied the Schwarzschild-de Sitter geometry in four dimensions, and showed that the dynamics is specified by a Pöshl-Teller effective potential. They calculated exact expressions for the quasinormal modes, demonstrating why a previous approximation made by Moss and Norman [15] hold in the near extreme regime.

In the present work, we generalize the method used by Cardoso and Lemos in [14]. We analyze a scalar field, massless and massive, in the d-dimensional Schwarzschild-de Sitter (SdS) and Reissner-Nordström-de Sitter (RNdS) geometry. The approach here is a bottom-up one. In the next section, the parameter space of the d-dimensional Schwarzschild-de Sitter is discussed. The geometry of the block outside the event horizon is then treated in the near extreme limit. In section 4, we show that the ideas developed in the previous sections can be applied to more general spacetimes, including the d-dimensional Reissner-Nordström-de Sitter black holes. In section 5, the dynamics of the scalar field in spherical backgrounds is discussed, and analytical formulas for the quasinormal frequencies are calculated. In the last section, some final remarks are made.

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II. SCHWARZSCHILD-DE SITTER METRIC

The metric describing a d-dimensional non-asymptotically flat spherical black hole was presented in [16]. Written in spherical coordinates, the Schwarzschild-de Sitter metric is given by

$$ds^2 = -h(r)dt^2 + h(r)^{-1}dr^2 + r^2 d\Omega_{d-2}^2 , \quad (1)$$

where the function $h(r)$ is

$$h(r) = 1 - \frac{2m}{r^{d-3}} - \frac{\Lambda r^2}{3} . \quad (2)$$

The integration constant m is proportional to the black hole mass, and $d\Omega_{d-2}^2$ is the line element of the $(d-2)$-dimensional unit sphere:

$$\begin{aligned} d\Omega_{d-2}^2 &= d\left(\theta^1\right)^2 + \sin^2\theta^1\, d\left(\theta^1\right)^2 + \cdots \\ &\quad + \sin^2\theta^1 \cdots \sin^2\theta^{d-2}\, d\left(\theta^{d-2}\right)^2 . \end{aligned} \quad (3)$$

If the cosmological constant is positive, the spacetime is asymptotically de Sitter. In this case, Λ is usually written in terms of a "cosmological radius" a as

$$\Lambda = \frac{3}{a^2} . \quad (4)$$

The causal structure of the spacetime described by the metric presented depends of the positive real roots of the function $h(r)$. This set in turn depends, for a given dimension, on the parameters m and Λ. By rescaling the metric and the coordinates t and r, it can be seen that parameter space of the metric has dimension one. Indeed, if the mass is non-vanishing, a convenient parameter is $m^2/a^{2(d-3)}$. We will classify the possible horizons of the spacetime in the following proposition.

Proposition 1 *Let $d \geq 4$, $a^2 > 0$ and $m > 0$.*

- *The spacetime has two horizons if and only if the condition*

$$\frac{m^2}{a^{2(d-3)}} < \frac{(d-3)^{d-3}}{(d-1)^{d-1}} \quad (5)$$

is satisfied. This is the usual d-dimensional Schwarzschild-de Sitter black hole.

- *The spacetime has one horizon if and only if the condition*

$$\frac{m^2}{a^{2(d-3)}} = \frac{(d-3)^{d-3}}{(d-1)^{d-1}} \quad (6)$$

is satisfied. This is the extreme Schwarzschild-de Sitter black hole.

- *The spacetime has no horizon if and only if the condition*

$$\frac{m^2}{a^{2(d-3)}} > \frac{(d-3)^{d-3}}{(d-1)^{d-1}} \quad (7)$$

is satisfied.

One way to demonstrate the proposition 1 is to write the function $h(r)$ as

$$h(r) = -\frac{P(r)}{a^2 r^{d-3}} , \quad (8)$$

where $P(r)$ is the polynomial

$$P(r) = r^{d-1} - a^2 r^{d-3} + 2ma^2 . \quad (9)$$

The characterization of the zeros of $h(r)$ is therefore equivalent to the characterization of the roots of $P(r)$. Here, we will just sketch the steps to be done. The maximum, minimum and inflection points of $P(r)$ can be calculated. To prove that the conditions are sufficient it is used the asymptotic behavior of $P(r)$, which depends of d being even or odd. Together with the information of the extreme points, it is possible to determine the number of real roots, their signs and multiplicities. With the condition (6), the two positive roots collapse in a double root. To prove that the condition is necessary it is used the fact that extremes are explicitly calculated.

From proposition 1 we learn that the global structure of the manifolds described by the Schwarzschild-de Sitter metric is largely independent of the dimension. With a small value of $m^2/a^{2(d-3)}$, this metric describes a spacetime with two horizons — an event horizon r_+ and a cosmological horizon r_c ($0 < r_+ < r_c$). The region of interest in this paper is the block

$$T_+ = \{(t, r, \theta_1, \cdots, \theta_{d-2}), r_+ < r < r_c\} . \quad (10)$$

In the critical value, the event horizon and the cosmological horizon coincide. This is the extreme Schwarzschild-de Sitter black hole. When $m^2/a^{2(d-3)}$ is larger than the critical value, the metric no longer describes a black hole.

III. NEAR EXTREME SCHWARZSCHILD-DE SITTER BLACK HOLE

Since we are interested in the limit where the event and cosmological horizons are very close, it is natural to define the dimensionless parameter δ' as

$$\delta' = \frac{r_c - r_+}{a} , \quad (11)$$

so that the near extreme limit is such that

$$0 < \delta' \ll 1 . \quad (12)$$

In the Schwarzschild-de Sitter black hole case however, is convenient to use another dimensionless parameter $\bar{\delta}$, defined as

$$\bar{\delta} = \sqrt{1 - \frac{m^2}{a^{2(d-3)}} \frac{(d-1)^{d-1}}{(d-3)^{d-3}}} . \quad (13)$$

It is clear from proposition 1 that the near extreme SdS black hole corresponds to the limit

$$0 < \bar{\delta} \ll 1 . \quad (14)$$

The limits (12) and (14) are equivalent. Furthermore, it can be shown that they go to zero at the same rate:

$$\bar{\delta} = \frac{d-1}{2}\delta' + O(\delta'^2) \ . \tag{15}$$

In the Schwarzschild-de Sitter scenario, the function $h(r)$ has one local maximum r_0 in the interval $[r_+, r_c]$. This point can be expressed in terms of the parameters m and a as

$$r_0 = \left[(d-3)ma^2\right]^{\frac{1}{d-1}} \ . \tag{16}$$

Near the extreme limit, the function $h(r)$ can be approximated by its Taylor expansion up to the second order in δ' around the local maximum $r = r_0$:

$$h(r) = h(r_0) + \frac{a^2}{2}\left.\frac{d^2h(r)}{dr^2}\right|_{r=r_0}\left(\frac{r-r_0}{a}\right)^2 + O\left(\delta'^3\right) \ . \tag{17}$$

To lowest order δ' and $\bar{\delta}$ are proportional, and the expression (17) can be written as

$$h(r) = \frac{d-1}{a^2}\left(r_c^{ap} - r\right)\left(r - r_+^{ap}\right) + O\left(\bar{\delta}^3\right) \ , \tag{18}$$

where the constants r_+^{ap} and r_-^{ap} are approximations of the event and cosmological horizons, given by

$$\begin{aligned} r_c^{ap} &= r_0 + a\sqrt{\frac{1-\left(1-\bar{\delta}^2\right)^{\frac{1}{d-1}}}{d-1}} \\ &= a\left(\sqrt{\frac{d-3}{d-1}} + \frac{\bar{\delta}}{d-1}\right) + O\left(\bar{\delta}^2\right) \ , \end{aligned} \tag{19}$$

$$\begin{aligned} r_+^{ap} &= r_0 - a\sqrt{\frac{1-\left(1-\bar{\delta}^2\right)^{\frac{1}{d-1}}}{d-1}} \\ &= a\left(\sqrt{\frac{d-3}{d-1}} - \frac{\bar{\delta}}{d-1}\right) + O\left(\bar{\delta}^2\right) \ . \end{aligned} \tag{20}$$

The next step is to calculate the tortoise radial function $x(r)$, defined at the block T_+ in the usual way:

$$\begin{aligned} x(r) &= \int \frac{dr}{h(r)} \\ &= -\frac{1}{2\kappa_c^{ap}}\ln\left(r_c^{ap} - r\right) + \frac{1}{2\kappa_+^{ap}}\ln\left(r - r_+^{ap}\right) \\ &\quad + O\left(\bar{\delta}^3\right) \ , \end{aligned} \tag{21}$$

where the constants κ_c^{ap} and κ_+^{ap} are

$$\kappa_c^{ap} = \kappa_+^{ap} = \frac{d-1}{2a^2}\left(r_c^{ap} - r_+^{ap}\right) = \frac{\bar{\delta}}{a} + O\left(\bar{\delta}^2\right) \ . \tag{22}$$

Observe that these constants are approximations to the surface gravities of the event and cosmological horizons, which tend to zero in the extreme limit.

The key point is that, in this limit, the function $x(r)$ can be analytically inverted:

$$r(x) = \frac{e^{2\kappa_+^{ap}x}r_c^{ap} + r_+^{ap}}{1 + e^{2\kappa_+^{ap}x}} + O\left(\bar{\delta}^3\right) \ . \tag{23}$$

With the expression (23), the function $h(x) \equiv h(r(x))$ can be explicitly calculated:

$$h(x) = \frac{\bar{\delta}^2}{d-1}\frac{1}{\cosh^2(\kappa_+^{ap}x)} + O\left(\bar{\delta}^3\right) \ . \tag{24}$$

All the constants in (23) and (24) can be obtained from the parameters m and Λ.

The approach used here to treat the near extreme Schwarzschild-de Sitter black hole can be generalized. In the next section, we will see that $r(r)$ and $h(x)$ in the expressions (23) and (24) have the same form in a broader class of near extreme spacetimes.

IV. MORE GENERAL SETTING

The fact that the function $h(r)$ for the d-dimensional Schwarzschild-de Sitter black hole can be written in the form (18) is not a particularity of this specific geometry. Basically, all we have used is that:

- The function $h(r)$ has at least two positive real roots r_1 and r_2 ($r_1 < r_2$). If r_1 and r_2 are consecutive roots, we are interested in the submanifold given by the block

$$T_1 = \{(t, r, \theta_1, \cdots, \theta_{d-2}), r_1 < r < r_2\} \ . \tag{25}$$

- The function $h(r)$ is, at least, a C^3 function at the interval $[r_1, r_2]$.
- The points r_1 and r_2 are simple roots of $h(r)$.
- There is a near extreme limit, a region of the parameter space where the horizons are arbitrarily close, such that

$$0 < \frac{r_2 - r_1}{r_1} \ll 1 \ . \tag{26}$$

Since $h(r)$ is continuous, there is a maximum or minimum point $r_0 \in]r_1, r_2[$. But in general the analytical determination of r_0 in terms of the parameters of the metric is not easy. It is convenient then to consider r_1, r_2 and κ_1 as the fundamental parameters of the spacetime, where κ_1 is the surface gravity at the horizon $r = r_1$.

In terms of these parameters, the function $h(r)$ can be approximated, in the near extreme limit, as

$$h(r) = \frac{2\kappa_1}{r_2 - r_1}(r_2 - r)(r - r_1) + O\left(\delta^3\right) \ , \tag{27}$$

with $\delta = (r_2 - r_1)/r_1$. The tortoise function $x(r)$, whose domain is the interval $]r_1, r_2[$, can be easily calculated from the expression (27). Its inverse $r(x)$ is

$$r(x) = \frac{e^{2\kappa_1 x} r_2 + r_1}{1 + e^{2\kappa_1 x}} + O\left(\delta^3\right) , \tag{28}$$

and from (28) it is straightforward to obtain $h(x)$:

$$h(x) = \frac{(r_2 - r_1)\kappa_1}{2\cosh^2(\kappa_1 x)} + O\left(\delta^3\right) . \tag{29}$$

One possible generalization of the d-dimensional Schwarzschild-de Sitter geometry is the metric in the form (1) with the function $h(r)$ given by

$$h(r) = 1 - \frac{2m}{r^{d-3}} + \frac{q^2}{r^{2d-6}} - \frac{\Lambda r^2}{3} . \tag{30}$$

This is the d-dimensional Reissner-Nordström-de Sitter metric [16], which describes a charged black hole asymptotically de Sitter. The integration constants m and q are proportional to the black hole mass and electric charge.

The parameter space for the RNdS metric is much more complex than the SdS case. It can be shown that the function $h(r)$, for arbitrary d, has at most three positive real roots (plus negative roots). And if the number of positive roots is three, they are simple. These roots are the Cauchy (r_-), event (r_+) and cosmological (r_c) horizons, with $0 < r_- < r_+ < r_c$. The function $h(r)$ is smooth in both intervals $]r_-, r_+[$ and $]r_+, r_c[$, and there are regions of the parameter space in which the intervals collapse to points.

The conditions shown in the beginning of this section therefore apply to the RNdS metric, and we have two possible near extreme situations for the RNdS metric, where $r_- \approx r_+$ and $r_+ \approx r_c$. Although the work developed in this section applies for both cases, we are mainly interested in the second one. We will therefore specify the work to the near extreme T_+ block of the Schwarzschild-de Sitter and Reissner-Nordström-de Sitter metric.

V. EFFECTIVE POTENTIAL AND QUASINORMAL MODES

We introduce in the d-dimensional SdS or RNdS spacetime a real scalar field Φ, with mass $\mu \geq 0$, described by the equation

$$\left(\Box - \mu^2\right)\Phi = 0 . \tag{31}$$

Expanding the field in (hyper)spherical harmonics, in the form

$$\Phi = \sum_{\ell m} r^{-\frac{d-2}{2}} \psi_\ell(t,r) \mathrm{Y}_{\ell m}(\{\theta_i\}) , \tag{32}$$

we get a decoupled wave equation for each value of ℓ. In terms of the coordinates t and x, this equation reads

$$-\frac{\partial^2 \psi_\ell}{\partial t^2} + \frac{\partial^2 \psi_\ell}{\partial x^2} = V(x)\psi_\ell . \tag{33}$$

The effective potential $V(x) \equiv V(r(x))$ is obtained using the function $r(x)$ and the effective potential in terms of the radial coordinate r:

$$V(r) = h(r)\Omega(r) , \tag{34}$$

with the function $\Omega(r)$ given by

$$\begin{aligned}\Omega(r) &= \mu^2 + \frac{\ell(\ell+d-3)}{r^2} + \frac{d-2}{2r}h'(r) \\ &+ \frac{(d-2)(d-4)}{4r^2}h(r) .\end{aligned} \tag{35}$$

In four dimensions, if the mass of the scalar field is set to $\mu^2 = 2/a^2$, then the effective potential above is also the effective potential for the electromagnetic field (as can be seen for example in [14]). So our results apply to this case as well.

Expanding the function $V(r)$ around the point $r = (r_+ + r_c)/2$, we have, for $\ell > 0$ or $\mu > 0$:

$$V(r) = \left[\frac{\ell(\ell+d-3)}{r_+^2} + \mu^2\right] \frac{2\kappa_+(r_c - r)(r - r_+)}{r_c - r_+} + O(\delta) . \tag{36}$$

The expression (36) for $V(r)$ is not useful when $\mu = \ell = 0$ because in this case $V(r) = 0 + O(\delta)$. Assuming $\ell > 0$ or $\mu > 0$, we calculate the function $V(x)$ as

$$V(x) = \frac{V_0}{\cosh^2(\kappa_+ x)} + O(\delta) , \tag{37}$$

where the constant V_0 is given by

$$V_0 = \left[\frac{\ell(\ell+d-3)}{r_+^2} + \mu^2\right] \frac{(r_c - r_+)\kappa_+}{2} . \tag{38}$$

The effective potential $V(x)$ in the expression (37) is the Pöshl-Teller potential [11], which has been extensively studied. In particular, attention has been dedicated to the quasinormal modes associated [12, 13]. One way to introduce the problem [1, 17] is Laplace transforming the wave equation (33), so the problem can be put as a Cauchy initial value problem. It is found that there is a discrete set of possible values to s such that the function $\hat{\psi}_\ell$, the Laplace transformed field, satisfies both boundary conditions:

$$\lim_{x \to -\infty} \hat{\psi}_\ell e^{sx} = 1 , \tag{39}$$

$$\lim_{x \to +\infty} \hat{\psi}_\ell e^{-sx} = 1 . \tag{40}$$

By making the formal replacement $s = i\omega$, we have the usual quasinormal mode boundary conditions. The frequencies ω (or s) are called quasinormal frequencies.

Using the result (38), we find that for both Schwarzschild-de Sitter and Reissner-Nordström-de Sitter cases we have:

$$\frac{\omega}{\kappa_+} = \sqrt{\left[\frac{\ell(\ell+d-3)}{r_+^2} + \mu^2\right] \frac{r_c - r_+}{2\kappa_+} - \frac{1}{4}} - i\left(n + \frac{1}{2}\right) . \tag{41}$$

For the SdS geometry, V_0 can be written in terms of m and Λ. The real and imaginary parts of the quasinormal frequencies in this case are

$$\mathrm{Re}(\omega) = \left[\frac{\Lambda}{3} - \frac{m^2 \Lambda^{d-2}}{3^{d-2}} \frac{(d-1)^{d-1}}{(d-3)^{d-3}}\right]^{\frac{1}{2}} \left[\frac{\ell(\ell+d-3)}{d-3} + \frac{3\mu^2}{(d-1)\Lambda} - \frac{1}{4}\right]^{\frac{1}{2}} , \tag{42}$$

$$\mathrm{Im}(\omega) = -\left(n + \frac{1}{2}\right) \left[\frac{\Lambda}{3} - \frac{m^2 \Lambda^{d-2}}{3^{d-2}} \frac{(d-1)^{d-1}}{(d-3)^{d-3}}\right]^{\frac{1}{2}} , \tag{43}$$

with $n \in \{0, 1, \ldots\}$ labelling the modes.

VI. CONCLUSIONS

We have studied a scalar field outside the event horizon of spherical d-dimensional black holes with near extreme cosmological constant. Its dynamics is determined by a Pöshl-Teller effective potential, which allows us to calculate analytic expressions for the quasinormal frequencies. In the Schwarzschild-de Sitter case, the parameter space can be precisely characterized. As a consequence, the quasinormal modes can be written in terms of the parameters m and Λ of the metric.

Our results generalize the previous conclusions obtained in [14] for the scalar and electromagnetic fields. In particular, we see that the real part of the quasinormal frequencies does not depend on the mode (n), and that the relaxation time of the field is independent of its mass. We also demonstrate that the addition of charge to the black hole or mass to the scalar field does not alter the basic characteristics of the field dynamics in the near extreme regime.

Since we are imposing spherical symmetry, it is not too surprising that the field evolution is qualitatively independent of the dimension. However, it is interesting to note that the explicit expressions of the frequencies are very similar for any value of d. It would be instructive to see if this happens in non-spherical geometries.

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